School on General Relativity, Astrophysics and Cosmology

List of lectures

Warsaw (July 24-29):

Abhay Ashtekar:
Gravitational waves: interplay between mathematical foundations and observations

This mini-course will have two parts. The first will be devoted to conceptual and mathematical issues associated with gravitational waves in full nonlinear general relativity. The second part will illustrate the use of these results as a diagnostic tool to improve waveform models. These discussions will complement those on approximation and numerical methods.

Maciej Dunajski:
Twistor methods in General Relativity

Twistor theory was originally proposed by Roger Penrose as a new geometric framework for physics that aims to unify general relativity and quantum mechanics. In the twistor approach, space–time is secondary with events being derived objects that correspond to compact holomorphic curves in a complex three–fold, the twistor space.

This mini course will provide an elementary introduction to twistor theory leading to applications of twistor methods to gravitational instantons.

Rod Gover:
Conformal and related techniques for applications in GR

Topics should include: Spacetime models and spacetime compactification; Conformal geometry and tractors; prolongation of some relevant overdetermined PDE; The geometry of scale; Geometric compactification and related boundary calculus; Applications to the massive wave equation and its scattering as an example; Conformal boundary conditions for the Einstein equations.

Ruth Gregory:
On the Nature and Mathematics of Black Holes

In this short course we will explore various of the fascinating aspects of black holes in four, and more, dimensions. We start with a review of a general Birkhoff theorem in arbitrary dimension and cosmological constant, then discuss the types of black objects that are possible. We will discuss no hair theorems and black hole perturbations, and the fascinating recent developments in black hole thermodynamics. We will then talk about black holes in cosmology, time dependence and finally link to the standard model via instantons and vacuum decay.

Piotr Jaranowski:
Post-Newtonian General Relativity and Gravitational Waves

Higher-order post-Newtonian (PN) corrections to the equations of motion of compact binary systems composed of black holes or neutron stars are fundamental to the development and success of gravitational-wave astronomy. In the series of lectures, I will present the application of the ADM Hamiltonian formalism of general relativity to deriving equations of motion of compact binary systems within the perturbative PN scheme. Both conservative and dissipative (related to the emission of gravitational waves) effects in the dynamics will be considered.

Adam Pound:
Self-force theory and the gravitational two-body problem

Gravitational self-force theory is the principal method of modelling compact binaries with small mass ratios. In these lectures, I describe the mathematical foundations of self-force theory, its place in the gravitational two-body problem, and its applications in gravitational-wave astronomy.

Chęciny (July 31-August 4):

Julian Adamek:
Relativistic Simulations for Cosmology

Cosmological N-body simulations are one of the most versatile tools for studying the evolution of large-scale structure in the Universe. While the Newtonian limit of general relativity can be used for most purposes within the basic LCDM model, the true nature of the dark components (dark matter and dark energy) is unknown and may ultimately require a relativistic description. Also the neutrinos from the standard model are relativistic for most of the cosmic history if they have a mass within the range allowed by cosmological and laboratory constraints. In this course I will introduce a framework for relativistic N-body simulations that can treat any relativistic degrees of freedom self-consistently. Furthermore, I will discuss the important aspect of how the simulation data are mapped to observables by constructing the past light cone of an observation event. A metric-based approach is presented that is also suitable for treating a wide range of models beyond LCDM.

Suggested reading:

- Chapters 1 and 2 of Baumgarte and Shapiro, "NUMERICAL RELATIVITY - Solving Einstein's Equations on the Computer"
- Sections 2 and 3 of "Relativistic N-body simulations with massive neutrinos" (Adamek, Durrer and Kunz, arXiv:1707.06938)

Sebastiano Bernuzzi:
An introduction to numerical relativity and relativistic hydrodynamics

Numerical (or computational) relativity is the art of solving Einstein field equations by numerical methods. Its main application is the computation of strong and dynamical spacetimes, including gravitational collapse and the collision of compact objects with the associated gravitational waves. These lectures introduce the 3+1 formulation of general relativity and the related formalisms currently used in numerical relativity. They provide an overview of the initial data construction, hyperbolic evolution schemes, gauge choice and numerical methods for compact object simulations. The lectures also showcase applications to strongly-gravitating astrophysical systems.

Vladimir Karas:
Relativistic effects in spectra and polarization from black hole accretion disks

Astrophysical black holes are often characterized by only two parameters, namely, mass and angular momentum. However, cosmic black holes are not completely isolated. Instead, they interact with their gaseous and stellar environment, and so the astrophysically realistic models may require additional information to describe the spacetime metric and to determine the state of the surrounding gaseous environment. We will review a fruitful approach to study variety of electromagnetic signatures from accretion disks in strong gravity regime. Transfer functions can be introduced, pre-computed, and then employed to generate model spectra and to fit them to the electromagnetic signal in X-rays. We have been developing this approach to analyse spectra and light curves and to predict the polarimetric properties. Other groups adopt different schemes which we will briefly outline, too. Models can be finally tested with the current and upcoming observations. Some of recent results challenge the expectations based on standard accretion scenarios.

Mikołaj Korzyński:
Redshift drift, position drift and parallax in general relativity

I will discuss the redshift and the position drifts in general relativity, i.e. the temporal variations of the redshift and the position on the sky of a light source, as registered by an arbitrary observer. With the recent advancements in astrometry, the drifts of distant sources are likely to become important observables in cosmology in the near future. In my lecture I will present the derivation of exact relativistic formulas for the drifts. I will show how the drifts may be expressed in terms of the kinematical variables characterizing the motions of the source and the observer, i.e. their momentary 4-velocities and 4-accelerations, as well as the spacetime curvature along the line of sight. The formulas we derive are completly general and involve automatically all possible GR effects. They may be regarded as the counterpart of the Sachs optical equations for temporal variations of the standard observables. I will discuss their physical consequences and their possible applications to the gravitational lensing theory, cosmology and pulsar timing. Building on the same formalism I will also consider the trigonometric parallax effect in general relativity, and show how we can measure the mass density along the line of sight by comparing the parallax distance and the angular diameter distance to a single source.

Literature:

- "Optical drift effects in general relativity", M. Korzyński, J. Kopiński, Journal of Cosmology and Astroparticle Physics 03 (2018) 012
- "Geometric optics in general relativity using bilocal operators", M. Grasso, M. Korzyński, J. Serbenta, Phys. Rev. D 99 (2019) no.6, 064038
- "Geometric optics in relativistic cosmology: new formulation and a new observable", M. Korzyński, E. Villa, Phys. Rev. D 101 (2020) no.6, 063506

Prerequisites:

- Basic general relativity: worldlines, null and timelike geodesics, parallel transport, curvature tensor, Einstein equations
- Basic cosmology: Friedmann equations, cosmological distances, redshift
- Somewhat more advanced topics in general relativity: geodesic deviation equation, optical Sachs equations (recommended but not necessary, I will introduce this material during the course)

Patryk Mach:
Matter around black holes - self-gravitating systems

Matter around black holes is usually modeled neglecting its self-gravity, i.e., assuming a fixed background metric. In these lectures I will focus on the opposite case, in which the self-gravity of matter is taken into account. Simple general-relativistic systems in which the effects of self-gravity can be studied include axially symmetric configurations - stationary disks (or tori) around black holes - or spherically symmetric steady accretion flows. I will consider mostly hydrodynamical or magentohydrodynamical models. The interplay between the structure of spacetime affected by the self-gravity of matter and the motion of matter around black holes leads to several interesting phenomena, which I will shortly discuss: bifurcations of solutions, occurrence of various ergoregions, changes in the phase-space of geodesic orbits.

Olivier Sarbach:
Particle motion and dynamics of a Vlasov gas in the exterior of a Kerr black hole

These lectures start with a discussion of some of the properties of the most important black hole solution in general relativity and relativistic astrophysics: the Kerr black hole. In particular, the notions of static and stationary observers, ergospheres, horizons, causal structure and the motion of free-falling massive and massless particles will be reviewed. Next, some tools are introduced to understand the geometry of the cotangent bundle associated with a (generic) curved spacetime (M,g). Based on these tools, a manifestly covariant theory is derived describing a relativistic Vlasov gas, that is, a gas consisting of collisionless particles propagating in (M,g). In the final part of the lectures, this theory is applied to the study of the dynamics of a Vlasov gas consisting of particles which follow spatially bound timelike geodesics in the exterior of a Kerr black hole. To this purpose, generalized action-angle variables are introduced in which the geodesic flow simplify considerably and the relativistic Vlasov equation can be solved analytically. Based on this representation, it is shown that - even though it is collisionless - the gas undergoes a relaxation process and settles down to a stationary, axisymmetric configuration. The underlying mechanism for this effect, which is due to phase mixing, will be explained.