

18th Symposium on Integrable Systems

Report of Contributions

Contribution ID: 1

Type: **not specified**

Cyclic Lie-Rinehart algebras

Monday 6 July 2026 17:55 (25 minutes)

We study Lie-Rinehart algebra structures in a framework provided by duality pairings of modules over unital commutative associative algebra. Thus, we construct new examples of Lie brackets corresponding to a fixed anchor map whose image is a cyclic submodule of the derivation module, and therefore we call them cyclic Lie-Rinehart algebras. Special cases of our construction include Lie algebroid structures on cotangent bundles of differential manifolds and also certain differential operators that occur in mathematical physics.

Presenter: DOBROGOWSKA, Alina (Faculty of Mathematics, University of Białystok)

Contribution ID: 2

Type: **not specified**

Rational interpolants and solutions of dispersionless Hirota system

Tuesday 7 July 2026 10:05 (25 minutes)

The aim of this talk is to construct a class of explicit nontrivial rational solutions of the dispersionless Hirota system of PDEs. All the solutions in this class are of homogeneity degree 1 and are quotients of homogeneous polynomials. It is well-known that the solutions of the Hirota dispersionless systems describe Veronese webs. By nontriviality of the solutions it is meant that the corresponding Veronese webs are nonflat at generic points.

Presenter: PANASYUK, Andriy (Faculty of Mathematical and Natural Sciences, Cardinal Stefan Wyszyński University)

Contribution ID: 3

Type: **not specified**

Elliptic Gaudin-type model in an external magnetic field and modified algebraic Bethe ansatz

Monday 6 July 2026 15:30 (30 minutes)

We will consider elliptic Gaudin-type model in an external magnetic field associated with non-skew-symmetric elliptic r -matrix. We will discuss a modified algebraic Bethe ansatz for the considered elliptic r -matrix and obtain the spectra of the corresponding Gaudin-type Hamiltonians in terms of solutions of modified Bethe equations. The applications of the obtained result to the diagonalization of the anisotropic quantum Euler top, quantum Zhukovsky-Volterra top, quantum Steklov and Rubanovsky tops will be given.

Presenter: SKRYPNYK, Taras (University of the National Education Commission, Krakow)

Contribution ID: 4

Type: **not specified**

Nonlinear wave superpositions obtained via Lie modules

Monday 6 July 2026 15:00 (30 minutes)

This talk presents a study of nonlinear superpositions of Riemann wave solutions admitted by hyperbolic first-order systems. We focus on the Euler system and non-elastic wave superpositions that cannot be decomposed into pairwise independent interactions of waves. The property of quasi-rectifiability of the families of vector fields imposes certain conditions on the commutators of these vector fields. They enable us to find a parametrization of the region of superpositions of Riemann waves which leads to a simplification of the initial system. In order to identify non-elastic superpositions we prove that a class of associated Lie modules can be uniquely transformed into a real Lie algebra through an angle-preserving transformation. We select a particular basis of vector fields associated with a given module which ensures the property of quasi-rectifiability. That, in turn, allows us to construct the reduced form of the Euler system for which a non-elastic superposition of two Riemann waves is then derived. A study of the geometry of the manifolds of non-elastic wave superpositions in terms of deformations of submanifolds corresponding to the Lie algebras is performed. Finally, we adapt the described approach to the general form of a hydrodynamic-type system i.e., to arbitrary Lie modules of vector fields, providing the criteria for their quasi-rectifiability. A geometric interpretation of non-elastic wave superpositions in this system is presented.

Presenter: GRUNDLAND, Alfred Michel (Centre de Recherches Mathématiques Université de Montréal AND Département de Mathématiques et d'Informatique Université du Québec à Trois-Rivières)

Contribution ID: 5

Type: **not specified**

Instantons from projective plane

Monday 6 July 2026 09:30 (50 minutes)

I will show how combining some 19th century projective geometry with integrable harmonic maps equations leads to a construction of a new instanton, ultimately answering (negatively!) a question of S. T. Yau.

Presenter: DUNAJSKI, Maciej (University of Cambridge)

Contribution ID: 6

Type: **not specified**

Research and Numerical Simulation of the Euler Top

Tuesday 7 July 2026 11:25 (25 minutes)

Classical tops, such as the Euler, Lagrange, and Kovalevskaya tops, are important examples of rigid body systems in classical mechanics which can serve as useful test cases for numerical simulations, since they preserve several quantities such as energy, the magnitude of angular momentum, and additional integrals of motion. Although the Euler top seems to be the simplest case, it already exhibits interesting phenomena, for example the Dzhanibekov effect. The phase space of the Euler top can be parameterized by three components of angular momentum. Its rotational symmetry is related to the group $SO(3)$, and its equations of motion can be derived from the Lie–Poisson bracket. The Euler top is an example of Nambu dynamics where, as opposed to conventional Hamiltonian dynamics, two “Hamiltonians” are used instead of one.

Presenter: DĄBROWSKI, Marek (University of Białystok)

Contribution ID: 7

Type: **not specified**

Analysis of the motion of particles in the Reissner-Nordström spacetime

Monday 6 July 2026 12:20 (25 minutes)

The motion of neutral and charged test particles in the Reissner-Nordström spacetime is analysed. Inspired by the recent application of the Biermann-Weierstrass theorem to obtain a general non-radial solution to the geodesic motion in the Schwarzschild spacetime [1], we apply the same method to the Reissner-Nordström spacetime. This approach allows us to describe all types of orbits except for the purely radial ones with a single formula in terms of the Weierstrass elliptic function $\wp$. Using the obtained solution, we analyse the qualitative influence of the charge of the black hole on the motion of neutral particles. Similarly, we study the motion of charged test particles.

References

[1] Adam Cieřlik and Patryk Mach, Revisiting timelike and null geodesics in the Schwarzschild spacetime: general expressions in terms of Weierstrass elliptic functions, *Classical and Quantum Gravity*, 39(22):225003, 2022.

Presenter: BEMBENEK, Jakub (Instytut Fizyki Uniwersytetu Zielonogórskiego, Szkoła Doktorska Nauk Ścisłych i Technicznych Uniwersytetu Zielonogórskiego)

Contribution ID: 8

Type: **not specified**

On aggregation-quantization permutability problem for discrete-time Markov chains

Tuesday 7 July 2026 12:25 (25 minutes)

Given random walk on a graph, the corresponding discrete-time quantum walk can be constructed using the method proposed by Szegedy. On the other hand, given a partition of the set of states of a Markov chain, one can study the corresponding aggregated process. We extend the aggregation technique to the level of quantum Markov chains. We provide conditions under which application of these two operations - Szegedy's quantization and aggregation - give the same result. In particular, we show that the conditions are satisfied in the case of the random walk on graphs equipped with equitable partitions. We illustrate the approach on example of discrete-time classical/quantum walks on N-dimensional hypercube and its connection the Ehrenfests urn model with N particles. We also discuss relation of the example to the Kravtchouk polynomials. The presentation is based on joint work with Artur Siemaszko and Adam Zalewski arXiv:2603.14269.

Presenter: DOLIWA, Adam (University of Warmia and Mazury)

Contribution ID: 9

Type: **not specified**

Integrability of a charged rigid body in a constant electromagnetic field

Monday 6 July 2026 16:55 (25 minutes)

We study the dynamics of a charged rigid body with stationary charge distribution in external constant electric and magnetic fields. The total charge of the body vanishes and the charge distribution is described by symmetric matrix of the 'electrostatic inertia' of the body. The equations of motion are derived and it is shown that they are Hamiltonian with respect to a certain degenerated Poisson structure. Integrability of this system is analysed using Kovalevskaya method, the Ziglin theorem concerning the splitting of separatrices and the differential Galois theory. The non-integrability theorems under general assumptions and some integrable cases are presented.

Work in collaboration with Andrzej J. Maciejewski

Presenter: PRZYBYLSKA, Maria (Institute of Physics, University of Zielona Góra)

Contribution ID: 10

Type: **not specified**

Energy-Preserving Mixed Finite Element Approximation of the Korteweg–de Vries Equation

Tuesday 7 July 2026 11:50 (25 minutes)

The preservation of energy is an important requirement in the long-time numerical simulation of Hamiltonian partial differential equations. This work develops an energy-preserving mixed finite element approximation of the Korteweg–de Vries (KdV) equation based on the discrete gradient methodology.

Starting from the Hamiltonian formulation of the equation, an energy - preserving time integration scheme is constructed using the discrete gradient approach. To accommodate the third-order spatial derivative, the KdV equation is rewritten as a system of first-order equations through the introduction of auxiliary variables, leading to a mixed variational formulation and a mixed finite element approximation in space. The resulting nonlinear systems are solved using iterative linearization techniques.

In addition, a splitting strategy based on the decomposition of the Hamiltonian into linear dispersive and nonlinear components is considered. This leads to a Strang-type splitting formulation in which simpler subproblems can be evolved separately.

Numerical experiments investigate long-time energy conservation and the propagation of solitary-wave solutions.

Presenter: JURGIELEWICZ, Maciej (University of Białystok)

Contribution ID: 11

Type: **not specified**

Quantum walks on Cayley graphs of finite Coxeter groups, and their aggregations

Tuesday 7 July 2026 12:50 (25 minutes)

In this talk we discuss classical and quantum walks on Cayley graphs of finite Coxeter groups and their reductions obtained by suitable aggregations of the state space. We begin with the definition and classification of finite Coxeter groups, together with the associated Cayley graphs determined by Coxeter generating sets. We then show how graph symmetries and suitable partition properties can be used to reduce classical random walks and to analyse the corresponding quantum walks in the sense of Szegedy's construction.

Special attention is devoted to the Coxeter group of type (H_3) . We describe the corresponding Cayley graph and present reductions leading to graphs associated with the fullerene, the dodecahedron, the icosahedron and the icosidodecahedron. For selected reductions we present transition matrices and compare classical and quantum simulations, with particular emphasis on the reduced walk on the icosahedral graph.

We also discuss the Coxeter group (H_4) , whose geometry is related to highly symmetric four-dimensional polytopes, in particular the 120-cell and the 600-cell. We explain how certain graphs and their reductions can be obtained using fundamental weights, and how the resulting models provide examples of quantum walks on highly symmetric discrete spaces. The examples presented in the talk illustrate that Cayley graphs of Coxeter groups form a natural framework for studying the interplay between symmetry, aggregation and discrete quantum dynamics.

Presenter: ZALEWSKI, Adam (University of Warmia and Mazury in Olsztyn)

Contribution ID: 12

Type: **not specified**

Symmetry reductions of the rdDym equation via a maximal finite-dimensional symmetry subalgebra

Monday 6 July 2026 18:20 (20 minutes)

We perform a group-theoretic analysis of the rdDym equation, extending the results previously obtained by Kumar and co-authors [1,2]. The Lie algebra of point symmetries of the rdDym equation is well known and infinite-dimensional. We identify a 13-dimensional subalgebra characterized by the property of being maximal within the class of finite-dimensional subalgebras that induce arbitrary projective transformations of the time variable (t).

We then carry out a complete classification of all one-dimensional subalgebras of this 13-dimensional algebra and use them to obtain symmetry reductions of the rdDym equation. Among the resulting reduced equations, we distinguish those that can be integrated by quadratures or reduced to solving the Riccati equation.

References

- [1] Kumar, S., Group invariant solutions of (2+1)-dimensional rdDym equation using optimal system of Lie subalgebras, S. Kumar, A.-M. Wazwaz, D. Kumar, A. Kumar, Physica Scripta. - 2019. - Vol. 94, No. 11. - P. 115202.
- [2] Kumar, S. Some closed-form invariant solutions and dynamical behavior of multiple solutions for the (2+1)-dimensional rdDym equation using the Lie symmetry approach, S. Kumar, Results in Physics. - 2021. - Vol. 28. - P. 104642.

Presenter: ZHALUKEVICH, Dzianis (Institute of Mathematics NAS of Belarus)

Contribution ID: 13

Type: **not specified**

A multi-purpose system of second-order ordinary differential equations with exponential nonlinearity

Monday 6 July 2026 11:55 (25 minutes)

A system of real ordinary differential equations $\ddot{y}_k = \sum_{l=1}^n M_{kl} \exp y_l$, $k = 1, \dots, n$, where M is a real $n \times n$ constant symmetric matrix, is analysed in detail. The motivation includes several physical applications, which are discussed for the general and some special n 's.

A simple symmetry analysis and a discussion of the Lagrangian-Hamiltonian structure is performed for arbitrary n and M . Then a search is done for special solutions and a test is performed for their stability. Finally, integrability of the system is examined by means of the Painlevé test.

The main results are: (1) existence of a one-parameter family of exact solutions. The solutions are found explicitly for general n and almost all matrices M (the exception is a set of measure zero in the space of real symmetric matrices). (2) The exact solution is proved to be unstable to small perturbations of the initial conditions. (3) For the integrability analysis, the system is tested for the Painlevé property. In spite of the existence of exact solutions, the analysis shows that the system is non-integrable in general. (4) A deeper insight into the Painlevé test shows that the only completely integrable cases of these systems are equivalent to the Toda lattice by similarity transformations of an auxiliary matrix related to M .

Presenter: GOLDSTEIN, Piotr (National Centre for Nuclear Research)

Contribution ID: 14

Type: **not specified**

Non-autonomous finite-dimensional constraints of soliton hierarchies and Painlevé type systems

Monday 6 July 2026 11:20 (25 minutes)

I will present results of joint work with Maciej Błaszak (UAM, Poznań) and Krzysztof Marciniak (Linköping University). Since the classical works of Novikov et al., there has been a tremendous amount of research devoted to connections between soliton hierarchies and their integrable finite-dimensional reductions, which was mainly focused on stationary flows. Recently, we have revisited this idea in a novel and systematic way [1–3]. We investigated not only stationary flows but also the so-called stationary systems, by which we mean a stationary flow together with all lower flows from the hierarchy, that is, finite-dimensional systems of evolutionary equations. As a result, we were able to show that, in the case of particular soliton hierarchies, the related stationary systems can be represented as classical separable Stäckel systems.

Here, we generalize the above concept of stationary systems to the so-called non-autonomous restrictions of soliton hierarchies. These restrictions are defined through invariant time-dependent constraints that are appropriate deformations of stationary flows through compositions of the so-called master symmetries and lower flows, an idea based on [4]. It turns out that this class of time-dependent restrictions of soliton hierarchies, at least in particular cases, is represented by non-autonomous Hamiltonian finite-dimensional dynamical systems of Painlevé type. Let us emphasize that the original Painlevé equations are non-autonomous nonlinear ODEs that, at the beginning of the 20th century, led to the definition of new transcendental special functions. I will illustrate our theory by considering the Korteweg–de Vries (KdV) hierarchy and its coupled generalizations, in particular the Dispersive Water Waves (DWW) hierarchy.

References

- [1] M. Błaszak, B.M. Szablikowski and K. Marciniak, Stäckel representations of stationary KdV systems, *Rep. Math. Phys.* 92 (2023) 323–346
- [2] B.M. Szablikowski, M. Błaszak and K. Marciniak, Stationary coupled KdV systems and their Stäckel representations, *Stud. Appl. Math.* 153 (2024) e12698
- [3] M. Błaszak, K. Marciniak, B.M. Szablikowski, Stationary systems of the AKNS hierarchy, *J. Non-linear Math. Phys.* 32 (2025) 1–22
- [4] M. Błaszak, K. Marciniak and B.M. Szablikowski, Non-autonomous soliton hierarchies, *Symmetry* 17 (2025) 1–27

Presenter: SZABLIKOWSKI, Błażej (Adam Mickiewicz University)

Contribution ID: 15

Type: **not specified**

Lyapunov Integrability Test (LIT): a numerical framework for exploring integrability in continuous dynamical systems. [online]

Tuesday 7 July 2026 16:05 (25 minutes)

We introduce the Lyapunov Integrability Test (LIT), a new numerical framework for exploring integrability in parameter-dependent nonlinear dynamical systems. Inspired by the philosophy underlying the Mandelbrot set, the proposed approach shifts the focus from analyzing individual trajectories to investigating the global organization of parameter space. Instead of identifying regions associated with stable periodic motion, the LIT searches for parameter values compatible with the existence of additional first integrals by analyzing the structure of the Lyapunov spectrum. The method is simple to implement, computationally efficient, and applicable to both Hamiltonian and non-Hamiltonian systems. Its performance is demonstrated on several benchmark models, including the ABC flow, the heavy top, quantum dots, a helically symmetric Hamiltonian system with three degrees of freedom, and the classical double pendulum. The results show that the LIT reliably identifies known integrable cases while providing a practical tool for discovering new candidate integrable regimes and investigating the transition from integrable to chaotic dynamics.

Presenter: SZUMIŃSKI, Wojciech (Institute of Physics, University of Zielona Góra)

Contribution ID: 16

Type: **not specified**

Transfer of integrable structures of soliton hierarchies to integrable Newton equations.

Monday 6 July 2026 14:30 (30 minutes)

In 1990-ties I have in collaboration with M.Antonowicz, P.Kulish, S.Orlov developed technique of transferring integrable structures of soliton hierarchies to finite dimensional invariant manifolds defined as stationary/restricted flows. They are systems of ODE's that, after suitable parametrisation, acquire form of Newton equations with velocity independent forces.

Structures of integrable hierarchies such as bi-hamiltonian formulation, Lax representation, Miura transformation, r-matrix give rise to corresponding structures for 2nd order ODE's being stationary and restricted flows. These structures were interesting by their own and gave rise to further, new classes of integrable ODE's of mechanical type.

These results have been largely forgotten and the purpose of this talk is to bring them again to light as there are still many new results about integrability of Newton equations to be found there. I shall illustrate this approach by discussing 2 simple examples: the stationary KDV equation and the Garnier system of motion of a particle in a quartic potential.

Reference.

“Mechanical systems related to the Schrödinger spectral problem”

S Rauch-Wojciechowski - Chaos, Solitons & Fractals, December 1995

Presenter: RAUCH, Stefan (Department of Mathematics, Linköping University)

Contribution ID: 17

Type: **not specified**

Free energy of the classical Toda chain in a generalised Gibbs ensemble

Monday 6 July 2026 10:50 (30 minutes)

Classical integrable systems exhibit a tower of conserved quantities having local densities built out of traces of powers of the model's Lax matrix. It is argued that this local structure, absent in general models, leads to peculiar thermalisation properties of integrable systems. In particular, their equilibrium properties are expected to be grasped by so-called Generalised Gibbs measures. The study of Generalised Gibbs ensembles' partition functions was initiated by Spohn. He focused on the N -particle Toda chain and managed to describe the $N \rightarrow +\infty$ limiting distribution of the Eigenvalues of the model's Lax matrix under a Generalised Gibbs distribution. He was also able to conjecture an expression for the associated free energy. A thorougher description of the Gibbs measure, in the form of a large deviation principle with an explicit rate function, was later conjectured by Doyon and, independently, Spohn.

In this talk, after reviewing the various motivations for the study of the problem, I will explain how one can establish, on rigorous grounds, the explicit form of the Generalised Gibbs ensemble Toda chain rate function and free energy by using the separated variables representation of the model's partition function. This result constitutes the first step towards studying the thermodynamic limit of the model's dynamical correlation functions in such a setting. This is a joint work with T. Grava, A. Guionnet and A. Little.

Presenter: KOZLOWSKI, Karol (ENS de Lyon and CNRS)

Contribution ID: **18**

Type: **not specified**

Integrability within the quantum current algebra formalism

Tuesday 7 July 2026 14:45 (30 minutes)

Presenter: PRYKARPATSKI, Anatolij (Cracov University of Technology and Lviv Polytechnic University)

Contribution ID: 19

Type: **not specified**

Voss surfaces in sine-Gordon hierarchies

Tuesday 7 July 2026 09:40 (25 minutes)

We shall explore a method, initiated by Guichard in 1890, which allows to generate sequences of Voss surfaces by quadratures, starting from an arbitrarily chosen pseudospherical surface and a seed solution of the Moutard equation, by means of two simple transformations. In this talk we

1) identify the Guichard transformations with the well-known mutually inverse recursion operators for symmetries of the sine-Gordon equation;

2) present a lemma which allows us to derive the length of Guichard's sequences from the invariance properties of the initial sine-Gordon solution;

3) introduce an extended class of inverted operators, increasing the number of Voss surfaces obtainable by quadratures.

A number of Voss nets will be presented explicitly.

Presenter: MARVAN, Michal (Mathematical Institute in Opava)

Contribution ID: 20

Type: **not specified**

Non-integrability of the planar elliptic photo-gravitational Hill problem.

Monday 6 July 2026 16:30 (25 minutes)

We prove that the elliptic photo-gravitational Hill problem is not integrable except in one case, when the gravitational force of the lighter primary acting on the infinitesimal mass is balanced by the radiation-pressure force of this primary. In this exceptional case, the infinitesimal mass moves under the influence of the more massive primary localised at infinity. We show that in this case the equations of motion are integrable and can be solved explicitly in terms of elementary functions. Moreover, we distinguish a three-dimensional subspace of initial conditions for which the solutions are periodic and the corresponding orbits are algebraic curves defined by a polynomial of degree four. All of them are of genus zero.

Presenter: MACIEJEWSKI, Andrzej (University of Zielona Gora)

Contribution ID: 21

Type: **not specified**

On inverse power cluster-size distributions generated by the Random Domino Automaton

Tuesday 7 July 2026 15:40 (20 minutes)

The Random Domino Automaton (RDA)—a slowly driven system in the form of a one-dimensional stochastic cellular automaton—was introduced as a stylized simple model of earthquake statistics to provide a basis for the interrelation of Gutenberg–Richter law and Omori law with the waiting time distribution for earthquakes. The Gutenberg–Richter distribution provides a universal relationship between the frequency of earthquakes and their size, and—if earthquake magnitude is measured by their energy (or seismic moment)—it has the form of an inverse power-law distribution.

In the RDA model, energy-related clusters can grow, merge, and disintegrate (trigger avalanches) depending on specific system parameters, which in stationary conditions is described in terms of the coupled recurrence relation for cluster-size statistics. This formulation proves appropriate for studying the role of these mechanisms in the formation of discrete inverse power-law distributions, or discrete Zeta distributions.

By asymptotic analysis of the relationship between the avalanche probability and the resulting stationary cluster-size distribution, we show that the convolution term in the governing equation, which encodes cluster merging, plays a decisive role in generating inverse power-law relations for a wide regime of parameters.

We conclude by pointing out an interesting connection of RDA-type systems with well-known Catalan-like integer sequences (Catalan, Motzkin, Schröder numbers) and also mention the generalization of RDA to the geometry of the Bethe lattice.

Presenter: BIAŁECKI, Mariusz (Instytut Geofizyki PAN)

Contribution ID: 23

Type: **not specified**

The differential geometry of the (modified²) Korteweg-de Vries equation and associated Miura transformations

Tuesday 7 July 2026 09:00 (40 minutes)

We present a framework in three-dimensional Minkowski space $\mathbb{R}^{1,2}$ which unifies the extended Dym, KdV, modified KdV and modified modified KdV equations via parallel, offset and midsurfaces. Each equation governs a class of surfaces, the members of which are foliated by geodesics of certain properties. These classes of surfaces are linked by reciprocal and Miura-type transformations. In particular, we obtain a novel geometric interpretation of the classical Miura transformation linking the KdV and mKdV equations. In total, there exist ten classes which may be associated both combinatorially and literally with the 4 vertices and 6 midpoints of the edges of a (moving) tetrahedron.

Presenter: SCHIEF, Wolfgang Karl (University of New South Wales)

Contribution ID: 24

Type: **not specified**

Phase-space representation of quantum computation in terms of Grassmann variables

Tuesday 7 July 2026 15:15 (25 minutes)

Phase-space methods provide a powerful alternative formulation of quantum mechanics, offering geometric insight into quantum dynamics through quasiprobability distributions and star products. While these techniques are well established for continuous-variable systems, their application to finite-dimensional quantum systems, particularly qubits, remains an active area of research. In this talk, we present a phase-space formulation of quantum computation based on Grassmann variables and deformation quantization.

We begin by reviewing the construction of fermionic phase spaces and the deformation of Grassmann algebras into Clifford algebras via the fermionic star product. This framework provides a natural phase-space description of spin-1/2 systems, where Pauli operators emerge as generators of the resulting Clifford algebra. We then discuss how multiple qubits can be represented by extending the Grassmann phase space with independent sets of anticommuting variables and how quantum states, observables, and unitary transformations can be described within this formalism.

Particular attention is devoted to the representation of quantum gates and multi-qubit interactions as Hamiltonian flows generated by Grassmann-valued functions. We discuss the correspondence between Clifford operations and transformations preserving the fermionic phase-space structure, as well as the role of higher-order Grassmann polynomials in describing entangling operations. Finally, we outline possible applications of this approach to the analysis of quantum circuits, the simulation of spin systems, and the development of phase-space methods for quantum information processing.

Presenter: DOMAŃSKI, Ziemowit (Poznań University of Technology)

Contribution ID: 25

Type: **not specified**

Solutions of the hyperbolic Ernst equation

Monday 6 July 2026 12:45 (20 minutes)

Ernst-type equations are elegant reformulations of Einstein's vacuum equations of general relativity when the existence of two commuting Killing vector fields is assumed. Axisymmetric, stationary spacetimes such as rotating black holes and planar gravitational waves are examples of solutions of the Ernst-type equations.

An important mathematical feature of the Ernst-type equations is that they are integrable nonlinear differential equations, allowing the application of methods developed for solving integrable systems. In particular, the inverse scattering method leading to the dressing method enabled to generate n -soliton solutions on the Kasner background [1,2].

We will use the method that dates back to Bianchi's work i.e. and is based on a Bäcklund transformation and a nonlinear superposition principle [3,4]. We will compare the two methods showing differences between two soliton solutions. We will also present some solutions without singularities.

References

- [1] V. A. Belinsky and V. E. Zakharov, Sov. Phys. JETP, 48:985–994, 1978.
- [2] Belinski V, Verdaguer E. Gravitational Solitons. Cambridge University Press; 2001.
- [3] L. Bianchi. Memorie della Societ' a Italiana delle Scienze, detta dei XL, 13:261–289, 1905.
- [4] M. Nieszporski. The multicomponent Ernst equation and the Moutard transformation. Physics Letters A, 272(1):74–79, 2000.

Presenter: CHOIŃSKA, Debora (University of Warsaw)

Contribution ID: 26

Type: **not specified**

On exact discretizations of the Kepler problem

Tuesday 7 July 2026 11:00 (25 minutes)

The Kepler problem is a classical superintegrable dynamical system, possessing five functionally independent integrals of motion. We present a new explicit integrator which preserves exactly (up to round-off errors) all these integrals (and, as a consequence, all phase space trajectories). What is more, our numerical scheme also provides the exact explicit time discretization. We shortly mention other possibilities to approach the exact discretization of the Kepler problem: the Kustaanheimo–Stiefel regularization (standard in celestial dynamics) and a method inspired by the discretization of the generalized Nambu mechanics.

Presenter: CIEŚLIŃSKI, Jan (University of Białystok, Faculty of Physics)

Contribution ID: 27

Type: **not specified**

From pencils of Novikov algebras of Stäckel type to soliton hierarchies

In this presentation we construct evolutionary soliton hierarchies from pencils of Novikov algebras of Stäckel type. We start by defining a special class of associative Novikov algebras, which we call Novikov algebras of Stäckel type, as they are associated with classical Stäckel metrics in Viète coordinates. We obtain sufficient conditions for pencils of these algebras so that the corresponding Dubrovin-Novikov Hamiltonian operators can be centrally extended, producing sets of pairwise compatible Poisson operators. These operators lead to coupled Korteweg-de Vries (cKdV) and coupled Harry Dym (cHD) hierarchies, as well as to a triangular cKdV hierarchy and a triangular cHD hierarchy.

Presenter: BŁASZAK, Maciej (Faculty of Physics and Astronomy, Adam Mickiewicz University)

Contribution ID: 28

Type: **not specified**

Hamiltonian integrability via Malgrange pseudo group.

Monday 6 July 2026 17:20 (25 minutes)

We consider a 2 degree of freedom rational Hamiltonian H . Arnold Liouville integrability requires the existence of an additional first integral. We want to relax the condition of existence of an additional first integral by requiring only the preservation of a codimension 1 foliation transverse to the levels of H . The Malgrange pseudo group is defined in terms of differential invariants, so when such a codimension 1 foliation is preserved, the Malgrange pseudo group is not maximal. We present differential Galois conditions on the variational equations near Darboux points for homogeneous potentials for proving this non maximality. We then apply it to prove maximality of the Malgrange pseudo group for most homogeneous polynomial potentials of degree 3. At last, we look for potentials preserving a codimension 1 foliation, by looking for compatible vector fields of degree 1 in momenta. Among non Arnold Liouville integrable potentials, we find exactly the bihomogeneous potentials, and homogeneous potentials of degree 0.

Presenter: COMBOT, Thierry (Université Bourgogne Europe)

Contribution ID: **29**

Type: **not specified**

Registration