

Phase-space representation of quantum computation in terms of Grassmann variables

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Phase-space methods provide a powerful alternative formulation of quantum mechanics, offering geometric insight into quantum dynamics through quasiprobability distributions and star products. While these techniques are well established for continuous-variable systems, their application to finite-dimensional quantum systems, particularly qubits, remains an active area of research. In this talk, we present a phase-space formulation of quantum computation based on Grassmann variables and deformation quantization.

We begin by reviewing the construction of fermionic phase spaces and the deformation of Grassmann algebras into Clifford algebras via the fermionic star product. This framework provides a natural phase-space description of spin-1/2 systems, where Pauli operators emerge as generators of the resulting Clifford algebra. We then discuss how multiple qubits can be represented by extending the Grassmann phase space with independent sets of anticommuting variables and how quantum states, observables, and unitary transformations can be described within this formalism.

Particular attention is devoted to the representation of quantum gates and multi-qubit interactions as Hamiltonian flows generated by Grassmann-valued functions. We discuss the correspondence between Clifford operations and transformations preserving the fermionic phase-space structure, as well as the role of higher-order Grassmann polynomials in describing entangling operations. Finally, we outline possible applications of this approach to the analysis of quantum circuits, the simulation of spin systems, and the development of phase-space methods for quantum information processing.

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