

Free energy of the classical Toda chain in a generalised Gibbs ensemble

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Classical integrable systems exhibit a tower of conserved quantities having local densities built out of traces of powers of the model's Lax matrix. It is argued that this local structure, absent in general models, leads to peculiar thermalisation properties of integrable systems. In particular, their equilibrium properties are expected to be grasped by so-called Generalised Gibbs measures. The study of Generalised Gibbs ensembles' partition functions was initiated by Spohn. He focused on the N -particle Toda chain and managed to describe the $N \rightarrow +\infty$ limiting distribution of the Eigenvalues of the model's Lax matrix under a Generalised Gibbs distribution. He was also able to conjecture an expression for the associated free energy. A thorougher description of the Gibbs measure, in the form of a large deviation principle with an explicit rate function, was later conjectured by Doyon and, independently, Spohn.

In this talk, after reviewing the various motivations for the study of the problem, I will explain how one can establish, on rigorous grounds, the explicit form of the Generalised Gibbs ensemble Toda chain rate function and free energy by using the separated variables representation of the model's partition function. This result constitutes the first step towards studying the thermodynamic limit of the model's dynamical correlation functions in such a setting. This is a joint work with T. Grava, A. Guionnet and A. Little.

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