

Non-autonomous finite-dimensional constraints of soliton hierarchies and Painlevé type systems

Monday 6 July 2026 11:20 (25 minutes)

I will present results of joint work with Maciej Błaszak (UAM, Poznań) and Krzysztof Marciniak (Linköping University). Since the classical works of Novikov et al., there has been a tremendous amount of research devoted to connections between soliton hierarchies and their integrable finite-dimensional reductions, which was mainly focused on stationary flows. Recently, we have revisited this idea in a novel and systematic way [1–3]. We investigated not only stationary flows but also the so-called stationary systems, by which we mean a stationary flow together with all lower flows from the hierarchy, that is, finite-dimensional systems of evolutionary equations. As a result, we were able to show that, in the case of particular soliton hierarchies, the related stationary systems can be represented as classical separable Stäckel systems.

Here, we generalize the above concept of stationary systems to the so-called non-autonomous restrictions of soliton hierarchies. These restrictions are defined through invariant time-dependent constraints that are appropriate deformations of stationary flows through compositions of the so-called master symmetries and lower flows, an idea based on [4]. It turns out that this class of time-dependent restrictions of soliton hierarchies, at least in particular cases, is represented by non-autonomous Hamiltonian finite-dimensional dynamical systems of Painlevé type. Let us emphasize that the original Painlevé equations are non-autonomous nonlinear ODEs that, at the beginning of the 20th century, led to the definition of new transcendental special functions. I will illustrate our theory by considering the Korteweg–de Vries (KdV) hierarchy and its coupled generalizations, in particular the Dispersive Water Waves (DWW) hierarchy.

References

- [1] M. Błaszak, B.M. Szablikowski and K. Marciniak, Stäckel representations of stationary KdV systems, Rep. Math. Phys. 92 (2023) 323–346
- [2] B.M. Szablikowski, M. Błaszak and K. Marciniak, Stationary coupled KdV systems and their Stäckel representations, Stud. Appl. Math. 153 (2024) e12698
- [3] M. Błaszak, K. Marciniak, B.M. Szablikowski, Stationary systems of the AKNS hierarchy, J. Nonlinear Math. Phys. 32 (2025) 1–22
- [4] M. Błaszak, K. Marciniak and B.M. Szablikowski, Non-autonomous soliton hierarchies, Symmetry 17 (2025) 1–27

Presenter: SZABLIKOWSKI, Błażej (Adam Mickiewicz University)