

A multi-purpose system of second-order ordinary differential equations with exponential nonlinearity

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A system of real ordinary differential equations $\ddot{y}_k = \sum_{l=1}^n M_{kl} \exp y_l$, $k = 1, \dots, n$, where M is a real $n \times n$ constant symmetric matrix, is analysed in detail. The motivation includes several physical applications, which are discussed for the general and some special n 's.

A simple symmetry analysis and a discussion of the Lagrangian-Hamiltonian structure is performed for arbitrary n and M . Then a search is done for special solutions and a test is performed for their stability. Finally, integrability of the system is examined by means of the Painlevé test.

The main results are: (1) existence of a one-parameter family of exact solutions. The solutions are found explicitly for general n and almost all matrices M (the exception is a set of measure zero in the space of real symmetric matrices). (2) The exact solution is proved to be unstable to small perturbations of the initial conditions. (3) For the integrability analysis, the system is tested for the Painlevé property. In spite of the existence of exact solutions, the analysis shows that the system is non-integrable in general. (4) A deeper insight into the Painlevé test shows that the only completely integrable cases of these systems are equivalent to the Toda lattice by similarity transformations of an auxiliary matrix related to M .

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