

Nonlinear wave superpositions obtained via Lie modules

Monday 6 July 2026 15:00 (30 minutes)

This talk presents a study of nonlinear superpositions of Riemann wave solutions admitted by hyperbolic first-order systems. We focus on the Euler system and non-elastic wave superpositions that cannot be decomposed into pairwise independent interactions of waves. The property of quasi-rectifiability of the families of vector fields imposes certain conditions on the commutators of these vector fields. They enable us to find a parametrization of the region of superpositions of Riemann waves which leads to a simplification of the initial system. In order to identify non-elastic superpositions we prove that a class of associated Lie modules can be uniquely transformed into a real Lie algebra through an angle-preserving transformation. We select a particular basis of vector fields associated with a given module which ensures the property of quasi-rectifiability. That, in turn, allows us to construct the reduced form of the Euler system for which a non-elastic superposition of two Riemann waves is then derived. A study of the geometry of the manifolds of non-elastic wave superpositions in terms of deformations of submanifolds corresponding to the Lie algebras is performed. Finally, we adapt the described approach to the general form of a hydrodynamic-type system i.e., to arbitrary Lie modules of vector fields, providing the criteria for their quasi-rectifiability. A geometric interpretation of non-elastic wave superpositions in this system is presented.

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