Plug and Play denoiser for deconvolving GRAND Electric field traces

Sara El Bouch, Simon Prunet

Université Côte d'Azur, OCA, CNRS, Nice, France

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Problem Statement

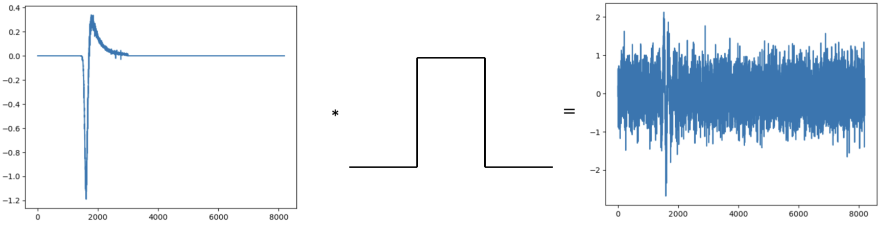


Figure: $Y(\omega) = A(\omega)X(\omega) + N(\omega)$

- Recover E-field signal from the noisy voltage measurement ?

Deep Plug-and-Play Vs. Wiener Filter?

Wiener Filter (Linear MMSE)

- Solves: $\min_{\mathbf{x}} \mathbb{E}[\|\mathbf{x} - \hat{\mathbf{x}}\|^2]$
- Assumes Gaussian priors: $\mathbf{x} \sim \mathcal{N}(0, \mathbf{C}_x)$
- Frequency-domain solution:

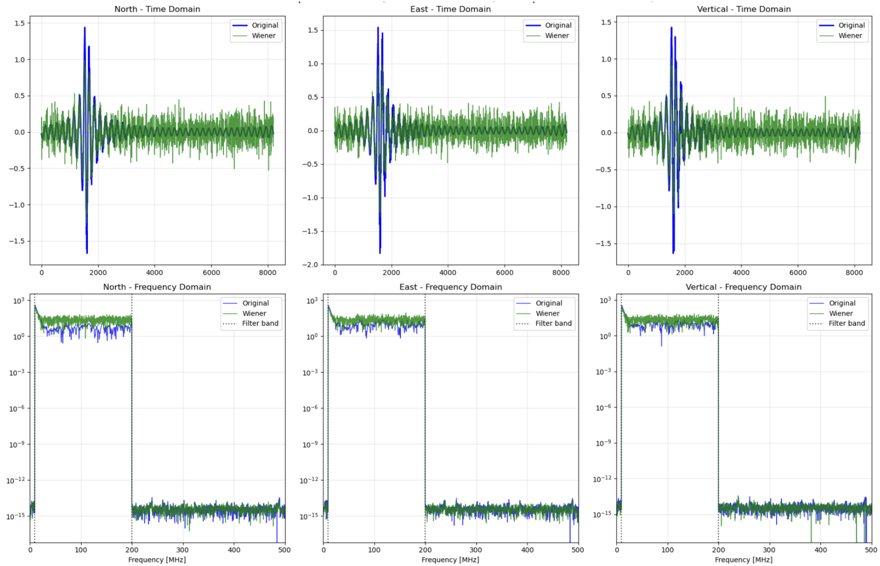
$$\hat{\mathbf{X}}(\omega) = \frac{H^*(\omega) S_x(\omega)}{|H(\omega)|^2 S_x(\omega) + S_n(\omega)} \mathbf{Y}(\omega)$$

- Linear transformation only
- Limited to second-order statistics

Deep PnP-HQS (Nonlinear Prior)

- Solves: $\min_{\mathbf{x}} \frac{1}{2}\|\mathbf{A}\mathbf{x} - \mathbf{y}\|^2 + \lambda R(\mathbf{x})$
- Nonlinear regularization via deep denoiser.
- Captures complex signal structure

Wiener filter: Optimal linear solution



Prior Approximation via Deep Learning

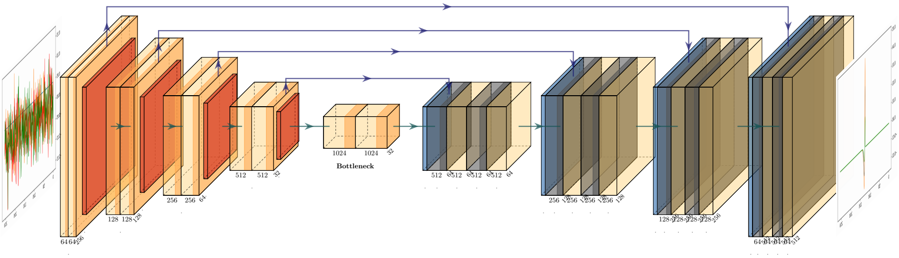
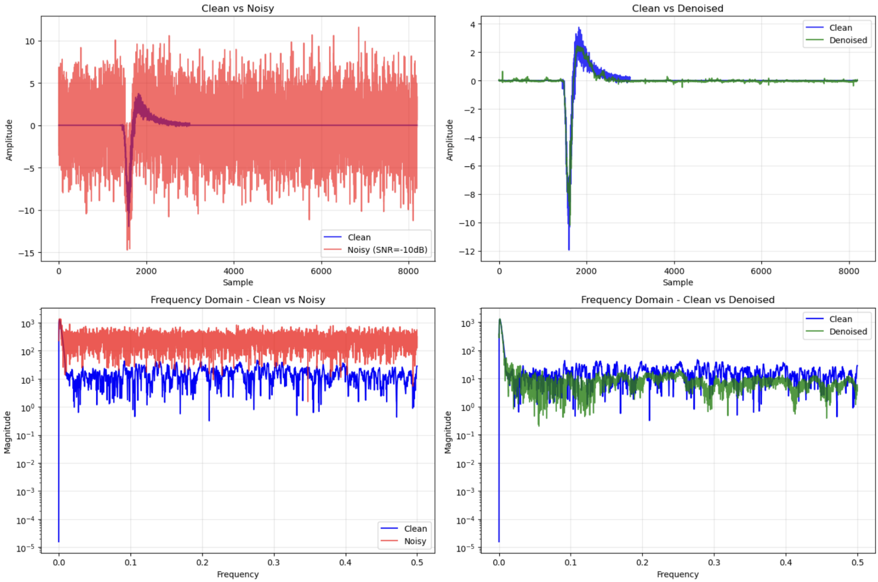


Figure: U-shaped Architecture

- Training Dataset: ZHAireS-NJ simulation files.
- This approximates the **negative log-prior** of clean E-field signals:

$$R(\mathbf{x}) \approx -\log p(\mathbf{x}_{\text{clean}})$$

Results



Clean vs Noisy
Amplitude
Sample
Clean
Noisy (SNR=-10dB)
Clean vs Denoised
Amplitude
Sample
Clean
Denoised
Frequency Domain - Clean vs Noisy
Magnitude
Frequency
Clean
Noisy
Frequency Domain - Clean vs Denoised
Magnitude
Frequency
Clean
Denoised

Results

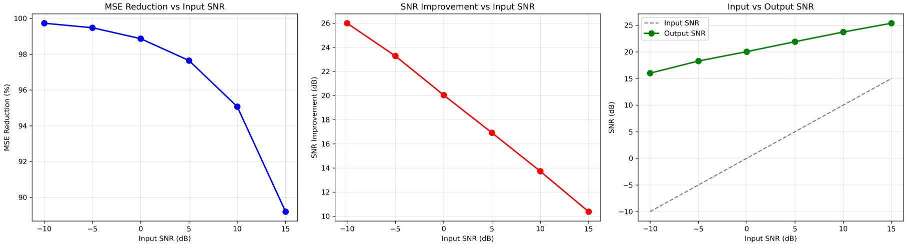


Figure: Summary of training results on 100 traces

Deep Plug-and-Play

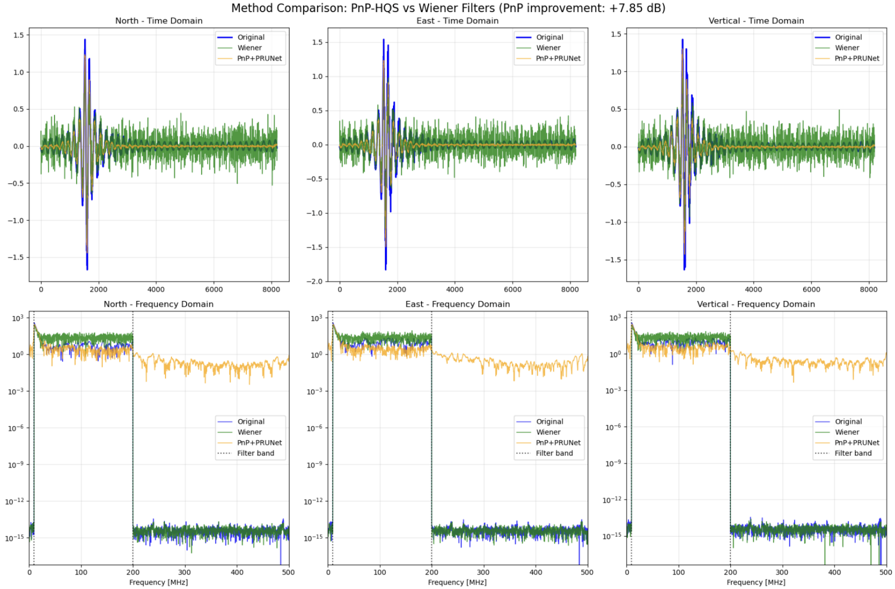
Core Idea: Decompose inverse problem into data fidelity + denoising subproblems

Algorithm 1 Deep PnP Algorithm

Initialize: $\mathbf{x}^{(0)} = \mathbf{A}^T\mathbf{y}$
for $k = 1, 2, \ldots, K$ **do**
  x-subproblem: $\mathbf{x}^{(k)} = \arg\min_{\mathbf{x}} \frac{1}{2}\|\mathbf{A}\mathbf{x} - \mathbf{y}\|^2 + \frac{\mu_k}{2}\|\mathbf{x} - \mathbf{z}^{(k-1)}\|^2$
  z-subproblem: $\mathbf{z}^{(k)} = \mathrm{DRUNet}(\mathbf{x}^{(k)}, \sigma_k)$
  Update parameters: σ_k, μ_k
end for

Deep Plug And Play: Replace traditional regularization with deep learning denoiser (DRUNet)

Deep PnP vs Wiener



Real Antenna Response Modeling

Complete Antenna-to-Voltage Transformation

The forward operator $\mathbf{A}$ models the complete signal chain from E-field to digitized voltage:

$$\mathbf{y}(\omega) = \mathbf{A}(\omega)\mathbf{x}(\omega) + \mathbf{n}(\omega)$$

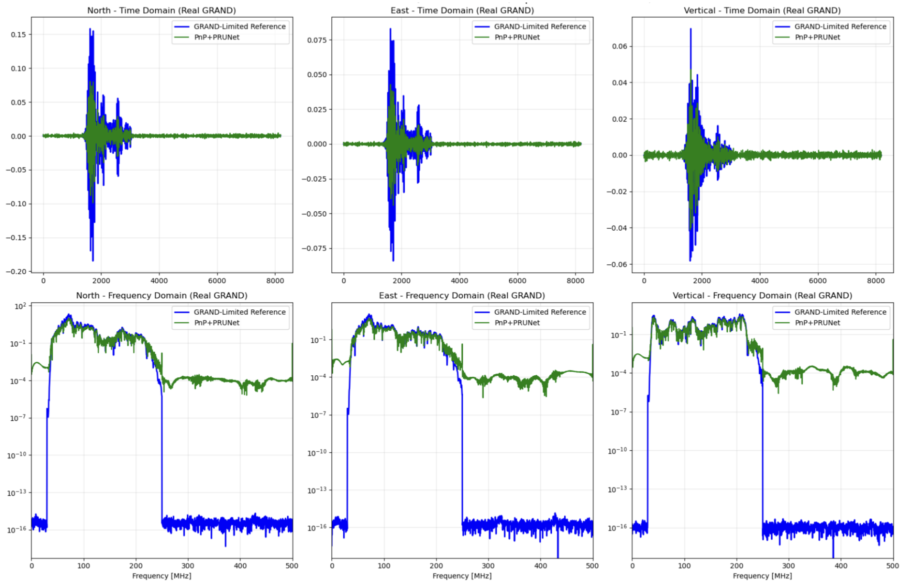
Mathematical Decomposition:

$$\mathbf{A}(\omega) = \mathbf{T}_{\text{elec}}(\omega) \cdot \mathbf{L}_{\text{eff}}(\theta, \phi, \omega)$$

where:

- $\mathbf{L}_{\text{eff}}(\theta, \phi, \omega)$: Antenna effective length tensor (3×3 complex matrix)
- $\mathbf{T}_{\text{elec}}(\omega)$: Electronics transfer function (3×1 complex vector)

Preliminary Results with GRAND full antenna response



Conclusion

PnP-HQS offers several benefits over traditional methods:

- **Modularity:** Decouples the physics model (A) and the learned prior ($Denoiser_\sigma(\cdot)$).
- **Flexibility:** - Handles varying noise levels through explicit conditioning on σ.