Supernova gravitational waves and protoneutron star asteroseismology

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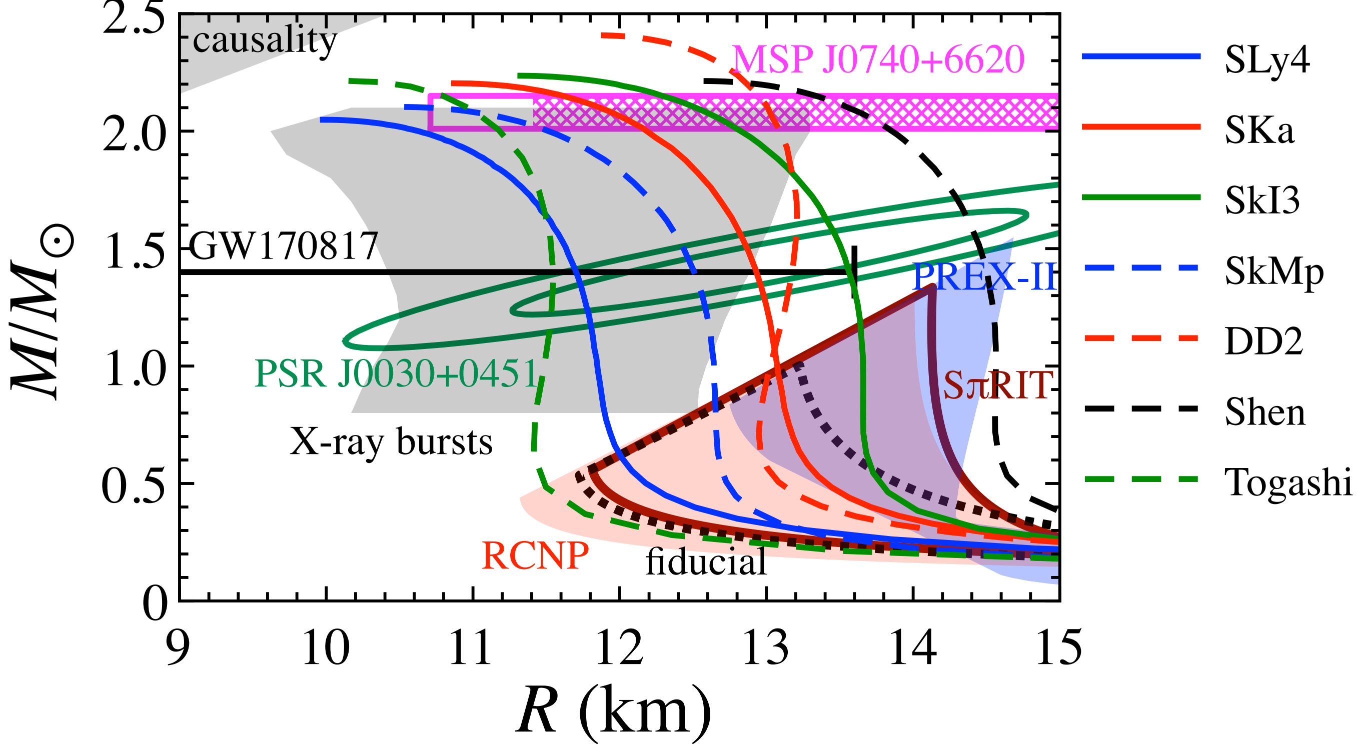
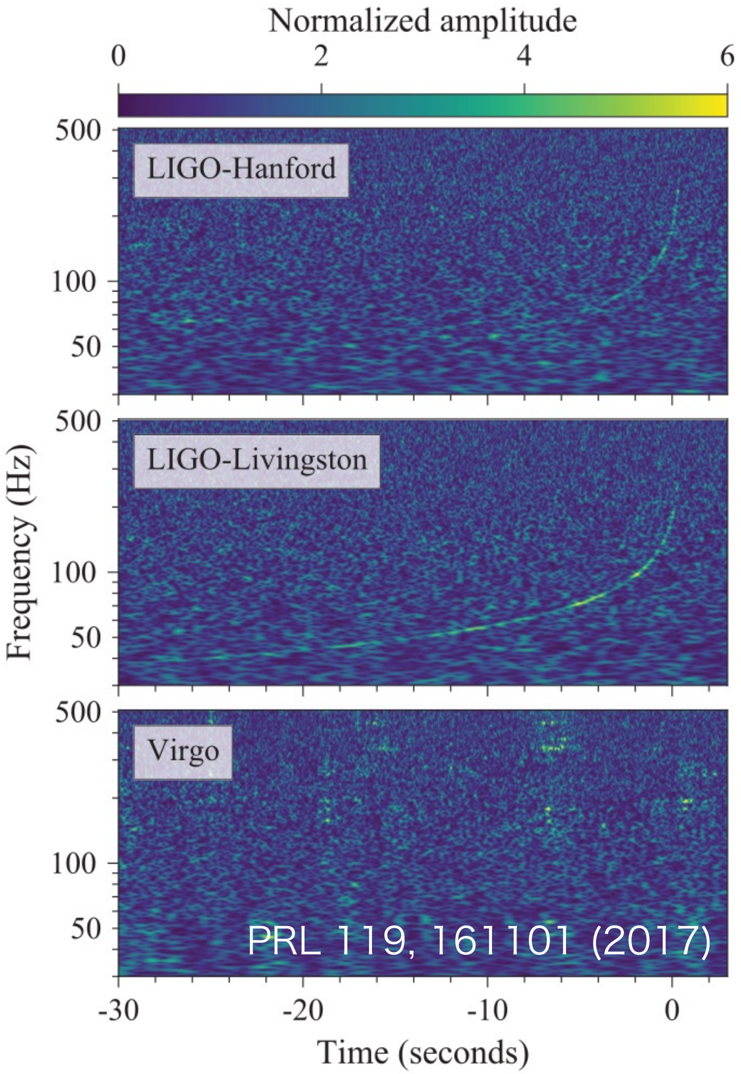
Dawn of GW astronomy

- GWs from compact binary mergers have been detected.
  - ✓ GWs become a new tool for extracting astronomical information.
- The next candidate must be a supernova explosion.

HS, Nishimura, Naito, 22





PRL 119, 161101 (2017)

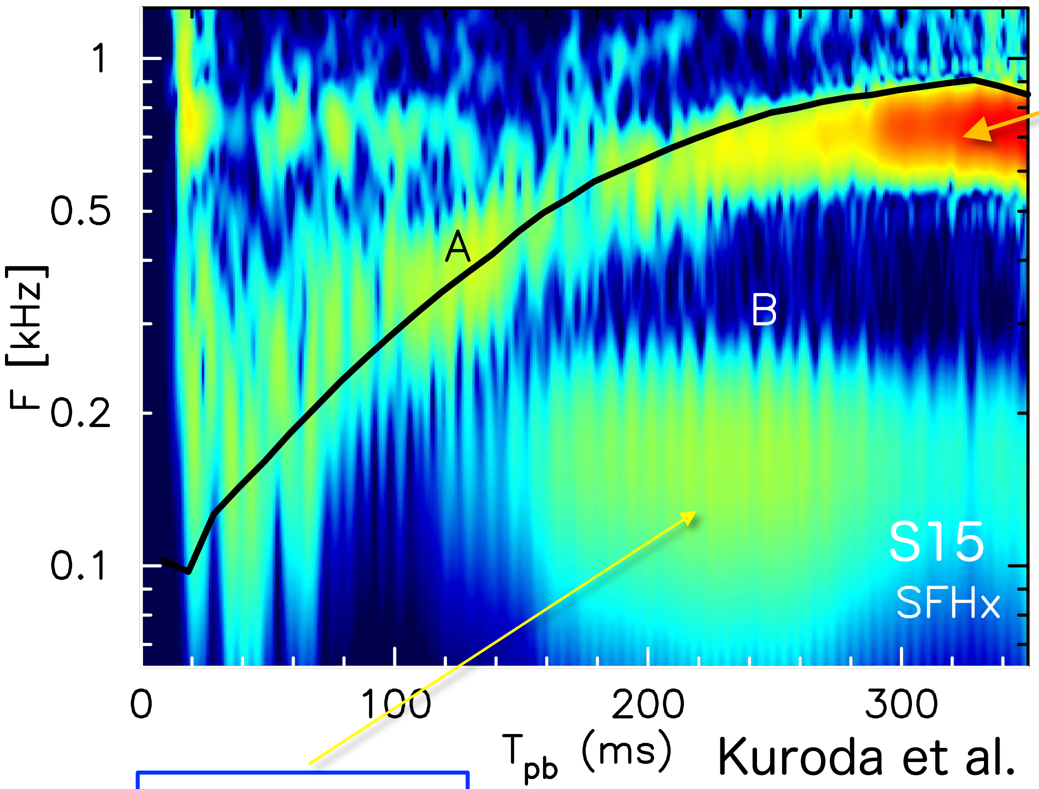
Next candidate of GW sources

- core-collapse supernovae
  - ✓ compared to the binary merger, the system is almost spherically symmetric
    - ➢ weak gravitational waves
    - ➢ we may be able to detect only the event happened in our galaxy
  - ✓ many numerical simulations show the existence of GW signals
    - ➢ SN GWs depend on the SN models, such as progenitor mass, EOS, and gravity
    - ➢ it may be difficult to extract physics of PNS from the GW signals.

- We adopt the perturbation approach, the so-called asteroseismology, to see the physic behind the GWs by identifying them with the specific frequency of PNS.

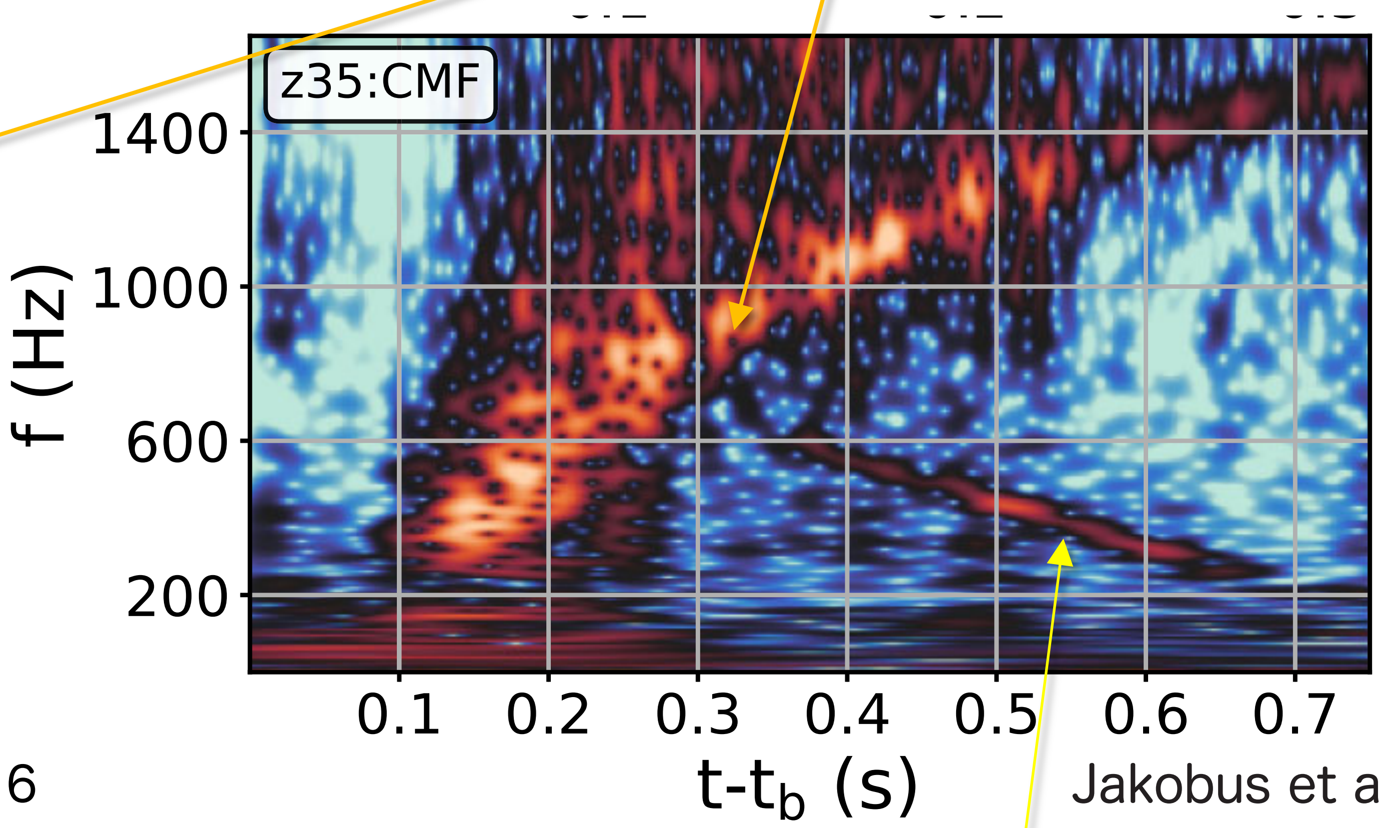
Supernova gravitational waves

ramp-up signals
a few 100Hz to kHz



Kuroda et al. 16

association
with SASI



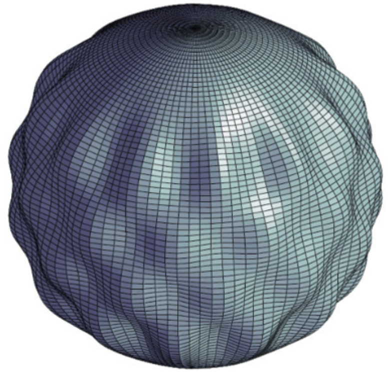
Jakobus et al 23

another distinct frequency band

Non-radial Oscillations in (proto)-NSs

- <u>axial type oscillations</u>
  - ✓ no stellar deformation, no density variation
    - ➢ w-modes (spacetime) : oscillations of specetime itself ~ M/R
    - ➢ t-modes (torsional) : due to the elasticity ~ v_s/R
    - ➢ r-modes (rotational) ~ mΩ
    - ➢ Alfven modes
- <u>polar type oscillations</u>
  - ✓ with density variation & stellar deformation
  - ✓ important for considering the GWs emission
    - ➢ f-mode (fundamental) ~ $(M/R^3)^{1/2}$
    - ➢ p-modes (pressure) : sound speed crossing ~ $(M/R^3)^{1/2}$
    - ➢ g-modes (gravity) : thermal/composition gradients ~ BV frequency
    - ➢ w-modes (spacetime) : oscillations of specetime itself ~ M/R
    - ➢ i/s-modes (interface/shear) : due to the elasticity
    - ➢ inertial modes (effect of rotation)
    - ➢ Alfven modes

we focus on in this talk



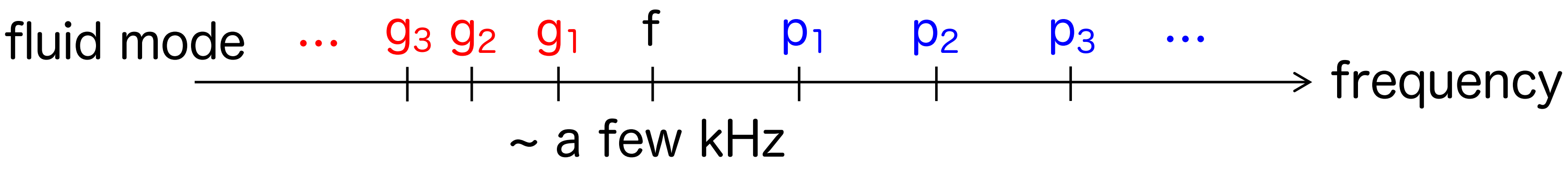
specific mode
↕
physics

Linear analysis

- eigenmodes are identified with linear analysis
- perturbation eqs. are derived from the linearized Einstein equations.
  - ✓ variables = background + perturbations $f = f_0 + \delta f$
  - ✓ decompose the perturbed variables

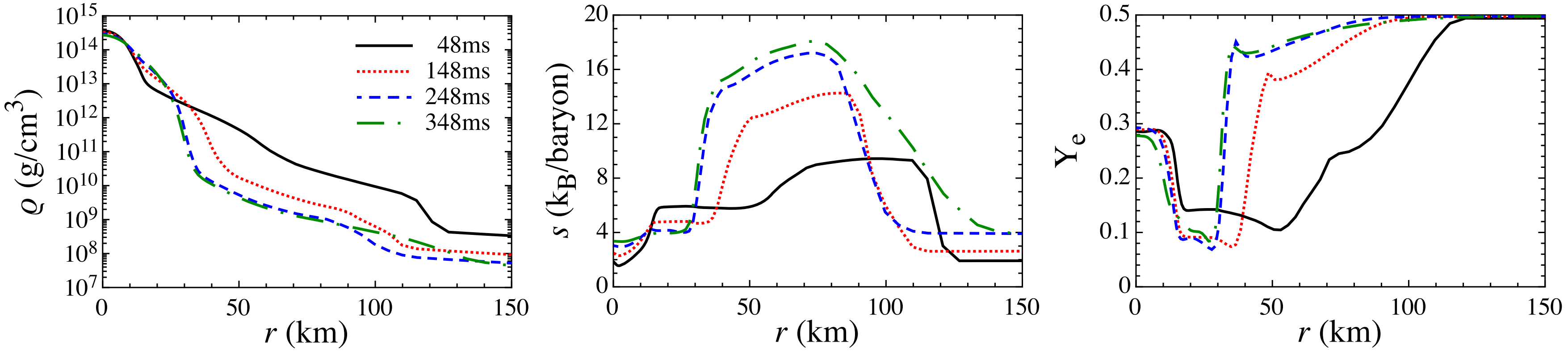
$$\delta f(t,r,\theta,\phi) = \delta f(r) e^{i\omega t} Y_{lm}(\theta,\phi)$$

- (GW) frequencies are determined by solving the eigenvalue problem.
  - ✓ appropriate boundary conditions
- if the background is spherically symmetric, the m-dependence is degenerate into m=0
  - ✓ ω are eigenfrequencies of the star for each l, where f = $\omega/2\pi$
  - ✓ subscript denotes the number of radial nodes in the eigenfunction

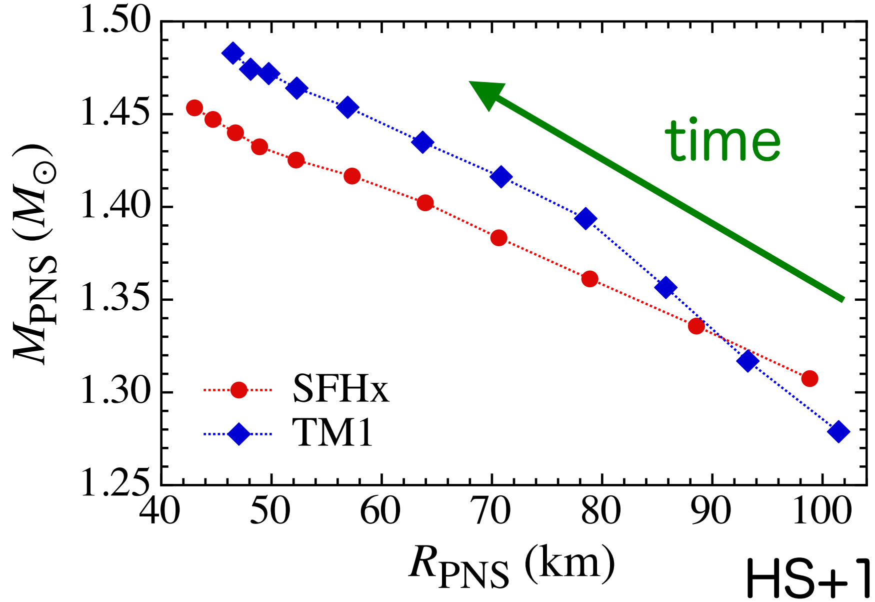


Asteroseismology on SN GWs

- PNS structure depends not only on the density and pressure profiles but also on the distribution of electron fraction and entropy (or temperature)



- Using the numerical data for core-collapse SNe, first one has to prepare background models, on which the linear perturbations are considered.
  - ✓ spherically symmetric background models are prepared by averaging in the angular direction
- GW frequencies are determined by solving the eigenvalue problem



HS+17

Simulation and linear analysis

- Linear analysis
  - ✓ we have done in the relativistic framework on the PNSs
    - ➢ Cowling approximation, neglecting the metric perturbations
    - ➢ with metric perturbations (non-Cowling)
    - ➢ in general, Cowling approximation overestimates the frequencies at most ~20%

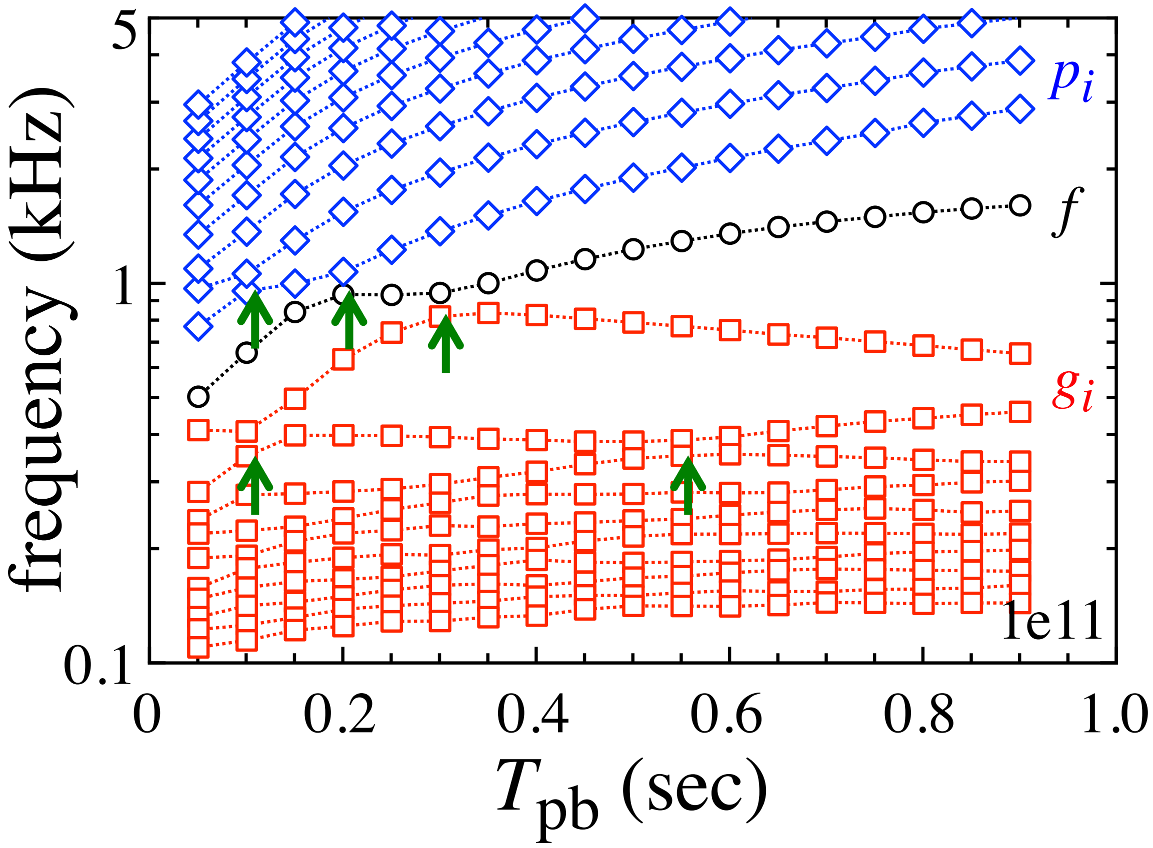
- Simulations
  - ✓ effective GR (Newtonian + effective potential) by T. Takiwaki
  - ✓ GR with monopole gravity by B. Muellar
  - ✓ GR with non-monopole gravity by B. Muellar

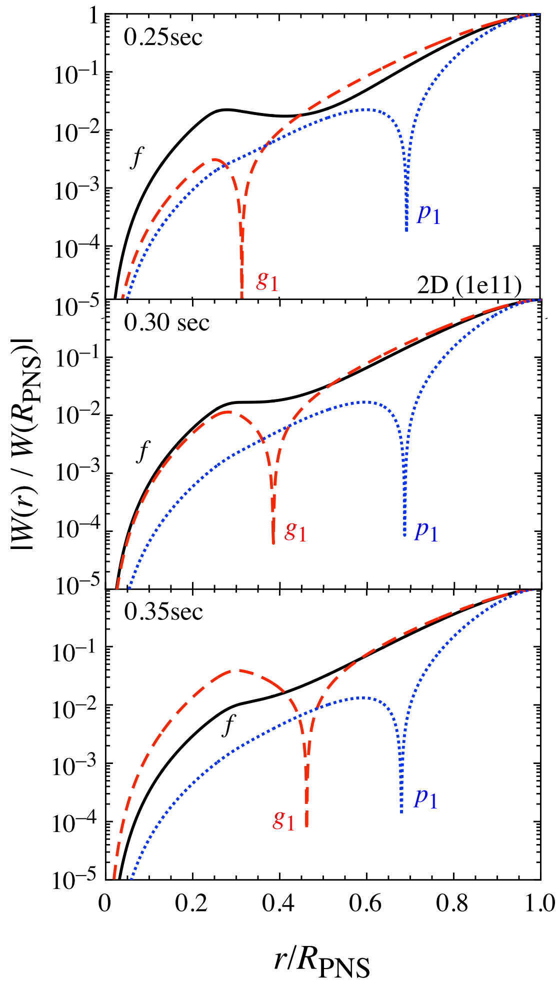
- with several EOSs and progenitor masses

Avoided crossing in GW frequency

(HS, Takiwaki 20b)

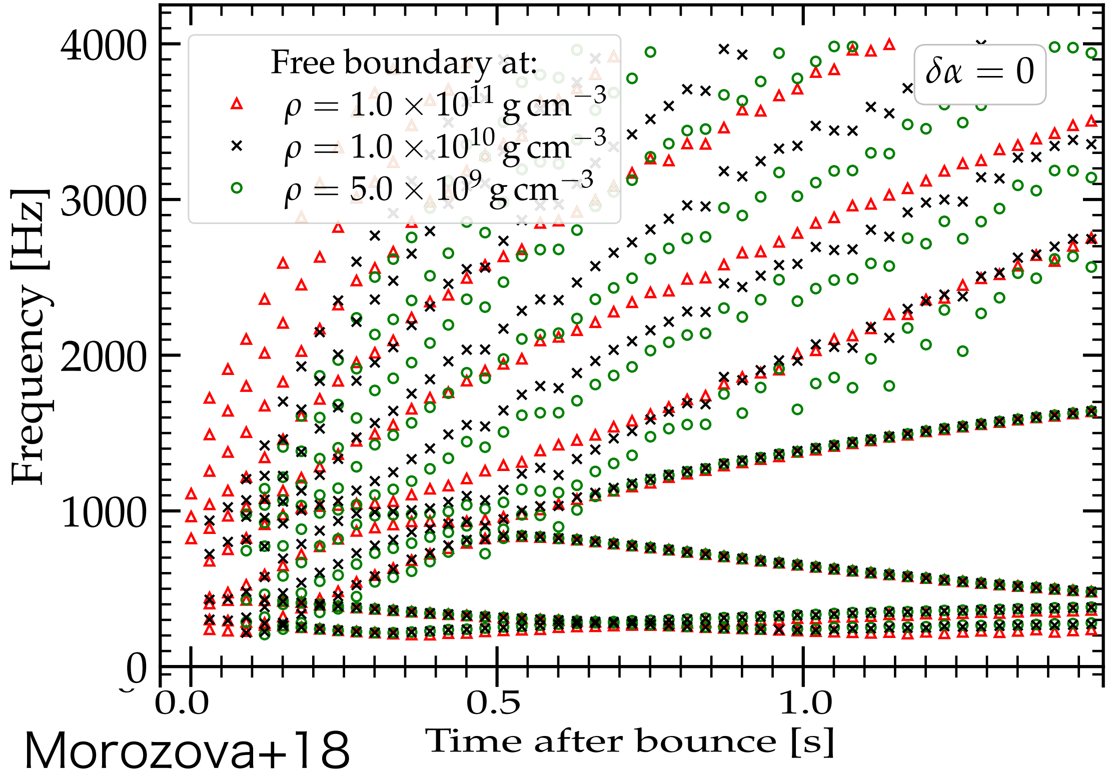
- in the early phase, one can observe the phenomena of avoided crossing between the eigenmodes.
- even in later phase, one can still observe between g_i-modes



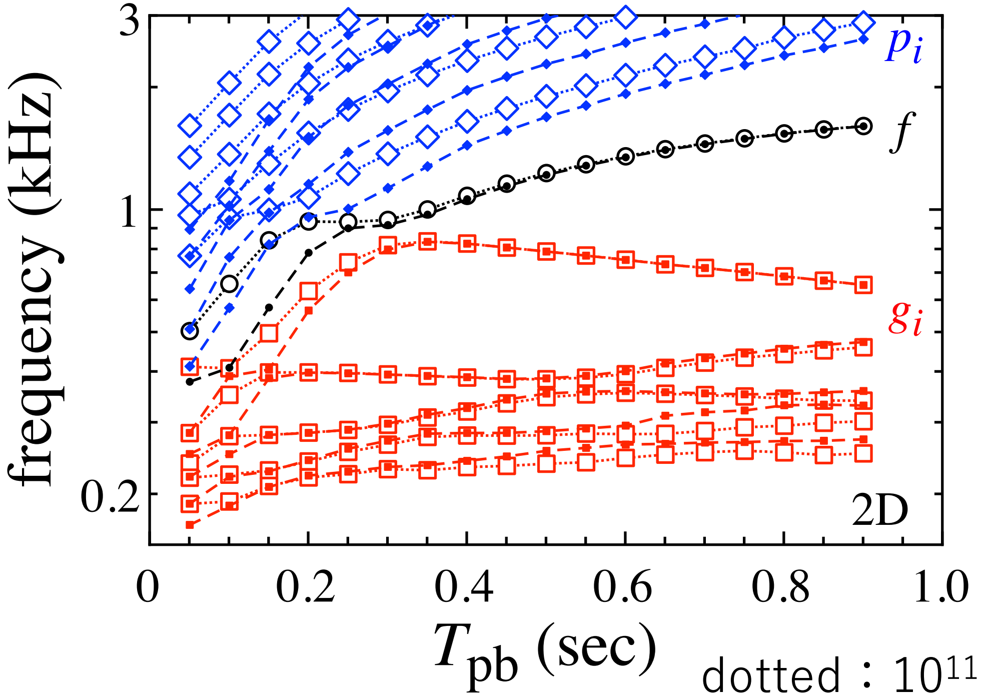


Comment on uncertainty in surface density

- in the late phase after core bounce, e.g., ~ 500ms, f-mode frequency becomes almost independent of the choice of surface density, ρ_s (Morozova+18)
- we also confirm this feature, i.e., f- & g_1-modes in later phase are almost independent of ρ_s, where g_1-mode decreases with time (Sotani & Takiwaki 20b).

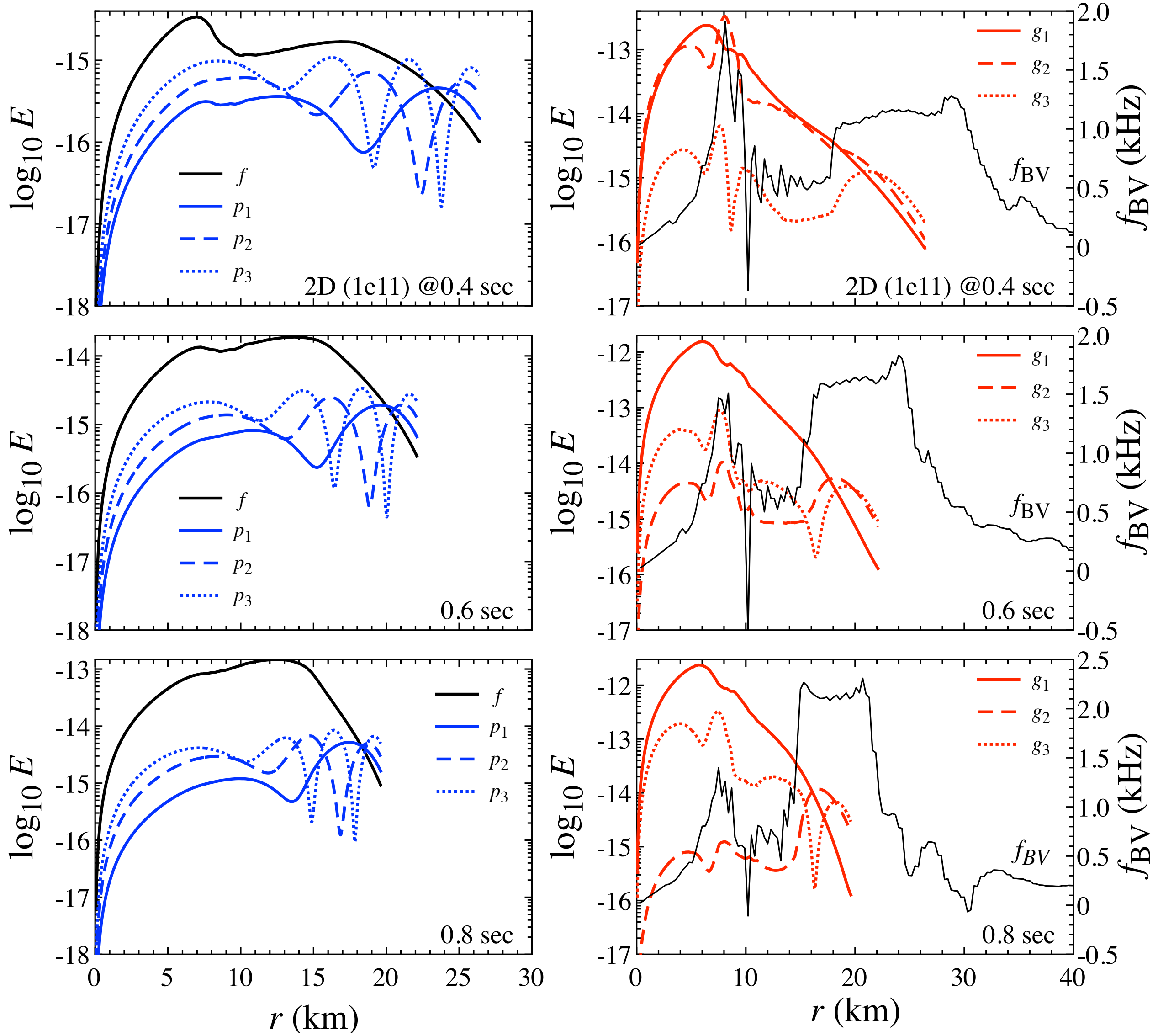


Morozova+18



dotted : 10^{11} g/cm^3
dashed : 10^{10} g/cm^3

pulsation energy density



$$E(r) \sim \frac{\omega^2 \varepsilon}{r^4}\left[W^2 + \ell(\ell+1) r^2 V^2\right]$$

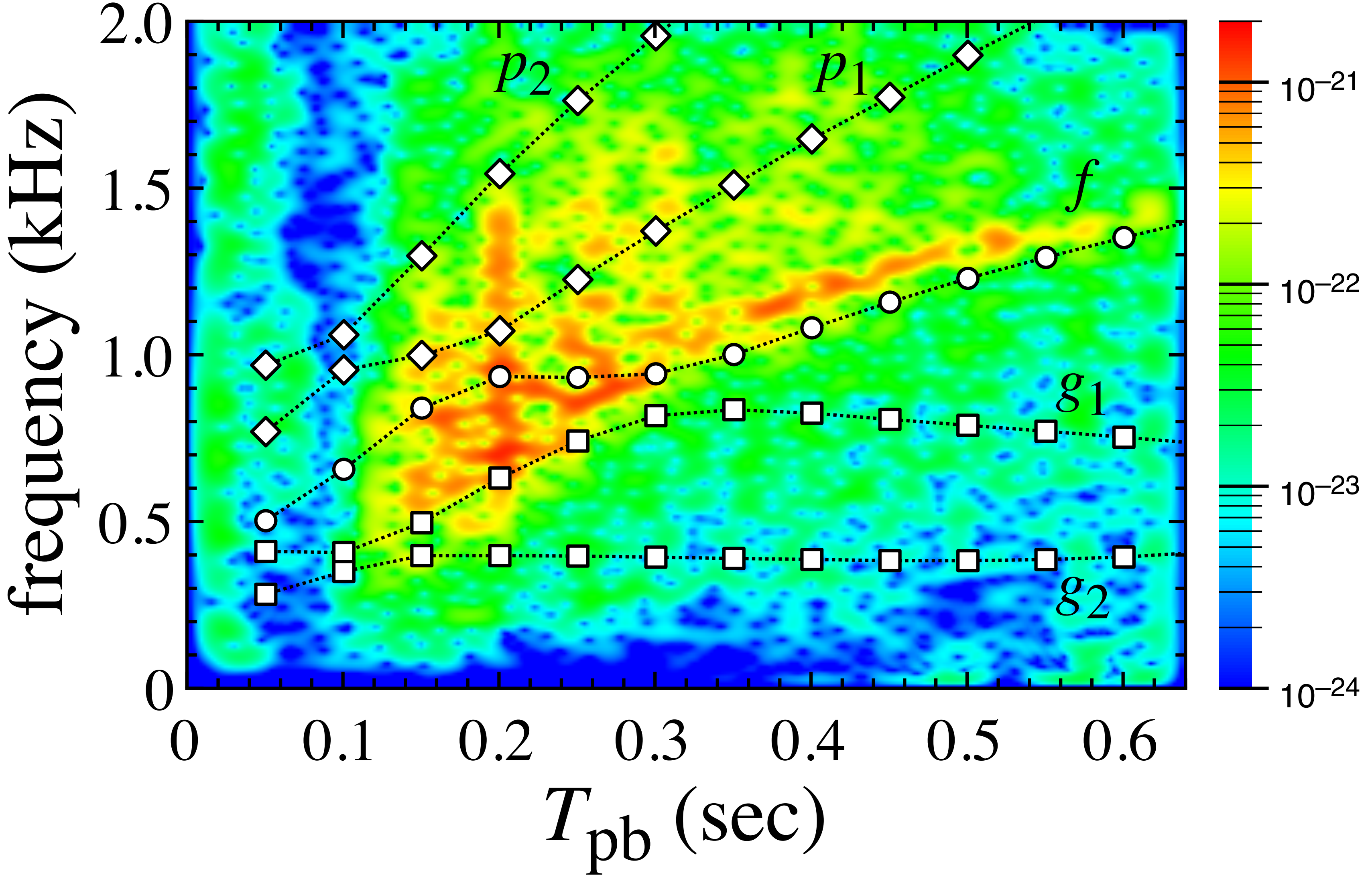
$$f_{\rm BV} = {\rm sgn}(\mathcal{N}^2)\sqrt{|\mathcal{N}^2|/2\pi}$$

$$\mathcal{N}^2 = -e^{2\Phi - 2\Lambda}\frac{\Phi'}{\varepsilon + p}\left(\varepsilon' - \frac{p'}{c_s^2}\right)$$

- f- & g_1-modes are not dominant @PNS surface
  → f- & g_1-modes weakly depend on ρ_s

- g_i-modes related to f_{BV}
- g_1-mode is strongly associated with BV freq. @r=8km, which decreases with time
  → decrease of g_1-mode

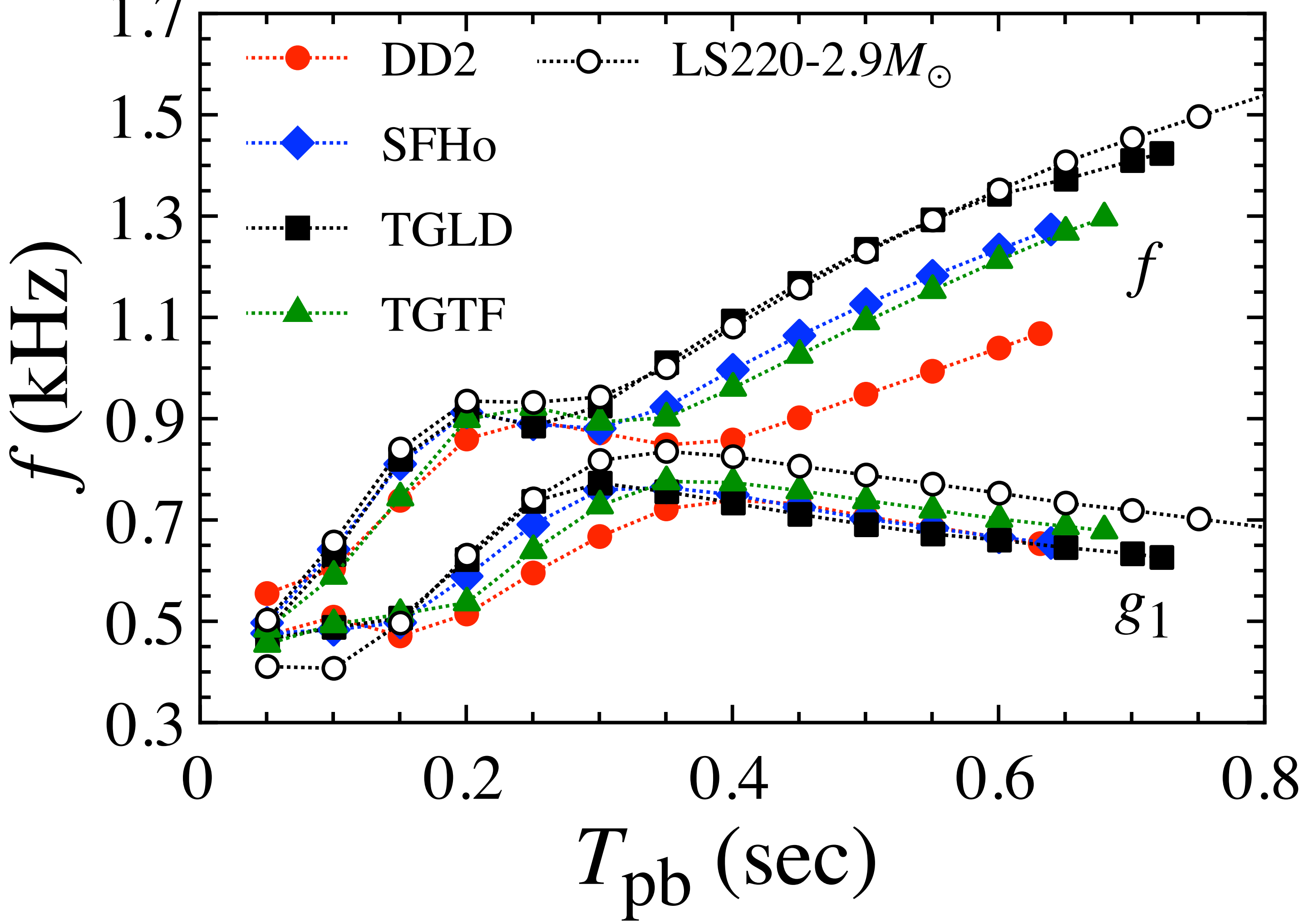
Comparison with GW signals in simulation



effective GR
+ Cowling

Dep. of GW signals on PNS models

HS, Takiwaki, Togashi, 2021



time evolution of GW frequencies strongly depend on the supernova models

TGTF : single nucleus approximation
TGLD : nuclear statistical equilibrium

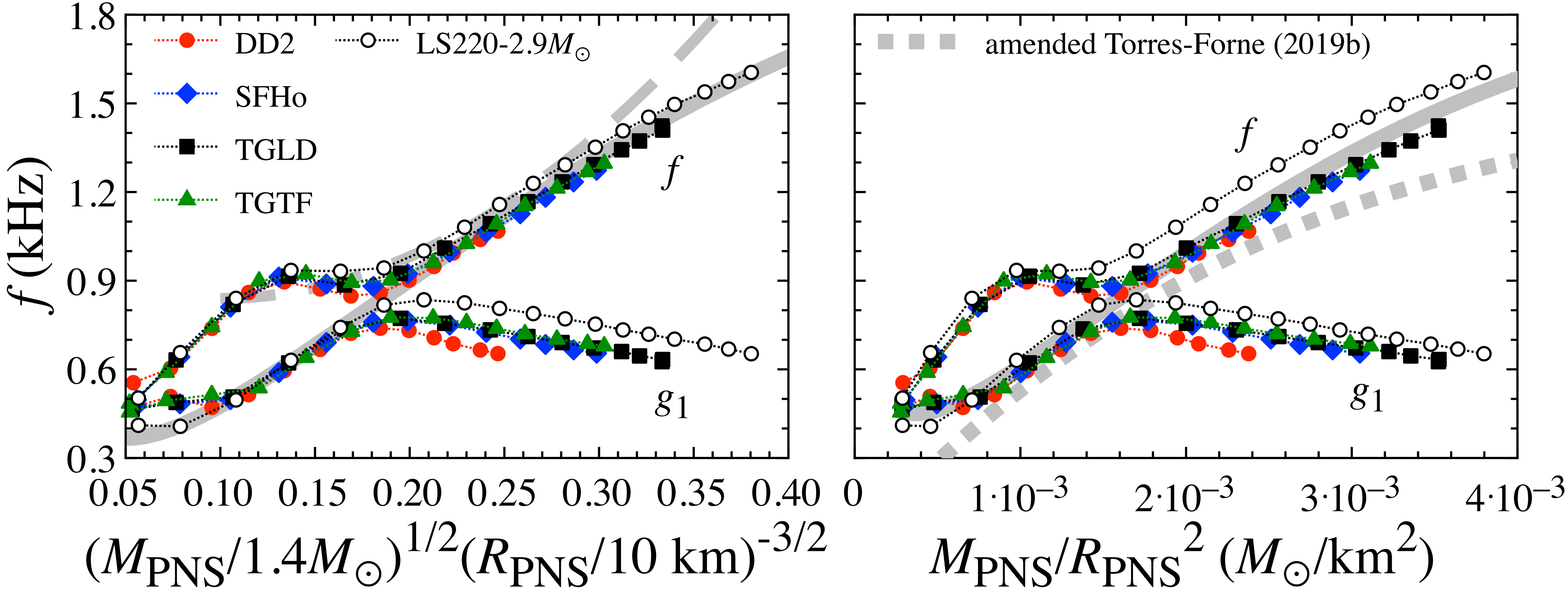
↓

different treatment in EOS for lower density region can change the GW spectra

Universal relations

HS, Takiwaki, Togashi, 2021

- The g_1- and f-mode frequencies can be well expressed as a PNS properties

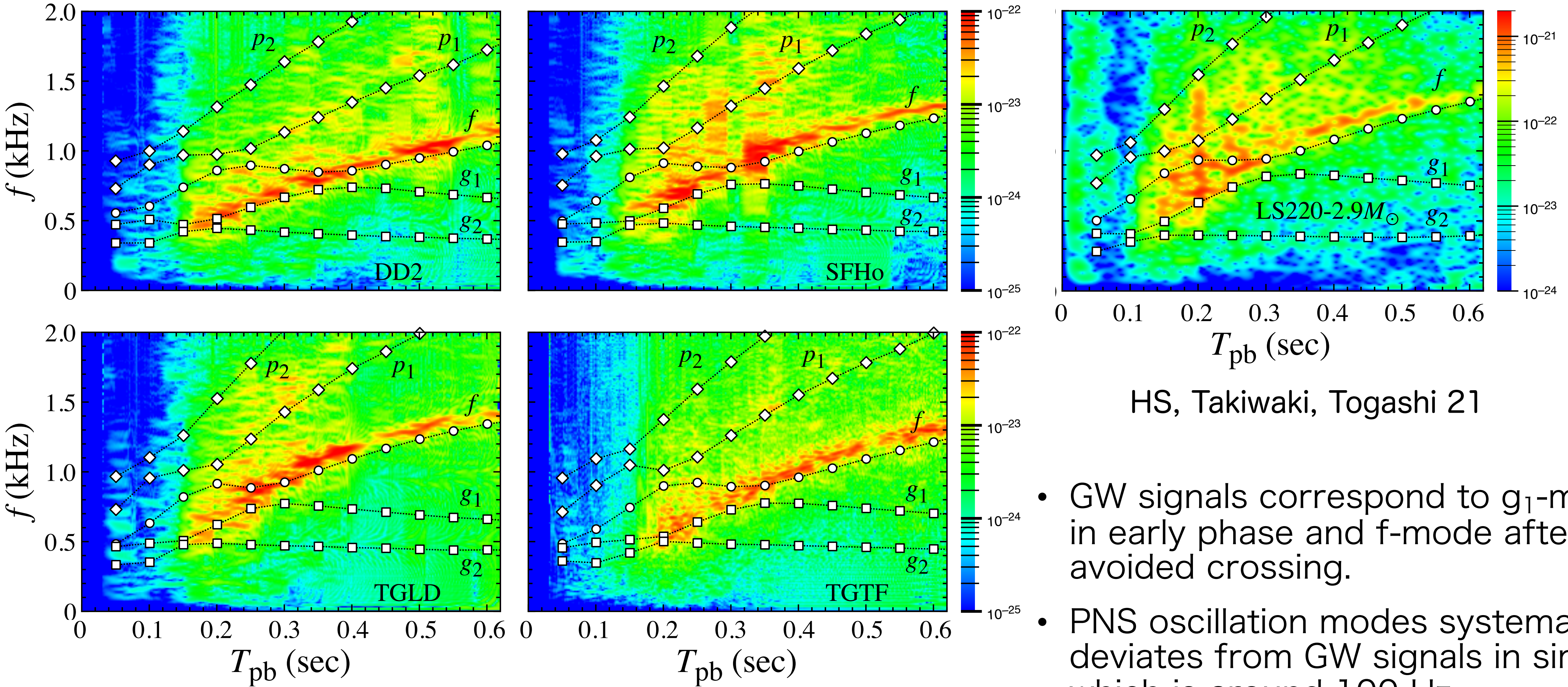


average density

surface gravity

$$f\ (\mathrm{kHz}) = -1.410 - 0.443\ln(x) + 9.337x - 6.714x^2$$

Systematical deviations



HS, Takiwaki, Togashi 21

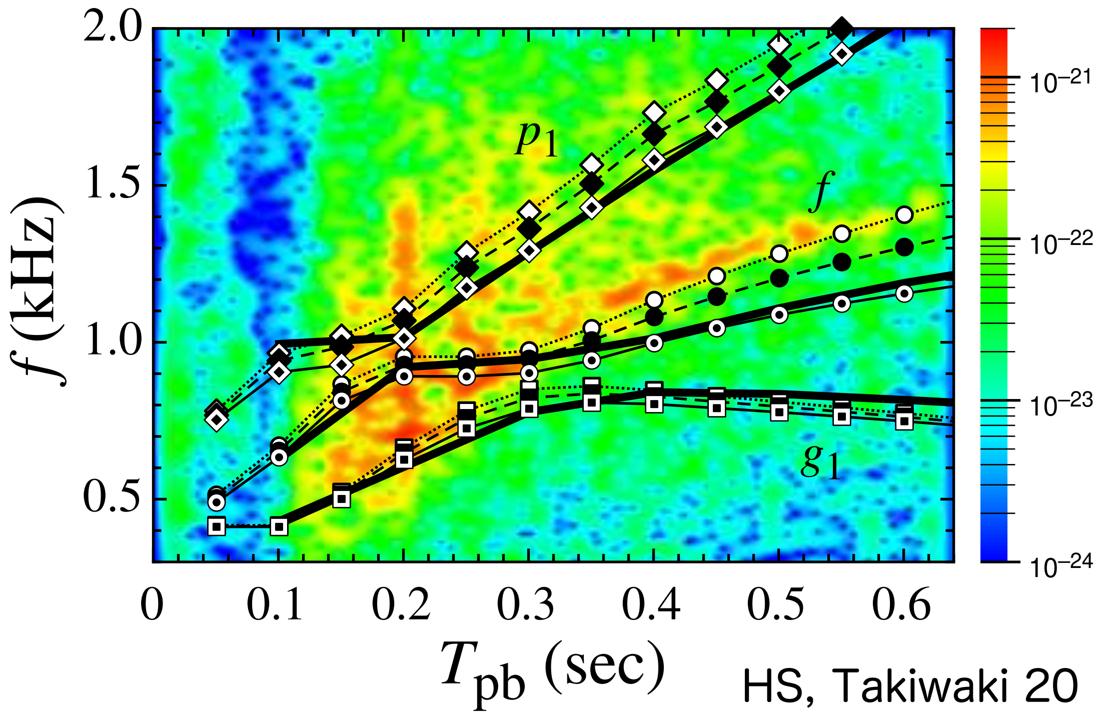
- GW signals correspond to g_1-mode in early phase and f-mode after avoided crossing.
- PNS oscillation modes systematically deviates from GW signals in simulation, which is around 100 Hz.

Possible Causes

- Effective GR + Cowling
  - ✓GW signals in simulations are higher than PNS oscillations
  - ✓With the metric perturbations, the deviation becomes more significant.

- Numerical simulations
  - ✓effective GR (Newtonian)
  - ✓monopole gravity

- Linear analysis
  - ✓GR framework

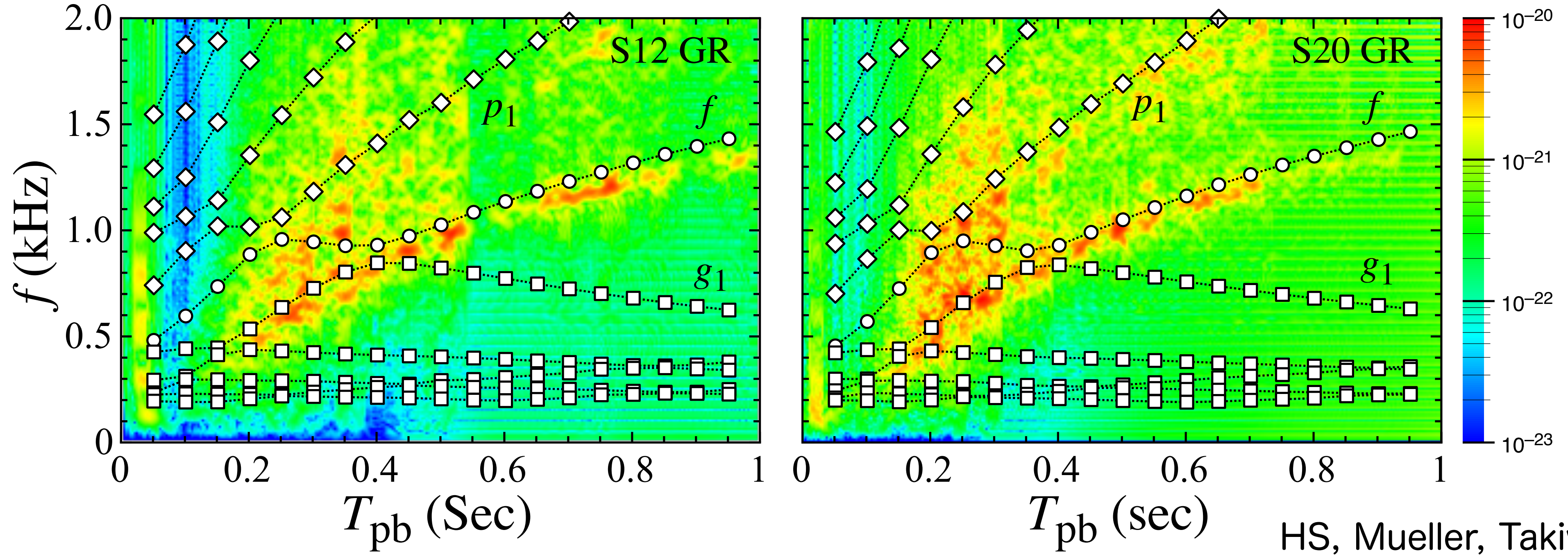


HS, Takiwaki 20

with GR simulations

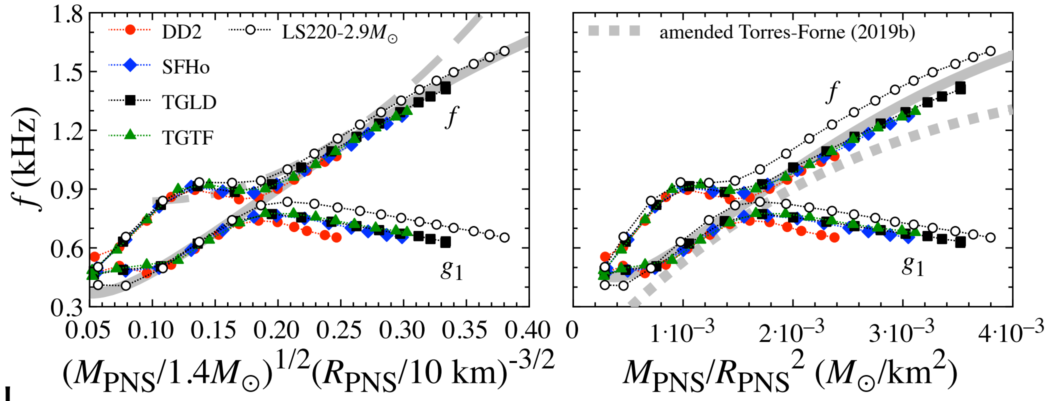
- GR simulations with a monopole approximation
- PNS oscillations with Cowling approximation

  → GW signals in the simulations agree well with the PNS oscillations.



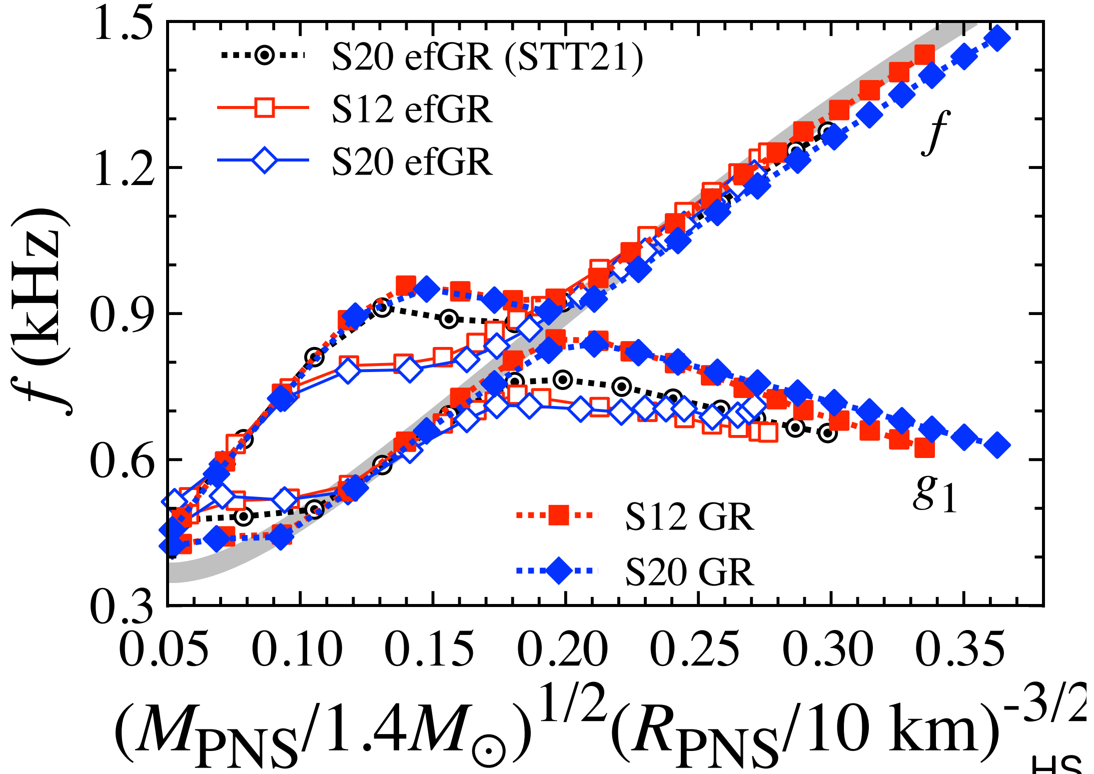
HS, Mueller, Takiwaki 24

Universal relations



STT21

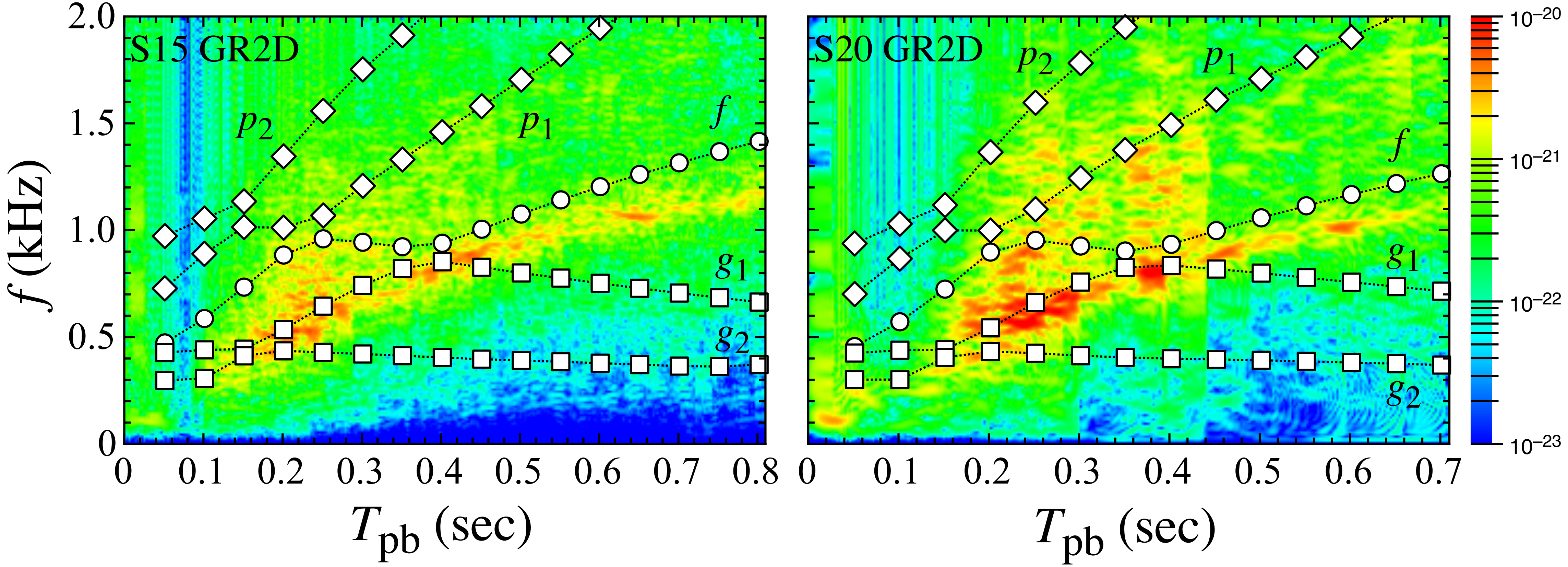
- the PNS oscillations with Cowling approximation, using the GR simulation with monopole gravity, are still on the universal relation derived with the effective GR simulations



HS, Muller, Takiwaki 24

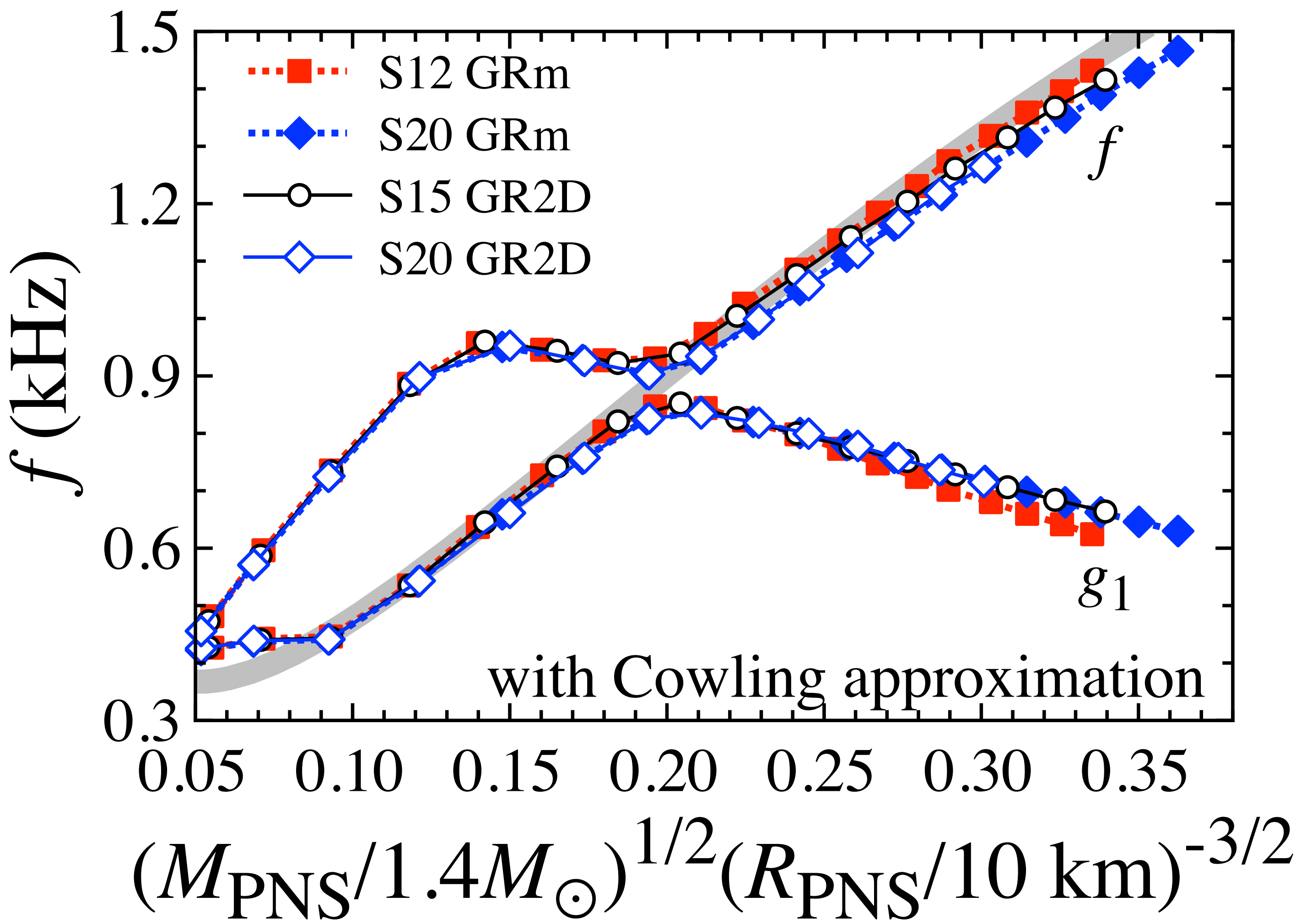
PNS oscillations with Cowling approximation

- PNS oscillation frequencies with Cowling is overestimated the GW frequencies, compared to the GW signals in the numerical simulation with non-monopole gravity.



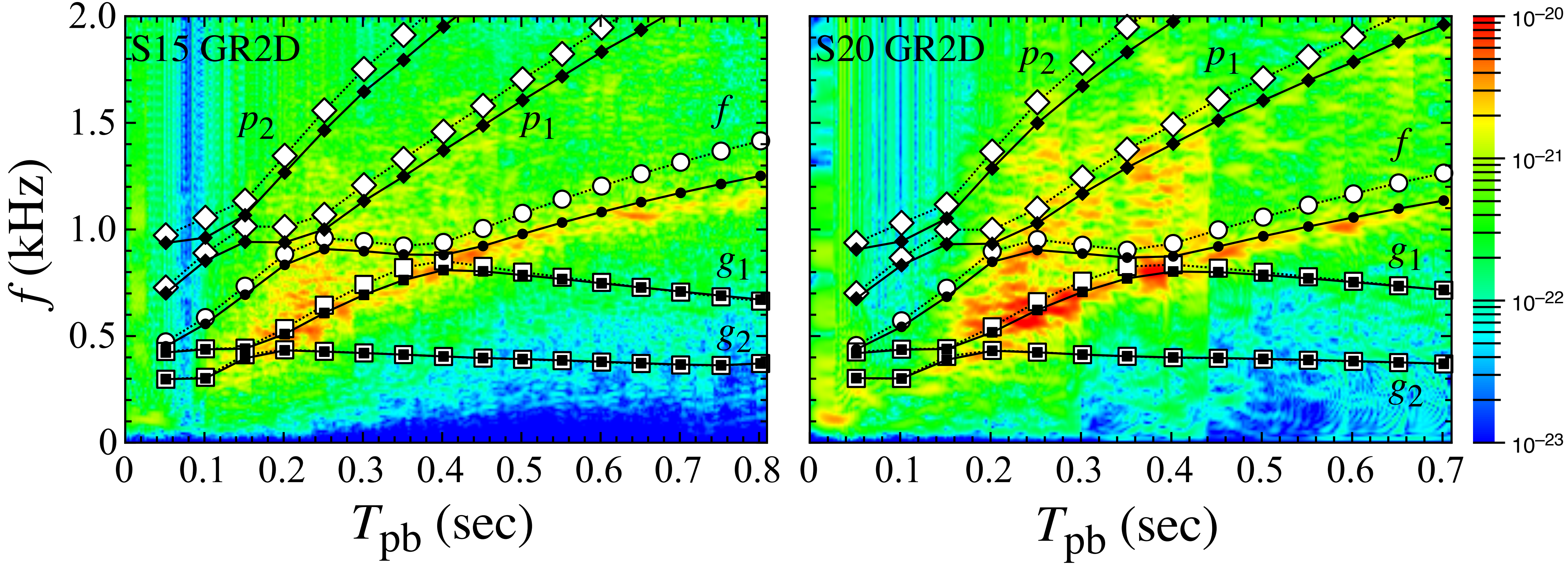
Universal relation

- PNS oscillation frequencies are still on the universal relation



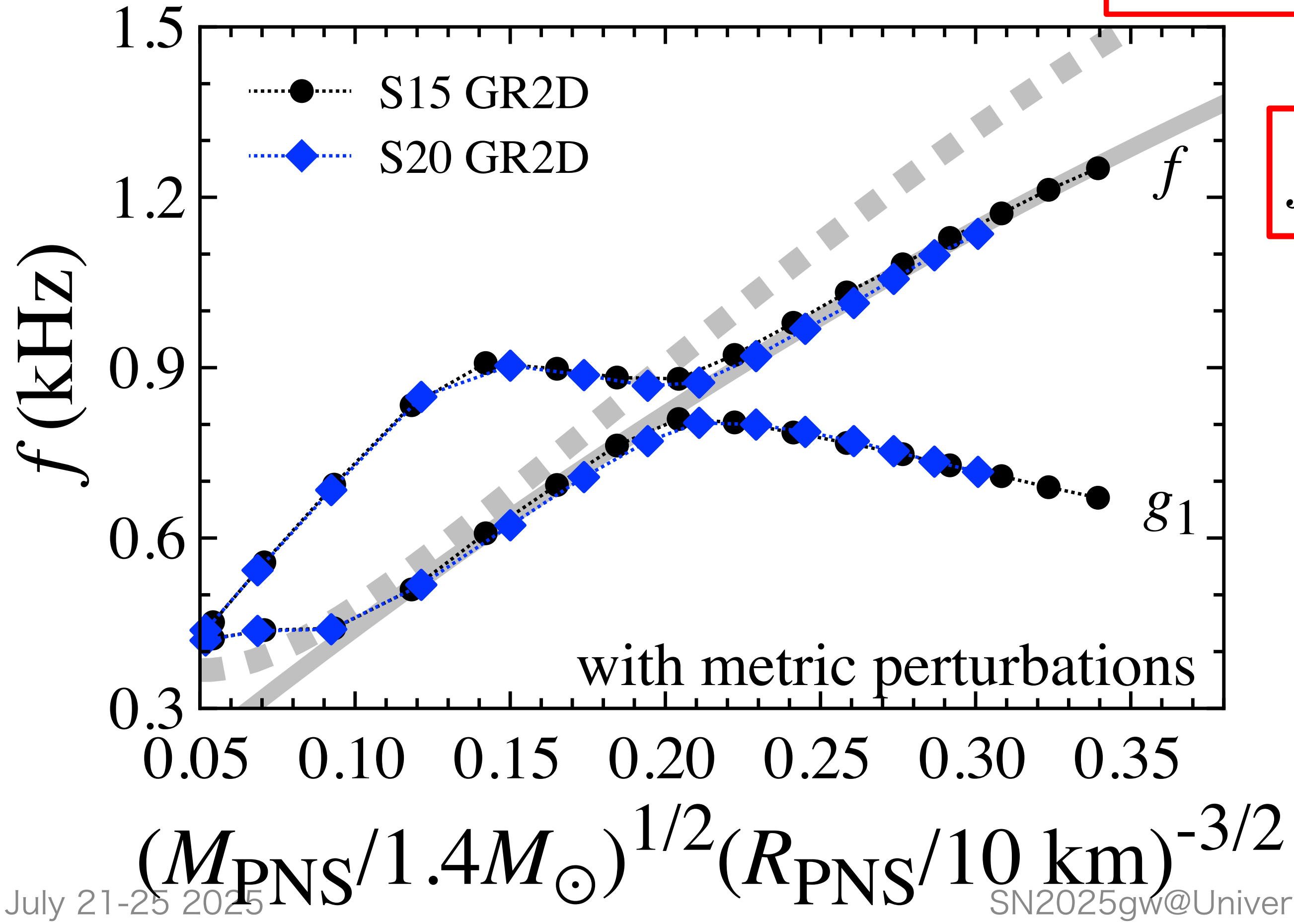
PNS oscillations with metric perturbations

- PNS oscillations frequencies with metric perturbations agree well with the GR simulations with non-monopole gravity.



Universal relations

- Universal relations should be modified.



with Cowling (GR with monopole gravity)

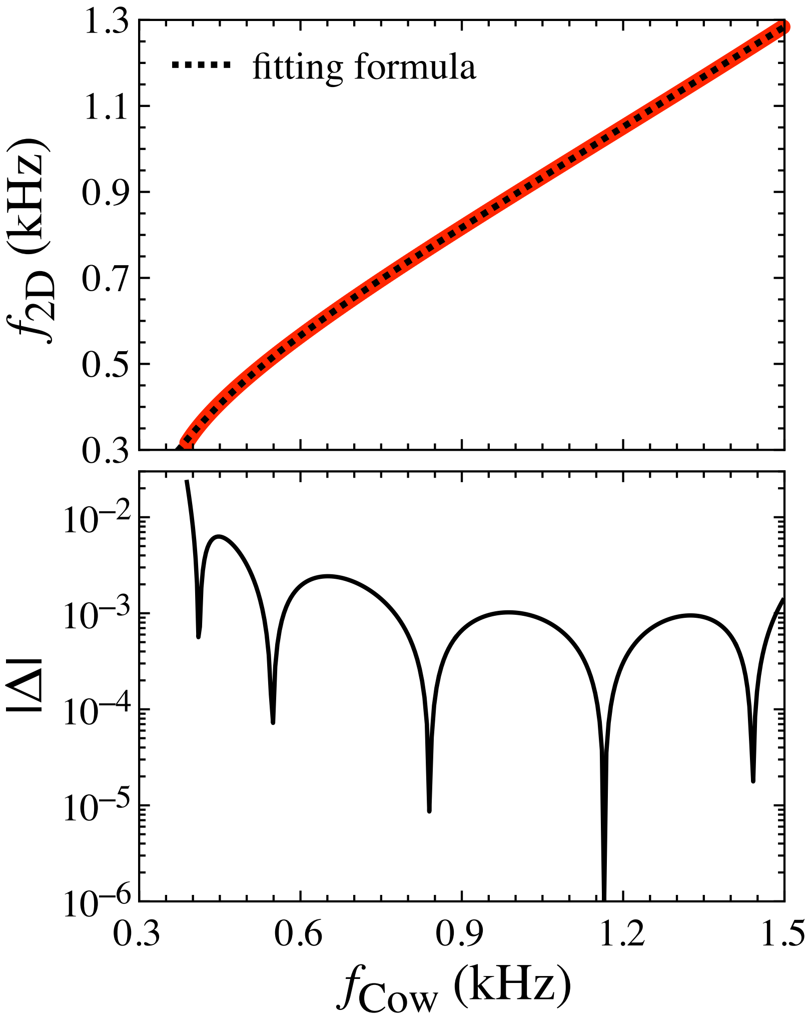
$$\boxed{f(\text{kHz}) = -1.410 - 0.443\ln(x) + 9.337x - 6.714x^2}$$

$$\boxed{f(\text{kHz}) = 0.0082 + 4.5908x - 2.6821x^2}$$

with metric perturbations
(GR with non-monopole gravity)

Estimation of the GW frequencies

- We derive two different universal relations
  - ✓ with monopole gravity (GR)
  - ✓ with non-monopole gravity (GR)
- Using the GW frequencies calculated with monopole gravity, one can estimate the GW freq. with non-monopole gravity.



$$\boxed{\begin{aligned} f_{2D}^{\mathrm{fit}} =& 1.7800 + 0.9676 \ln(f_{\mathrm{Cow}}) - 1.8052 f_{\mathrm{Cow}} \\ &+ 1.1441 f_{\mathrm{Cow}}^2 - 0.2236 f_{\mathrm{Cow}}^3, \end{aligned}}$$

Conclusion

simulations	Cowling approximation	With metric perturbations
Effective GR (monopole gravity)	f_GW > f_PNS	f_GW > f_PNS
GR (monopole gravity)	**f_GW ~ f_PNS**	
GR (non-monopole gravity)	f_GW < f_PNS	**f_GW ~ f_PNS**

f_GW : GW frequencies in the simulations
f_PNS : PNS frequencies

- With the Cowling approximation

$$f(\mathrm{kHz}) = -1.410 - 0.443\ln(x) + 9.337x - 6.714x^2$$

- GR non-monopole gravity

$$f(\mathrm{kHz}) = 0.0082 + 4.5908x - 2.6821x^2$$

- one can estimate the f_GW in GR with non-monopole gravity from f_GW in GR with monopole gravity

$$f_{\mathrm{2D}}^{\mathrm{fit}} = 1.7800 + 0.9676\ln(f_{\mathrm{Cow}}) - 1.8052 f_{\mathrm{Cow}} + 1.1441 f_{\mathrm{Cow}}^2 - 0.2236 f_{\mathrm{Cow}}^3,$$