SN2025gw: First IGWN Symposium on Core Collapse Supernova Gravitational Wave Theory and Detection

Understanding proto-neutron star oscillations in core-collapse supernova

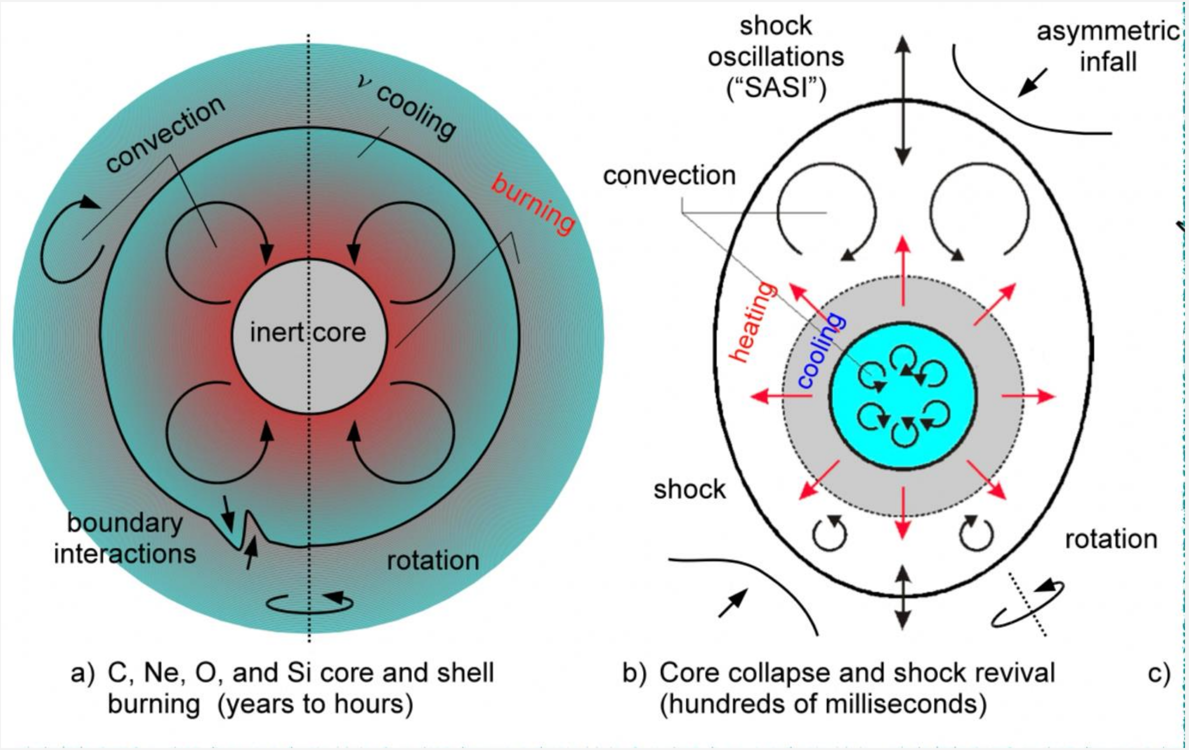
A brief overview of asteroseismology

Alejandro Torres-Forné

Dimitra Tseneklidou, Noah Wolf, Tristan Bruel, Marie-Anne Bizouard, Patricio Maturana, Renate Meyer, Carla Fröhlich, Bernhard Müller, Martin Obergaunlinguer, Andrea Pasamonti, Pablo Cerdá-Durán and Toni Font

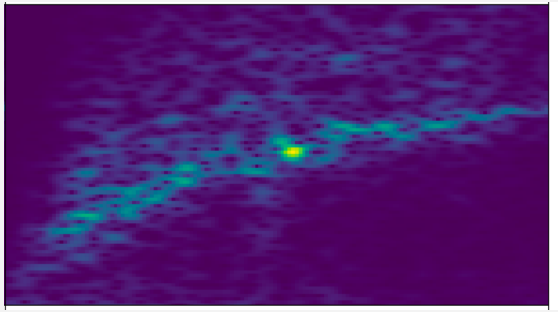
ANATOMY OF CCSN

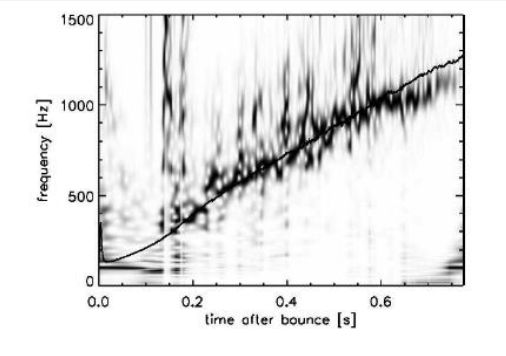
See Bernhard's Talk



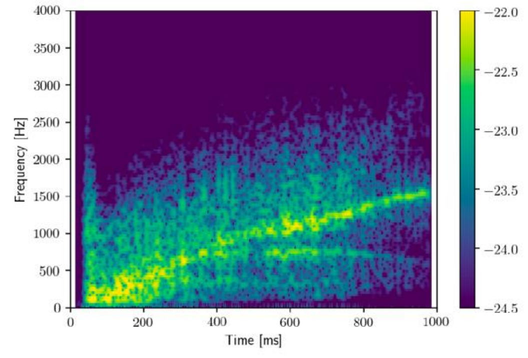
GW EMISSION FROM PNS

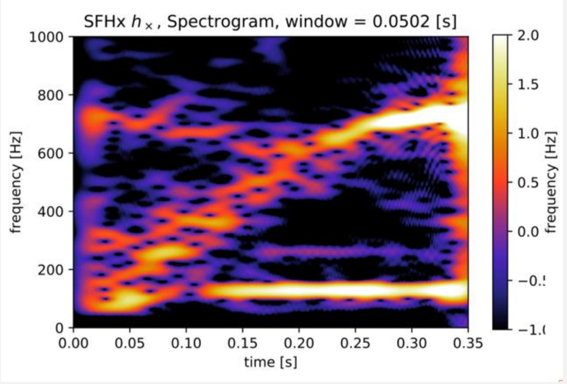
Highly stochastic





Time evolving frequencies (g-modes, SASI)





INTRODUCTION TO ASTEROSEISMOLOGY

- Simplified background: Reisenegger & Goldreich 1992, Ferrari et al 2003, 2004, Passamonti et al 2005, Krüger et al 2015, Camelio et al 2017
- Background based on simulations (f, p and w modes): Sotani et al 2018
- Background from simulations + Cowling approximation: TF et al 2018a
- Background from simulations + lapse perturbations: Morozova et al 2018
- Background from simulations + space-time perturbations (lapse and conformal factor): TF et al 2018b → GREAT code: General Relativistic Eigenmode Analysis Tool. https://www.uv.es/cerdupa/codes/GREAT/
- Pseudo-Newtonian analysis: Westernacher-Schneider 2020
- Universal Relations: TF et al 2019, Sotani 2021 See Sotani's talk
- Inclusion of accretion: Tseneklidou et al 2025 See Dimitra's talk

LINEAR PERTURBATION ANALYSIS

The system of equations:

σ: eigenfrequencies

$$[\mathbf{A}] \cdot [\eta] = \sigma^2 [\mathbf{B}] \cdot [\eta]$$

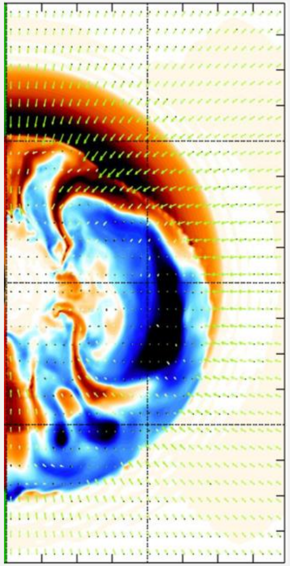
$A, \ B$: coefficient matrices

η: perturbed variables matrix

Boundary Conditions at different locations

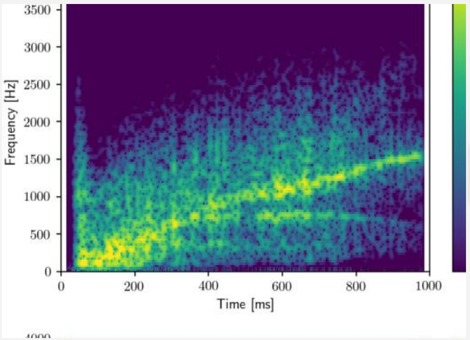
INTRODUCTION TO ASTEROSEISMOLOGY

Multi-dimensional numerical simulation



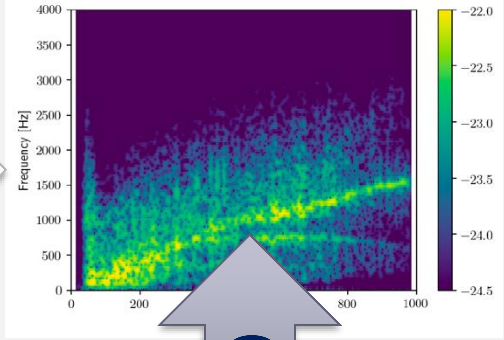
Obergaulinger et al 2013

waveform

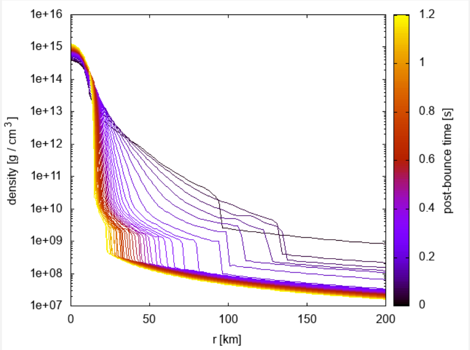


TF et al 2019

spectrogram

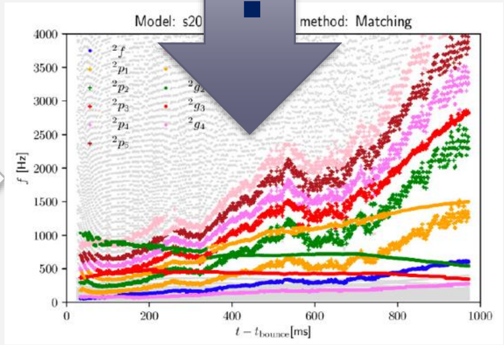


1D angular average



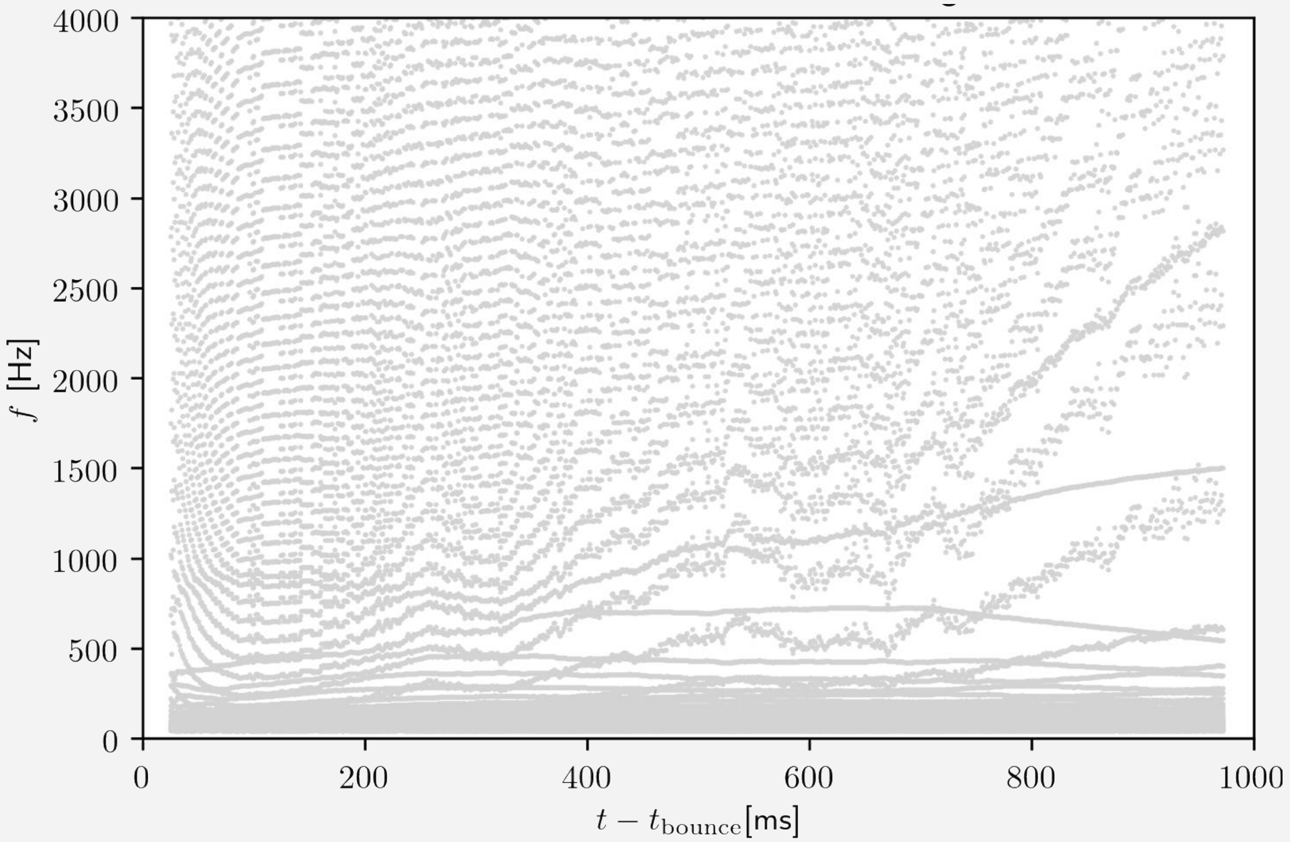
?

l=2 eigenmodes



TF et al 2018

INTRODUCTION TO ASTEROSEISMOLOGY

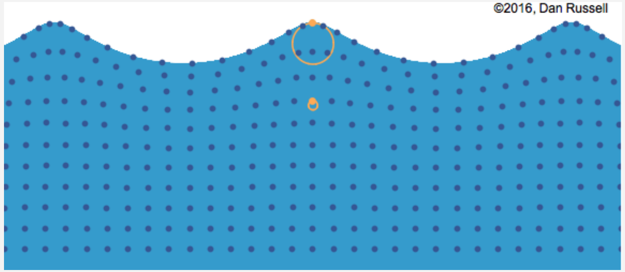


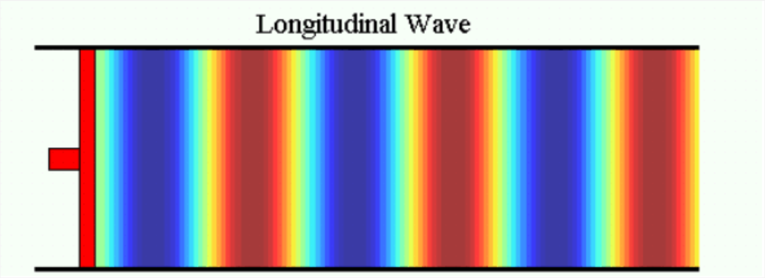
WHAT ARE P/G/F MODES?

Gravity (buoyancy) waves vs pressure waves

Transverse waves

Longitudinal waves

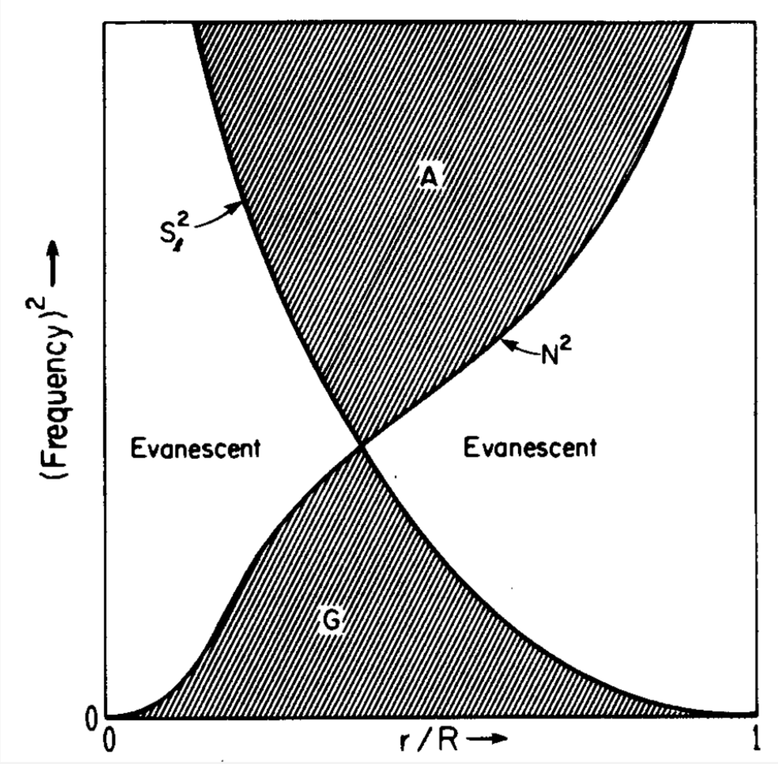




g-modes

p-modes

WHAT ARE P/G/F MODES?



Cox 1980

Propagation diagram

N^2 : Brunt-Väisälä frequency

$\mathcal{L}^2$: Lamb frequency

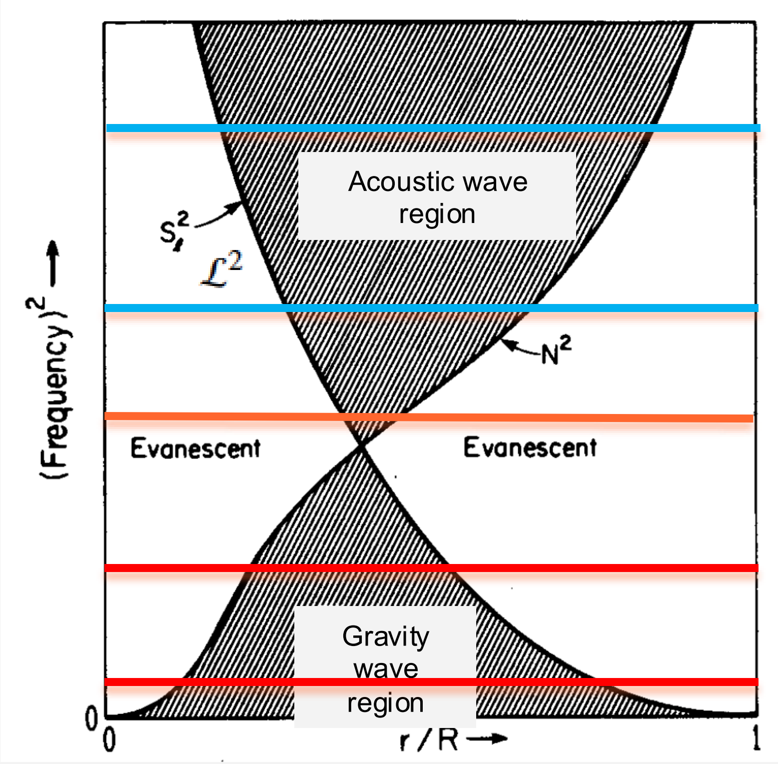
Acoustic wave region:

$$\sigma^2 > N^2, \mathcal{L}^2$$

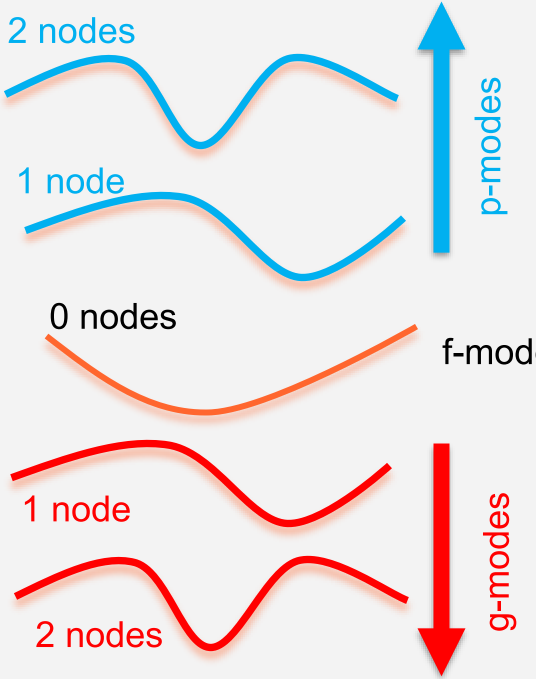
Gravity wave region:

$$\sigma^2 < N^2, \mathcal{L}^2$$

WHAT ARE P/G/F MODES?

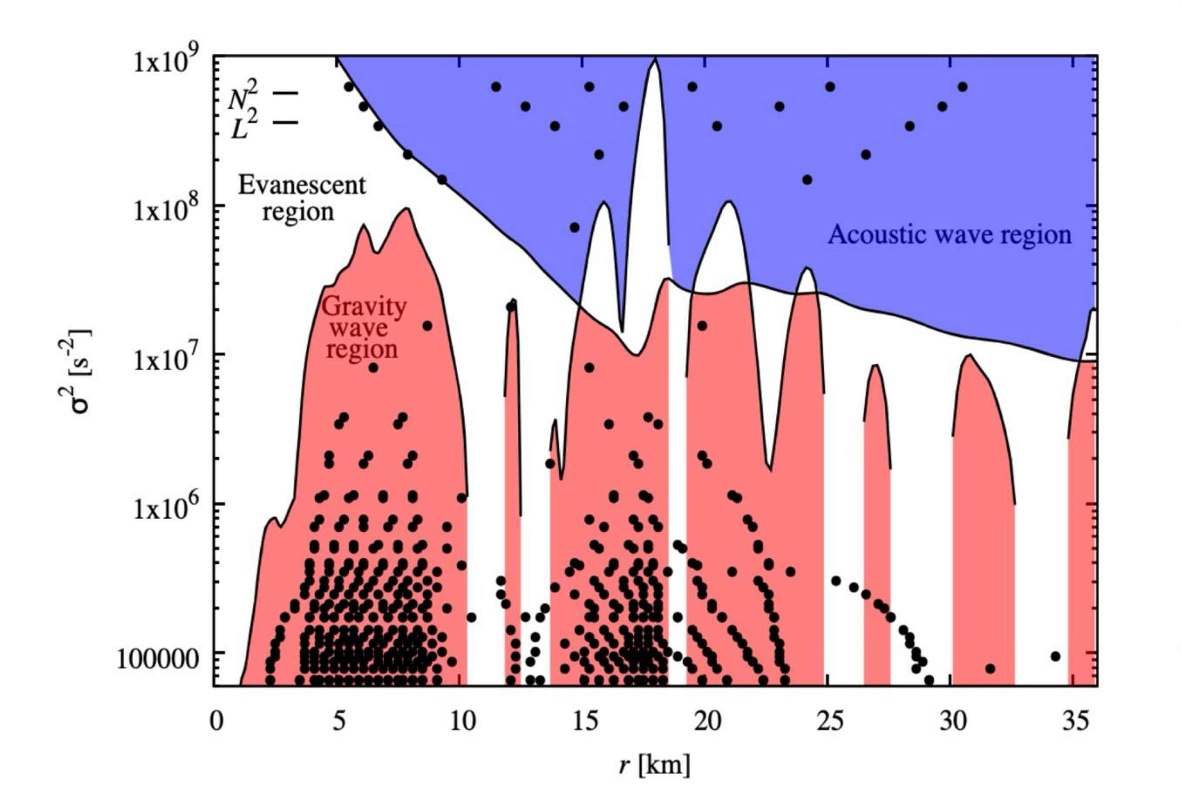


Cox 1980



Cowling 1941 classification

WHAT ARE P/G/F MODES?



Torres-Forné et al. (2019)

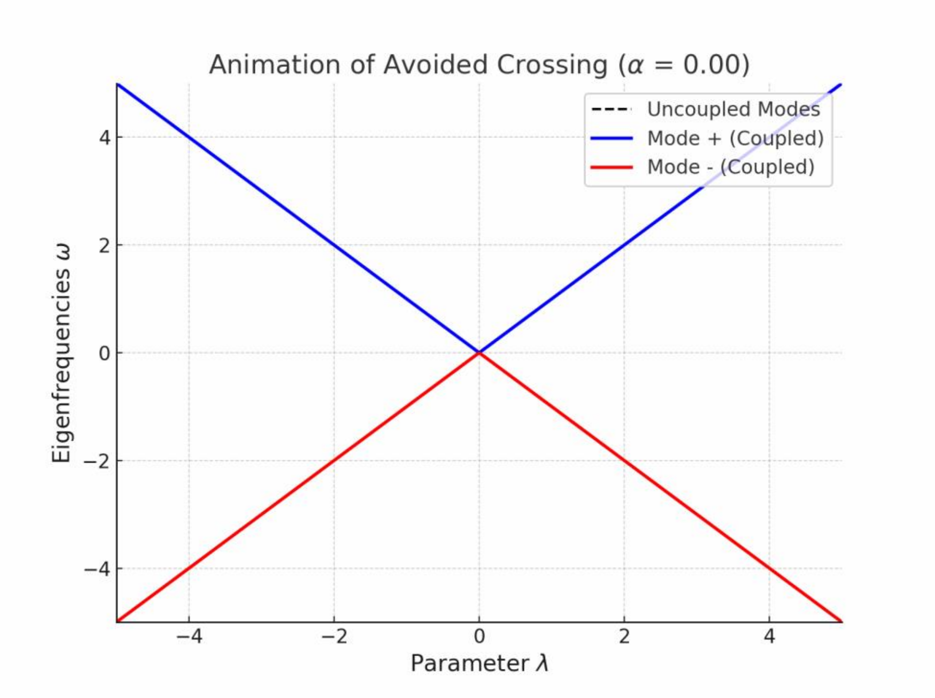
AVOIDED CROSSINGS

Coupled Oscillators:

$$\begin{pmatrix} \omega_1^2(\lambda) & \alpha \\ \alpha & \omega_2^2(\lambda) \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \omega^2 \begin{pmatrix} y_1 \\ y_2 \end{pmatrix}$$

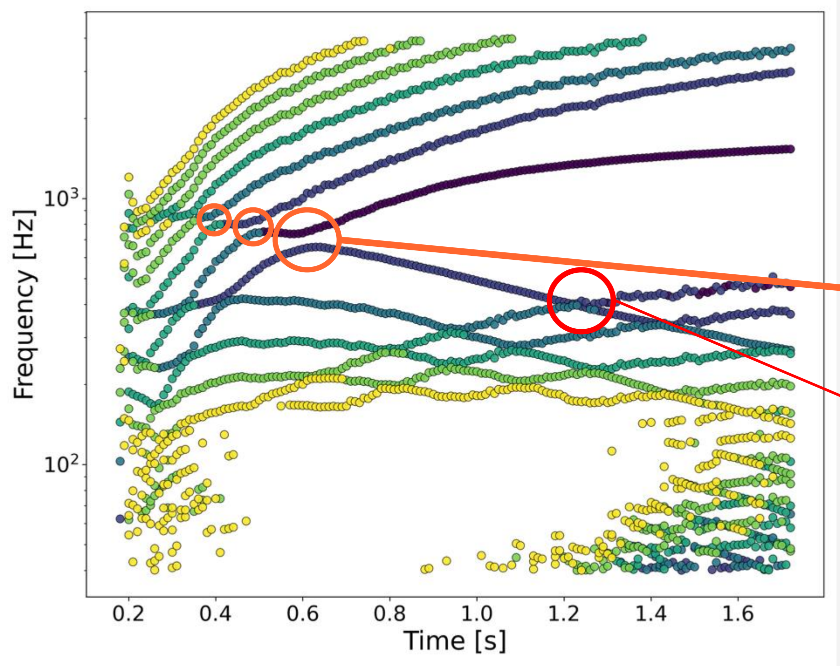
$$\omega_\pm^2(\lambda) = \frac{\omega_1^2(\lambda) + \omega_2^2(\lambda)}{2} \pm \frac{1}{2}\sqrt{\left(\omega_1^2(\lambda) - \omega_2^2(\lambda)\right)^2 + 4\alpha^2}$$

“ At the avoided crossing the two modes exchange nature, while still maintaining the original labelling… “



Aerts, C., Christensen-Dalsgaard, J., & Kurtz, D. W. (2010).

AVOIDED CROSSINGS

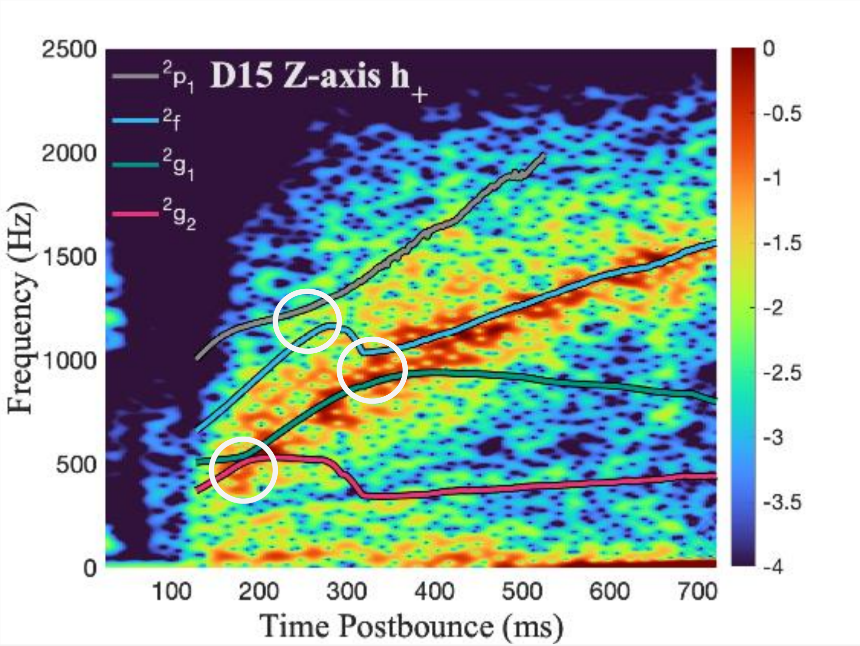


A series of avoided crossings

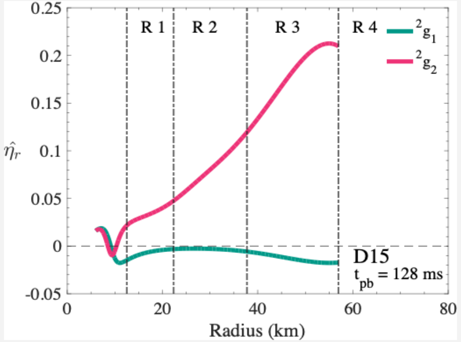
A series crossings: change in the number of nodes

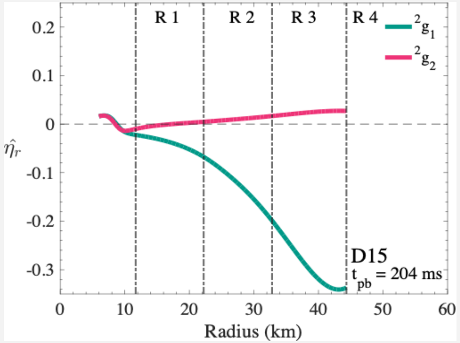
AVOIDED CROSSINGS

See Daniel's talk



Before the AC



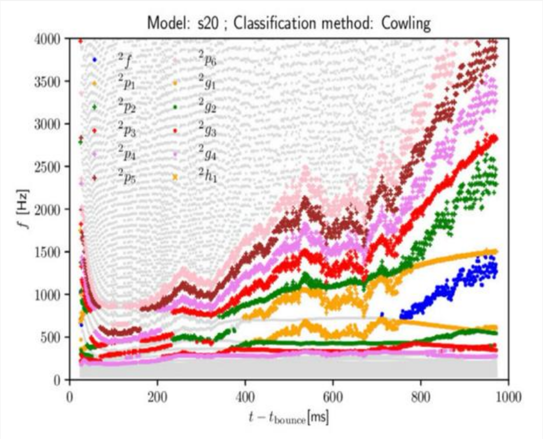


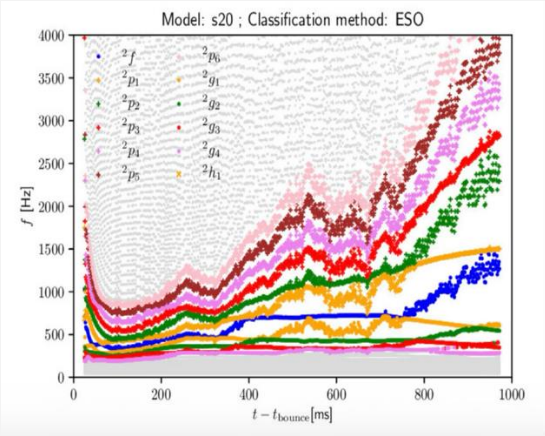
After the AC

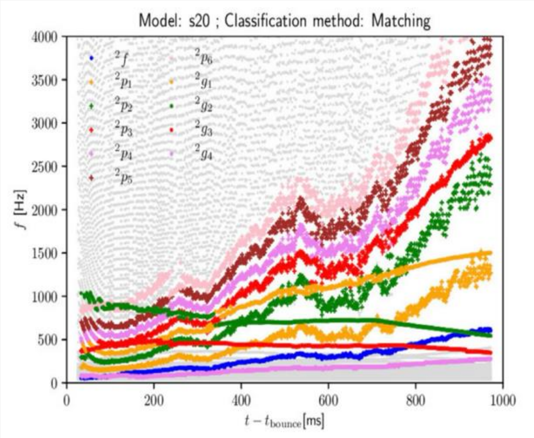
Murphy et al. Phys. Rev. D (2025) arXiv:2503.06406

P/G/F CLASSIFICATION

See also Rodriguez et al. (2023)







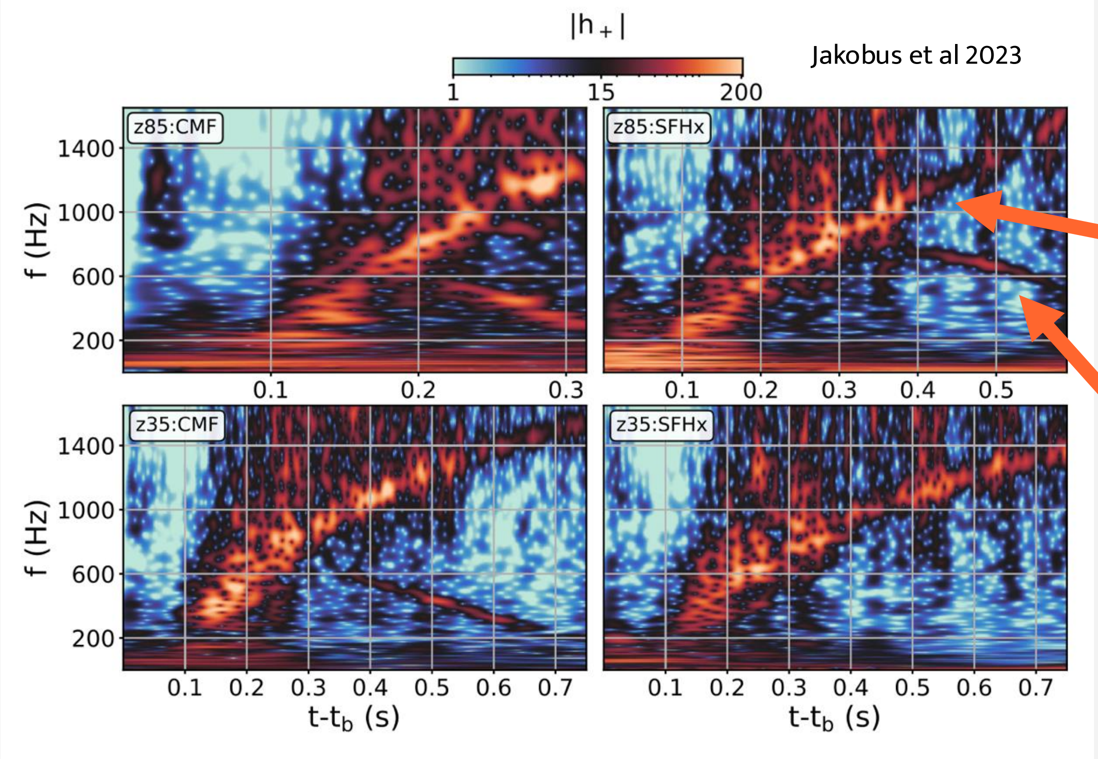
Classifies modes by radial nodes
Ambiguous fundamental mode limits early-time reliability.

Uses eigenfunction phase to track modes and avoided crossings.
Early-time mode mixing is complex.

Track mode evolution including avoided crossings.
Early-time mode mixing complicates interpretation.

WHAT WE CAN LEARN FROM ASTEROSEISMOLOGY?

MODE ORIGIN



Dominant mode

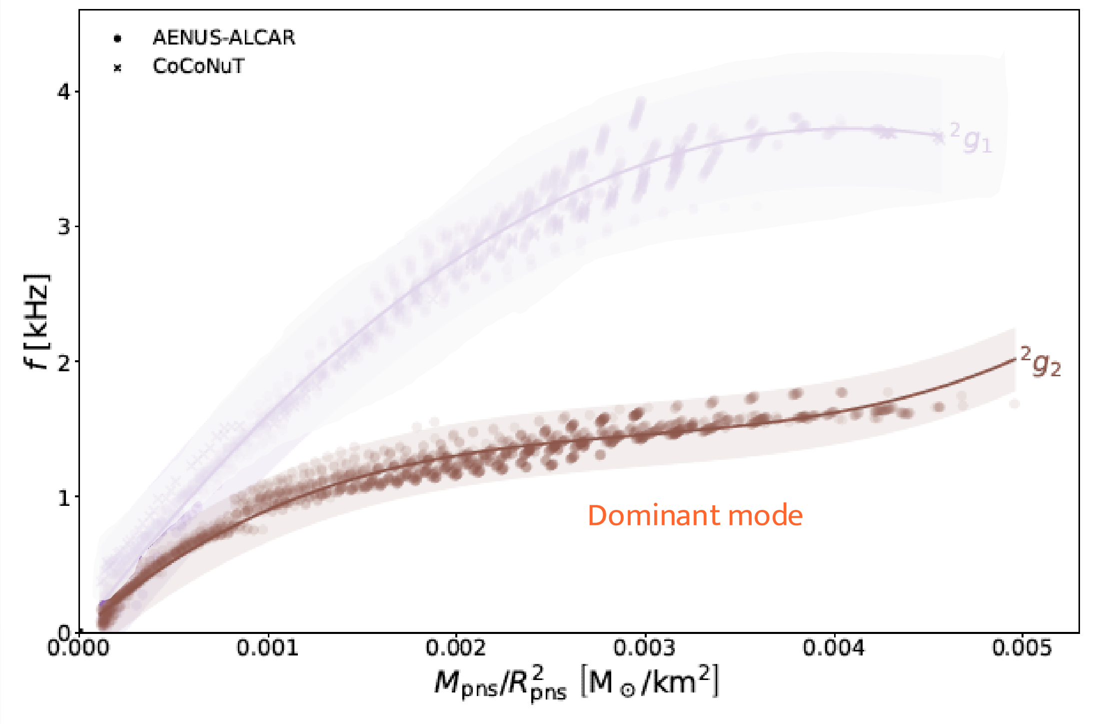
Lives at the PNS surface and depends on bulk properties

Core g-mode

Comes from the core of the PNS.

It is determined by the EoS

UNIVERSAL RELATIONS



26 1D simulations

2 codes (Alcar-Aenus and CoCoNuT)

6 EOS (LS220, Gshen-NL3, Hshen, SFHo, BHB-L, Hshen-L)

8 progenitors (11.2 – 75 $M_\odot$)

g-modes scale with PNS surface gravity

No dependence on EOS

Torres-Forné et al 2019

UNIVERSAL RELATIONS

Sotani & Takiwaki 2020

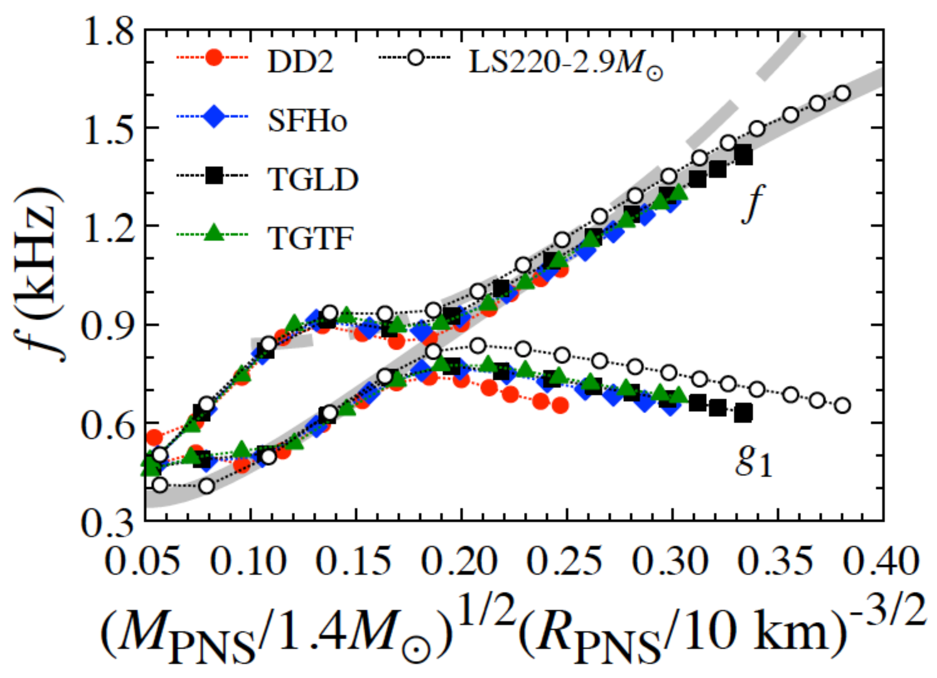
See Sotani's talk

Where the differences come from?

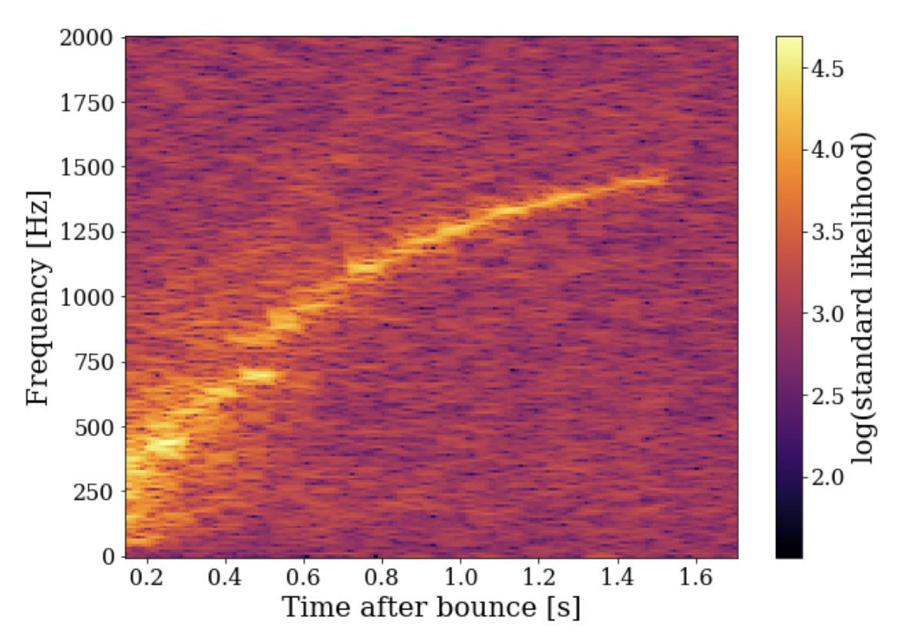
Different classification method

Difference between models introduce errors in the fits

Several combinations of M over R with different exponents produce similar fits



DOMINANT MODE - INFERENCE



LVK network, source at 5 kpc

Injections:

- Simulations: 10 x 2D & 3D
- Code: Aenus-Alcar
- Progenitors: 11.2-40 $M_\odot$
- EOS: LS220, Gshen, SFHo

Noise:

- Detector network 2nd gen (HL, HLV, HLVK, HLVKA..) and 3rd gen (ET, CE)
- Simulated noise

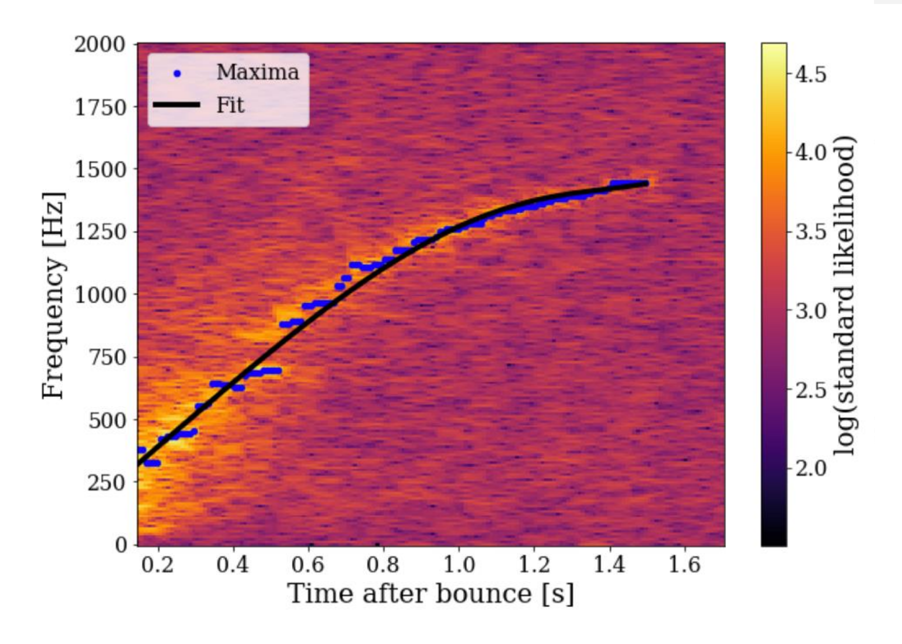
Observational scenario

- Neutrino trigger (time of bounce within ~10 ms)
- EM observation → accurate sky localization

Spectrograms:

- Dominant polarization frame (similar to X-pipeline)
- Time shifted data

DOMINANT MODE - INFERENCE



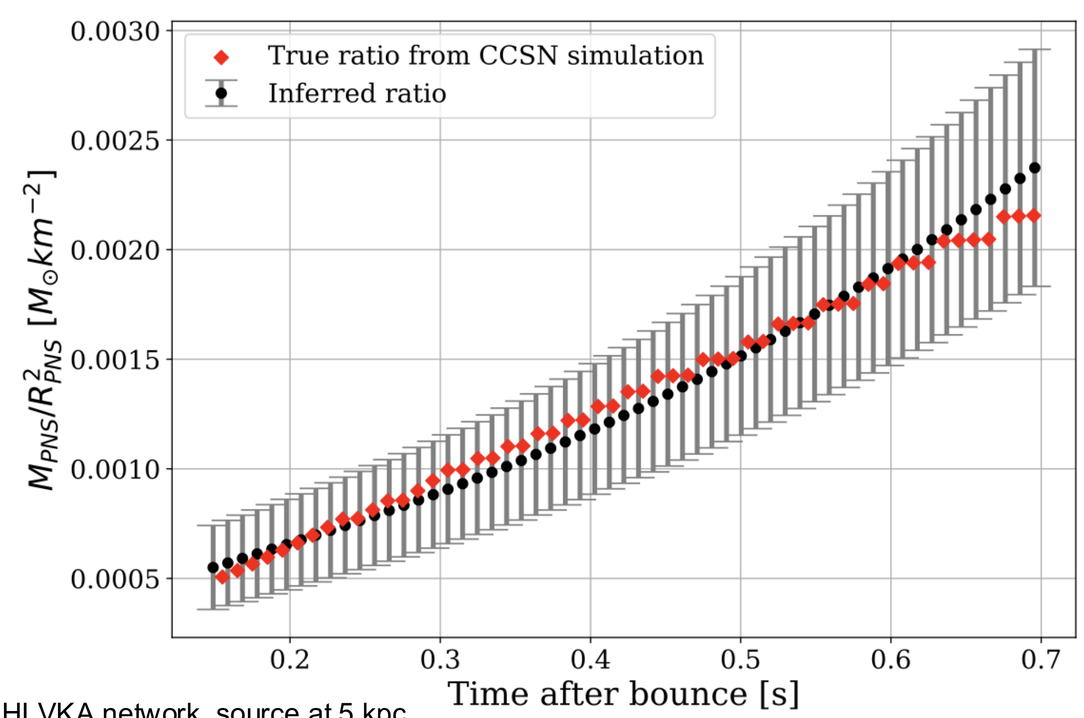
LVK network, source at 5 kpc

Time-frequency tracking of the main ridge

f(t)

DOMINANT MODE - INFERENCE

Bruel et al 2023

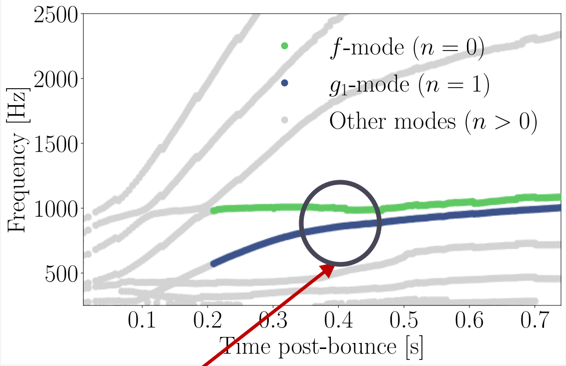


HLVKA network, source at 5 kpc

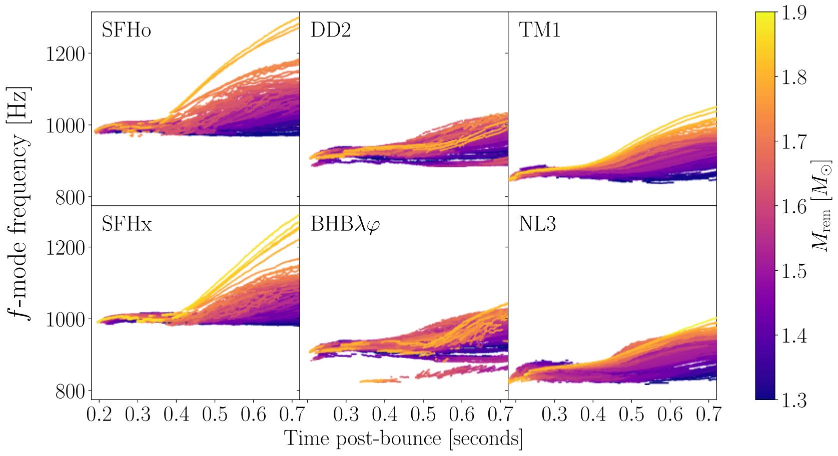
AVOIDED CROSSING FREQUENCY VS EOS

Wolfe et al (2023)

We analyze the gravitational wave eigen-frequencies of 1684 CCSN simulations.



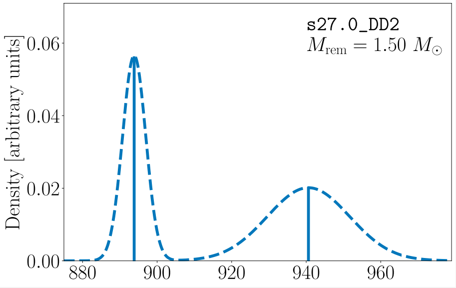
Avoided crossing

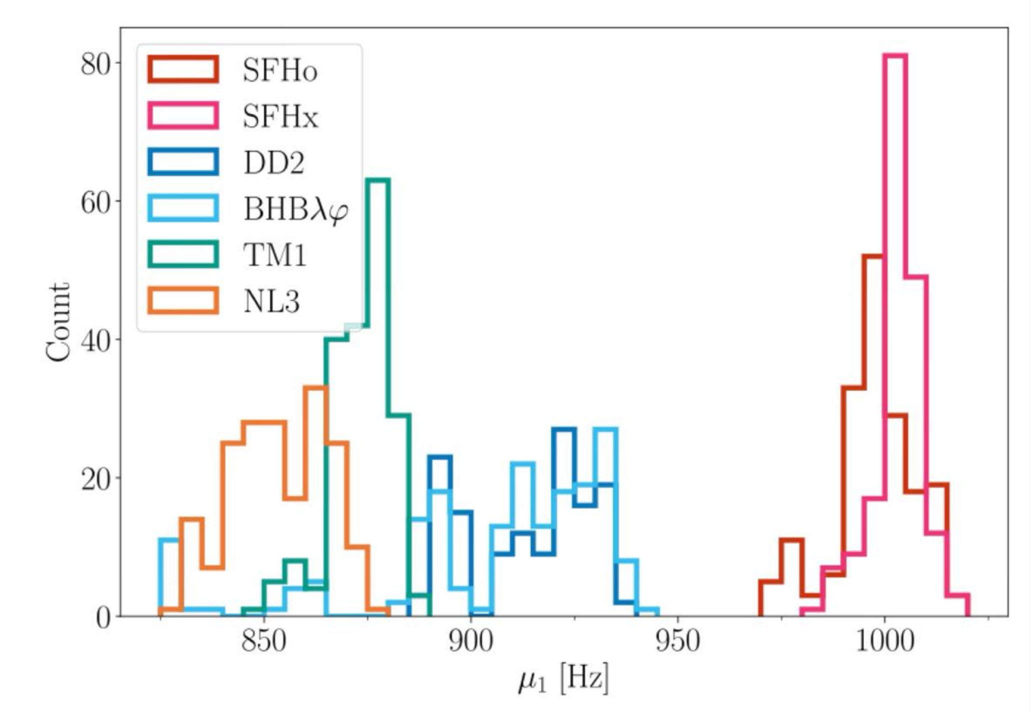


- Dominant mode frequencies show similar trends across all equations of state.
- Dominant mode frequencies are tightly clustered at early times for all models using the same EOS.
- Spread in frequencies observed at later times correlates with the remnant mass and depends on the EOS.
- Dominant mode monotonically increases with time for most of its evolution.
- Early-time period identified with low f-mode frequencies and late-time period with higher frequencies.

AVOIDED CROSSING FREQUENCY VS EOS

Gaussian mixture





TM1 & NL3 f<900 Hz.
DD2 & BHB f ∈ [875, 950] Hz.
SFHo & SFHx f ∈ [975, 1025] Hz.

CONCLUSIONS

- A nearby supernova would give us a unique opportunity to learn about ...
  - ... explosion mechanism
  - ... EOS at high densities and temperatures
  - ... rotation

- There is still a lot of work to do ...
  - ... eigenmode calculation: SASI, rotation ...
  - ... eigenmode classification
  - ... inference from other features (GW gap, secondary g-modes, SASI, bar-mode)
  - ... phenomenological templates including rotation

THANK YOU FOR YOU ATTENTION!