Roles of self-induced neutrino flavor conversions in core-collapse supernova theory

Hiroki Nagakura

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Multi-physics elements in CCSN theory

General Relativity

Numerical Relativity

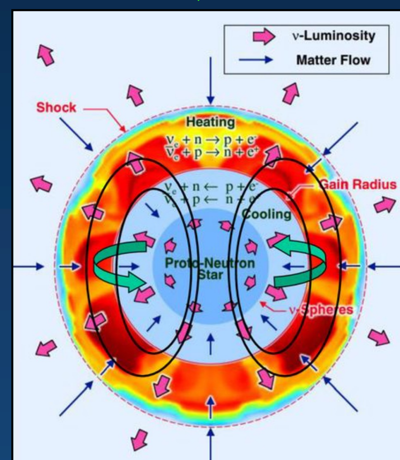
(M)Hydro dynamics

High-resolution
Shock-capturing Scheme

Fluid instabilities
(Convection, SASI, etc.)

Neutrino Transport

Boltzmann Transport
or alternative method
(e.g., M1 scheme)



Credit: Mezzacappa

Nuclear Network

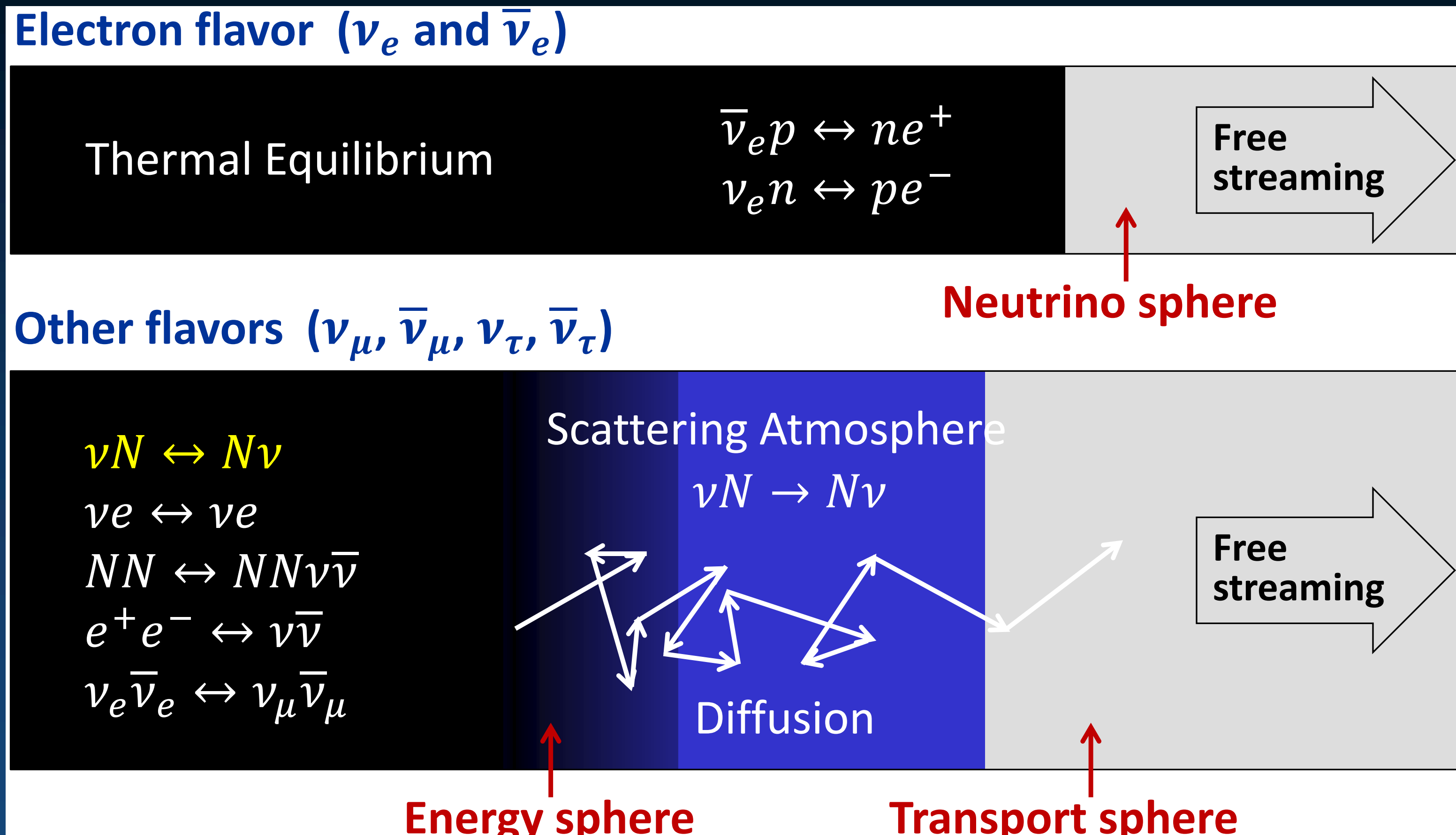
NSE at high ρ and T
Nucleosynthesis

Weak Interaction

Emission/absorption
Scatterings
Many-body corrections

A kinetic framework is essential for modeling of neutrino radiation field

Figure from Janka 2017



Optically thick $\longleftrightarrow$ Optically thin

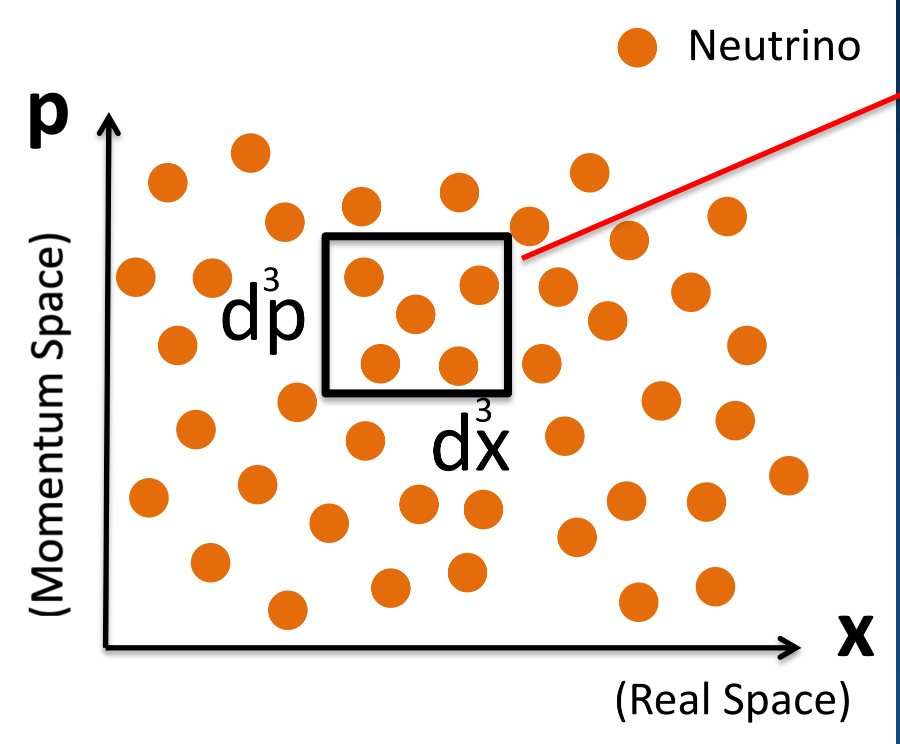
Boltzmann neutrino transport

See also a review by Mezzacappa et al. 2020

$$p^\mu \frac{\partial f}{\partial x^\mu} + \frac{dp^i}{d\tau}\frac{\partial f}{\partial p^i} = \left(\frac{\delta f}{\delta \tau}\right)_{\rm col},$$

(Time evolution + Advection Term) (Collision Term)

6-dimensional phase space



$$dN = f(t, \boldsymbol{p}, \boldsymbol{x}) d^3p d^3x$$

Conservative form of GR Boltzmann eq.

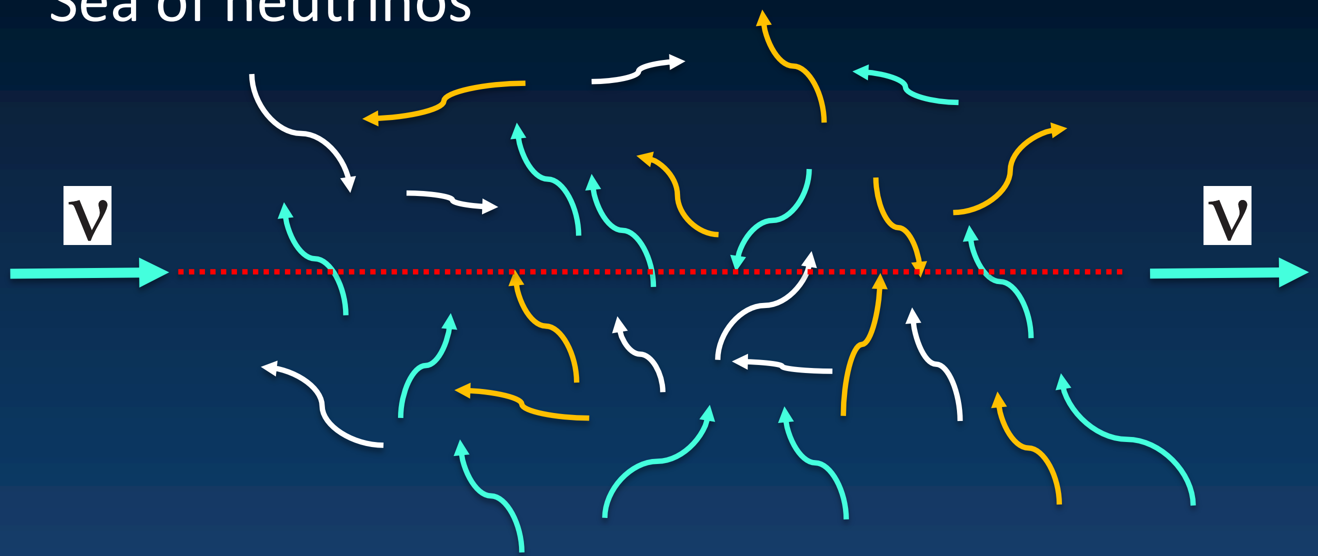
$$\frac{1}{\sqrt{-g}}\frac{\partial(\sqrt{-g}\nu^{-1}p^\alpha f)}{\partial x^\alpha}\bigg|_{q_{(i)}} + \frac{1}{\nu^2}\frac{\partial}{\partial \nu}(-\nu f p^\alpha p_\beta \nabla_\alpha e^\beta_{(0)})$$
$$+\frac{1}{\sin\bar\theta}\frac{\partial}{\partial\bar\theta}\left(\nu^{-2}\sin\bar\theta f\sum_{j=1}^{3}p^\alpha p_\beta\nabla_\alpha e^\beta_{(j)}\frac{\partial \ell_{(j)}}{\partial\bar\theta}\right)$$
$$+\frac{1}{\sin^2\bar\theta}\frac{\partial}{\partial\bar\varphi}\left(\nu^{-2} f\sum_{j=2}^{3}p^\alpha p_\beta\nabla_\alpha e^\beta_{(j)}\frac{\partial \ell_{(j)}}{\partial\bar\varphi}\right) = S_{\rm rad},$$

Shibata, Nagakura, Sekiguchi, and Yamada (2014)

Neutrino oscillation induced by self-interactions

Pantalone 1992

Sea of neutrinos



1. Refractions by self-interactions induce neutrino flavor conversions, which is analogy to matter effects (e.g., MSW resonance).

2. The oscillation timescale is much shorter than the global scale of CCSN/BNSM.

3. Collective neutrino oscillation induced by neutrino-self interactions commonly occurs in CCSNe environments.

Rich flavor-conversion phenomena driven by neutrino self-interactions

See reviews by
Duan 2010, Tambora + 2020,
Capozzi + 2022, Richers + 2022,
Volpe 2023, Fisher + 2024,
John + 2024, Yamada 2024

- Slow-mode (Duan et al. 2010)

- Energy-dependent flavor conversion occurs.
- The frequency of the flavor conversion is proportional to $\sqrt{\omega\mu}$

Vacuum: $\omega = \frac{\Delta m^2}{2E_\nu}$,
Matter: $\lambda = \sqrt{2} G_F n_e$,
Self-int: $\mu = \sqrt{2} G_F n_\nu$,

- Fast-mode (FFC) (Sawyer 2005)

- Collective neutrino oscillation in the limit of ω → 0.
- The frequency of the flavor conversion is proportional to μ
- Anisotropy of neutrino angular distributions drives the fast flavor-conversion.

- Collisional flavor instability (CFI) (Johns 2021)

- Asymmetries of matter interactions between neutrinos and anti-neutrinos drive flavor conversion.

$$\mathrm{Im}\ \Omega \cong \pm \frac{\Gamma - \bar{\Gamma}}{2} \frac{\mu S}{\sqrt{(\mu D)^2 + 4\omega\mu S}} - \frac{\Gamma + \bar{\Gamma}}{2}$$

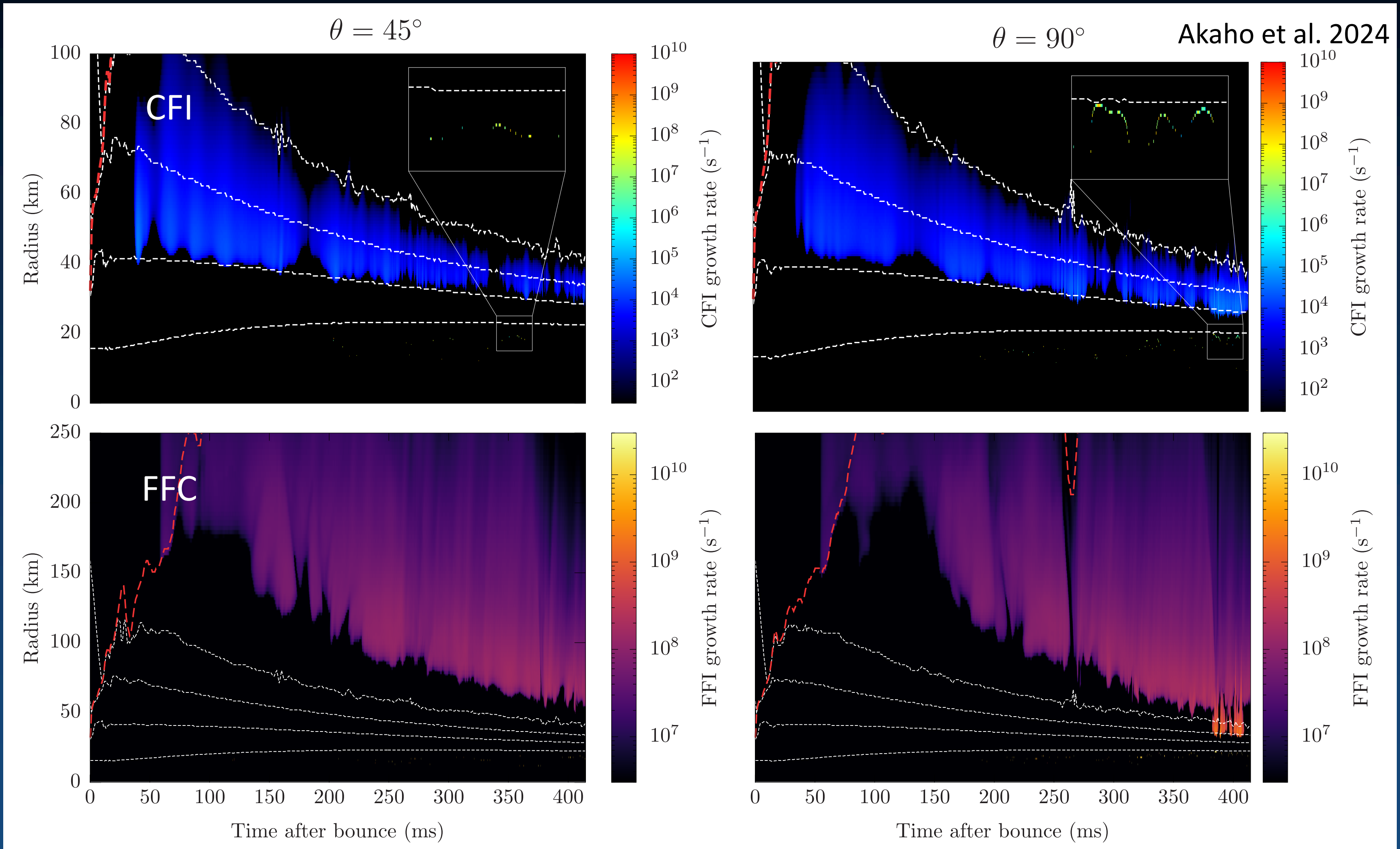
Γ: Matter-interaction rate

- Matter-neutrino resonance (Malkus et al. 2012)

- The resonance potentially occur in BNSM/Collapsar environment (but not in CCSN).
- Essentially the same mechanism as MSW resonance.

$$|\lambda + \mu| \sim |\omega|$$

FFC and CFI in CCSNe



Quantum Kinetics neutrino transport:

Vlasenko et al. 2014, Volpe 2015,
Blaschke et al. 2016, Richers et al. 2019

$$\underbrace{p^{\mu}\frac{\partial \overset{(-)}{f}}{\partial x^{\mu}} + \frac{dp^{i}}{d\tau}\frac{\partial \overset{(-)}{f}}{\partial p^{i}}} = \underbrace{-p^{\mu}u_{\mu}\overset{(-)}{S}_{\mathrm{col}}} + \underbrace{ip^{\mu}n_{\mu}[\overset{(-)}{H}, \overset{(-)}{f}]},$$

Advection terms
(Same as Boltz eq.)

Collision term

Oscillation term

f is not a
"distribution function"

Density matrix

$$\overset{(-)}{f} = \begin{bmatrix} \overset{(-)}{f}_{ee} & \overset{(-)}{f}_{e\mu} & \overset{(-)}{f}_{e\tau} \\ \overset{(-)}{f}_{\mu e} & \overset{(-)}{f}_{\mu\mu} & \overset{(-)}{f}_{\mu\tau} \\ \overset{(-)}{f}_{\tau e} & \overset{(-)}{f}_{\tau\mu} & \overset{(-)}{f}_{\tau\tau} \end{bmatrix}$$

Hamiltonian

$$\overset{(-)}{H} = \overset{(-)}{H}_{\mathrm{vac}} + \overset{(-)}{H}_{\mathrm{mat}} + \overset{(-)}{H}_{\nu\nu},$$

$$H_{\mathrm{vac}} = \frac{1}{2\nu}U\begin{bmatrix} m_1^2 & 0 & 0 \\ 0 & m_2^2 & 0 \\ 0 & 0 & m_3^2 \end{bmatrix}U^{\dagger},$$

$$H_{\mathrm{mat}} = D\begin{bmatrix} V_e & 0 & 0 \\ 0 & V_\mu & 0 \\ 0 & 0 & V_\tau + V_{\mu\tau} \end{bmatrix},$$

$$H_{\nu\nu} = \sqrt{2}G_F\int\frac{d^3q'}{(2\pi)^3}\left(1 - \sum_{i=1}^{3}\ell'_{(i)}\ell_{(i)}\right)\left(f(q') - \bar{f}^*(q')\right),$$

Global simulations: I. QKE simulation with attenuation method

Nagakura and Zaizen 2022, 2023

- Technical Issue:

$$\ell_{\mathrm{n}_\nu} \equiv c\,\mathrm{T}_{\mathrm{n}_\nu} = 0.235\ \mathrm{cm} \left(\frac{L_\nu}{4 \times 10^{52}\mathrm{erg/s}} \right)^{-1} \left(\frac{E_{\mathrm{ave}}}{12\mathrm{MeV}} \right) \left(\frac{R}{50\mathrm{km}} \right)^{2} \left(\frac{\kappa}{1/3} \right)$$

Oscillation wavelength is an order of sub-centimeter.

Too short !!!!

How can we make FFC simulations tractable???

- Attenuation prescription:

$$p^\mu \frac{\partial f}{\partial x^\mu} + \frac{dp^i}{d\tau} \frac{\partial f}{\partial p^i} = -p^\mu u_\mu S + i\,\xi\, p^\mu n_\mu [H, f],$$

Attenuation parameter ($0 \leqq \xi \leqq 1$)

- Attenuating Hamiltonian makes global QKE simulations tractable.
- Realistic features can be extracted by a convergence study of ξ ($\rightarrow$ 1).

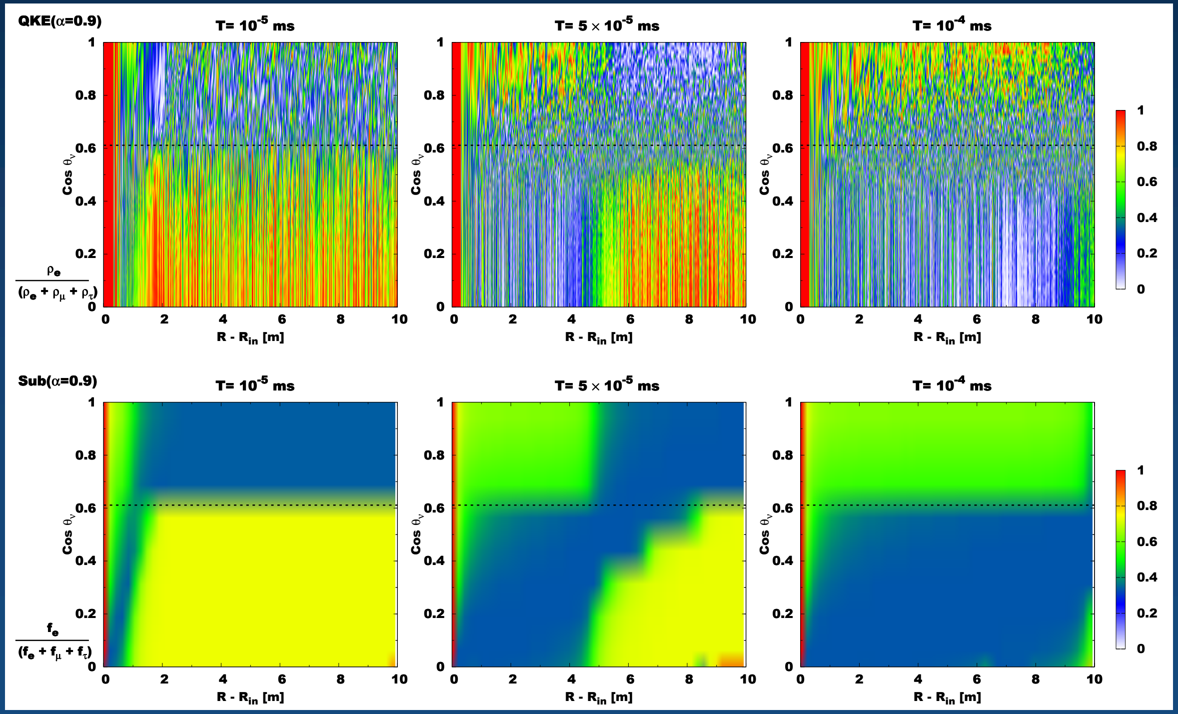
Global simulations: II. Subgrid model (BGK prescription)

Nagakura et al. 2024

$$p^\mu \frac{\partial f}{\partial x^\mu} + \frac{dp^i}{d\tau}\frac{\partial f}{\partial p^i} = -p^\mu u_\mu S + i p^\mu n_\mu [H, f]$$: Full QKE

$$p^\mu \frac{\partial f}{\partial x^\mu} + \frac{dp^i}{d\tau}\frac{\partial f}{\partial p^i} = -p^\mu u_\mu S + p^\mu n_\mu \frac{1}{\tau_a}(f - f^a)$$: Relaxation-time approximation

Radial-angular distributions for survival probability of electron-type neutrinos



Take-home messages:
Potential impacts of flavor conversion on CCSNe

- Shock dynamics

- Neutrino signal

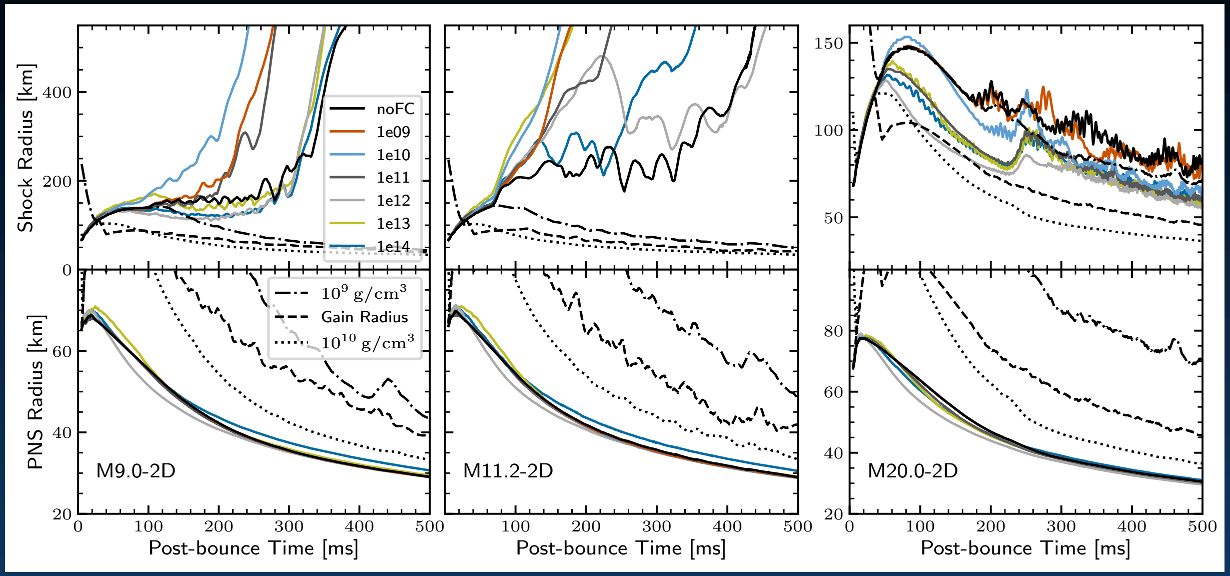
- PNS cooling

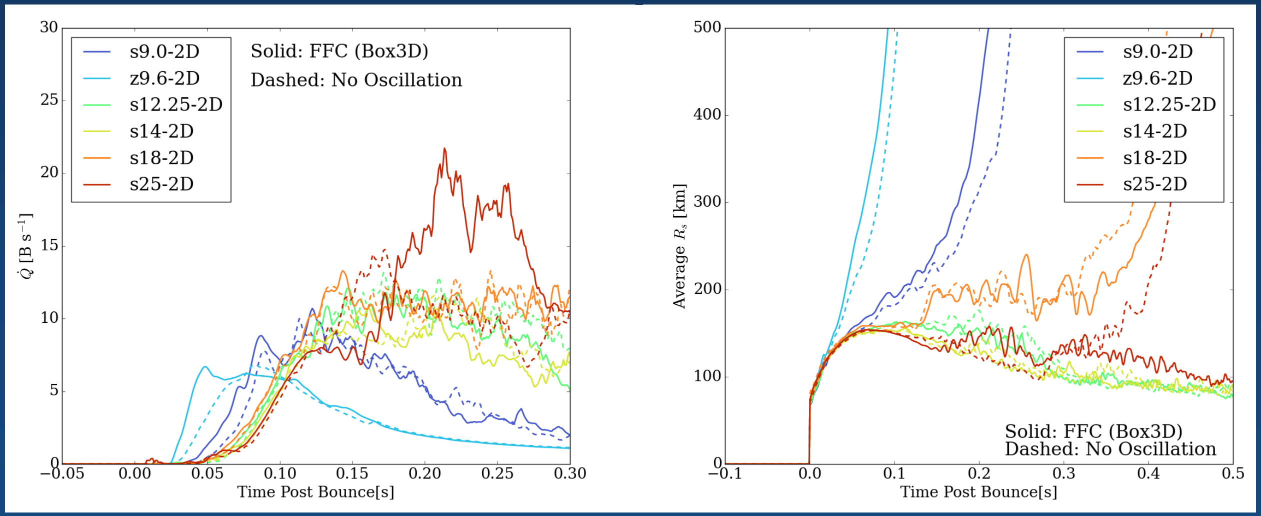
- Explosive nucleosynthesis

- Neutron star kick

ν Shock dynamics

Ehring et al. 2023

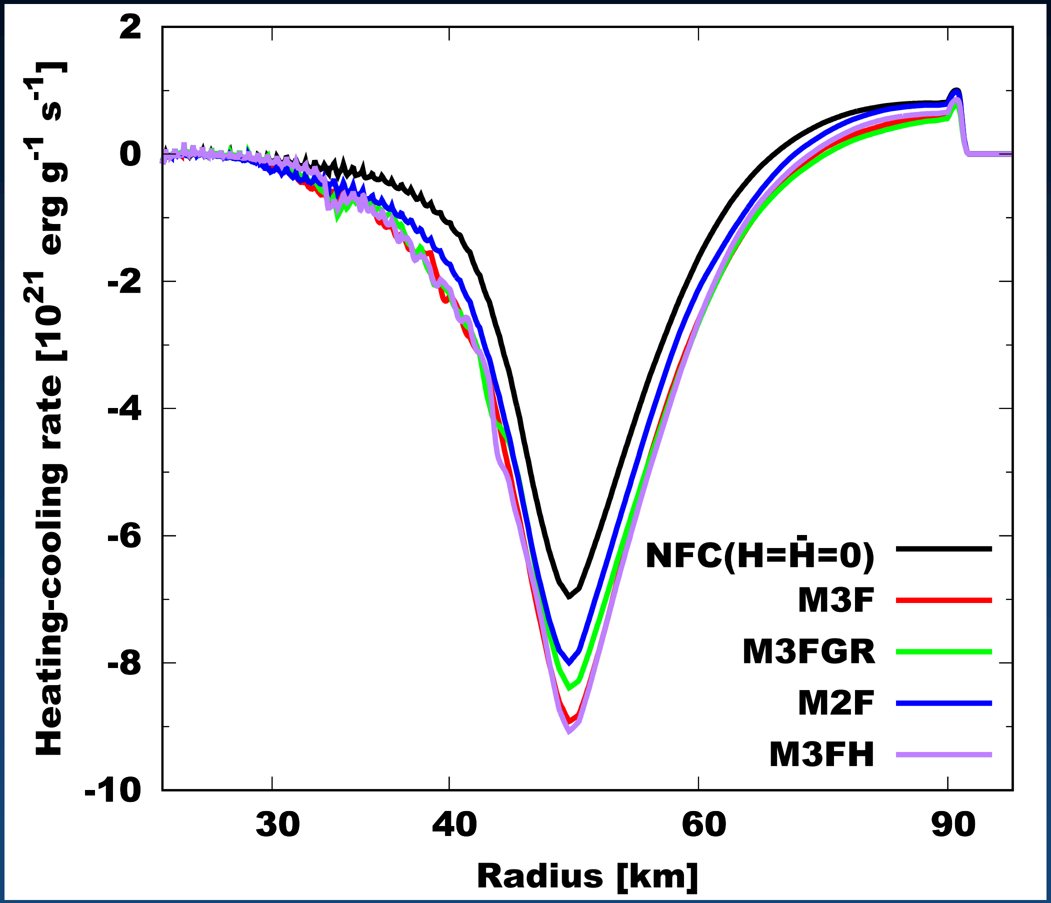




Wang and Burrows 2023

- Direct quantum kinetic neutrino transport simulation in CCSNe

Nagakura 2023
See also Xiong et al. 2024



Numerical setup:

Fluid-profiles are taken from a CCSN simulation.

General relativistic effects are taken into account.

A wide spatial region is covered.

Three-flavor framework

Neutrino-cooling is enhanced by FFCs
Neutrino-heating is suppressed by FFCs

Impacts on the explodability of CCSN

Take-home messages:
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  - Flavor conversions can both facilitate and hinder shock revival.
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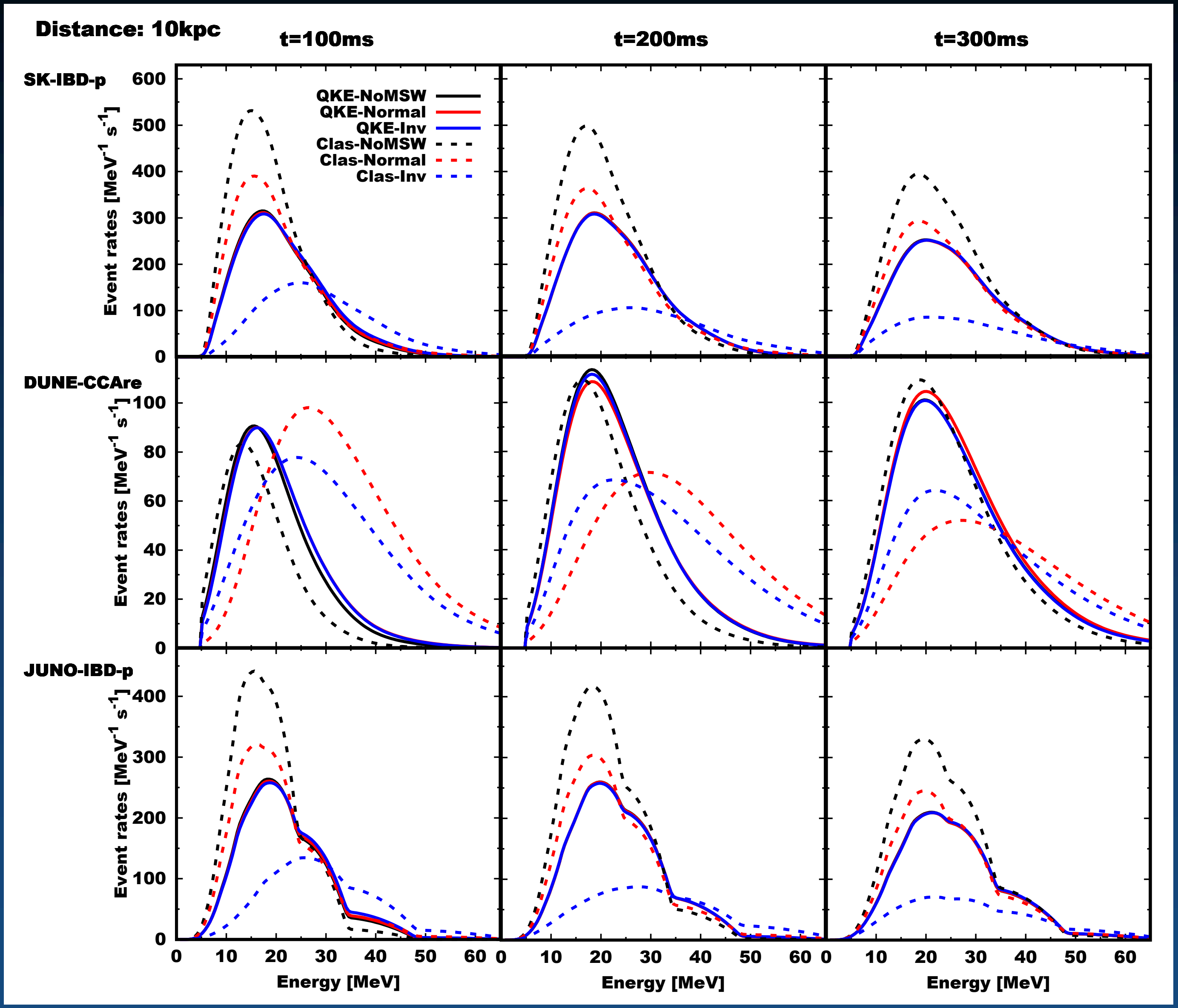
- Explosive nucleosynthesis
- Neutron star kick

ν Neutrino signal

Nagakura and Zaizen 2023



Take-home messages:
Potential impacts of flavor conversion on CCSNe

- Shock dynamics
  - Flavor conversions can both facilitate and hinder shock revival.
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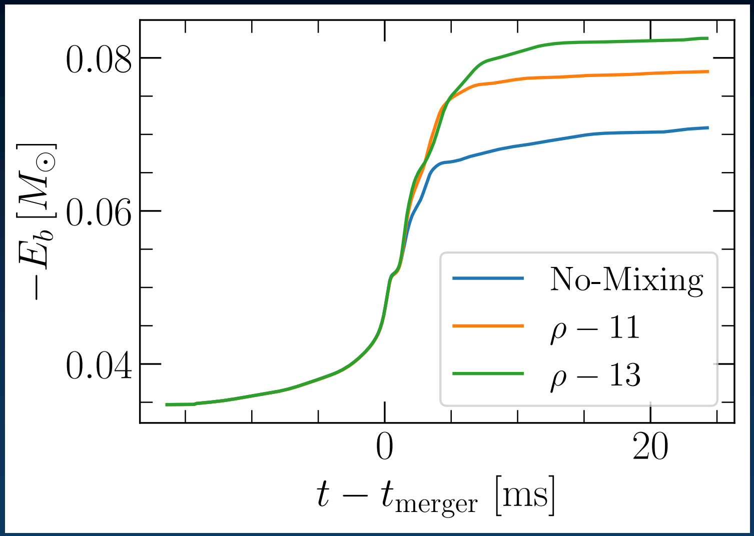
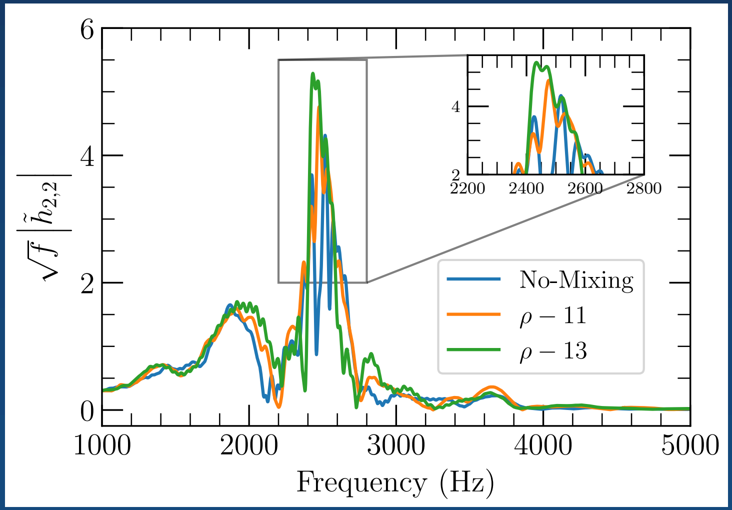
  - Flavor equipartition can be achieved.
- PNS cooling
- Explosive nucleosynthesis
- Neutron star kick

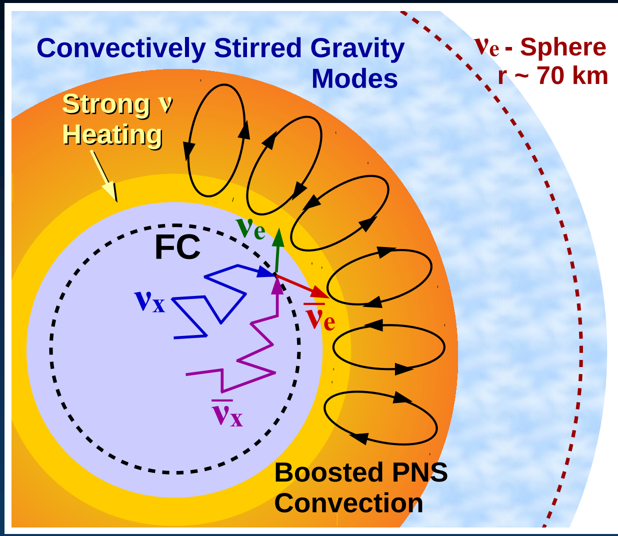
- PNS cooling => Impact on Gravitational Wave emission

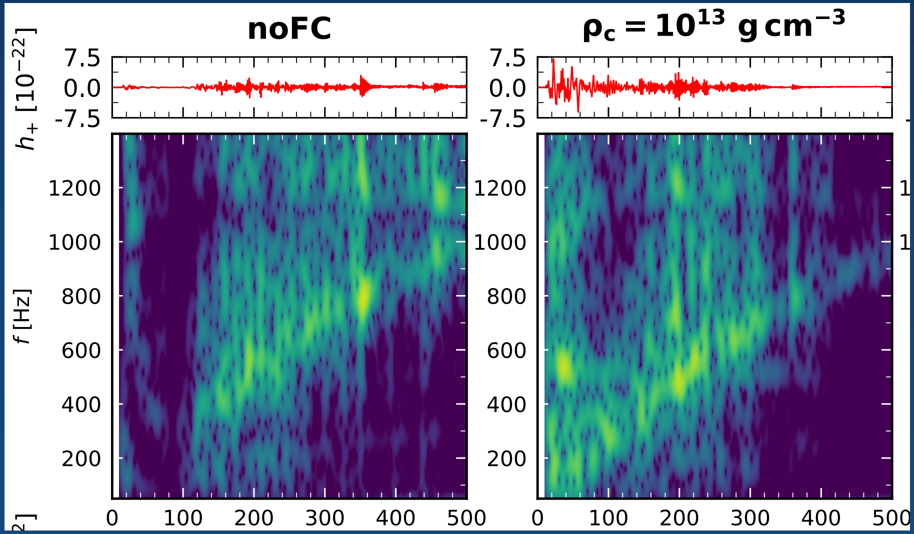
Qiu et al. 2025 (Binary neutron star merger remnant)





Ehring et al. 2025



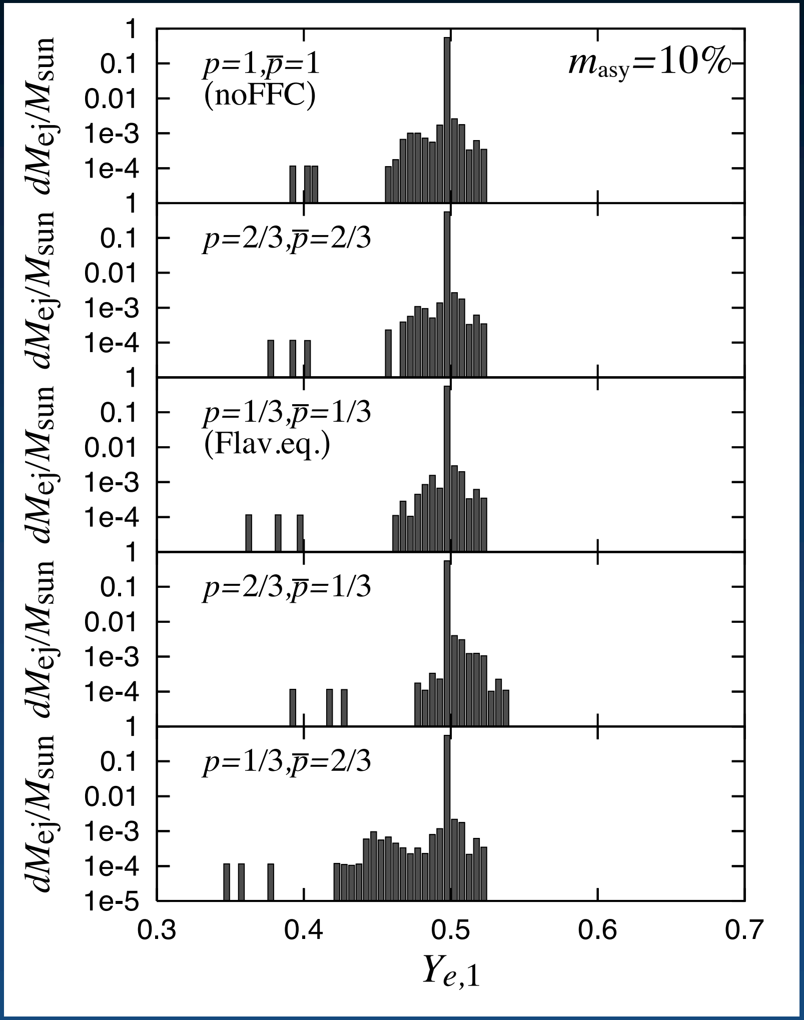


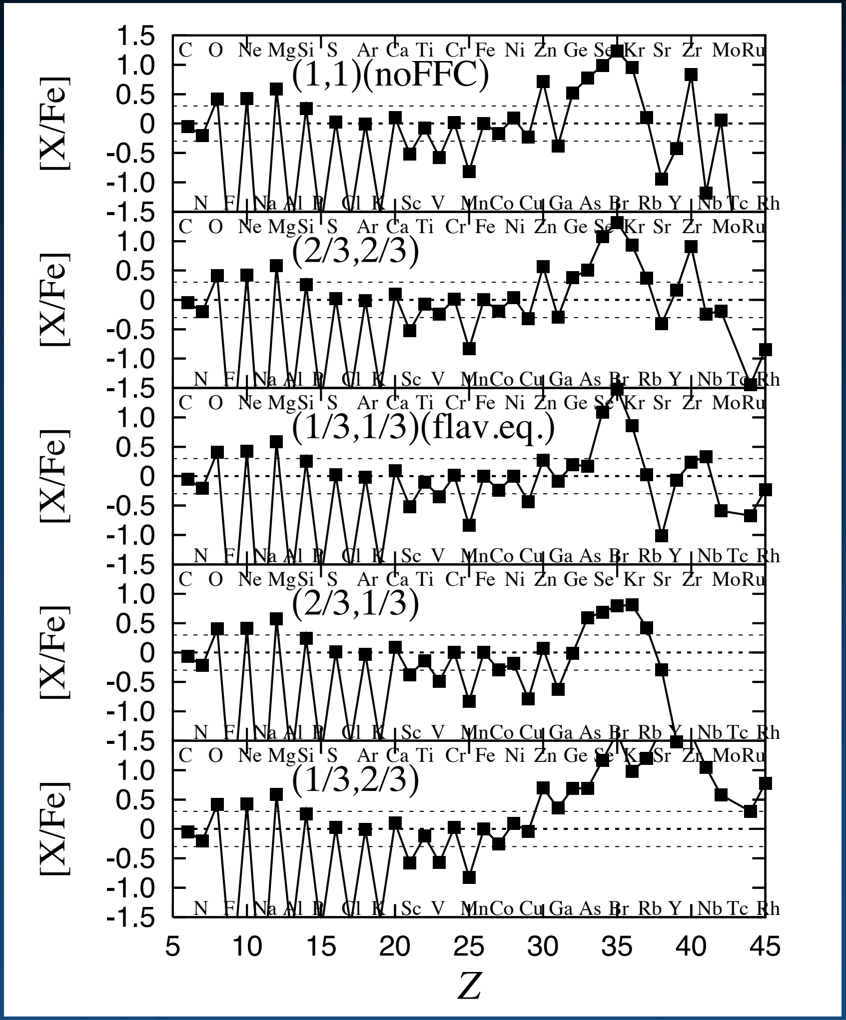
Take-home messages:
Potential impacts of flavor conversion on CCSNe

- Shock dynamics
  - Flavor conversions can both facilitate and hinder shock revival.
- Neutrino signal
  - Flavor equipartition can be achieved.
- PNS cooling
  - Likely accelerate PNS cooling, which give impacts on GW emission.
- Explosive nucleosynthesis
- Neutron star kick

∨ Explosive nucleosynthesis

Fujimoto and Nagakura 2023





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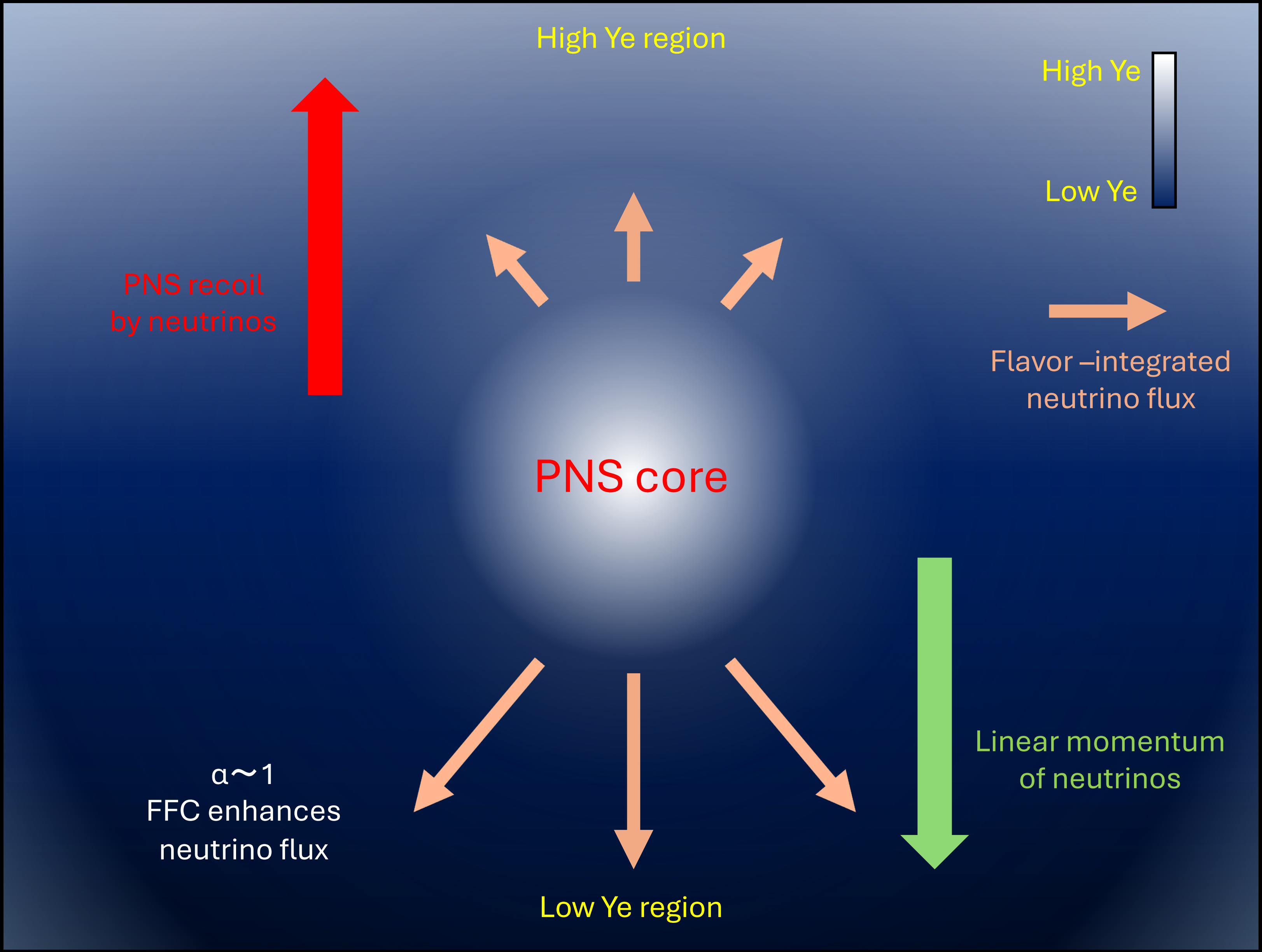
- Explosive nucleosynthesis
  - Heavier elements than Co (Z>27) can be changed.
- Neutron star kick

V Neutron star kick

Nagakura and Sumiyoshi 2024



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- PNS cooling
  - Likely accelerate PNS cooling, which give impacts on GW emission.
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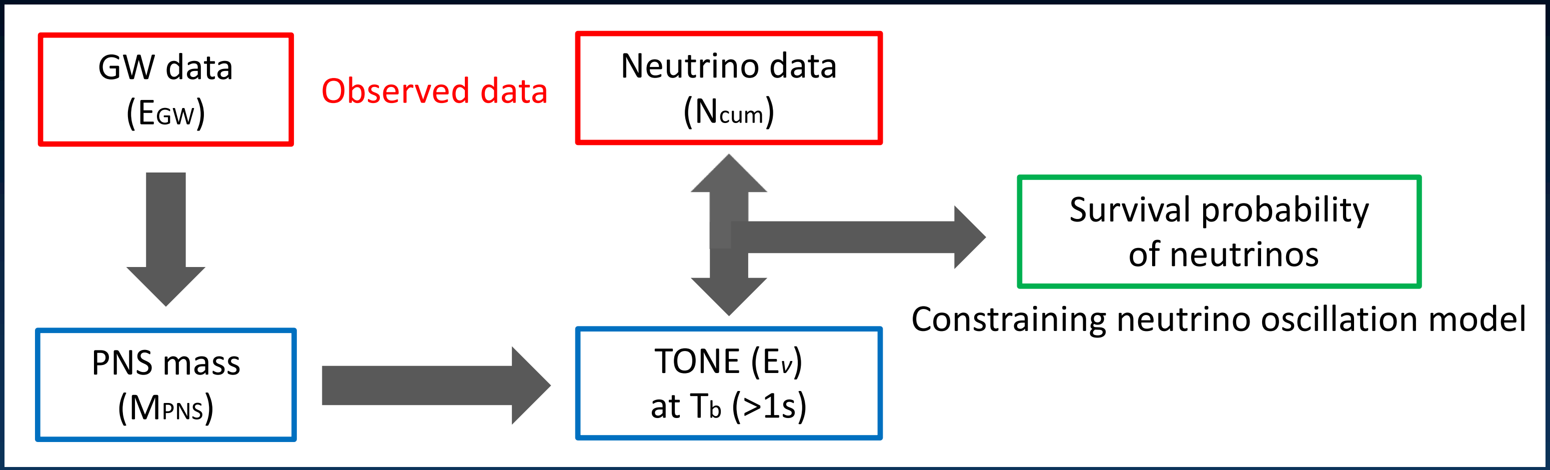
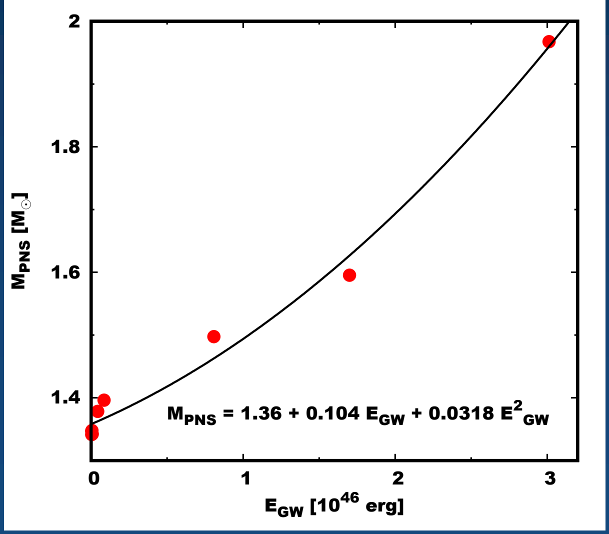
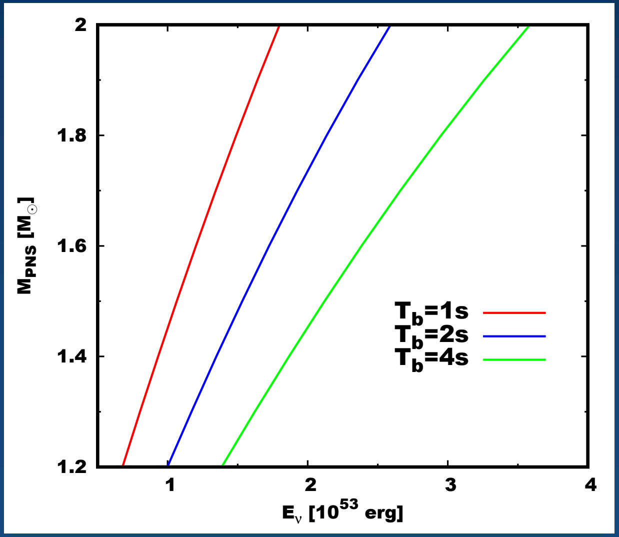
  - Heavier elements than Co (Z>27) can be changed.
- Neutron star kick
  - Asymmetric neutrino flavor conversions can accelerate NS kick.

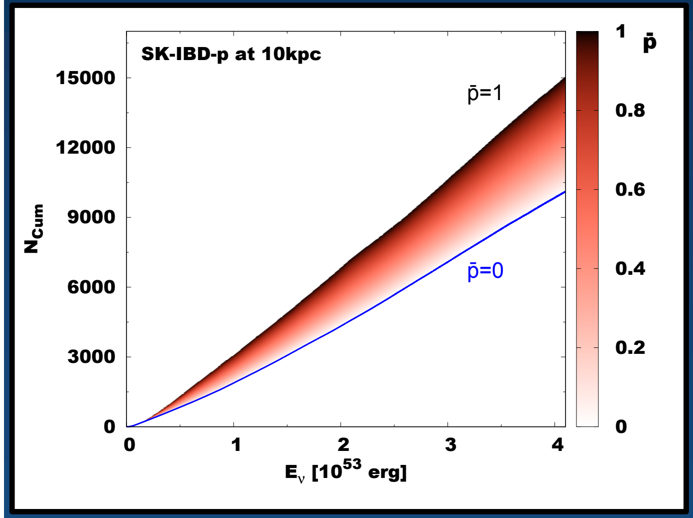
Constraining neutrino oscillation model in CCSNe by joint analysis of GW and neutrino signal

Nagakura and Vartanyan 2023









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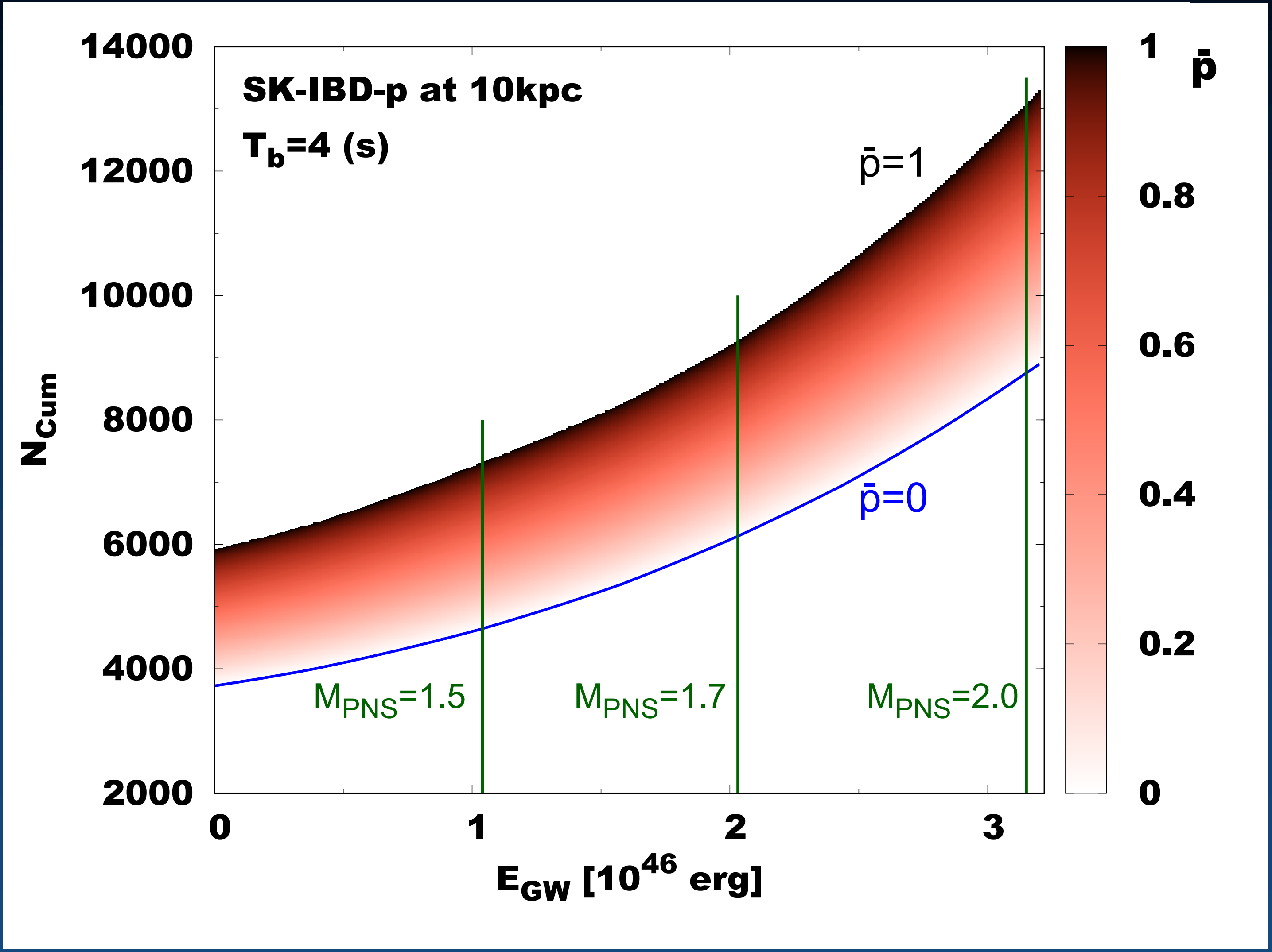


Summary

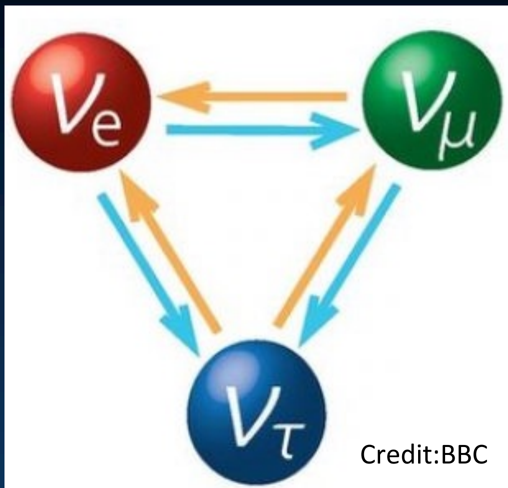
- Neutrino self-interactions induce strong flavor conversions
- Flavor instabilities occur in CCSN core ubiquitously
- Neutrino transport modeling needs to be replaced from classical to quantum kinetics
- Neutrino flavor conversion can change shock dynamics, neutrino signal, PNS cooling, nucleosynthesis, and NS kick

Future prospects

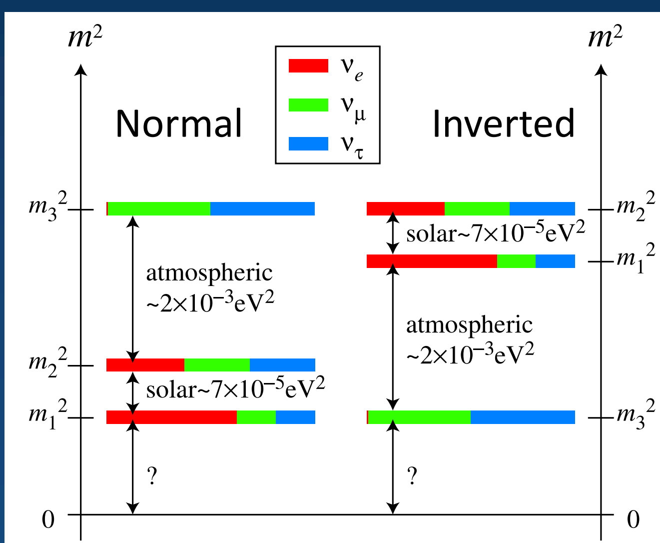
- Improving subgrid models of neutrino flavor conversions
- Developing approximate code for quantum kinetic neutrino transport
- Providing theoretical predictions of observable signals (neutrino, GW signal, etc.)

Backup

Neutrino oscillations



- There are many experimental evidences that neutrinos can go through flavor conversion.
- Neutrinos have at least three different masses.
- Flavor eigenstates are different from mass eigenstates.



Feruglio et al. 2003

$$|\nu_i\rangle = \sum_\alpha U^*_{\alpha i}\,|\nu_\alpha\rangle\,,$$

Mass state

$$|\nu_\alpha\rangle = \sum_i U_{\alpha i}\,|\nu_i\rangle\,,$$

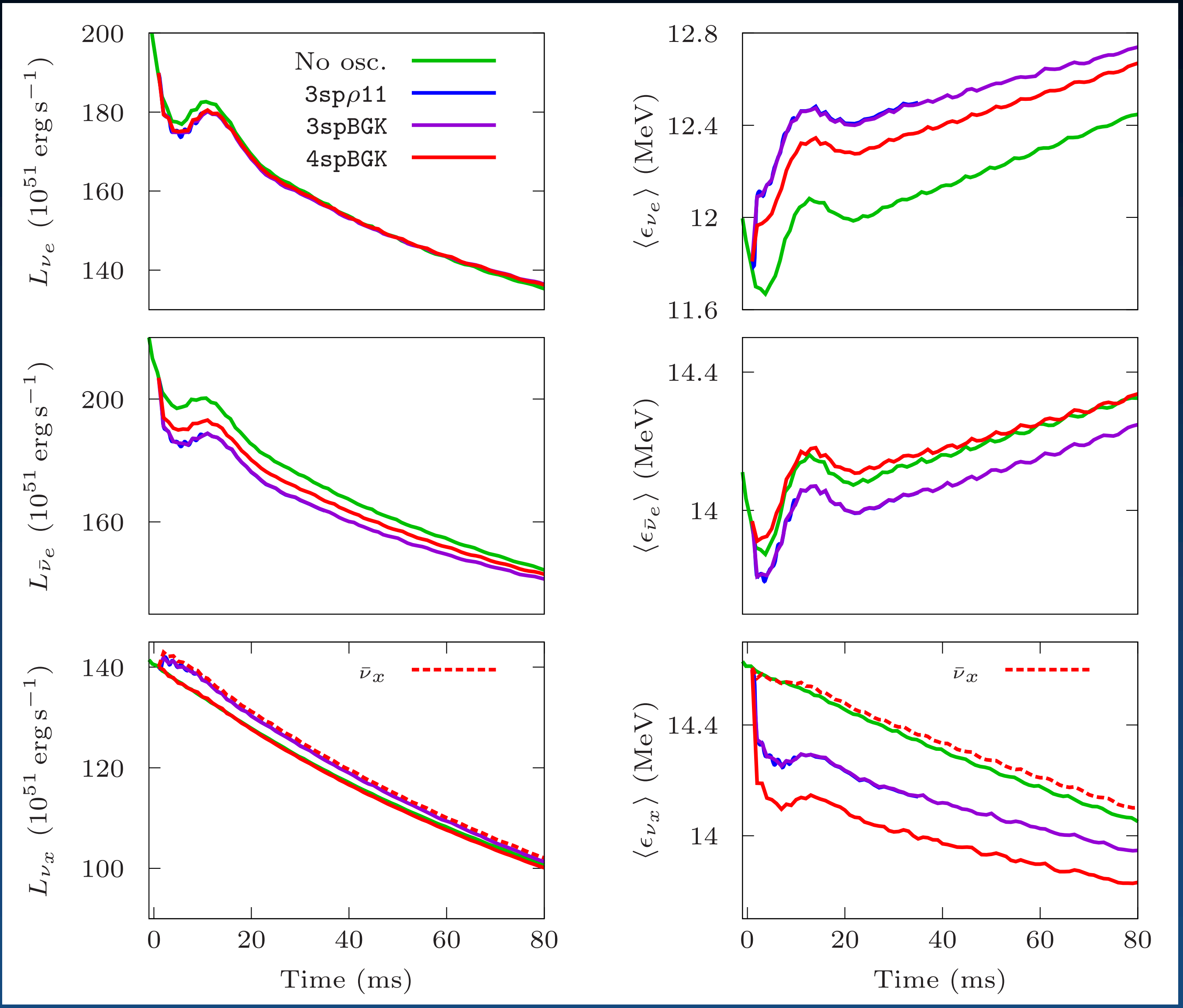
Flavor state

U: Pontecorvo–Maki–Nakagawa–Sakata matrix (PMNS matrix)

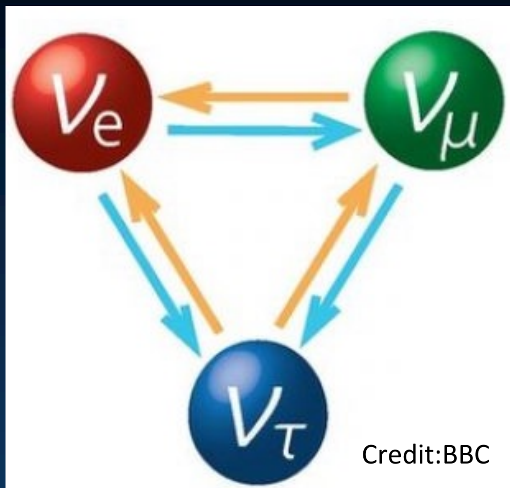
$$\begin{aligned}
U &= \begin{bmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{bmatrix} \\
&= \begin{bmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{bmatrix} \begin{bmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{bmatrix} \begin{bmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} e^{i\alpha_1/2} & 0 & 0 \\ 0 & e^{i\alpha_2/2} & 0 \\ 0 & 0 & 1 \end{bmatrix} \\
&= \begin{bmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{bmatrix} \begin{bmatrix} e^{i\alpha_1/2} & 0 & 0 \\ 0 & e^{i\alpha_2/2} & 0 \\ 0 & 0 & 1 \end{bmatrix},
\end{aligned}$$

ν Neutrino signal

Akaho, Nagakura, Yamada 2025: arXiv:2506.07017
(Based on BGK subgrid model)



Neutrino oscillations



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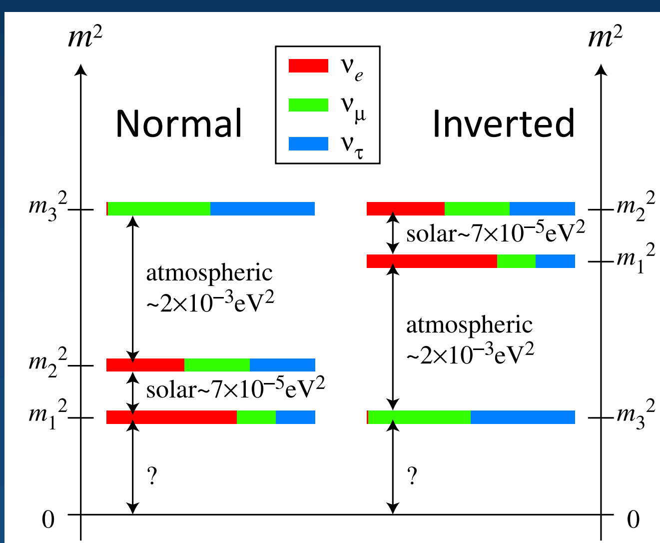

Feruglio et al. 2003

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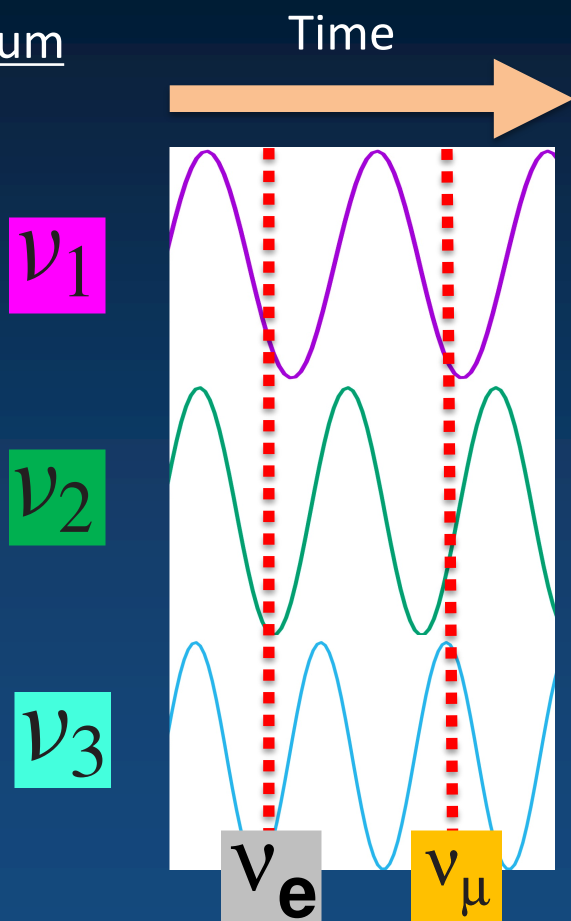
\end{aligned}$$

Neutrino oscillation with a plane-wave picture

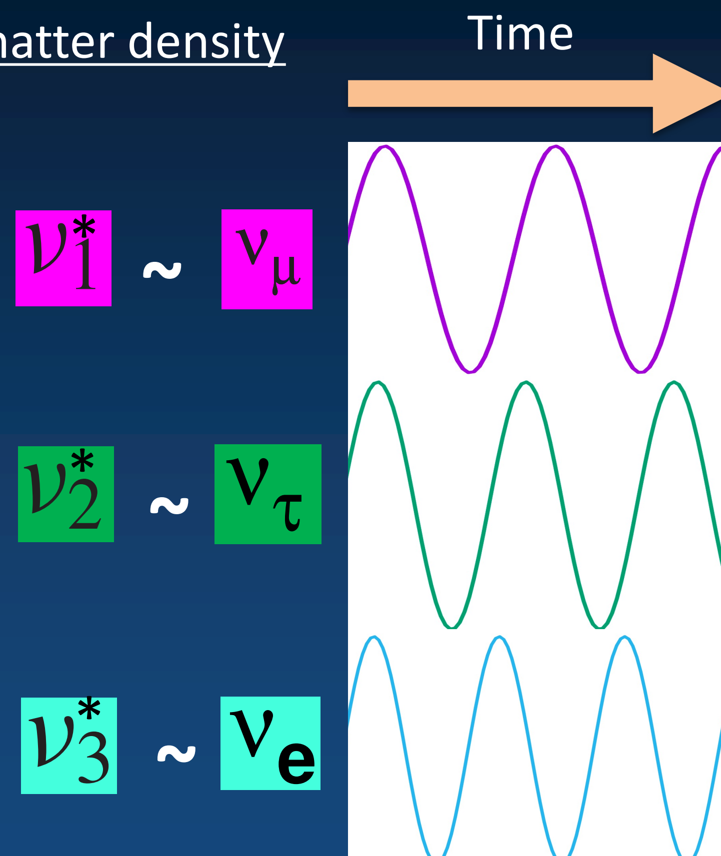
$$\psi_\ell(0,0) \equiv |\nu_\ell\rangle = \sum_{i=1}^{3} U_{\ell i}^* \, |\nu_i\rangle$$

$$\psi_\ell(t,x) = \sum_{i=1}^{3} U_{\ell i}^* \, |\nu_i\rangle \, e^{-i\phi_i}, \qquad \text{phase } \phi_i = E_i \, t - p_i \, x$$

Vacuum



High matter density



Boltzmann transport becomes a reasonable approximation.

- Global simulations:

General-relativistic quantum-kinetic neutrino transport (GRQKNT)

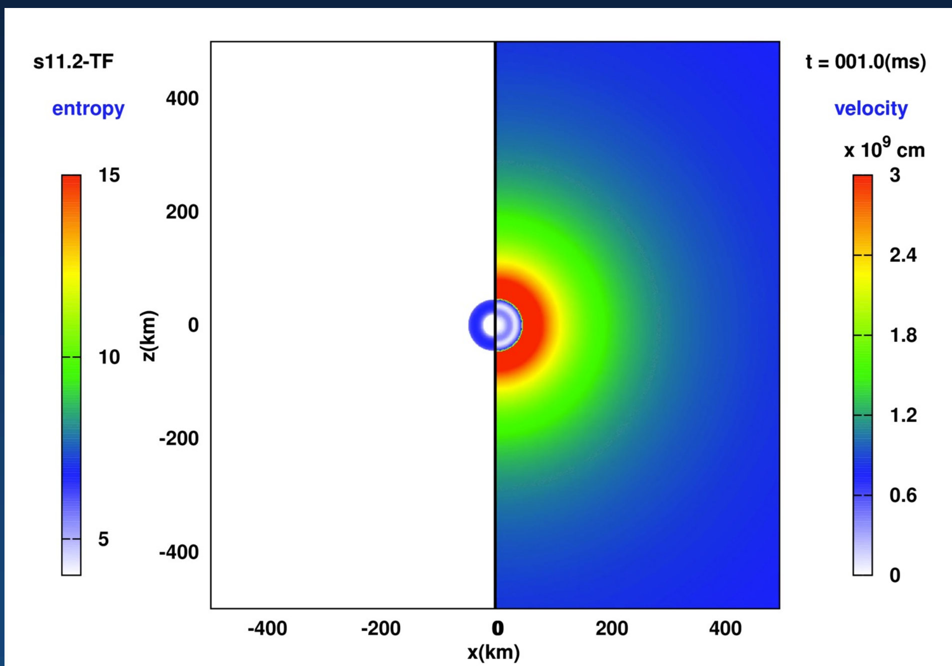
Nagakura 2022

$$p^{\mu}\frac{\partial \overset{(-)}{f}}{\partial x^{\mu}} + \frac{dp^{i}}{d\tau}\frac{\partial \overset{(-)}{f}}{\partial p^{i}} = -p^{\mu}u_{\mu}\overset{(-)}{S}_{\rm col} + ip^{\mu}n_{\mu}[\overset{(-)}{H}, \overset{(-)}{f}],$$

- Fully general relativistic (3+1 formalism) neutrino transport
- Multi-Dimension (6-dimensional phase space)
- Neutrino matter interactions (emission, absorption, and scatterings)
- Neutrino Hamiltonian potential of vacuum, matter, and self-interaction
- 3 flavors + their anti-neutrinos
- Solving the equation with Sn method (explicit evolution: WENO-5th order)
- Hybrid OpenMP/MPI parallelization

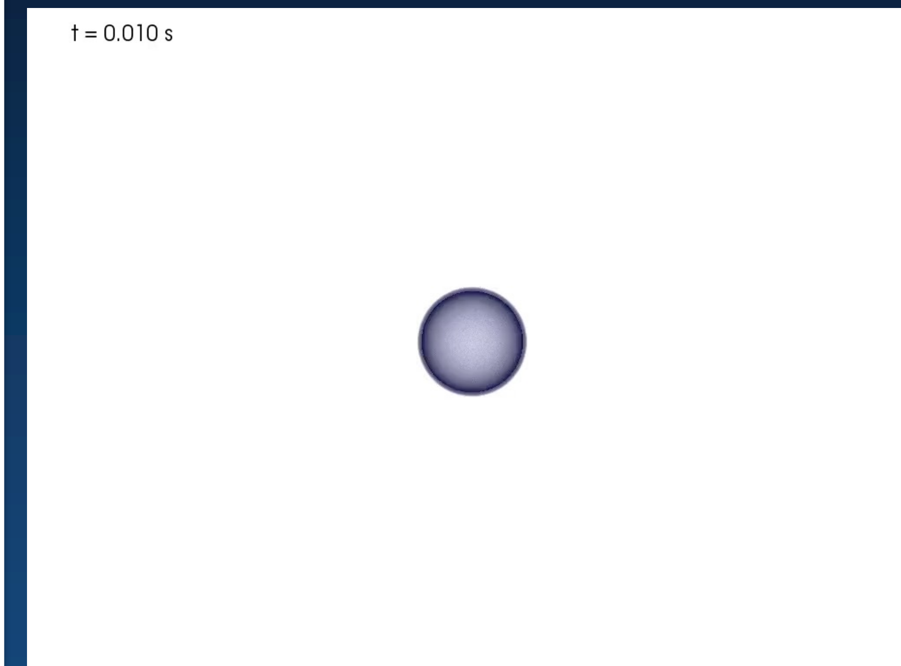
Multi-dimensional core-collapse supernova simulations (Neutrino-radiation-hydrodynamic simulations) 2014～

Full Boltzmann neutrino transport



Nagakura et al. ApJL 2019

Two-moment neutrino transport



Nagakura et al. MNRAS 2019

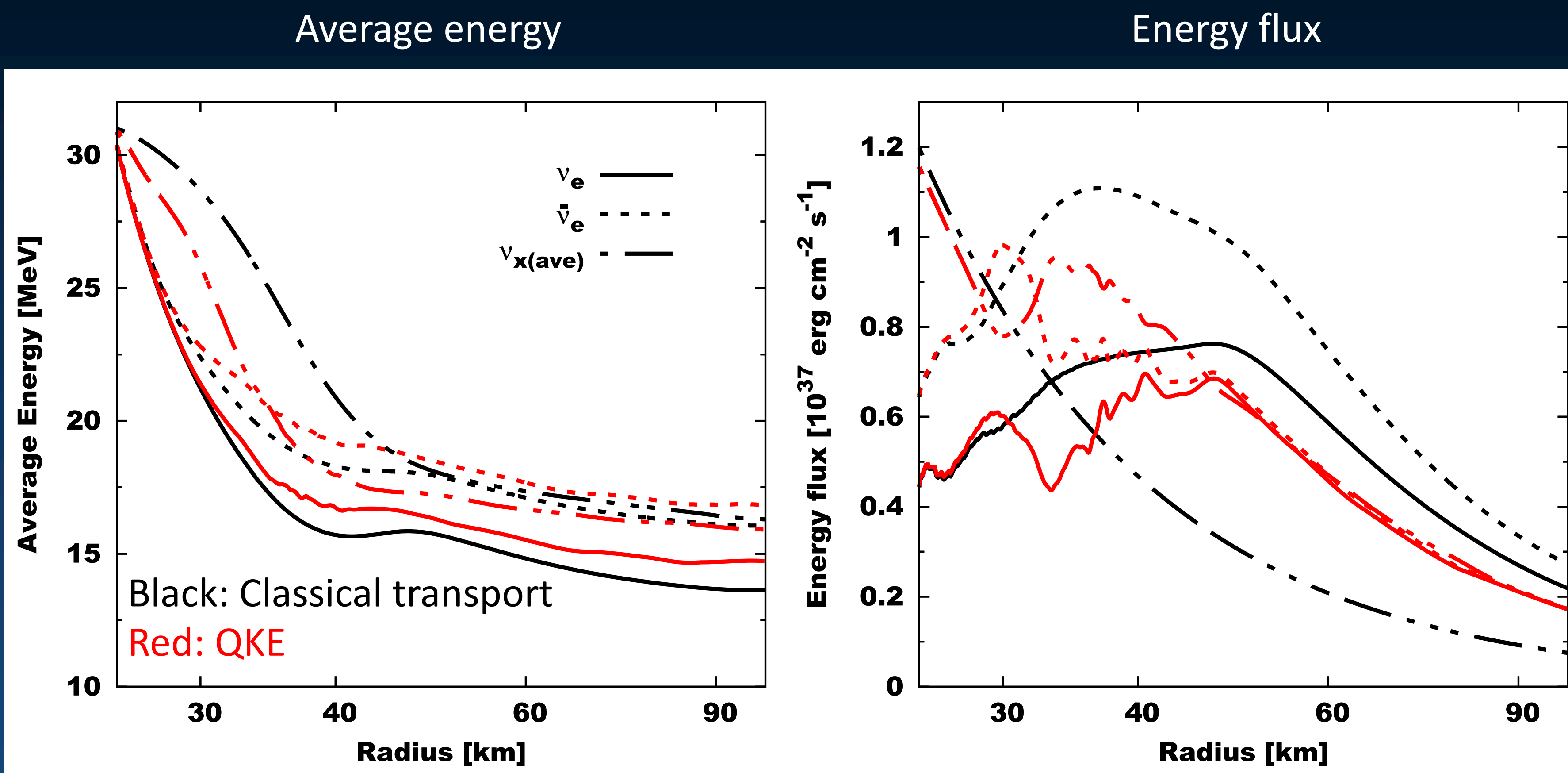
Today's topic:

"Quantum kinetics of neutrinos in dense astrophysical environments"

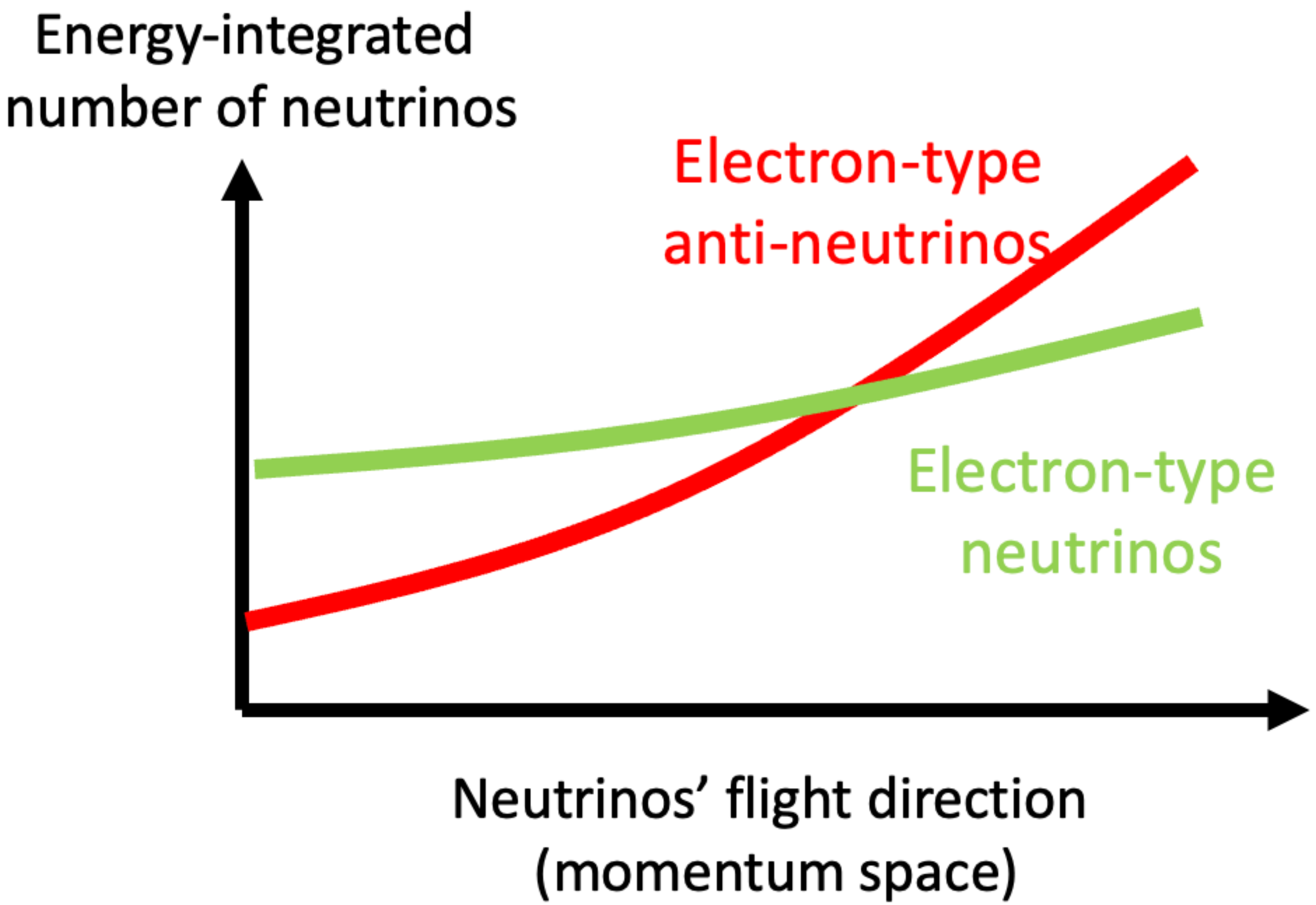
Neutrino transport with
neutrino oscillation (flavor conversion)

Core collapse supernova (CCSN)
Binary neutron star merger (BNSM)

- FFC in CCSN Nagakura PRL 2023



Instability criterion of FFC (ELN angular crossing)



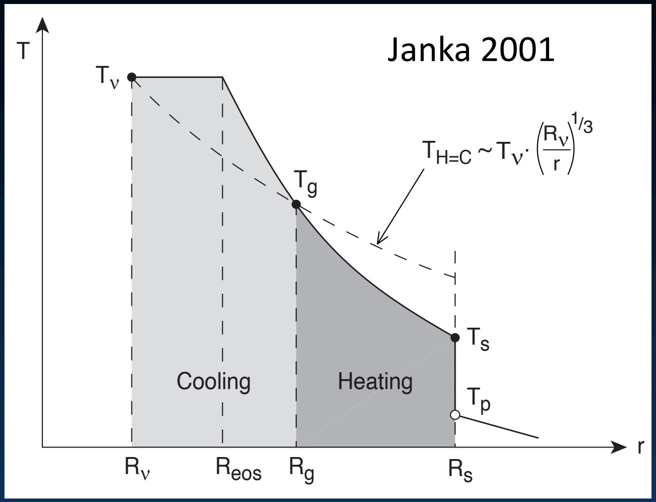
Neutrino-heating mechanism for CCSN explosions

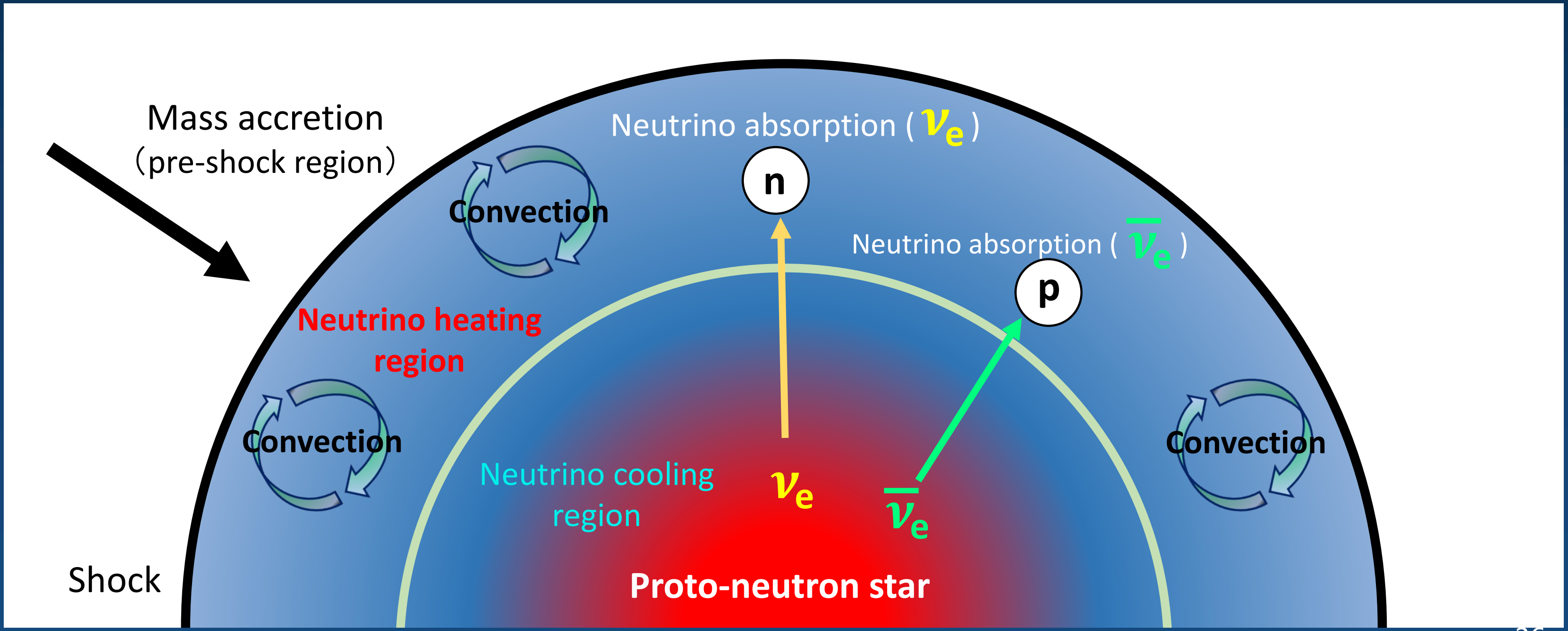
✓ Neutrino Heating Rate

$$Q_\nu^+ \approx 160\mathrm{Mev/s}\frac{\rho}{m_a}\frac{L_{\nu_e,52}}{r_7^2\langle\mu_\nu\rangle}\left(\frac{T_{\nu_e}}{4\mathrm{MeV}}\right)^2$$

✓ Neutrino Cooling Rate

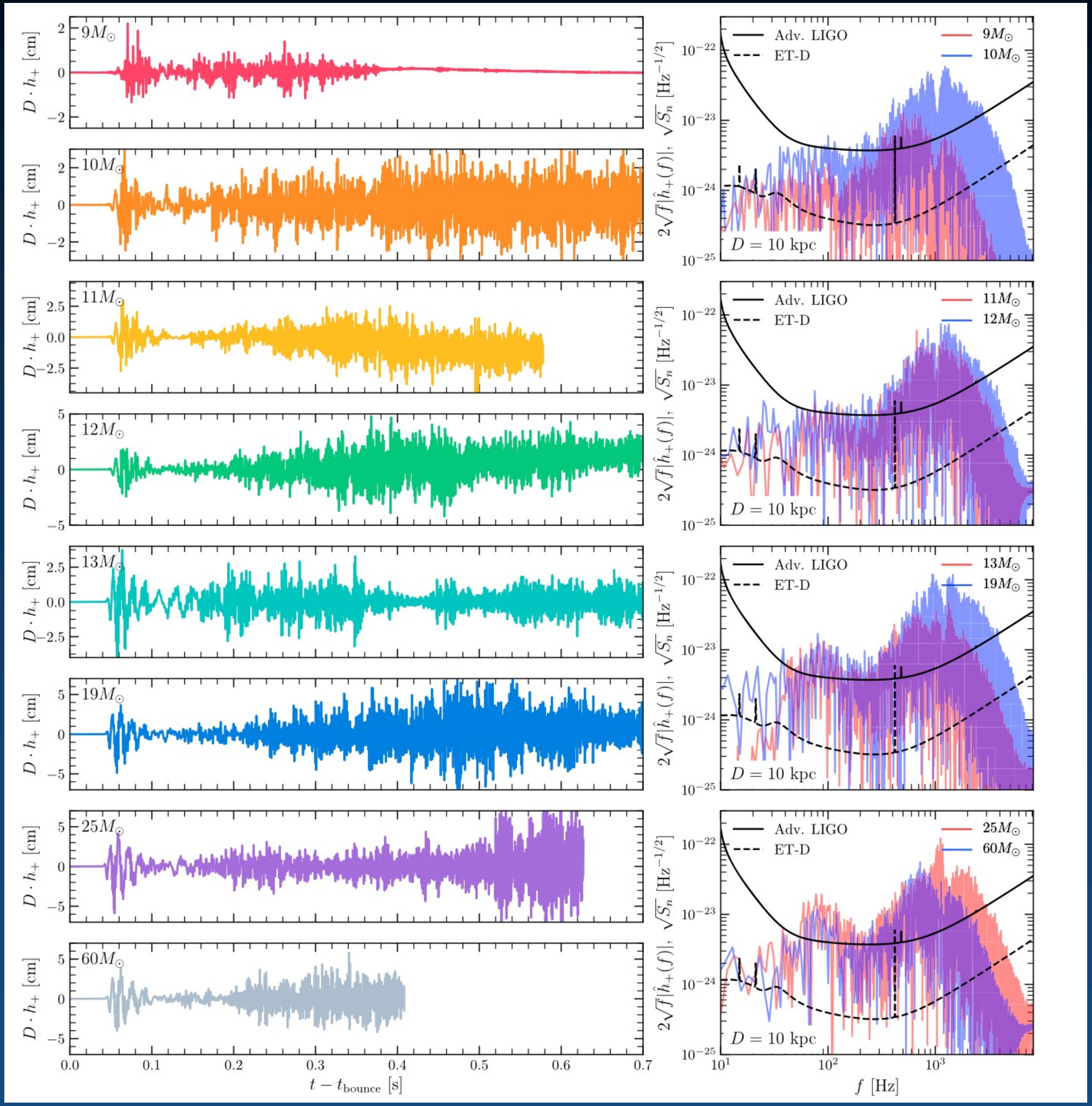
$$Q_\nu^- \approx 145\mathrm{Mev/s}\frac{\rho}{m_a}\left(\frac{T}{2\mathrm{MeV}}\right)^6$$





Gravitational Waves from CCSNe

Radice et al. 2019



Rich progenitor dependence

Most of GW energy is > 100Hz

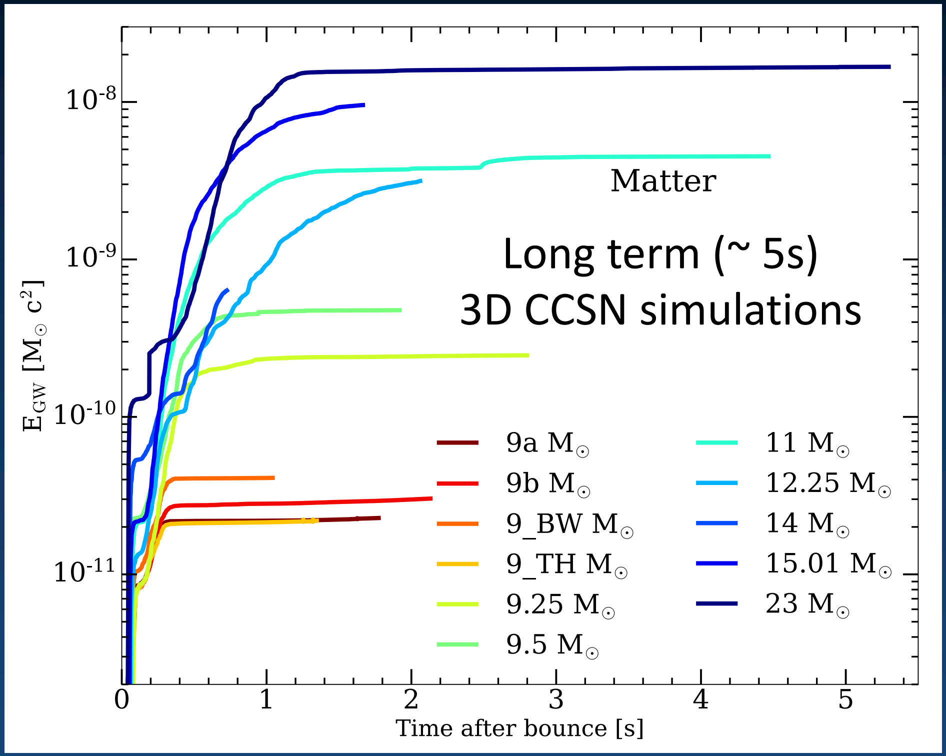
Detectability: under debate

Any correlations to others ?

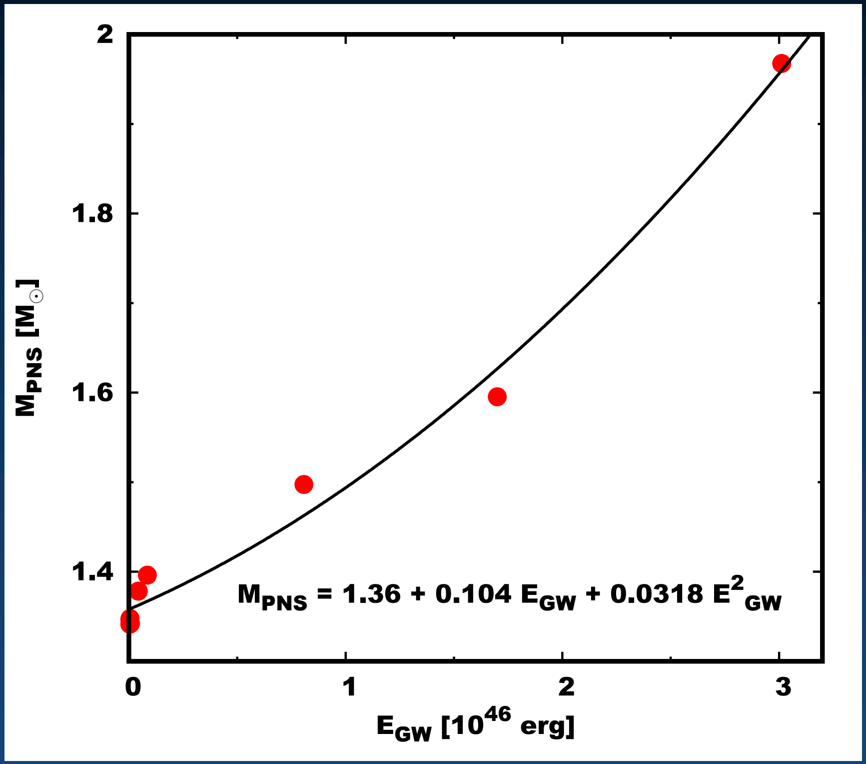
Yes, neutrinos!

GWs from long-term 3D CCSN simulations

Strong correlation between GWs and Proto-neutron star mass



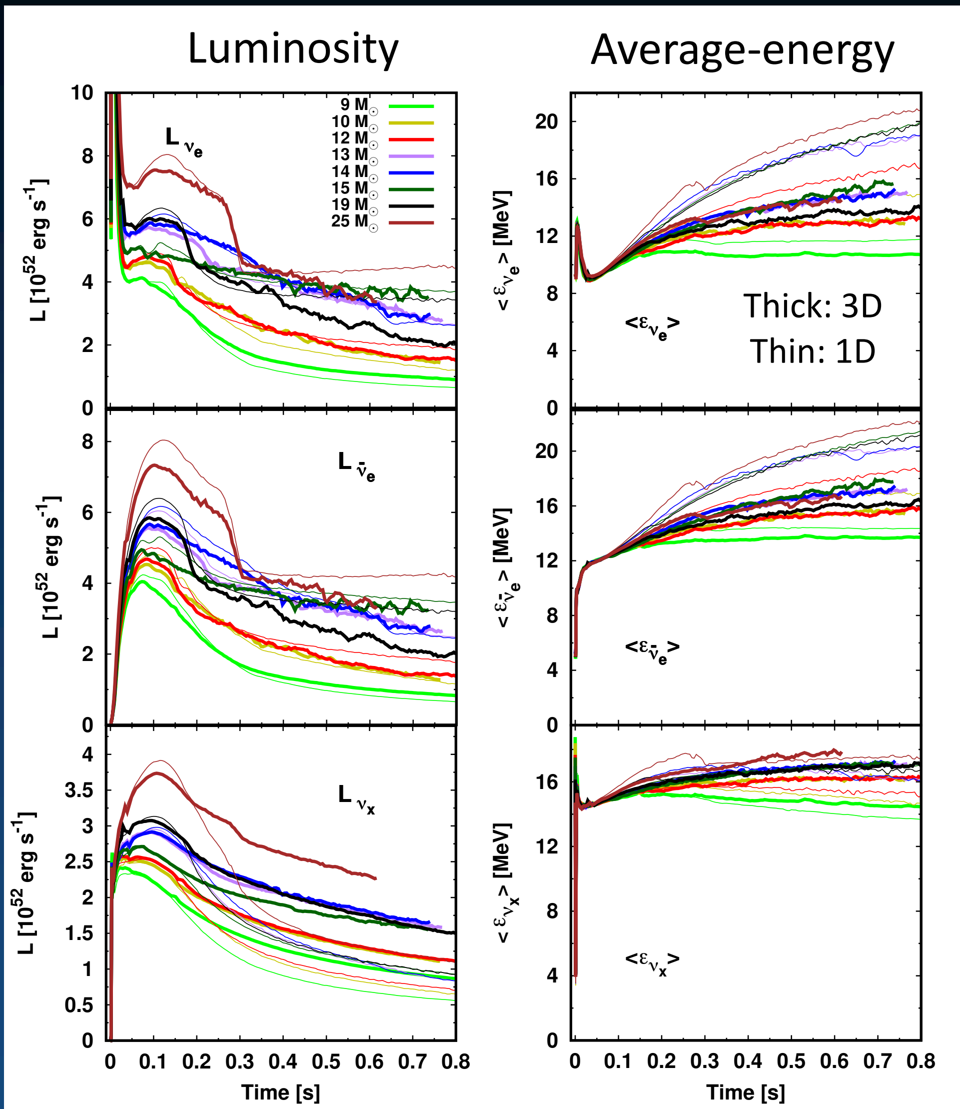
Vartanyan et al. 2023



Nagakura and Vartanyan 2023

Proto-neutron star mass can be estimated from GWs

Neutrino signals



Nagakura et al. 2021

Some new features emerge in 3D explosion models

1. Explosion models have low neutrino luminosity than those with non-explosions (due to weak mass accretion)

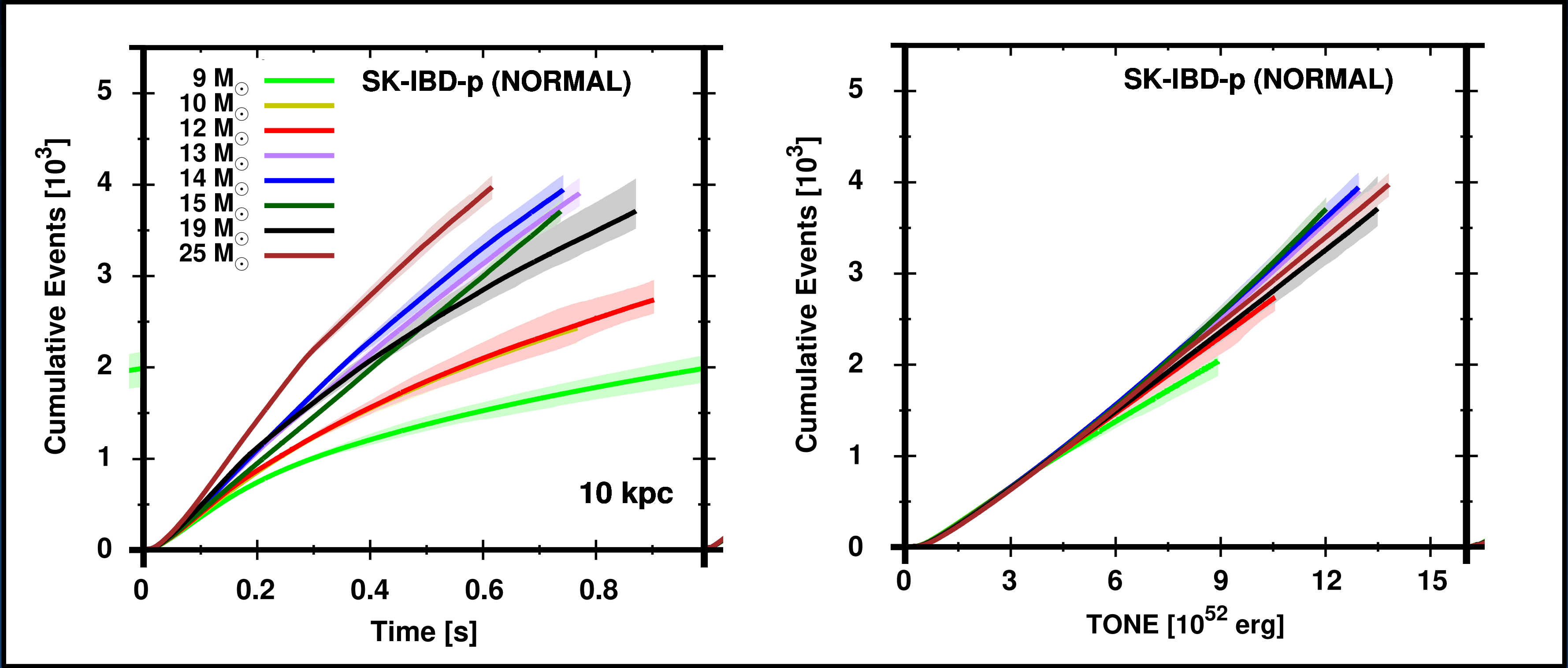
2. The average energy of electro-type neutrinos and their anti-partners are lower in 3D than 1D.

3. Neutrino luminosity of heavy-leptonic neutrinos are higher in 3D than 1D. (due to PNS convection)

Useful formula: $$\frac{L_{\nu 3D}}{L_{\nu 1D}} \sim \frac{\mathrm{T}^4_{\nu 3D}\mathrm{R}^2_{\nu 3D}}{\mathrm{T}^4_{\nu 1D}\mathrm{R}^2_{\nu 1D}}$$

Correlation between TONE (E_ν) and N_{cum} in neutrino detector

Nagakura et al. 2021



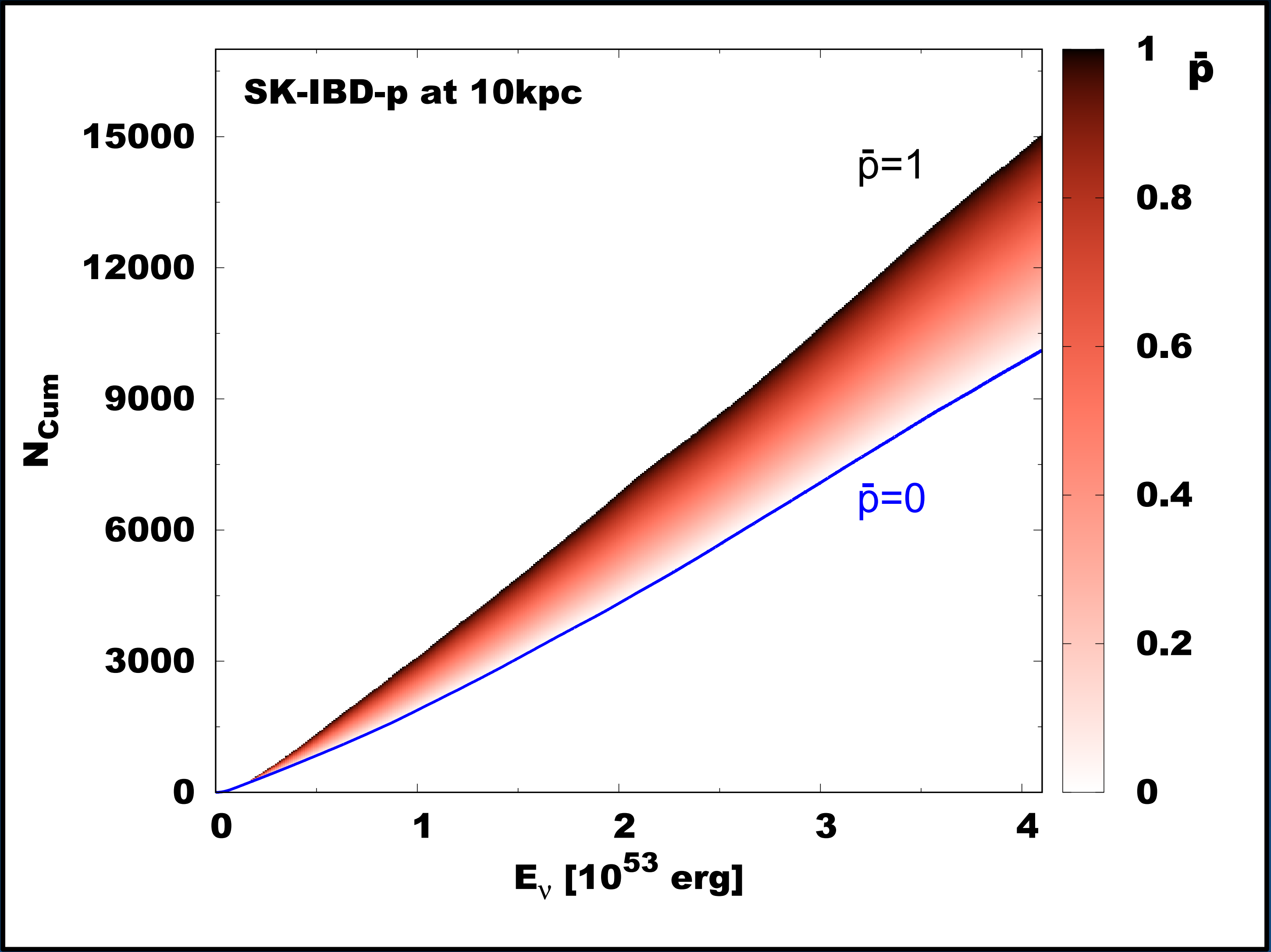
$$[\mathrm{SK-IBDp-NORMAL}]$$

$$N_{\mathrm{Cum}} = \left(220\, E_{52} + 5\, E_{52}^2 - 0.074\, E_{52}^3 + 0.0003\, E_{52}^4\right) \left(\frac{V}{32.5\,\mathrm{ktons}}\right)\left(\frac{d}{10\,\mathrm{kpc}}\right)^{-2},$$

E_ν can be estimated from N_{cum}

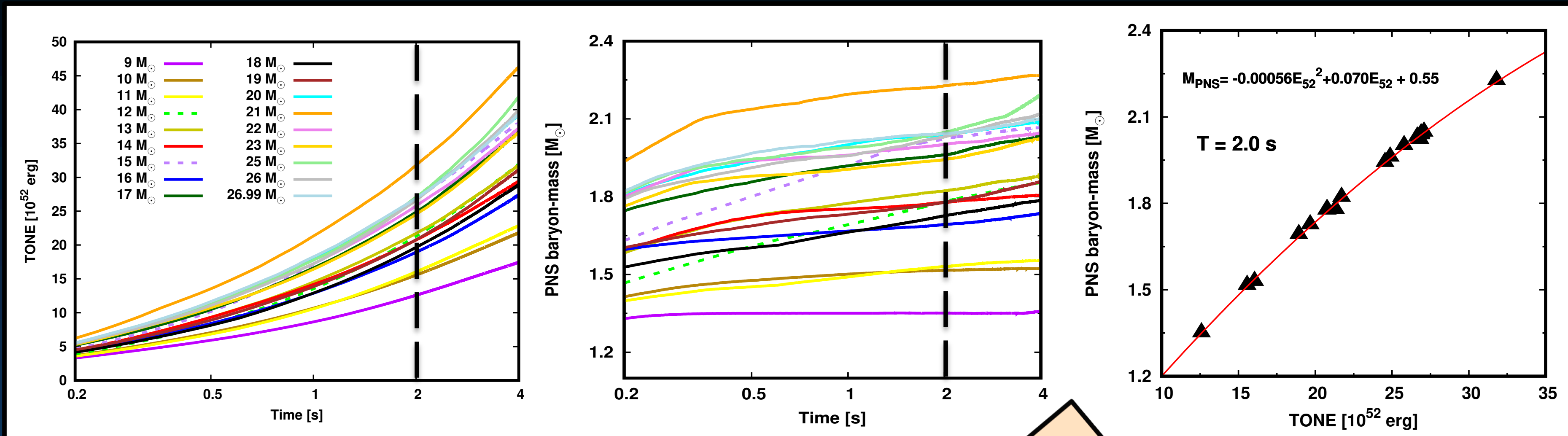
Neutrino detection counts depend on total neutrino emitted energy and neutrino oscillation.

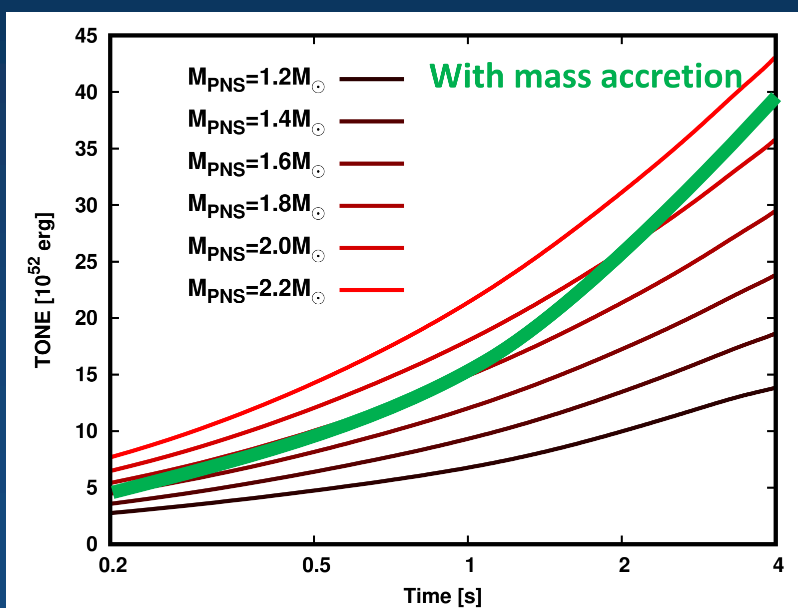
The relation is universal (less progenitor dependent)



E_ν has a strong correlation to M_{PNS}

Nagakura and Vartanyan 2022





The time evolution of radiated energy of neutrinos is characterized by PNS mass