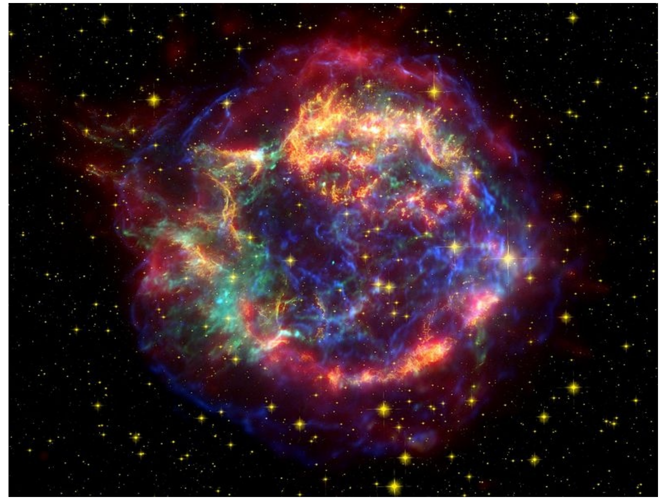

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Core-Collapse Supernovae & Gravitational Waves Part I: Theory & Modelling

Australian Government
Australian Research Council

Bernhard Müller
Monash University

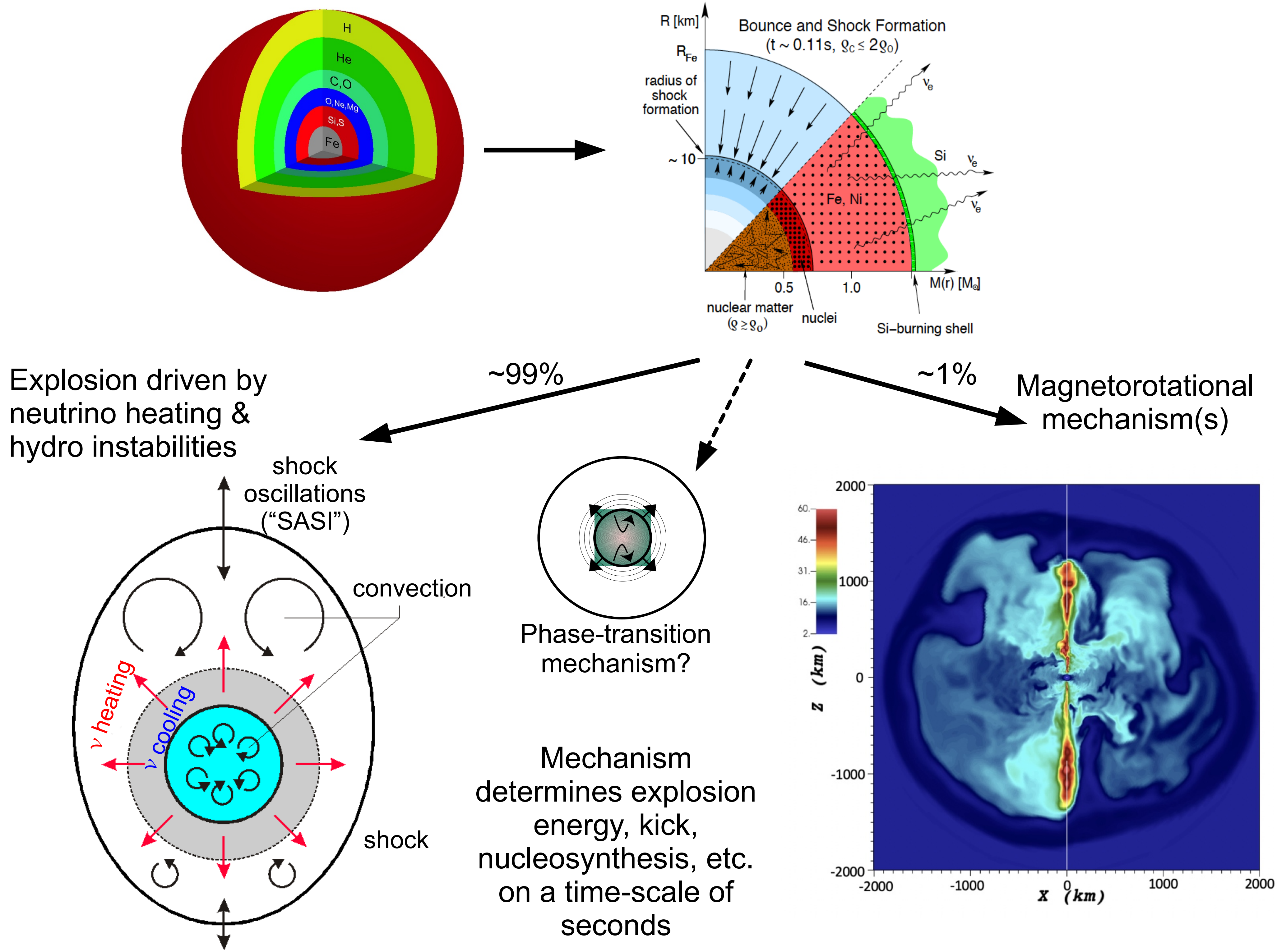
First IGWN Symposium on Core Collapse Supernova Gravitational Wave Theory and Detection

The Task

“to convey the necessary fundamentals in each case, the current state of the art and its achievements and shortcomings, and the progress needed to reach our common goal: to prepare for the next Galactic core collapse supernova, for both detection and discovery.”

The following references may do more justice to this task than I can in 90 min:

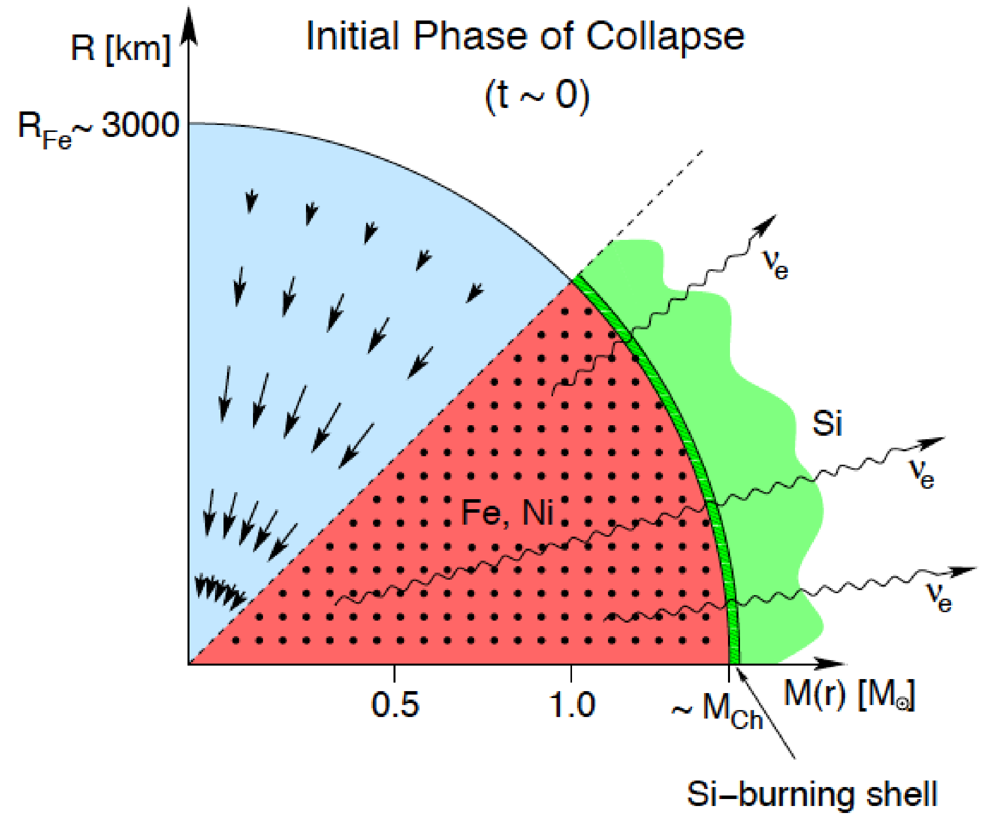
- Mueller 2020, Hydrodynamics of core-collapse supernovae and their progenitors, LRCA 6, 3
- Mezzacappa et al. 2020, Physical, numerical, and computational challenges of modeling neutrino transport in core-collapse supernovae, LRCA 6, 4
- Janka 2017. Neutrino-Driven Explosions (Handbook of Supernovae)
- Müller 2025. Supernova Simulations (New Frontiers in GRMHD Simulations) – focus on MHD
- Abdikamalov et al. 2021. Gravitational Waves from Core-Collapse Supernovae (Handbook of Gravitational Wave Astronomy)



H
He
C,O
O,Ne,Mg
Si,S
Fe
R [km]
Bounce and Shock Formation
(t ~ 0.11s, ϱc ≤ 2ϱ0)
RFe
radius of shock formation
~ 10
νe
Si
Fe, Ni
0.5
1.0
M(r) [M⊙]
nuclear matter (ϱ ≥ ϱ0)
nuclei
Si–burning shell
~99%
~1%
Explosion driven by neutrino heating & hydro instabilities
Magnetorotational mechanism(s)
shock oscillations (“SASI”)
convection
ν heating
ν cooling
shock
Phase-transition mechanism?
Mechanism determines explosion energy, kick, nucleosynthesis, etc. on a time-scale of seconds
2000
1000
0
-1000
-2000
z (km)
x (km)
60.
46.
31.
16.
2.

Collapse and Bounce

- Deleptonisation and photodisintegration trigger collapse on free-fall time
- Effective Chandrasekhar mass $M_{ch} \approx 5.8 Y_e^2 M_\odot$ shrinks due to deleptonisation
- Neutrino trapping at $\sim 5 \times 10^{11}$ g cm^{-3}
- Bounce of inner core, shock formation and stalling



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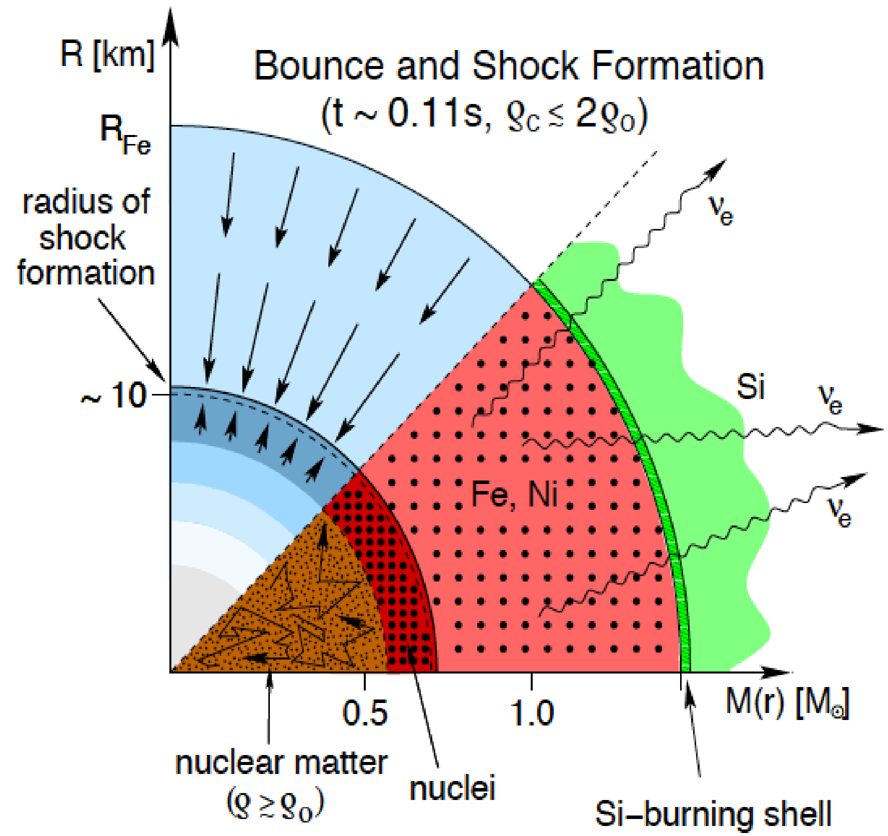

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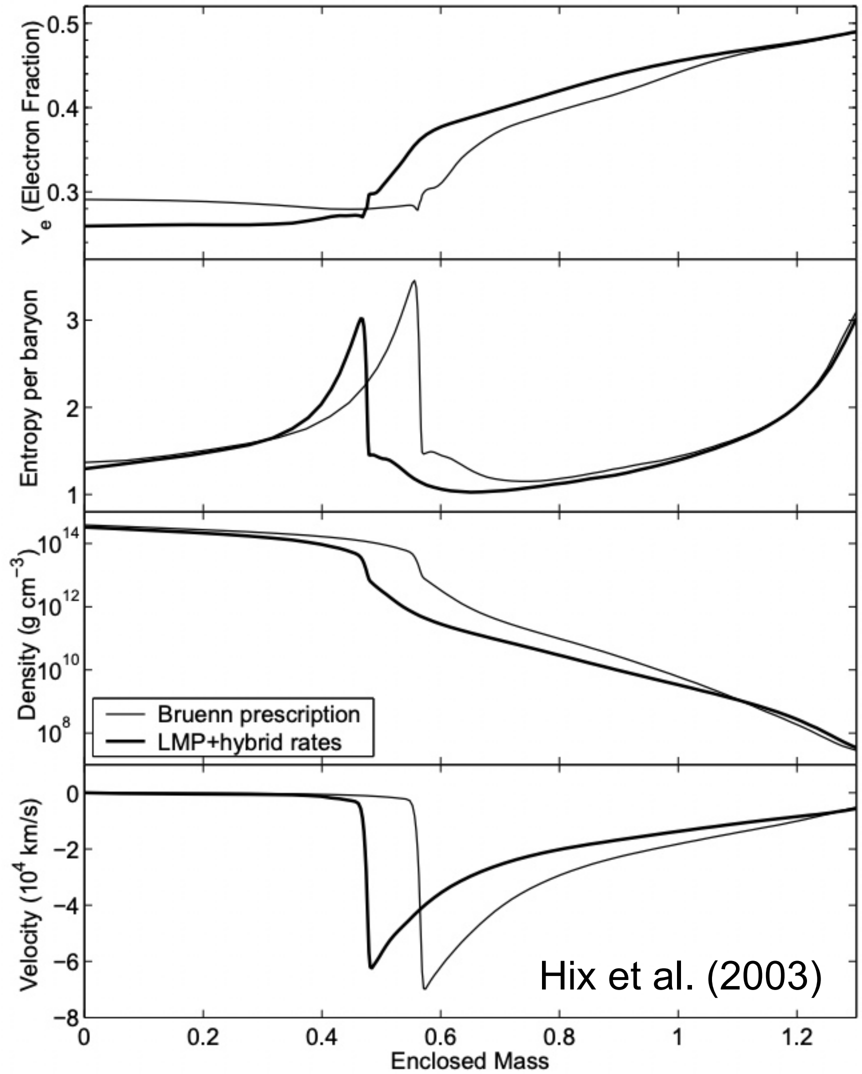
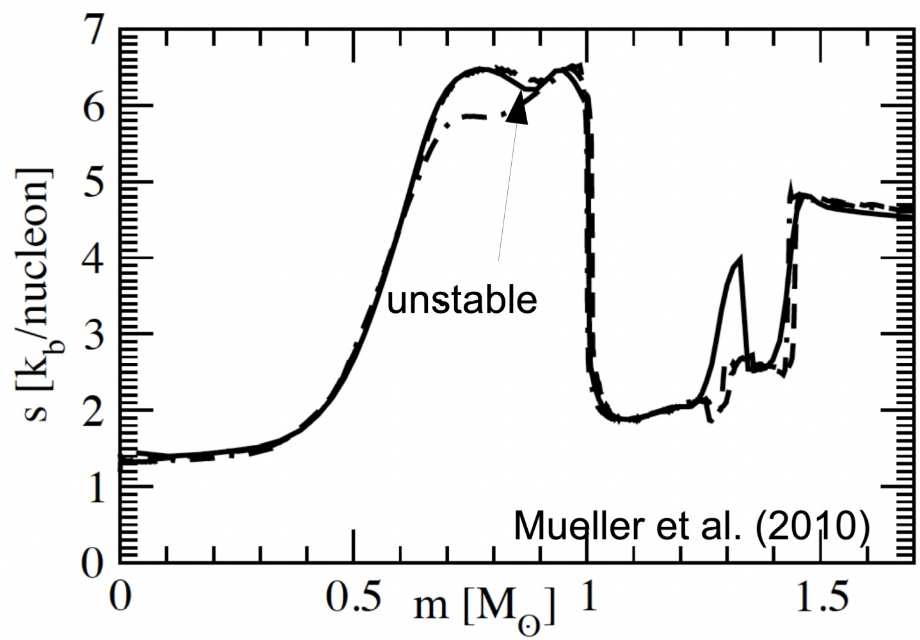


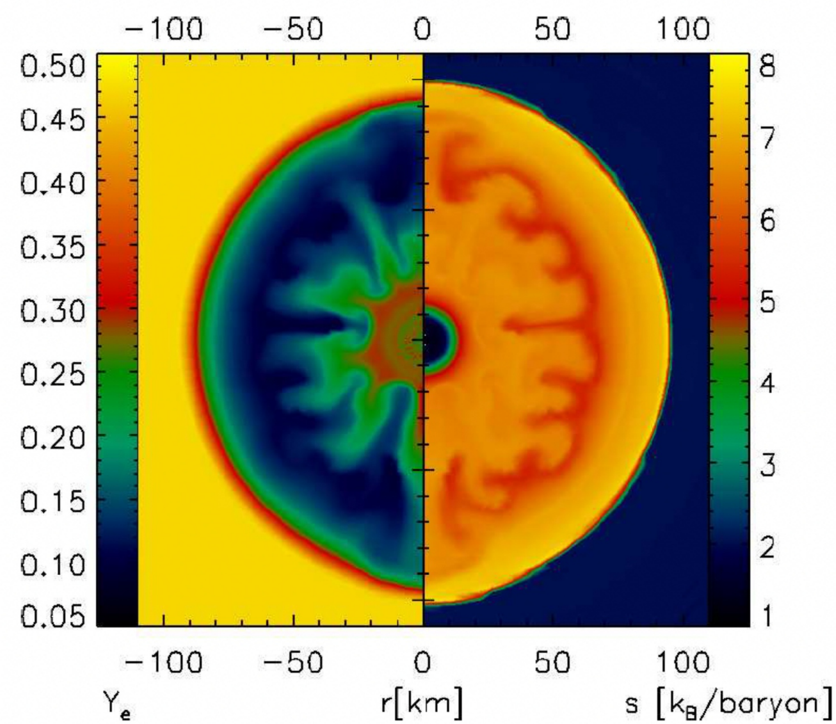
Collapse and bounce relatively **uniform across progenitors**, but dependent on **rotation**, **equation of state** and **neutrino rates**

Immediate Post-bounce Phase

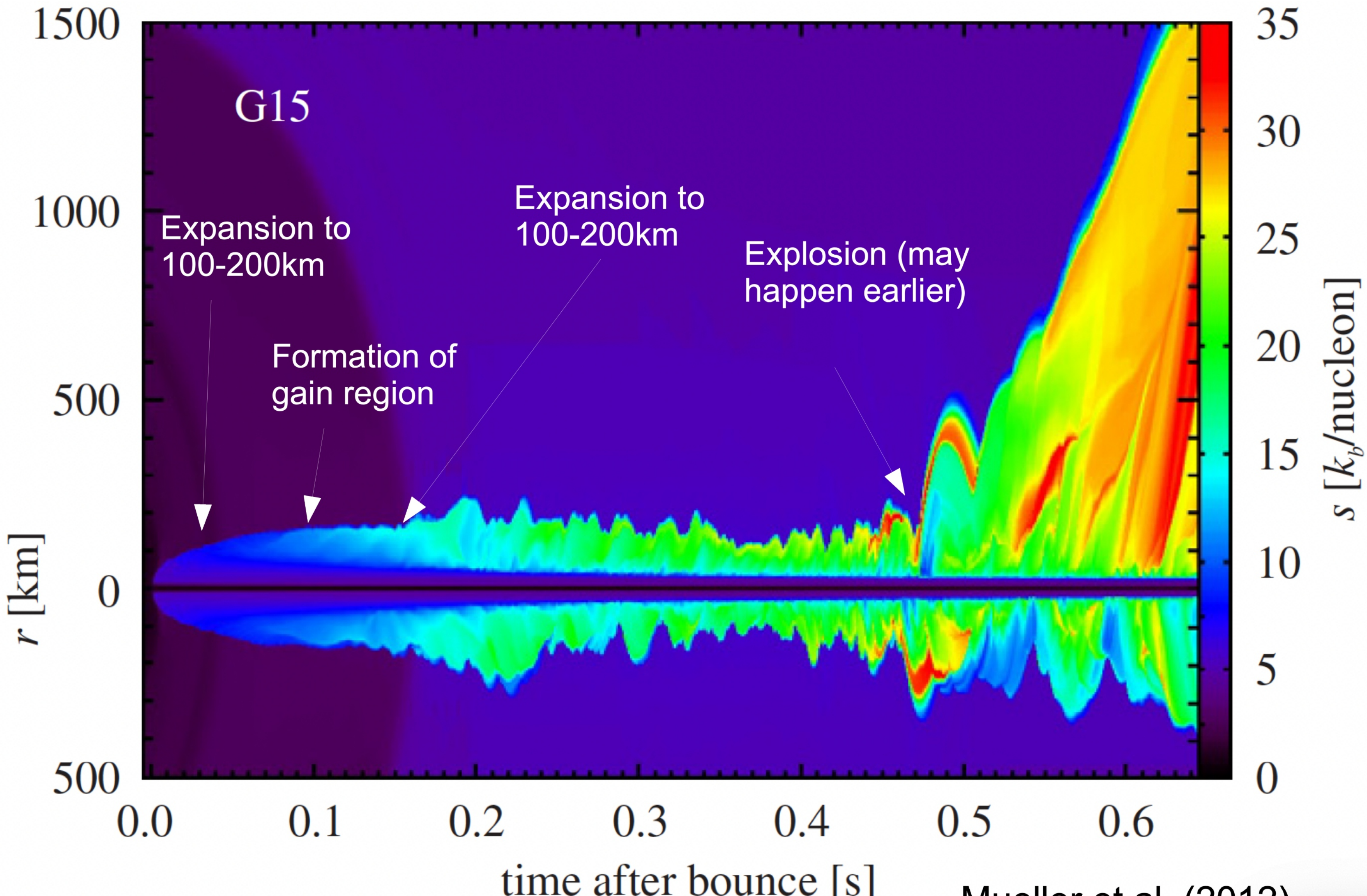
- “Ringing” of deformed core
- Prompt convective overturn due to weakening of shock and neutrino losses
- Accretion shock driven out to ~150km over tens of ms

  → quasi-stationary structure emerges



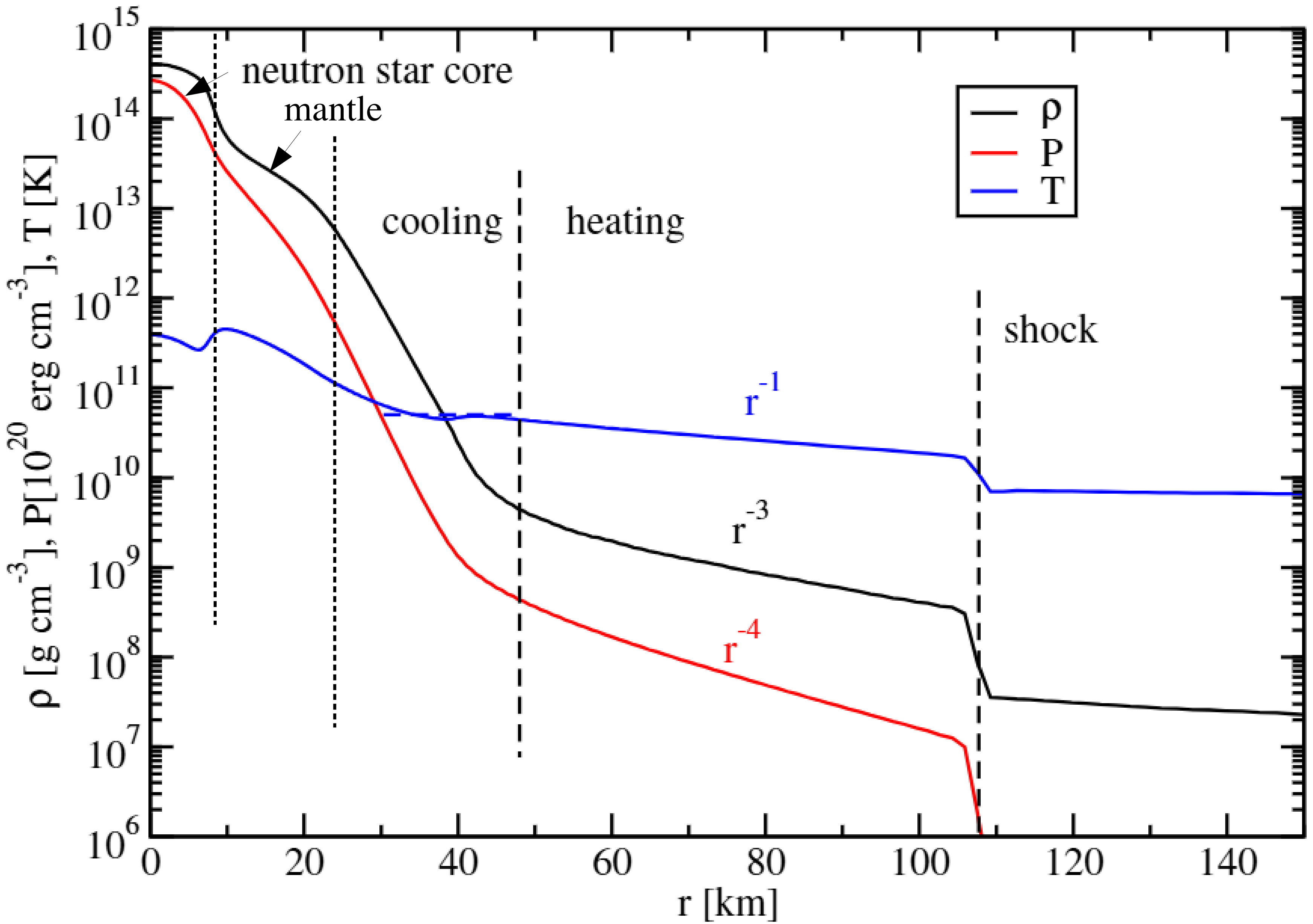


Shock Propagation

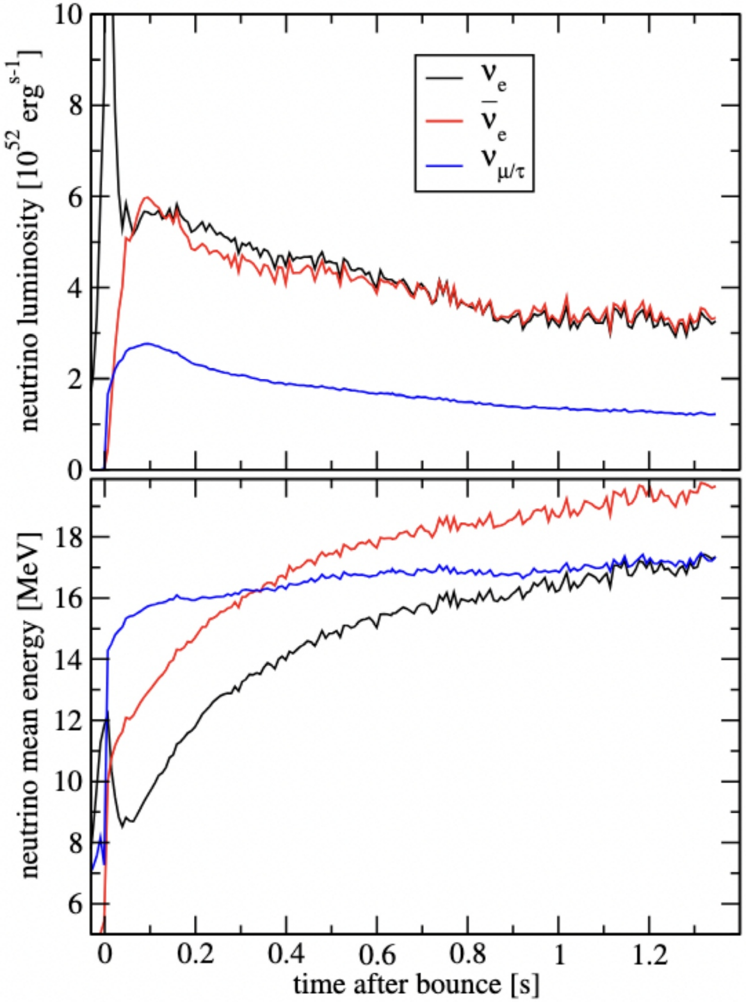


Mueller et al. (2012)

Supernova Core Structure During Accretion Phase



Neutrino Heating and Cooling



Time evolution of neutrino luminosities and mean energies, $15 M_\odot$ star.

For larger r neutrino absorption/emission is slow and the accreted mater undergoes *almost* adiabatic contraction, resulting in a stratification

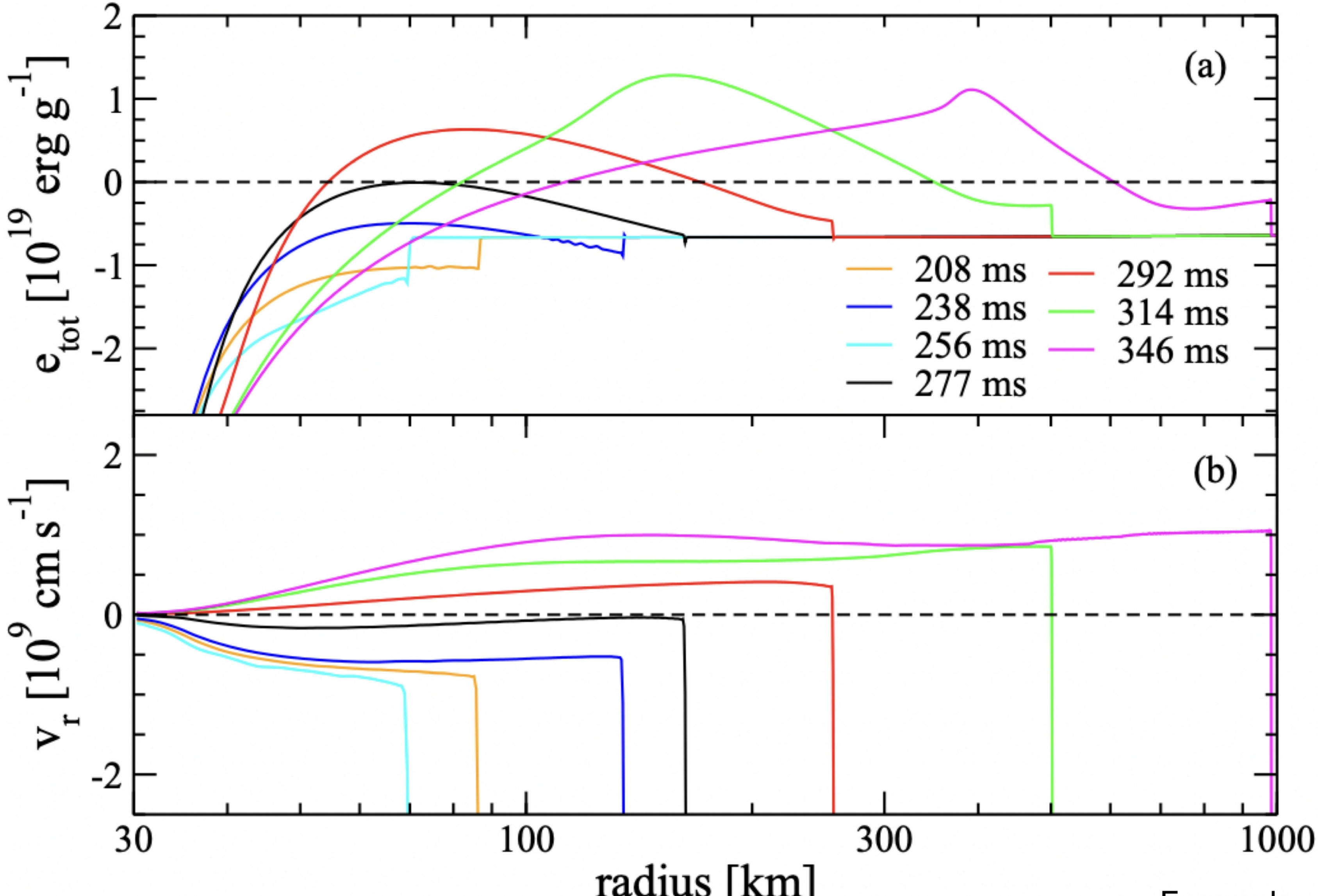
$$T \propto r^{-1}, \quad \rho \propto r^{-3}, \quad P \propto r^{-4}.$$

The resulting charged-current heating and cooling rates ($\nu_e + n \rightleftharpoons e^- + p$, $\bar{\nu}_e + p \rightleftharpoons e^+ + n$) scale as

$$\dot{q}_{\mathrm{heat}} \propto \frac{L_\nu \langle E_\nu^2 \rangle}{4\pi f r^2} \propto r^{-2},$$
$$\dot{q}_{\mathrm{cool}} \propto T^{-6} \propto r^{-6}$$

As the cooling rate decreases faster with radius, a region of net heating (*gain region*) eventually develops.

Explosions Conditions



Fernandez (2012)
(Cp. also earlier work by Janka 2000)

Explosion Conditions

How much neutrino heating is necessary to instigate an explosive runaway? A rough estimate is furnished by the comparison of two time-scales:

- the advection time-scale τ_{adv} (average time spent in the gain region by accreted matter)
- the heating time-scale $\tau_{heat} = E_{bind,gain}/Q_{heat}$ required to inject the binding energy $E_{bind,gain}$ into the gain region for a volume-integrated heating rate Q_{heat}
- $\tau_{adv}/\tau_{heat} \gtrsim 1$: gain region expands and pushes the shock out

Using the Rankine-Hugoniot jump conditions at the shock, balance between heating and cooling at the gain radius R_{gain}, spherical symmetry, and a few other approximations, one can translate this into a condition on the neutrino luminosities and mean energies:

$$\frac{\tau_{adv}}{\tau_{heat}} \propto \frac{(L_\nu \langle E_\nu \rangle^2)^{5/3} R_{gain}^{2/3}}{\dot{M} M}$$

Janka (2000); Mueller & Janka (2015)

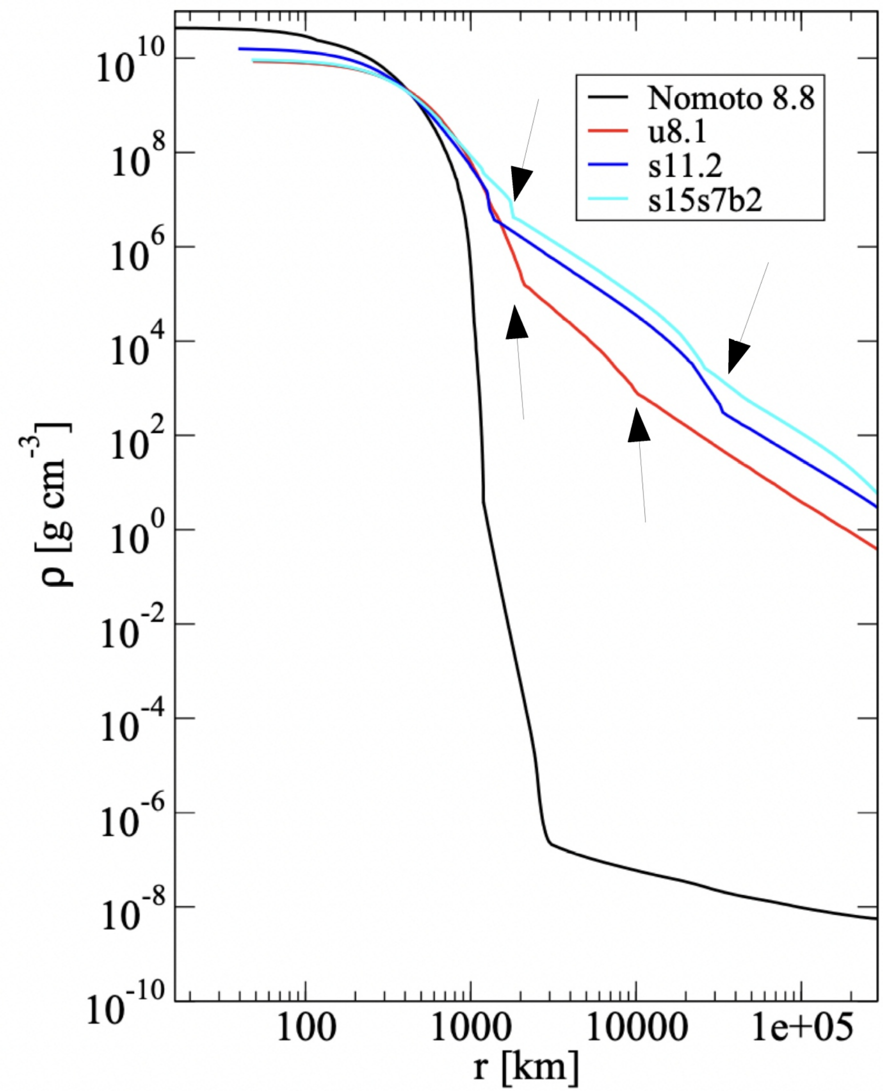
Impact of Progenitor Structure on Post-Bounce Phase

- **Density profile ρ(r)** → accretion rate after collapse phase

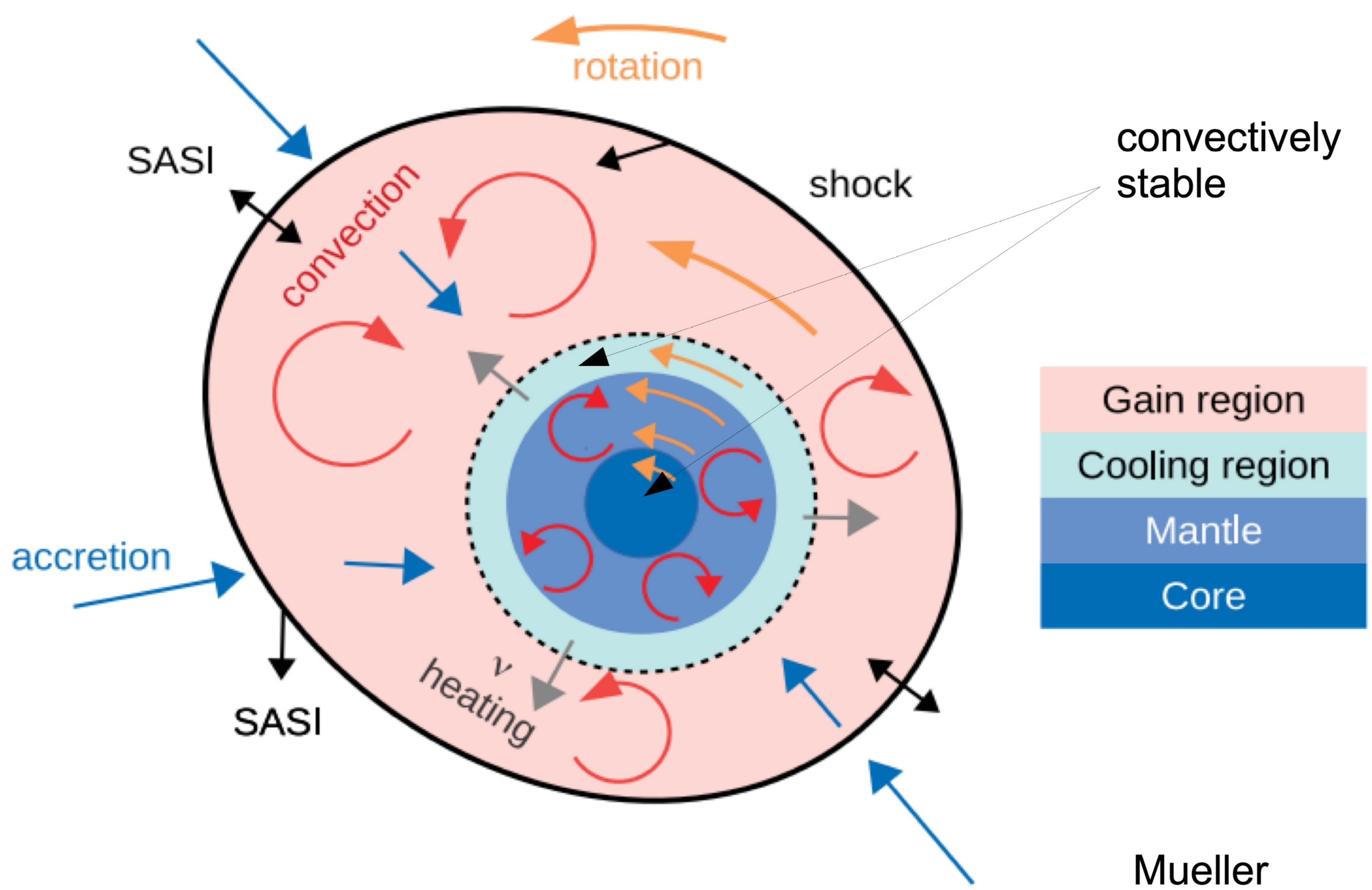
$$\dot{M} \approx \frac{2m}{t_{\mathrm{infall}}} \frac{\rho}{\bar{\rho}} \approx \frac{8\pi}{3} \sqrt{3Gmr^3}$$

- Accretion rate *also* influences neutrino emission
- Jumps in accretion rate due to **she interfaces** in progenitor (especially Si/O interface) often important for dynamics & explodability
- Popular explodability parameters like compactness (O'Connor & Ott 2011) reflect this:

$$\xi_M = \frac{M/M_\odot}{R/1000\ \mathrm{km}}$$

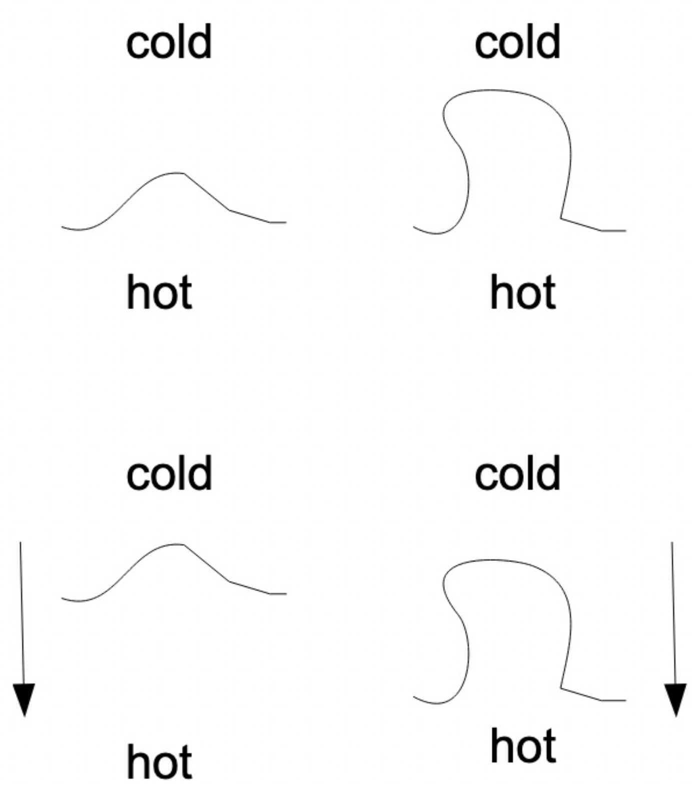


Structure of Supernova Core: Hydrodynamics Instabilities



Mueller (2025)

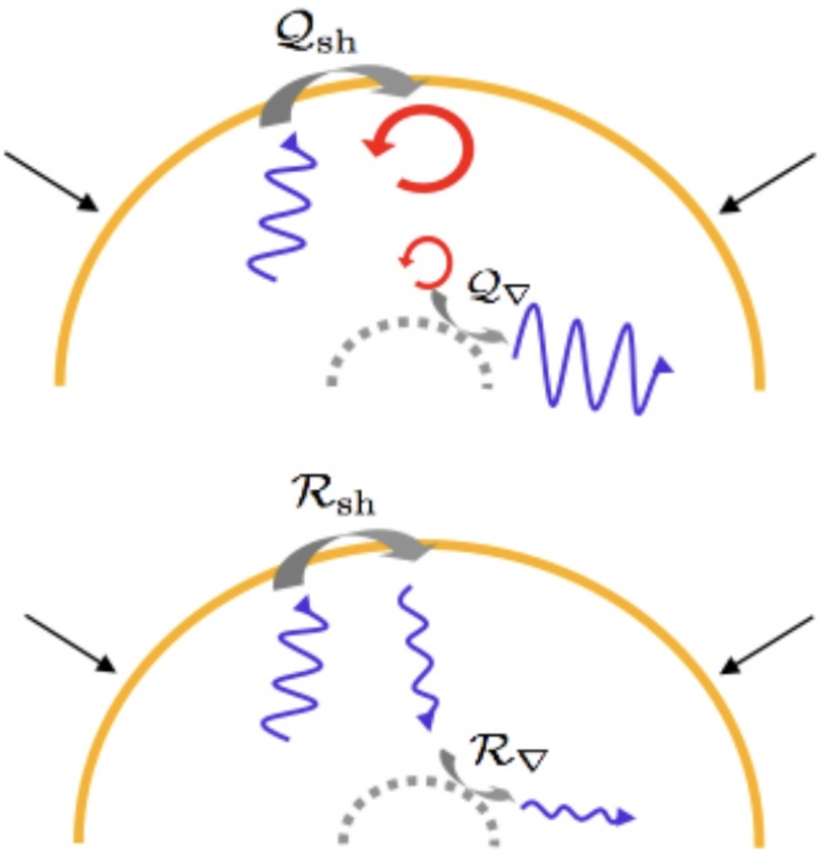
Neutrino-Driven Convection



- Heating in gain region results in an *entropy increase* as material is advected to the gain radius:
$$\frac{\mathrm{d}s}{\mathrm{d}r} < 0$$
- We can think of convection as a heat engine: heating $\rightarrow P\,dV$ work $\rightarrow$kinetic energy $\rightarrow$turbulent dissipation.
- In the non-linear phase energy input and dissipation balance each other, and the convective velocities reaches about (Müller & Janka 2015):
$$v_{\mathrm{conv}} \sim (\dot{q}_{\mathrm{heat}}(R_{\mathrm{sh}} - R_{\mathrm{gain}})^{1/3}.$$

Importance of convection realized since the 1990s (Herant et al. 1994; Burrows et al., 1995; Janka and Muller 1995)

Standing Accretion Shock Instability



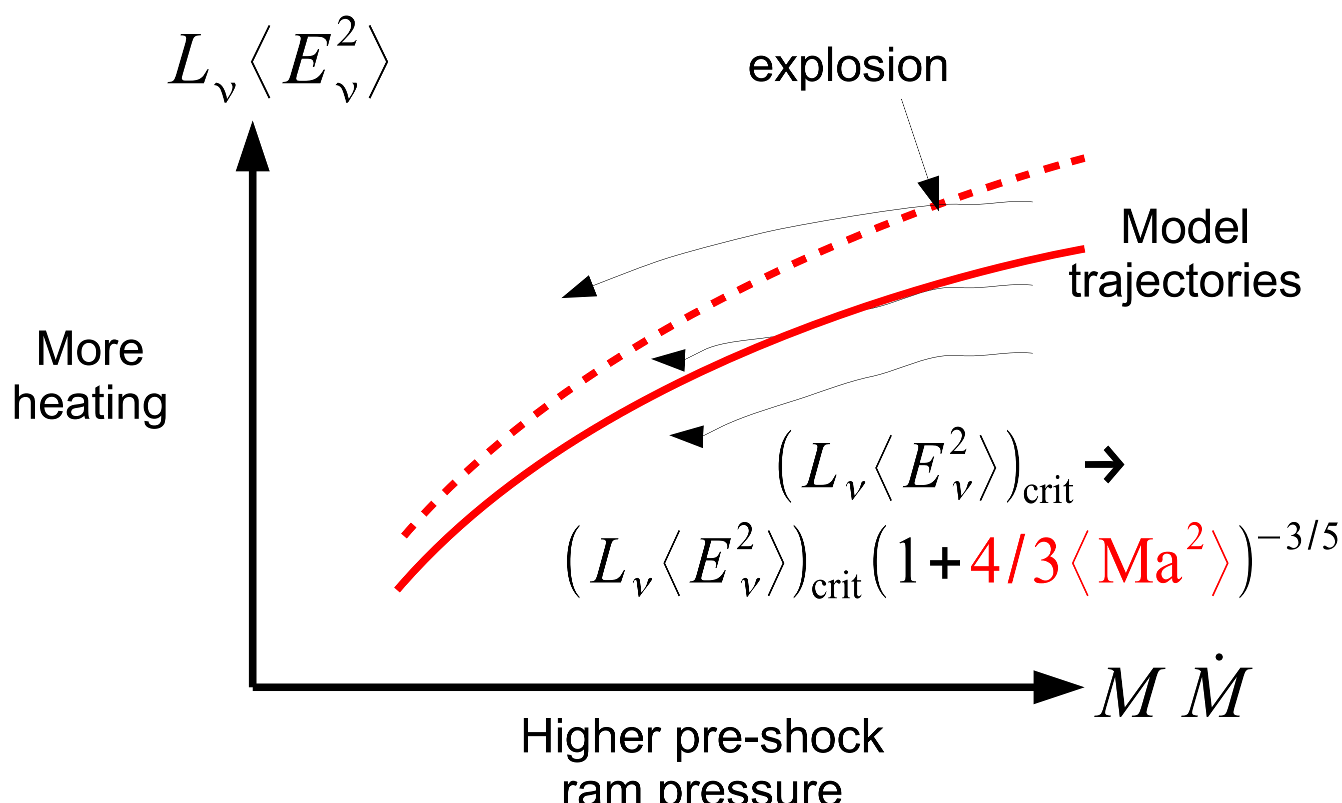
Guilet and Foglizzo (2012)

$$\chi = \int_{r_g}^{r_{sh}} \frac{\mathrm{Im}\,\omega_{\rm BV}}{|v_r|} \mathrm{d}r,$$

- Standing accretion shock instability" can grow even without convective instability (Blondin & Mezzacappa 2003)
- Mediated by a feedback loop of vorticity and acoustic waves between shock and neutron star surface (e.g. Guilet & Foglizzo 2012)
- Low-ℓ instability: dipole and quadrupole mode dominate
- Oscillatory instability: regular periodicity during linear phase.
- Requires sufficiently small unstable gradient (χ<3)
- Saturation by parasitic instabilities will lead to velocity perturbations $\delta v \sim \ln Q\, |v_r|$

Interaction of Instabilities and Neutrino Heating

- Turbulent “pressure”, turbulent viscosity, mixing, etc. modify quasi-hydrostatic structure of gain region & enlarge shock radius (Murphy et al. 2012 & others)
- Heating in larger volume → reduction of critical luminosity
- Reduction depends on average “turbulent Mach number” (Müller & Janka 2015) and is ~25% in 2D/3D compared to 1D



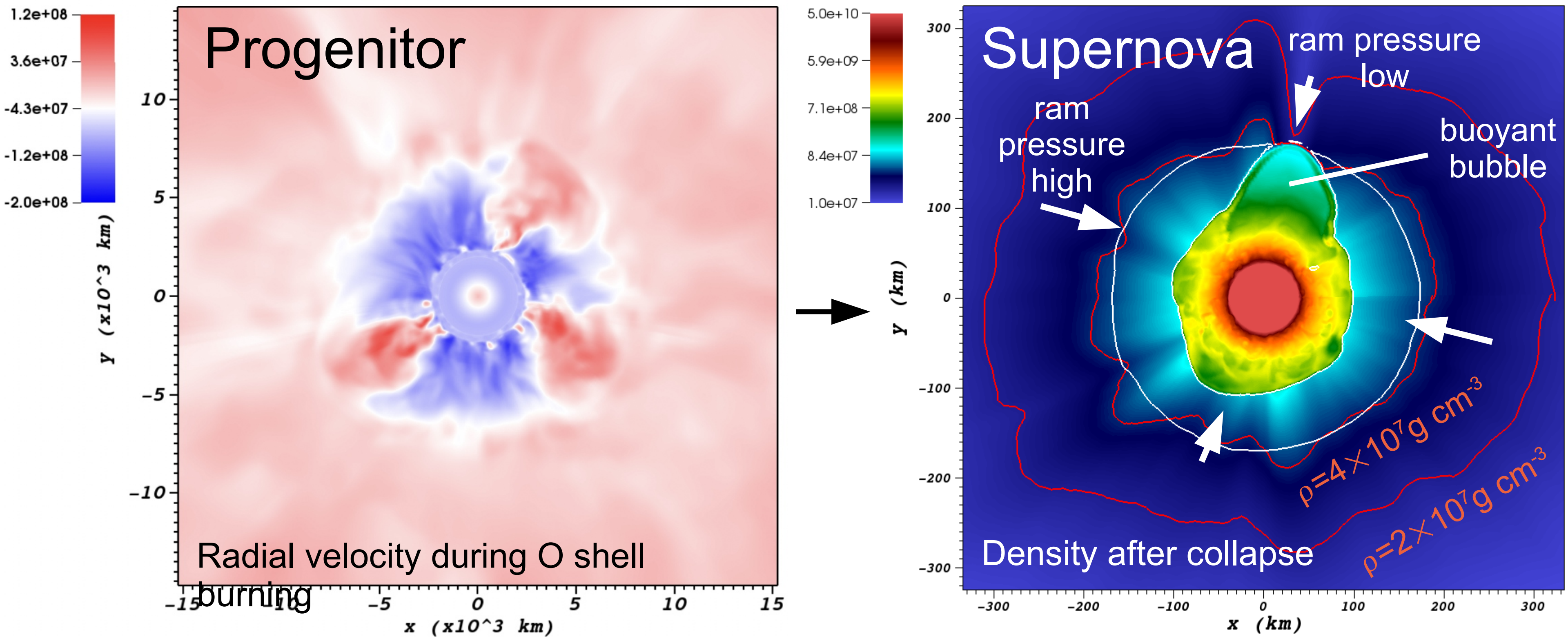
Turbulent velocity (→ ⟨Ma⟩) regulated by avg. neutrino heating rate:

$$v_{\mathrm{turb}} \sim \left[\dot{q}_\nu \left(r_{\mathrm{shock}} - r_{\mathrm{rgain}}\right)\right]^{1/3}$$

avg. heating rate per unit mass

Steady state applies because $t_{\mathrm{conv}}, t_{\mathrm{SASI}} < t_{\mathrm{evol}}$

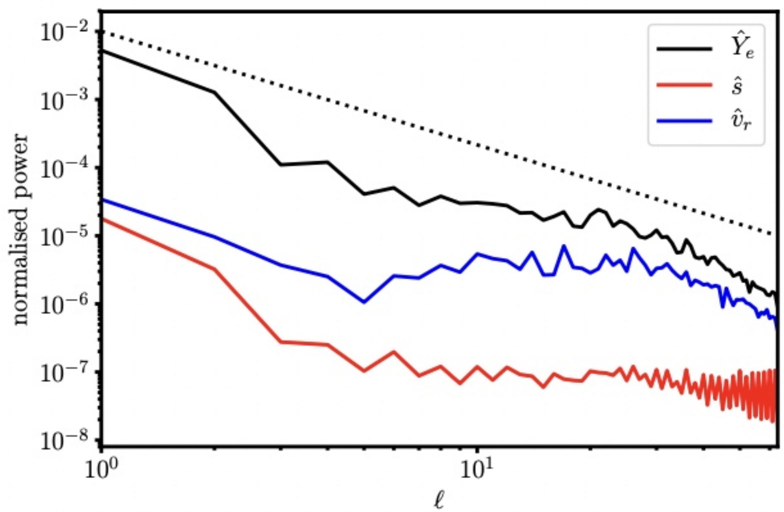
Perturbation-aided explosions



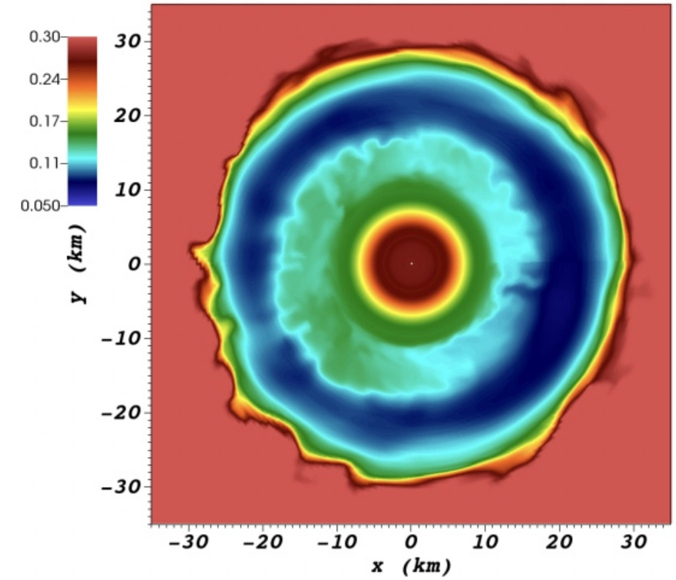
- Pre-collapse perturbations from O, Ne or Si shell burning often dynamically relevant (Couch et al. ‘15, Mueller et al. ‘15, ‘17)
- Subsonic convective motions Ma~0.1 translate into sizeable density and ram-pressure perturbations and lower critical luminosity:

$$\frac{\Delta L_{\mathrm{crit}}}{L_{\mathrm{crit}}} \sim \frac{(2\ldots4)\times \mathrm{Ma}_{\mathrm{prog}}}{\ell}$$

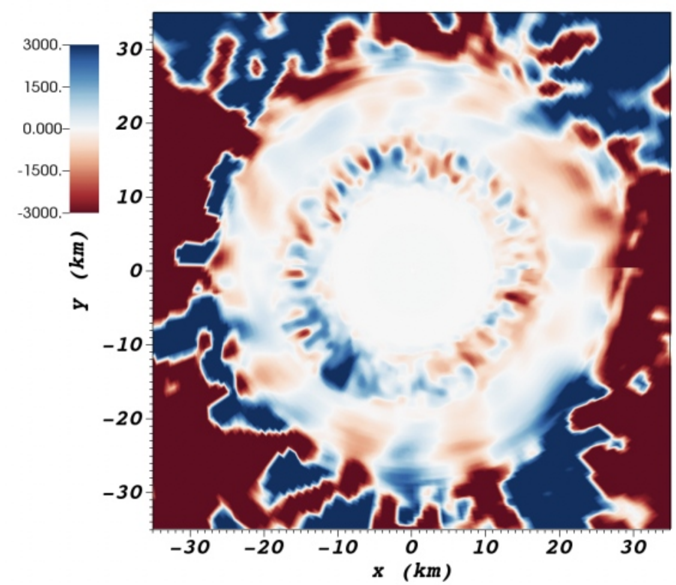
Proto-Neutron Star Convection



Turbulence spectra in PNS convection zone



Electron fraction



Radial velocity

Powell & Mueller (2019)

- Energy and lepton-number losses from PNS surface drive convection in the mantle
- Steady-state flow involves delicate interplay of entropy and lepton number gradients and convective and diffusive transport
- Low-mode lepton-number asymmetry (LESA, Tamborra et al. ‘14) can be present
- Indirect effect on gain region i) via the PNS radius and ii) modest effect on neutrino emission

Magnetic Fields

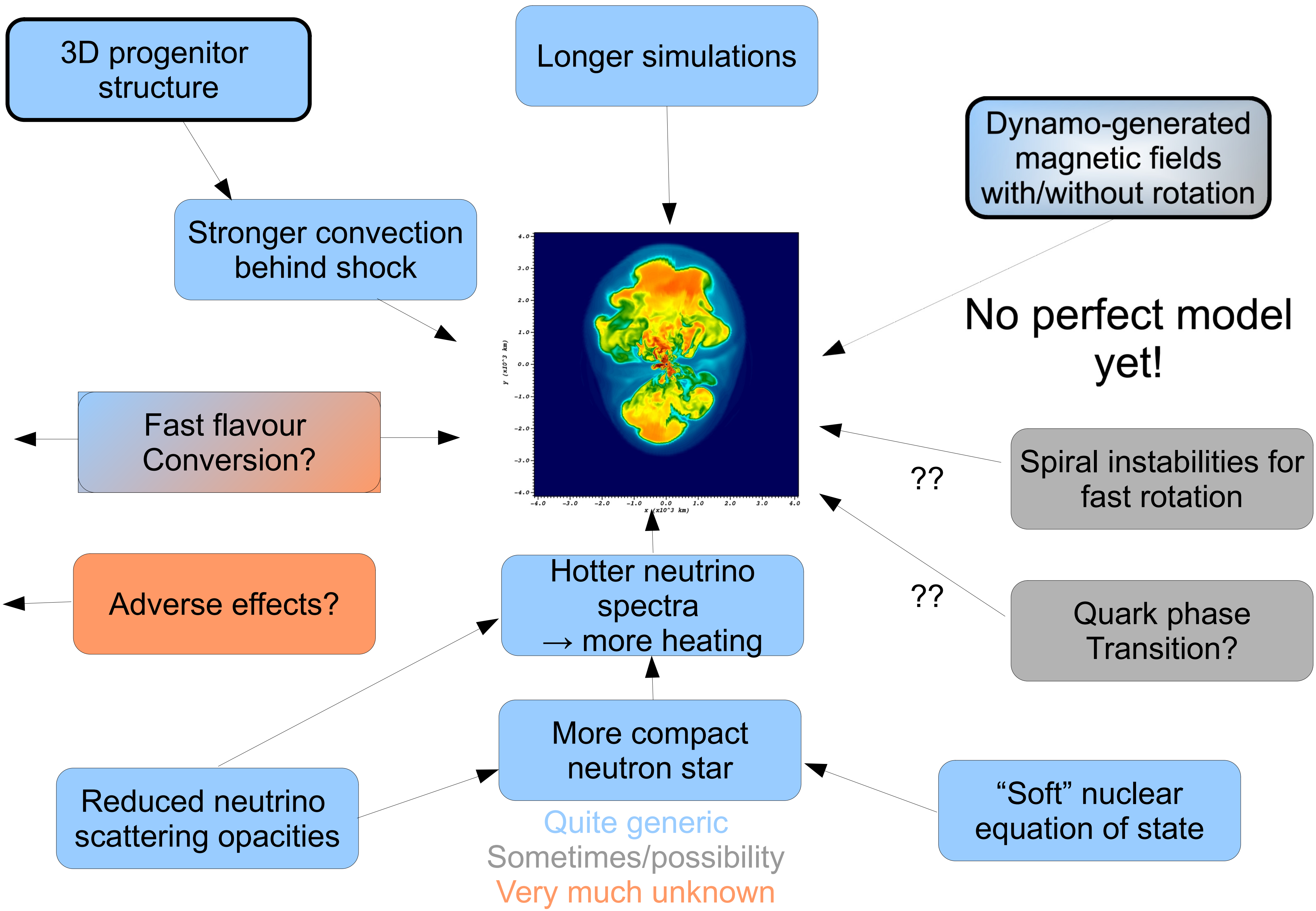
Post-collapse amplification usually critical to impact dynamics:

- **Without rotation:** Turbulent dynamo (up to ~40% of kinetic equipartition, Mueller & Varma 2020)
- **With rotation:**
  - Magnetorotational instability for fast amplification (→ difficult to resolve) with expected saturation fields of order (Akiyama et al. 2003):

$$B^2 \sim 4\pi\rho r^2 \omega^2 \frac{d\ln\omega}{d\ln r}$$

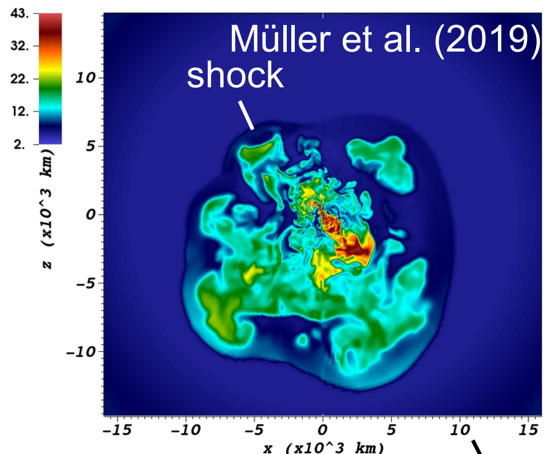
  - $\alpha-\Omega$ dynamo or other processes in PNS (Raynaud et al. 2020...)
- But initial fields may still be high enough (>10^{10}G in O shell) to decide time for amplification
- Strong pre-collapse fields ~10^{12}G may be present in some progenitors (merger products)

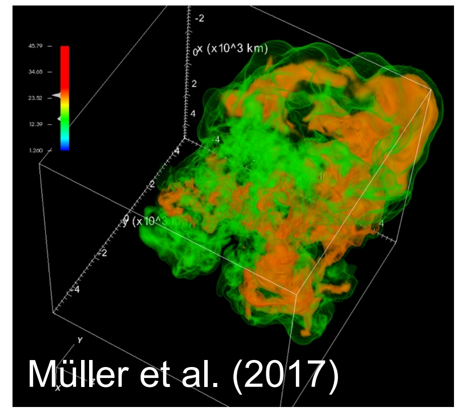
Putting it all together

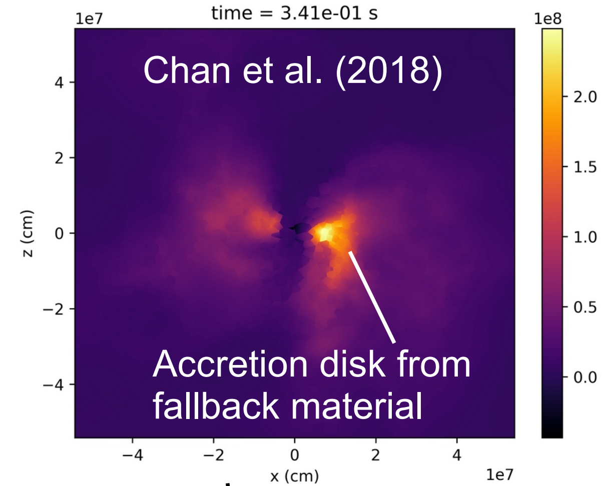


Current Status of 3D Explosion Models: An Emerging Consensus?

Monash & MPA groups







9.5$M_\odot$ **10.09$M_\odot$** 11.5$M_\odot$ **12$M_\odot$** **14$M_\odot$**` **18$M_\odot$**`` 18$M_\odot$` 20$M_\odot$ (40$M_\odot$) (85$M_\odot$) 100$M_\odot$

9.6$M_\odot$ ~~11.2$M_\odot$~~ **11.8$M_\odot$** **12.5$M_\odot$** **14.1$M_\odot$** **18.9$M_\odot$** ~~27$M_\odot$~~ (initial mass)

Hydrogen rich giants

Stripped-envelope progenitors (25-35% of all events)

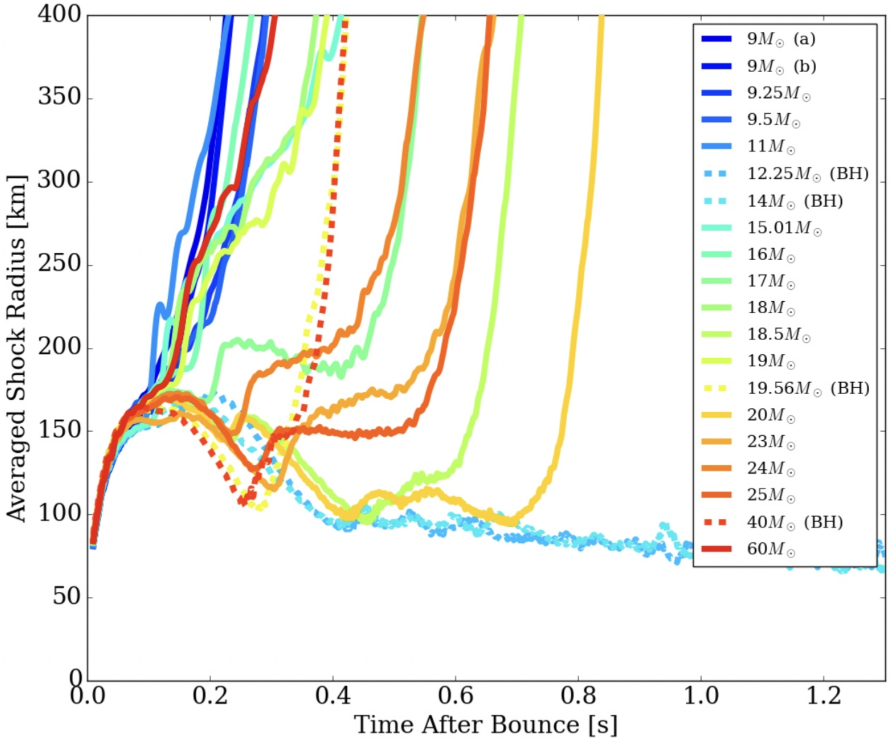
1.5$M_\odot$ 2.5$M_\odot$ **3$M_\odot$** 10$M_\odot$ 22$M_\odot$ (pre-collapse mass)

(Note: physics in models has evolved over 5 years)

Blue: shock revival & explosion
Orange: Fallback (explosion + black hole)
Red: failure
Bold: 3D initial conditions

Fornax code (Princeton) with relatively similar results, but no perfect agreement
VERTEX code (MPA) wither fewer explosions (e.g. with 3D initial models)
Oakridge: Explosions, but smaller 3D sets yet
FLASH code: No/few explosions

Black Holes in the Wrong Mass Range?



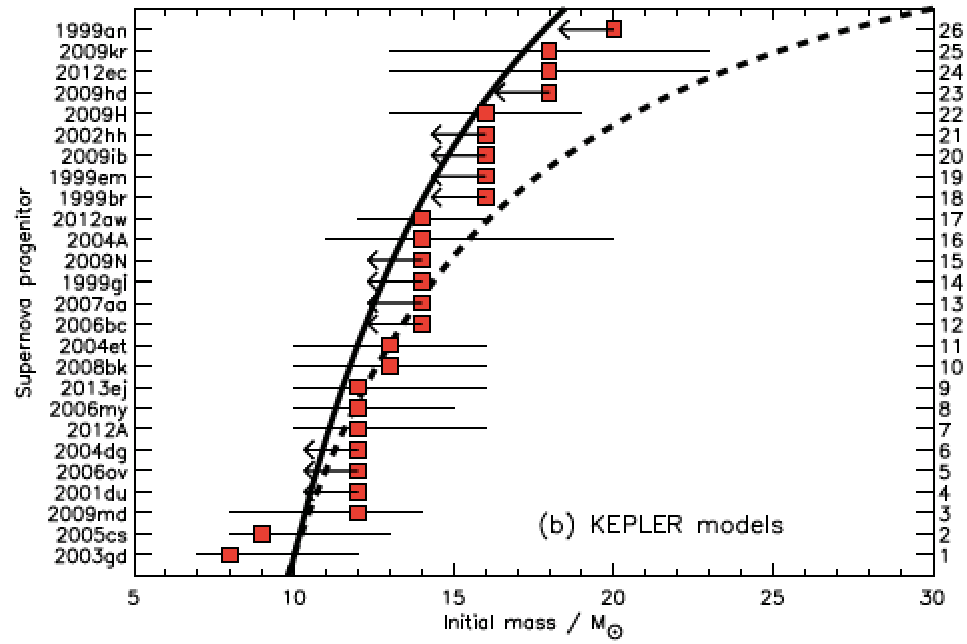


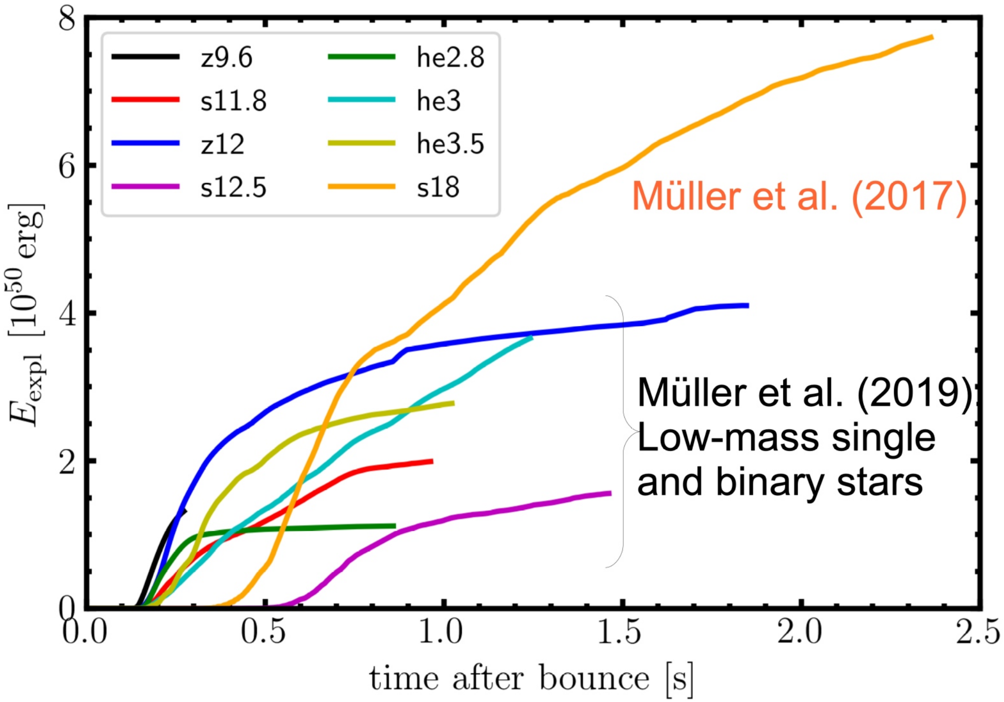
Figure 6. The progenitor detections are marked with error bars (data from Table 1 and the limits are marked with arrows (data from Table 2). The lines are cumulative IMFs with different minimum and maximum masses.

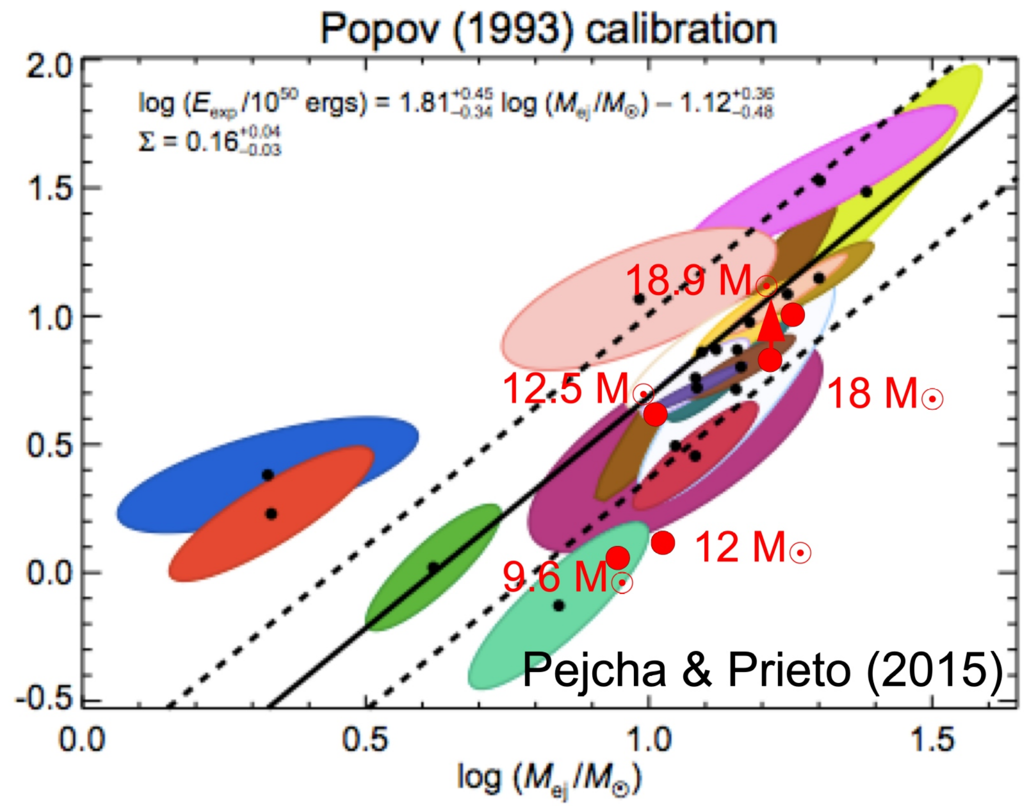
Burrows, Wang & Vartanyan (2024):
Black holes in progenitors of **moderate** mass

Cumulative distribution function of inferred progenitor masses from Smartt (2015)

- More large model sets by different group required
- Possible tensions with observations?
- **Robust:** Neutrino-driven explosions possible in 3D
- **Not yet robust:** Range of explosions from 3D models

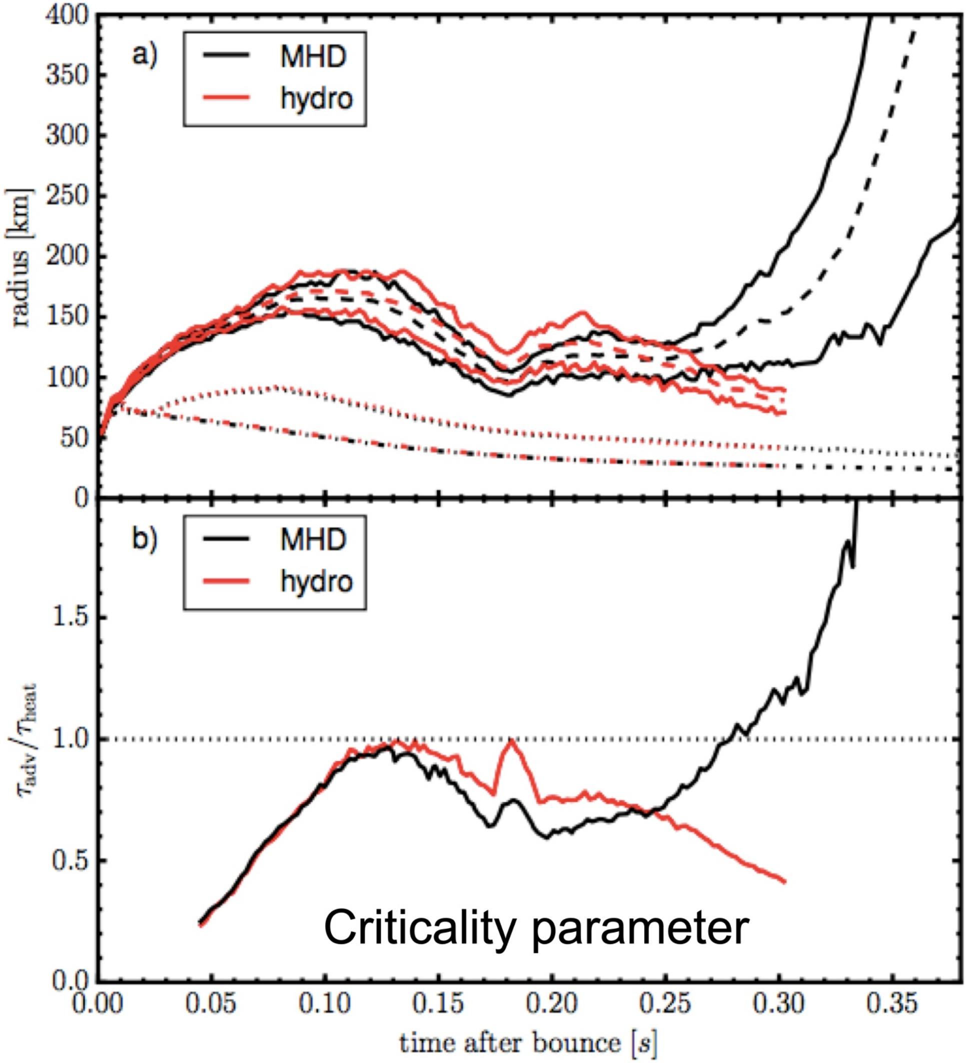
Explosion Properties – Tentative Picture



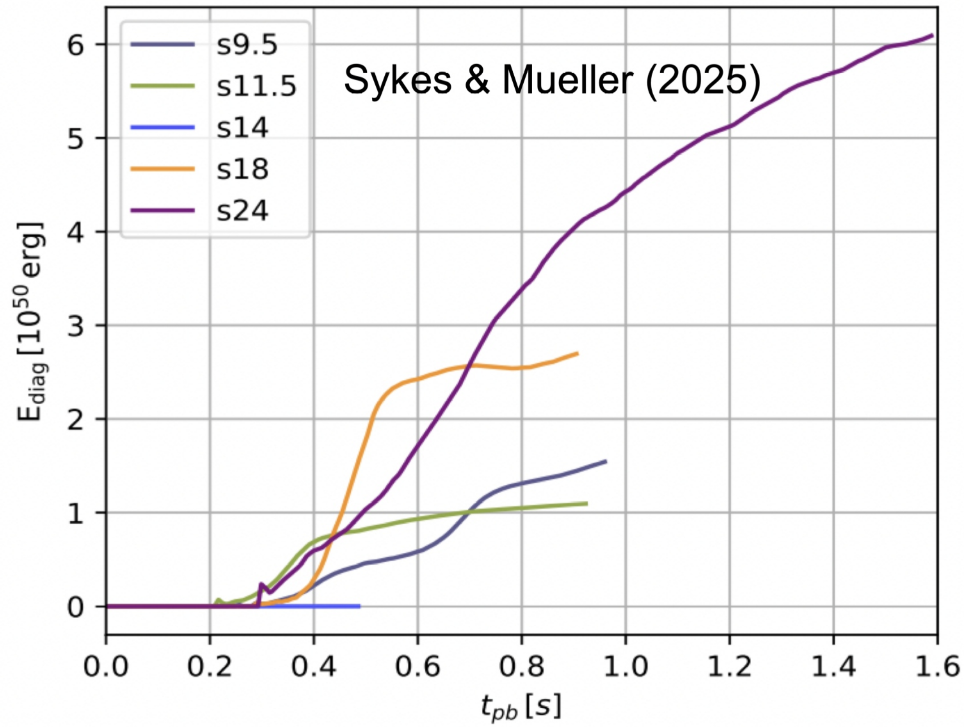


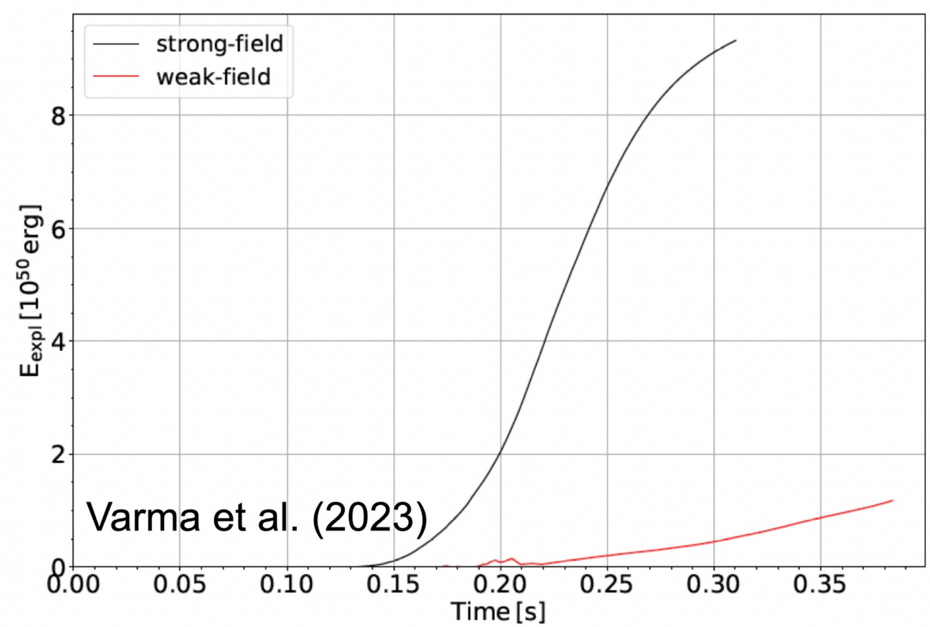
- Models explain correlation between progenitor mass and explosion energy
- Up to 10^{51}erg achievable for explosions from red supergiants (Bollig et al. 2021)
- Nickel masses ≤0.09M⊙ roughly compatible with observed range in SNe IIP
- Kicks up to ~1000km s^{-1} and spin periods between 1s and ~10ms as observed

Magnetic Fields in Neutrino-Driven Supernovae



Müller & Varma (2020), Similar results by Matsumoto, Takiwaki et al.

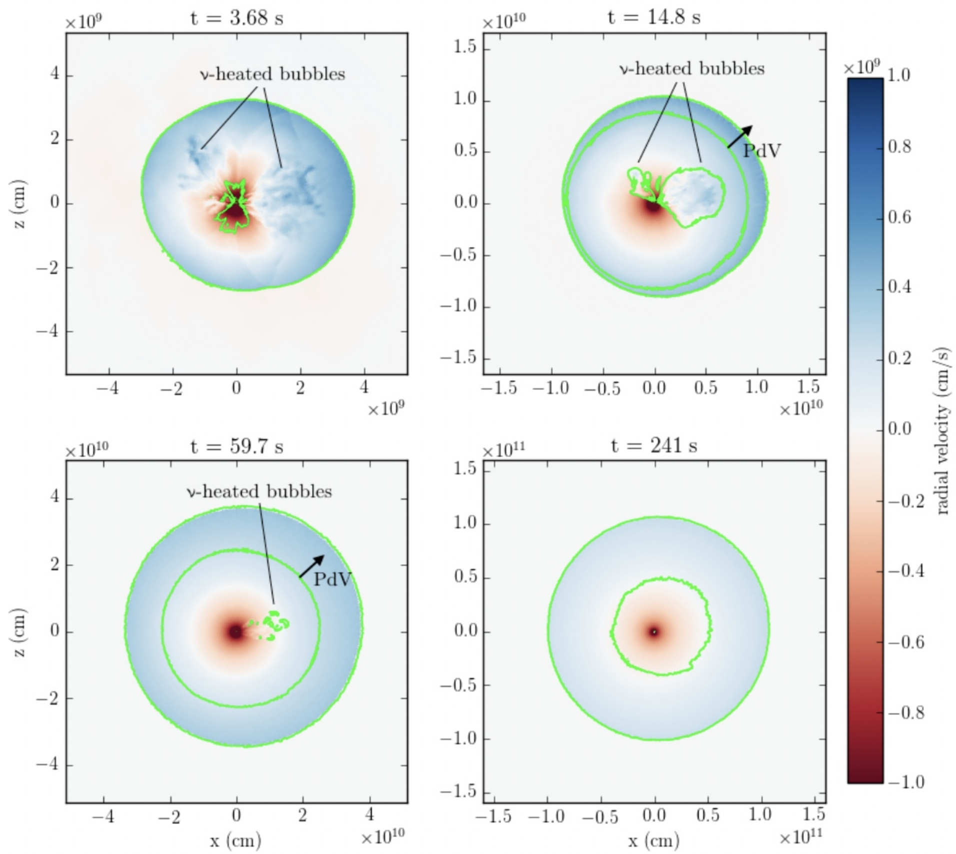


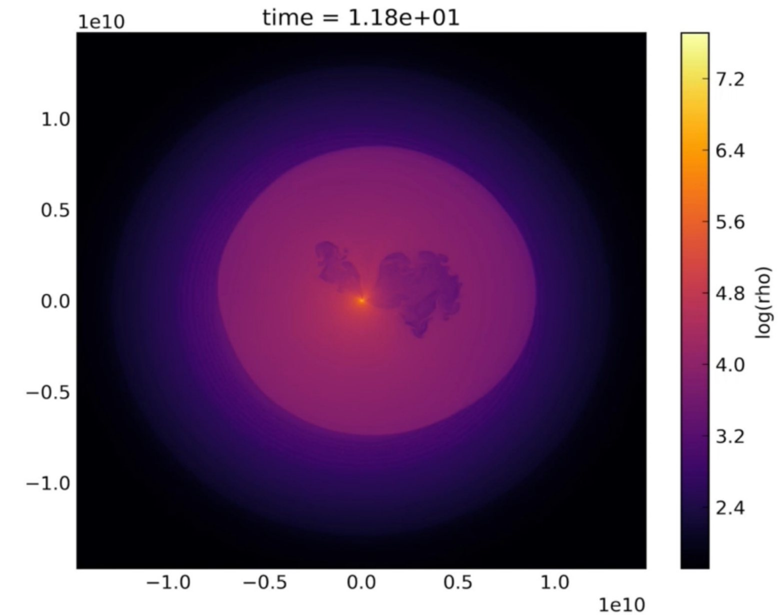


- Effect on shock revival: perhaps, small
- Effect on energetics: less likely
- Bigger effect for initial fields ~10^{12}G

Black-Hole Forming Explosions

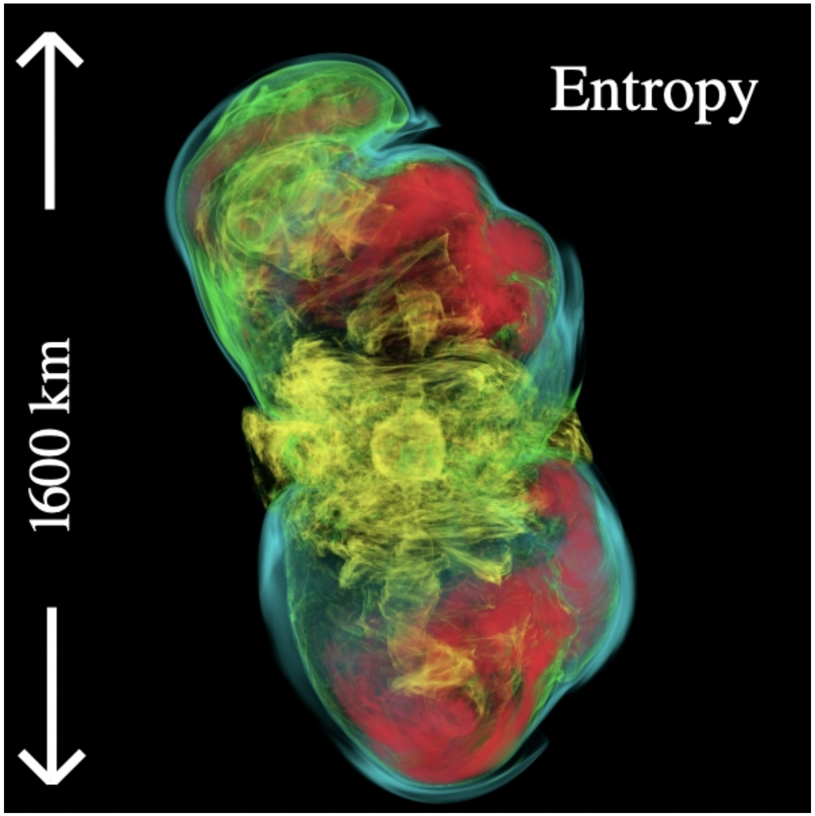
Chan et al. (2018)



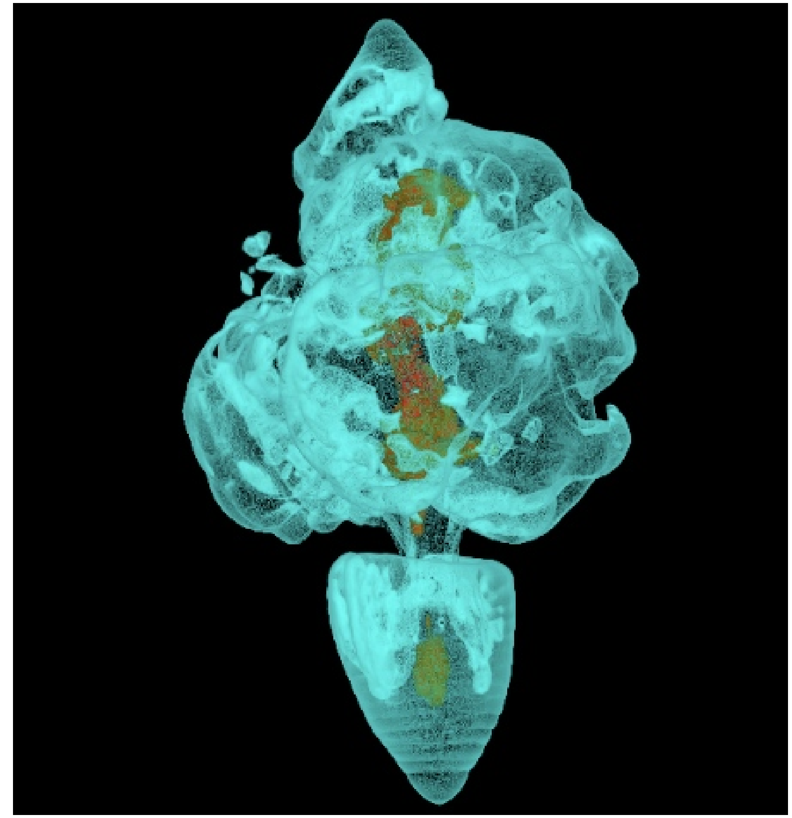


- Sufficiently developed explosions can avoid complete fallback after black hole formation and produce kicked and rotating BHs (Chan et al. ‘18, ‘20, Rahman et al. ‘22, Janka & Kresse ‘24, Burrows et al ’25)
  → Important for LIGO systems – black holes in former “mass gap”
- Sensitivity to equation-of-state physics and neutrino transport must be better explored
- Implications for multi-messenger astronomy will be challenging to model

Magnetorotational Explosions

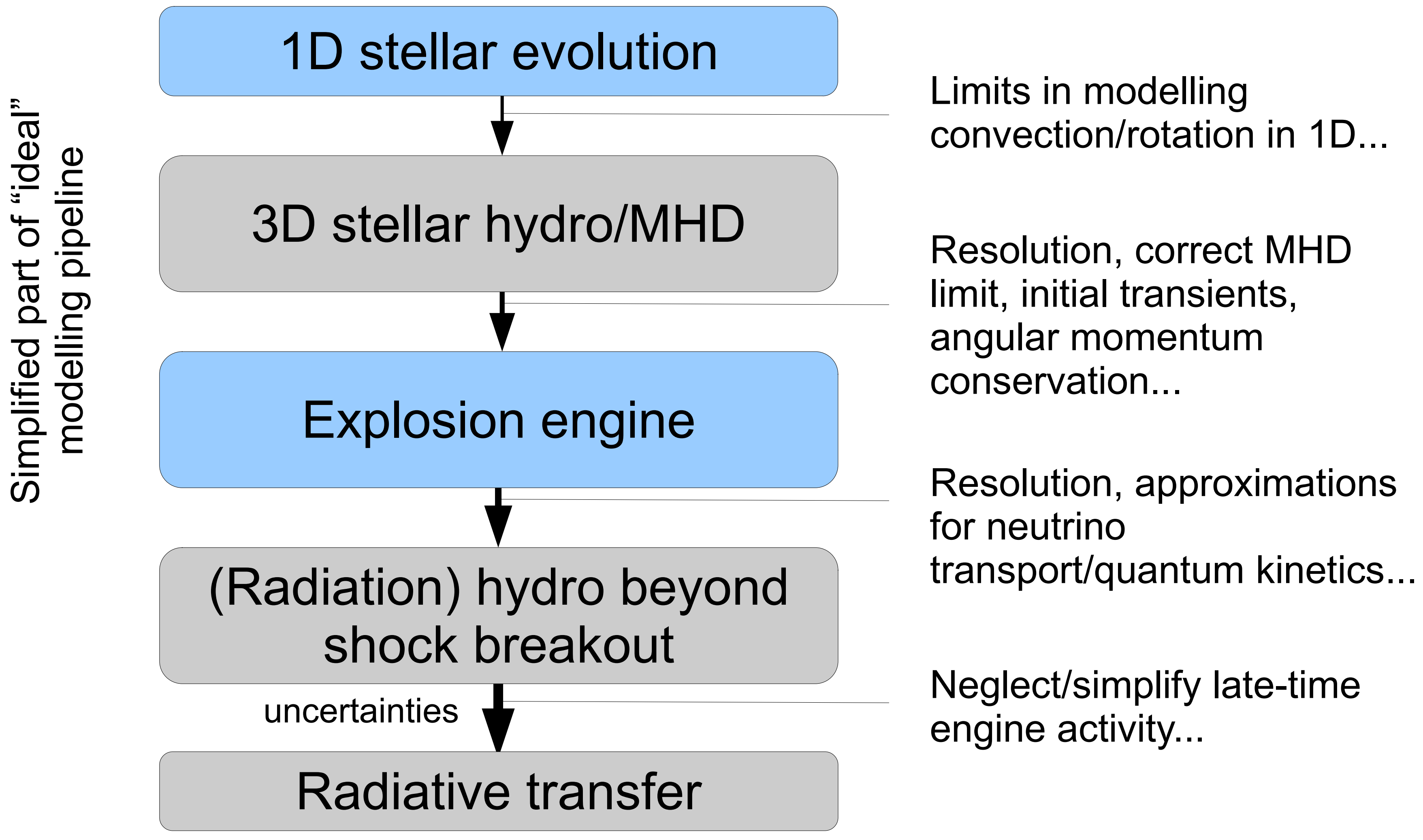


Moesta et al. ‘14


Powell et al. ‘23

- Many 3D simulations now available
- Differences in outcomes despite similarities (r-process vs. no r-process...)
- **Uncertainties in progenitor structure remain *critical***
- Code comparisons being performed - but this process is *tedious* in 2D/3D!

Pipeline Losses



- Development and automation of pipeline tools is **not** complete
- **Accumulation of uncertainties** limits first-principle approach, especially with current tools
- Long-range development must not be neglected
- Replication, model ensembles, integrated teams, and a continuum from high-end simulations to reduced models become more important
  → Requires adaptions to organisation and publication culture

Conclusions

- Many of the *components* of the physics behind shock revival and the explosion dynamics are well understood.
- 3D explosion models are now routine for many groups, but this does not mean the problem is solved.
- Many uncertainties (progenitor structure...) and hidden assumptions (transport treatment...) are baked into the multi-physics supernova problem.
- Important to consider how supernova modelling needs to evolve technically and organisationally to make further progress and aid gravitational wave and multi-messenger astronomy.