UNIVERSIDAD DE GUADALAJARA

CENTRO UNIVERSOTARIO DE CIENCIAS EXACTAS E INGENIERÍAS

DEPARTAMENTO DE FÍSICA

PARAMETER ESTIMATION FROM THE CORE-BOUNCE PHASE OF ROTATING CORE COLLAPSE SUPERNOVAE IN REAL INTERFEROMETER NOISE

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1rst IGWN Symposium CCSN GW

Motivation:

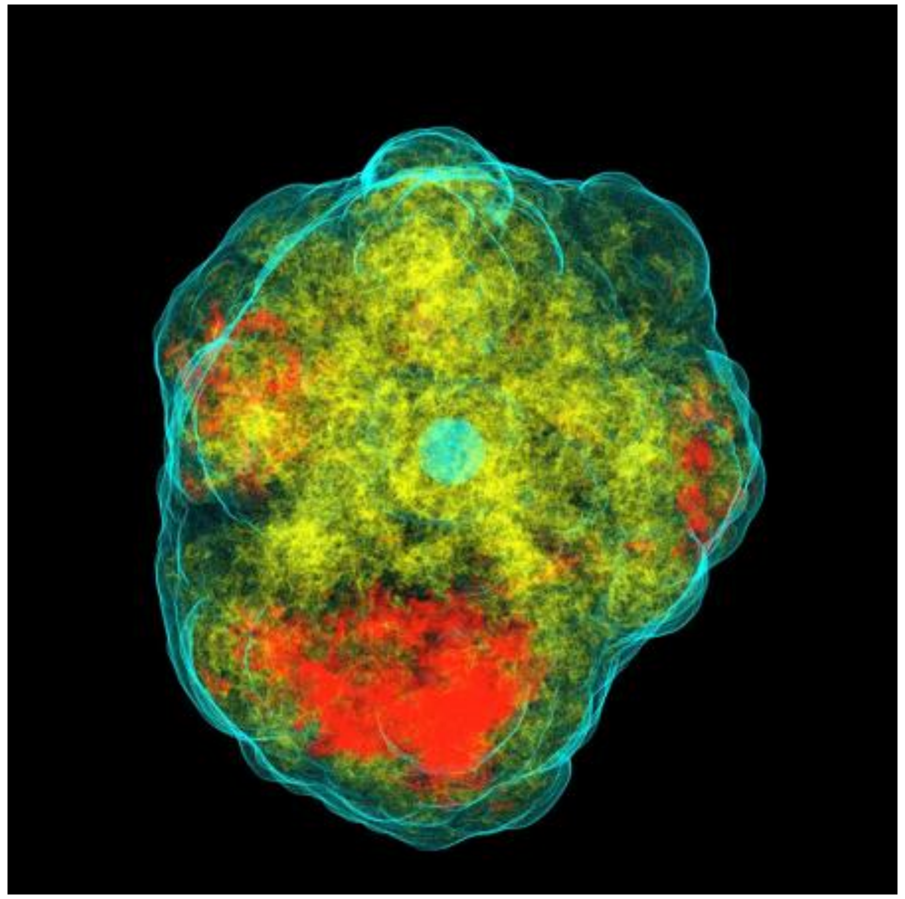
We propose an analytical model based on simulation for rotating core collapse supernovae from Richers´s catolog to:

- To propouse a parameter estimation methodology for rotating core collapse supernovae

- Analyse gravitational signals inmerse in simulate noise data.

- Compute theoretical minima in the error of the ratio β between rotational kinetic to potential energy

L. O. Villegas *et al* 2025 *Class. Quantum Grav.* **42** 115001



Christian Ott/Caltech et. al.

Differential rotation at Core-Bounce

- There is a dependence on the degree of differential rotation with the ratio of rotational kinetic energy and potential energy

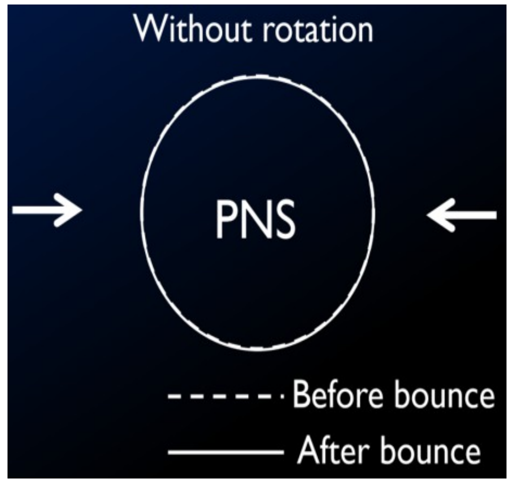
$$\beta = \frac{T}{|W|} = \frac{I\,\Omega^2}{2|W|}$$

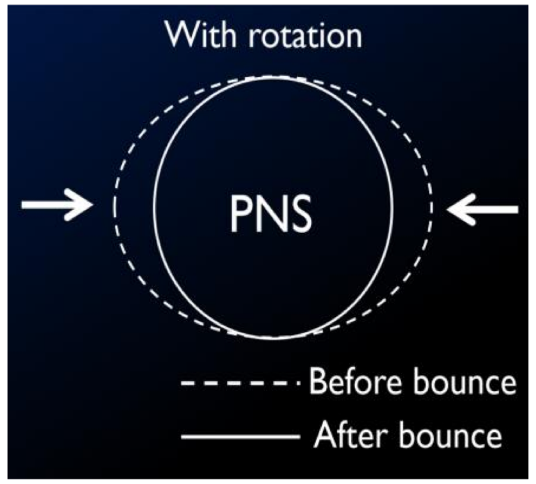
Rotation profile (before collapse)

$$\Omega(\varpi) = \Omega_0 \left[1 + \left(\frac{\varpi}{A}\right)^2\right]^{-1} ,$$

- Ω_0 maximum initial rotation rate
- ϖ is the distance from the axis of rotation
- A is a measure of the degree of differential rotation

Richers, S. et.al. 2017





To illustrate the outcomes derived from the estimation of the parameters β, we show three different scenarios:

1) β < 0.08 slow rotation

A1268w2.00_GShenFSU2.1 (β = 0.067)

2) 0.08 < β < 0.12 rapid rotation

A300w6.00_BHBLP (β = 0.083)

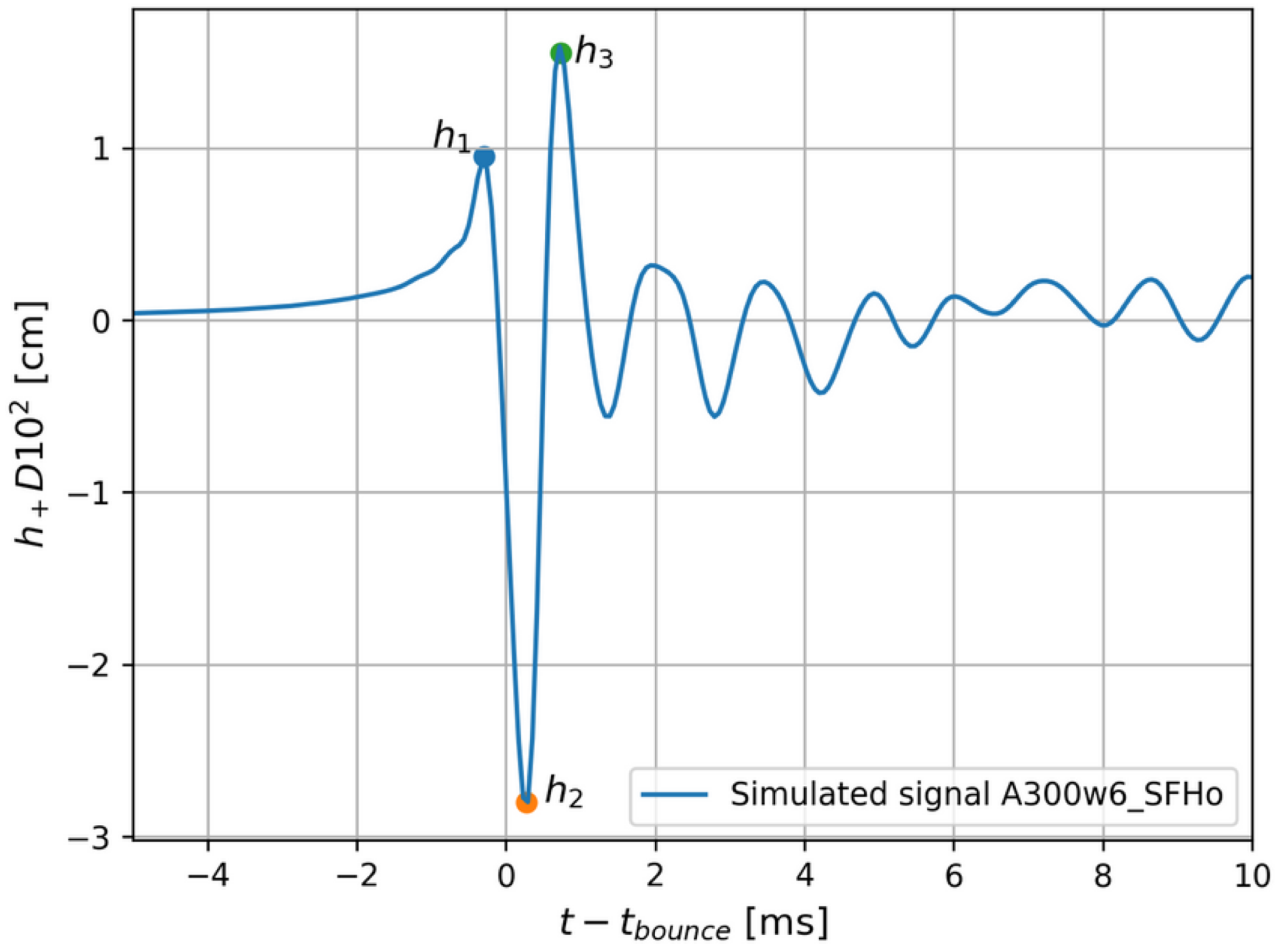
A634w6.00_SFHx (β = 0.108)

3) β > 0.12 extremely rapid rotation

1) Gravitational waves for rapid rotating core collapse supernovae analysis

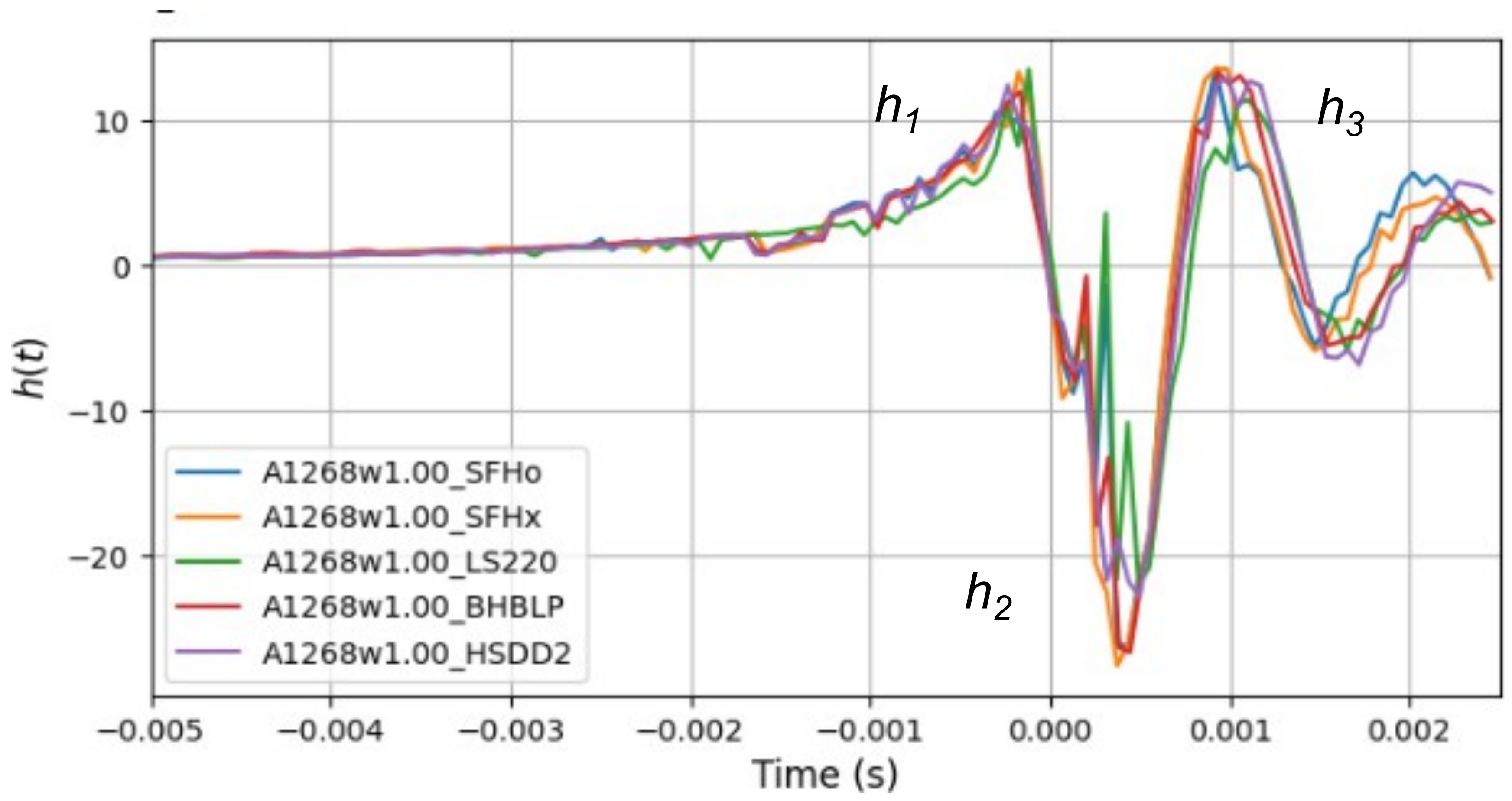
Richers´s catalog

- 1824 axisymmetric general-relativistic hydrodynamics simulations
- 2D simulations
- 12 M⊙ progenitor
- 98 different rotation profiles
- 18 different EOS
- 10kpc distance



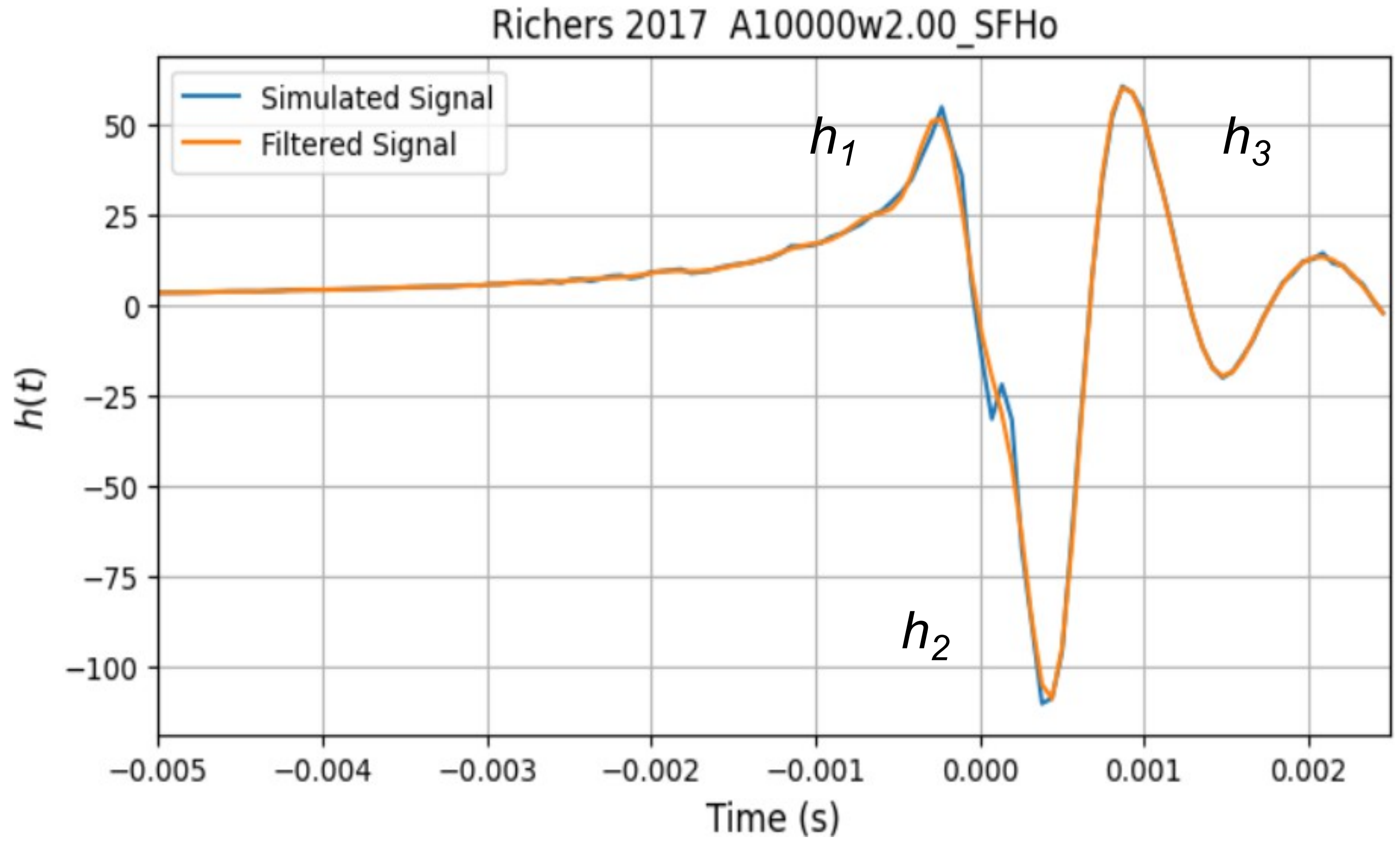
Richers, S. et.al. 2017

Waveforms Analysis



126 waveforms
16 rotation profiles
5 state equations

Waveforms with low pass filter

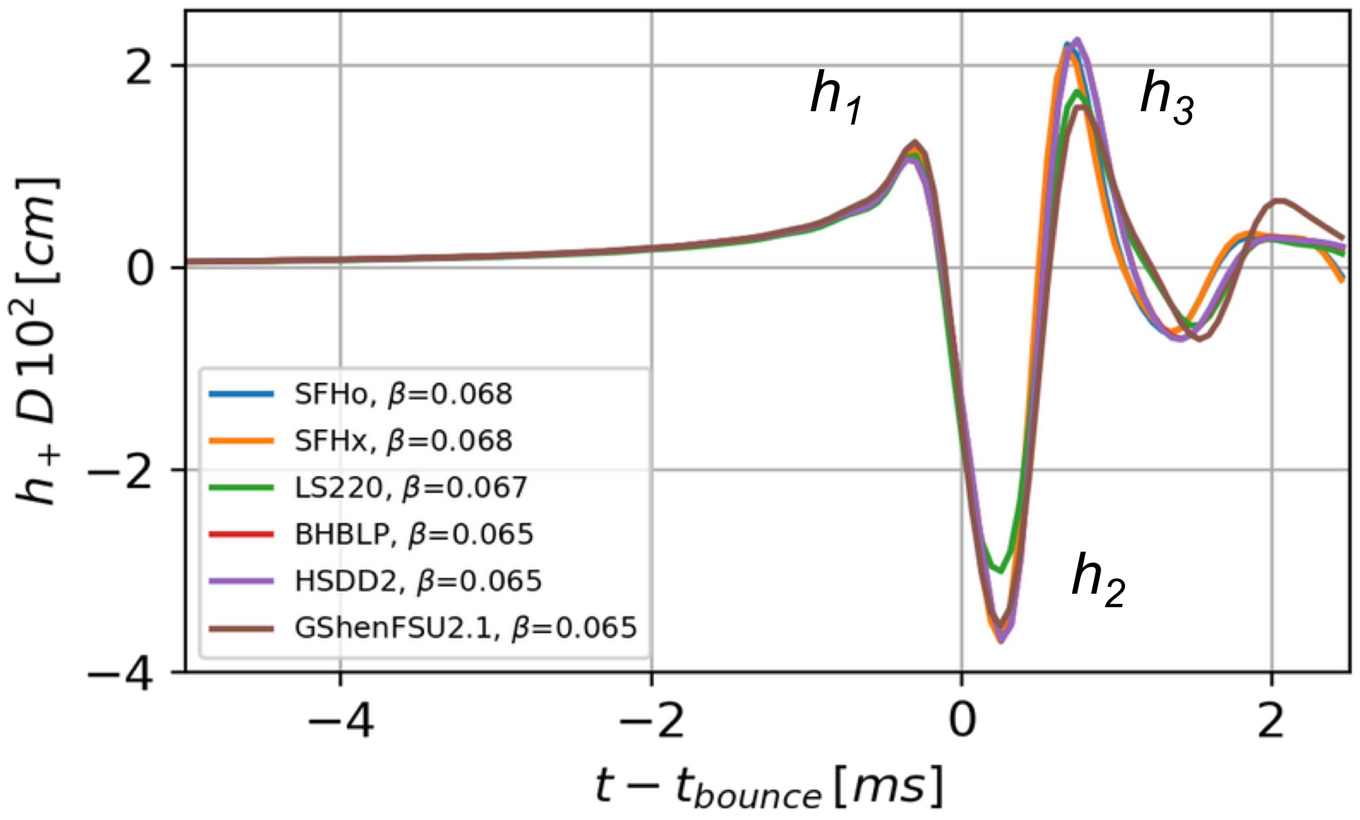


Models considered

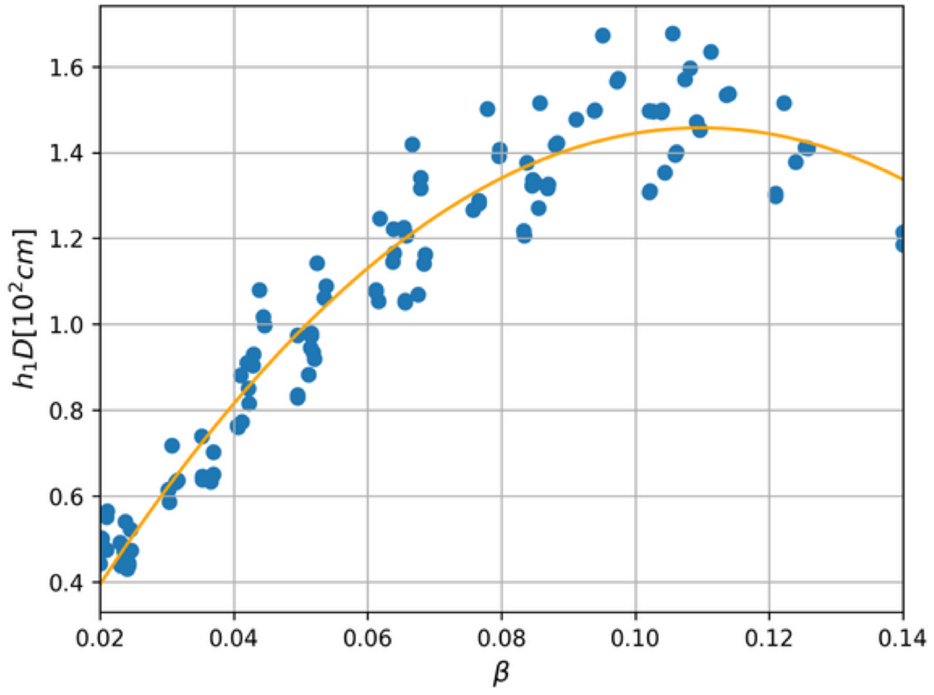
Name	A (km)	Ω_0 (rad s^{-1})
A1	300	3.0–11.0
A2	467	3.0–6.0
A3	634	2.0–6.0
A4	1268	1.0–5.0
A5	10 000	1.0–3.0

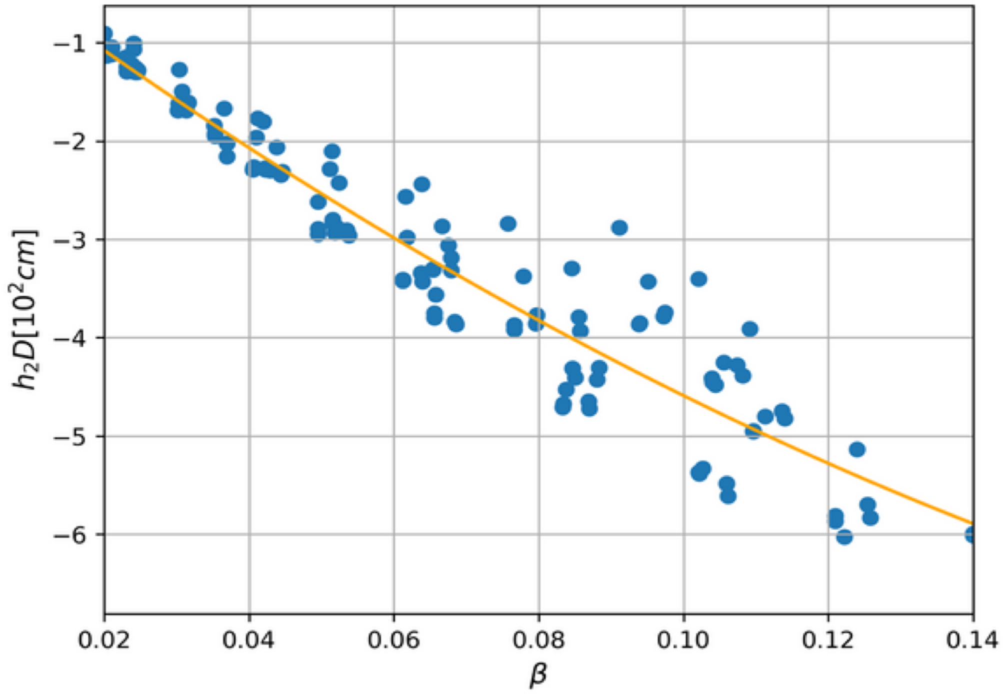
Signal analysis

For the amplitude of each peak of the core-bounce $h_1(\beta)$, $h_2(\beta)$ and $h_3(\beta,\alpha)$ we use the relationship that exists between them and the parameters β and α.



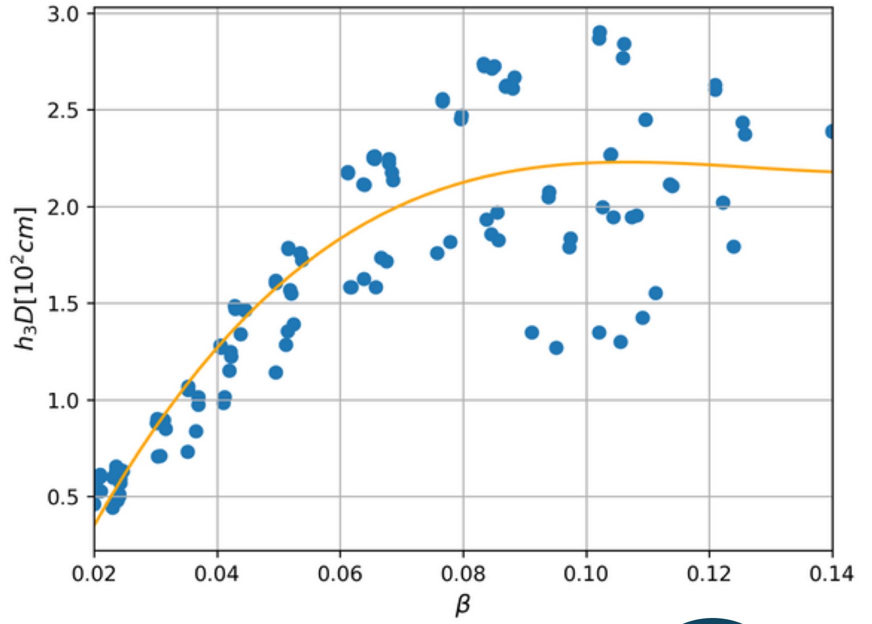
$$h_1(\beta) = -13.2 + 2.89 \times 10^3 \beta - 1.31 \times 10^4 \beta^2$$



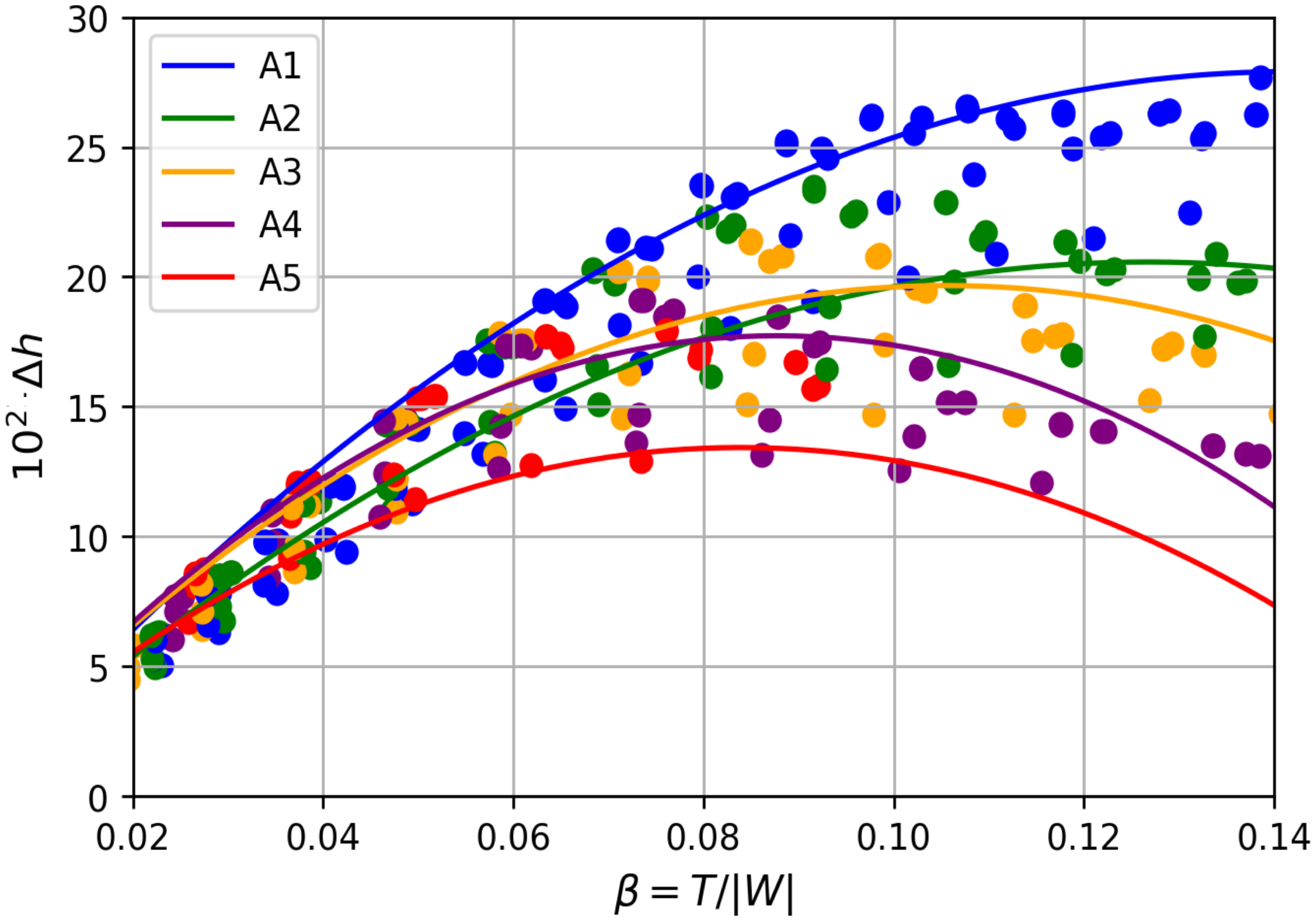


$$h_2(\beta) = -1.03 - 5.52 \times 10^3 \beta + 9.43 \times 10^3 \beta^2$$

$$h_3(\alpha, \beta) = 17.20 + \alpha \left(\frac{\beta}{0.06} \right)^2$$



For $h_3(\alpha, \beta)$ we consider the difference of amplitude when we have different EOS



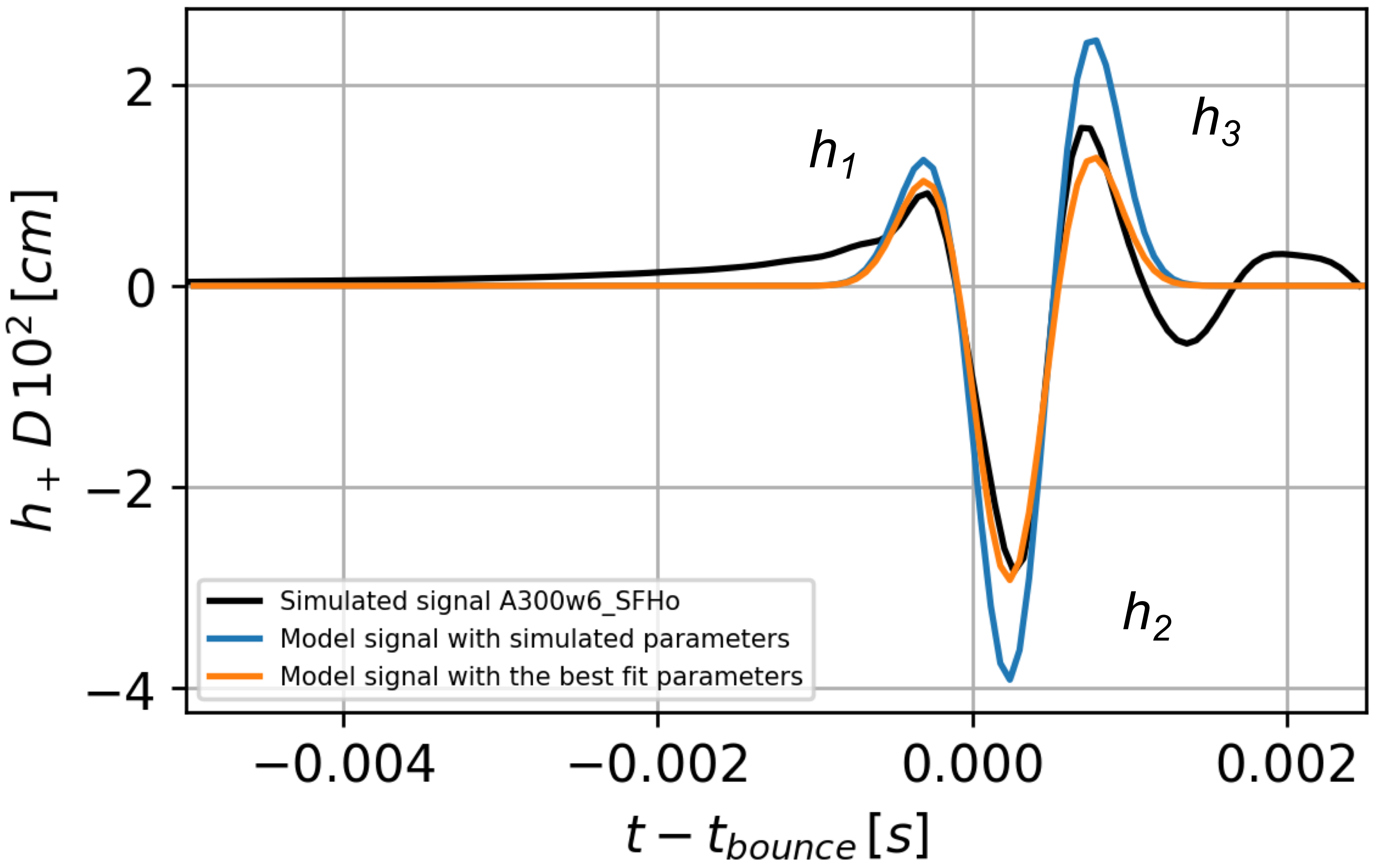
$$h_3(\alpha, \beta) = 17.20 + \alpha \left(\frac{\beta}{0.06} \right)^2$$

Scaling the parameter h
in terms of the three peaks,
we have the observable metric

$\Delta h = \max(h_1, h_3) - h_2$

2) Analytical model

$$h(t) = h_1(\beta)\exp\left[-\frac{(t-\tau)^2}{2\eta^2}\right] + h_2(\beta)\exp\left[-\frac{(t-\tau_a)^2}{2\eta^2}\right] + h_3(\alpha,\beta)\exp\left[-\frac{(t-\tau_b)^2}{2\eta^2}\right]$$



η = 0.2 ms

$\alpha = [30, 380]$

$\beta = [0.02, 0.14]$

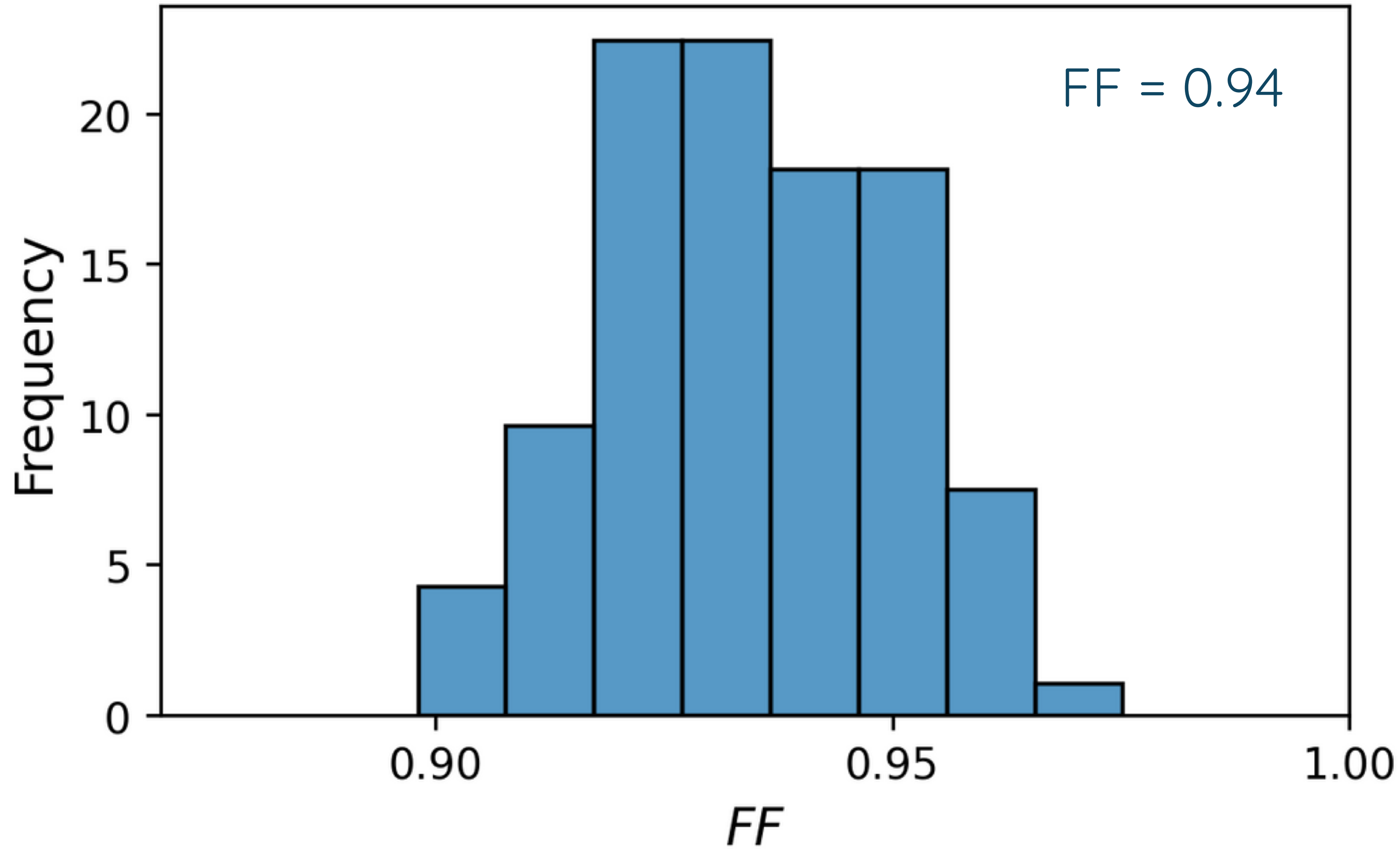
Arrival time of the peaks

$\tau_a = \tau + 0.5$ ms

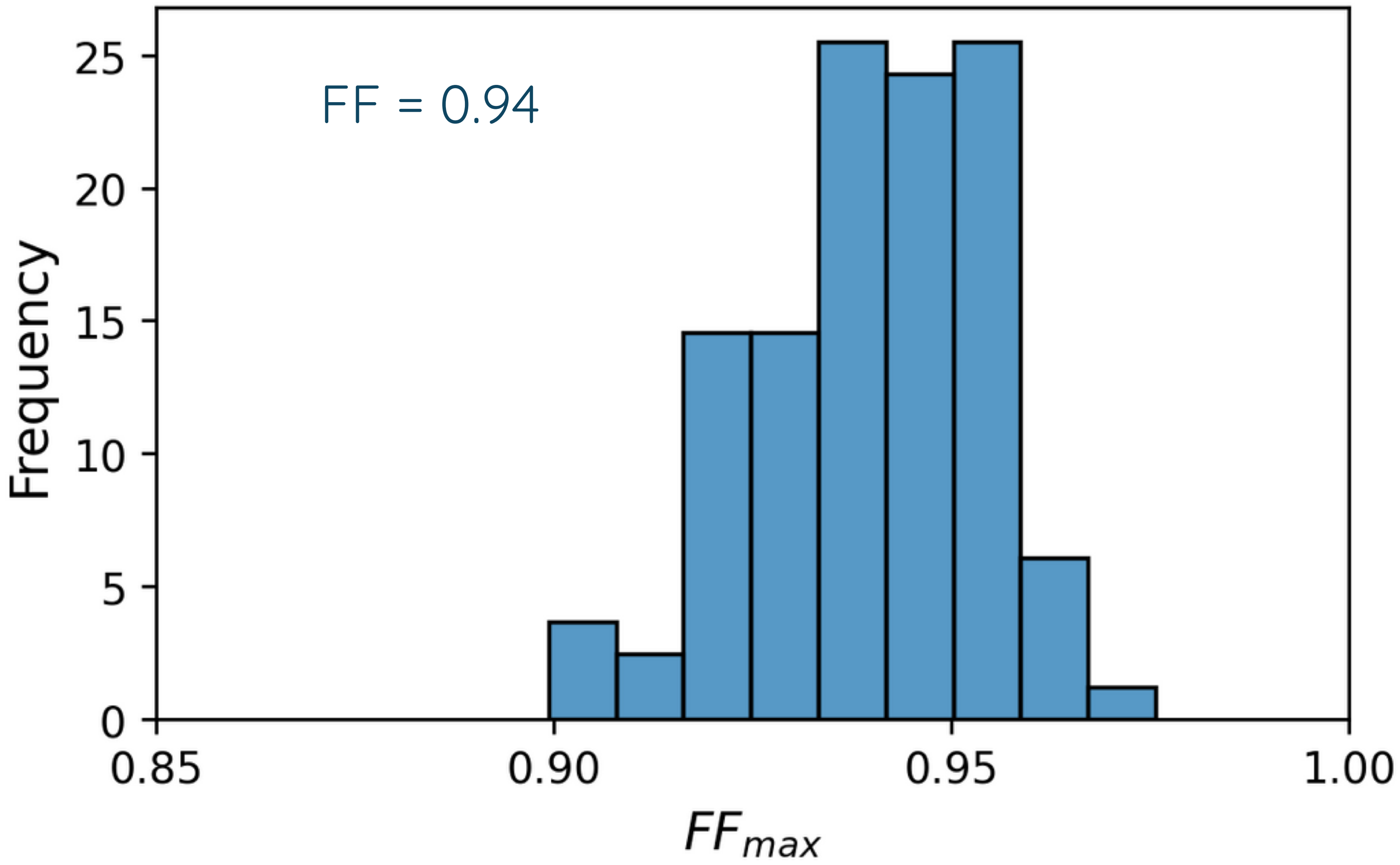
$\tau_b = \tau + 1.0$ ms

Fitting Factor (FF)

For analyze the similarity between analytical and numerical signals, we use FF technique



We compared the simulated waveforms with its corresponding analytical waveform using real value of β and the best fitting for α

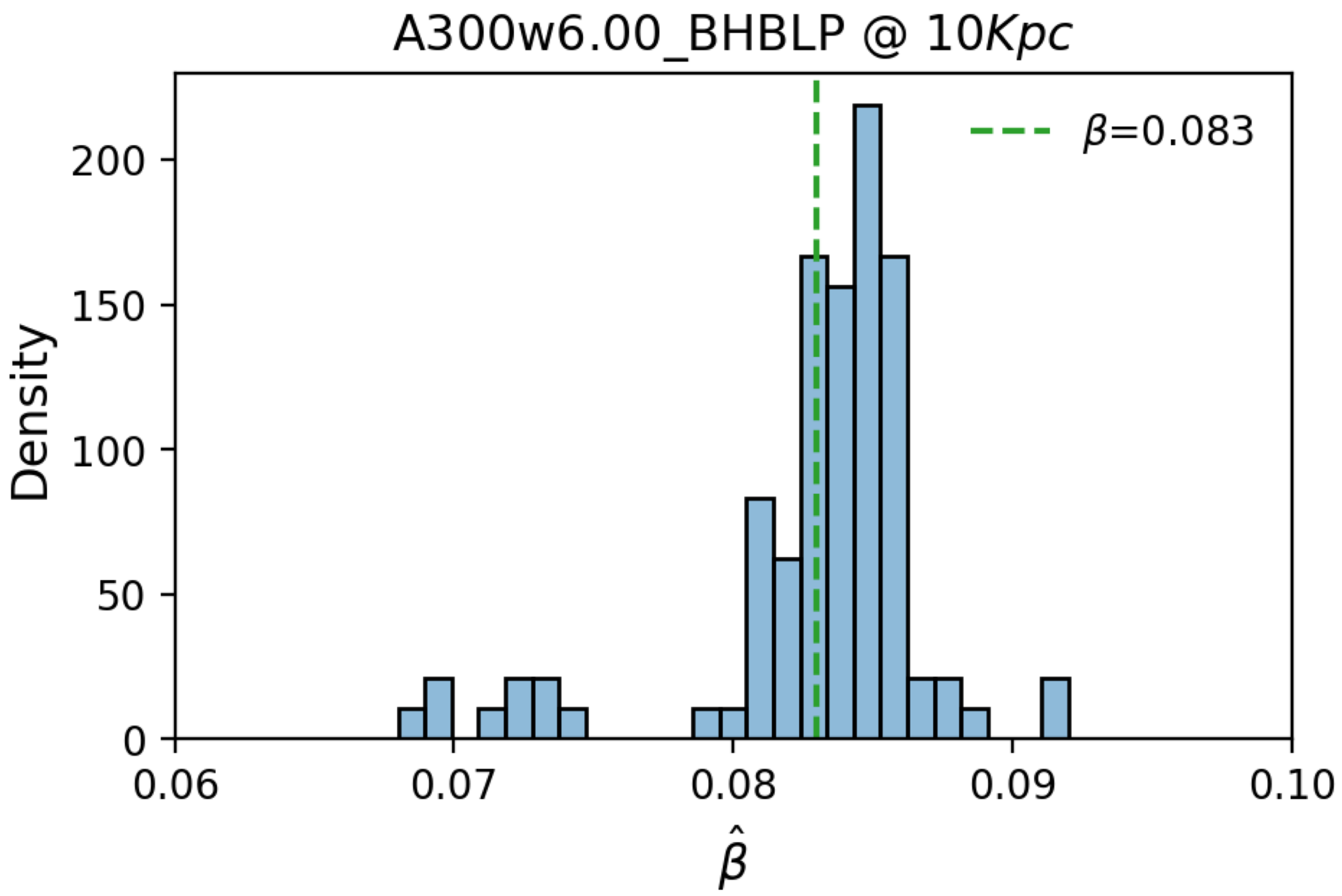


Histogram of the best fitting factor with the combination of different parameters

3) Matched Filter (MF)

We add noise in the numerical signal and estimate β

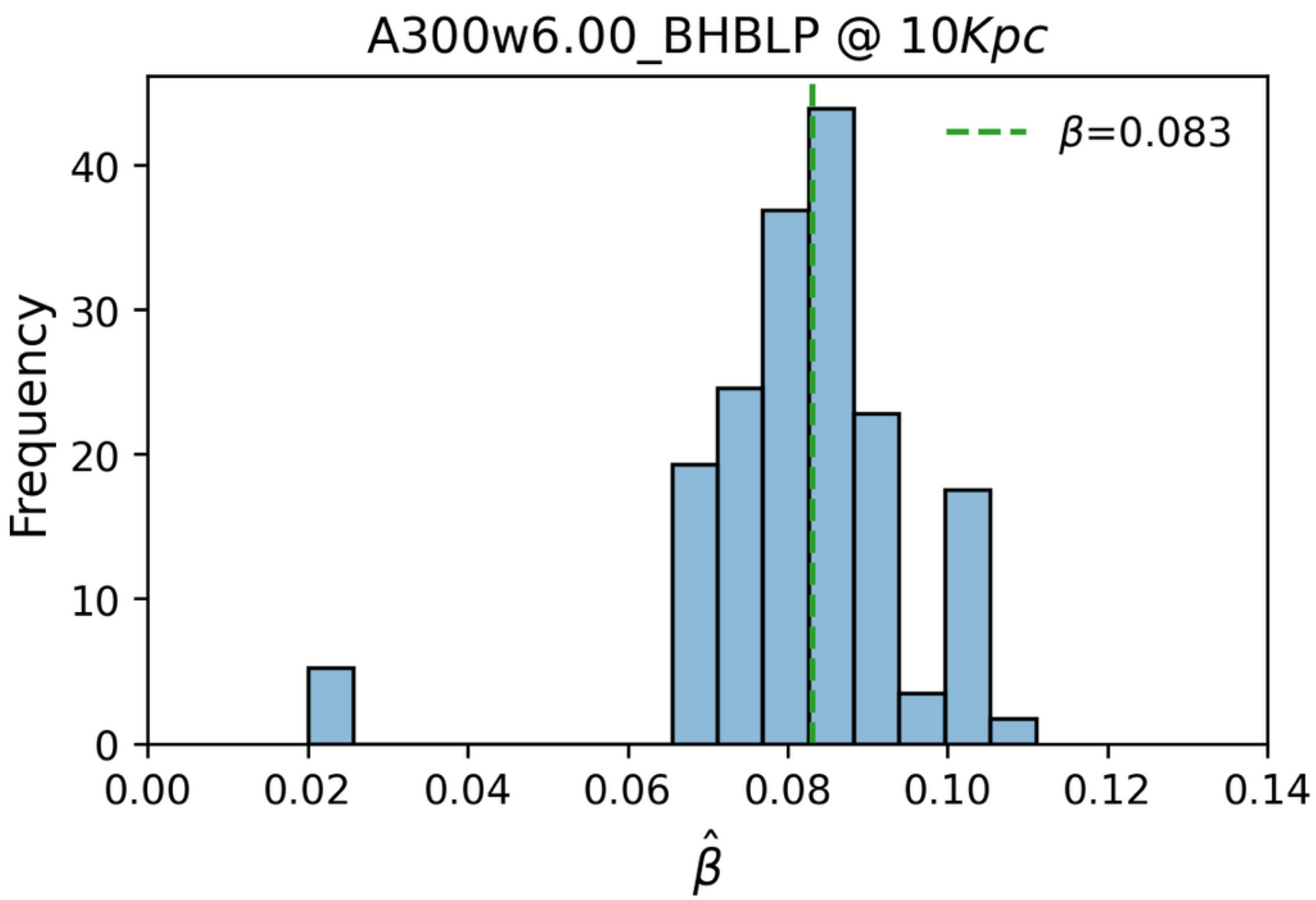
Colored simulated gaussian noise



$\hat{\beta}$ = 0.083

σ = 3.43 × 10^{-3}

Real Noise LIGO O3



$\hat{\beta}$ = 0.083

σ = 1.46 × 10^{-2}

Asymptotic expansion for the error estimation

Cramer-Rao Lower Bound (CRLB):
Expresses a lower bound on the covariance of the unbiased estimators of a parameter:

$$C_{\hat{\boldsymbol{\theta}}} = \boldsymbol{I}^{-1}(\boldsymbol{\theta})$$

Fisher Matrix information:
It is a way of measuring the amount of information that an observable random variable carries about an unknown parameter,

$$I_{ij} = -E\left[\frac{\partial^2 \ell\,(\boldsymbol{x};\boldsymbol{\theta})}{\partial\theta_i\theta_j}\right]$$

$$I_{ij} = \langle h_i(f), h_j(f)\rangle = 4\,R\int \frac{h_i(f)h_j^*(f)}{S_h(f)}df$$

where $S_h(f)$ is the Power Spectral Density (noise)

Covariance asymptotic expansion:
Asymptotic expansion of the variance in terms of the inverse of the SNR

$$\sigma^2(\hat{\theta}) = \frac{C_1^2(\hat{\theta})}{SNR} + \frac{C_2^2(\hat{\theta})}{SNR} + \dots$$

We define the first-order variance,

$$\sigma_1^2(\hat{\theta}) = \frac{C_1^2(\hat{\theta})}{SNR} = I^{-1}(\hat{\theta})$$

For the second order variance we use the expression:

$$\sigma_2^2(\hat{\theta}) = \frac{C_1^2(\hat{\theta})}{SNR} = I^{jm}I^{jn}I^{pq}\left(v_{nmpq} + 3\langle h_{nq}, h_{pm}\rangle + 2v_{nmp,q} + v_{mpq,n}\right)$$
$$+ I^{jm}I^{jn}I^{pz}I^{qt}\Big(v_{npm}v_{qzt} + \frac{5}{2}v_{npq}v_{mzt} + 2v_{qz,n}v_{mtp} + 2v_{qp,z}v_{nmt}$$
$$+ 6v_{mqp}v_{nt,z} + v_{pqz}v_{nt,m} + 2v_{mq,z}v_{pt,n} + 2v_{pt,z}v_{mq,n}{}' + v_{mz,t}\,v_{nq,p}\Big)$$

Zanolin, M et.al. 2010

Relative error estimating the parameter β

The estimated error can be found with the covariance

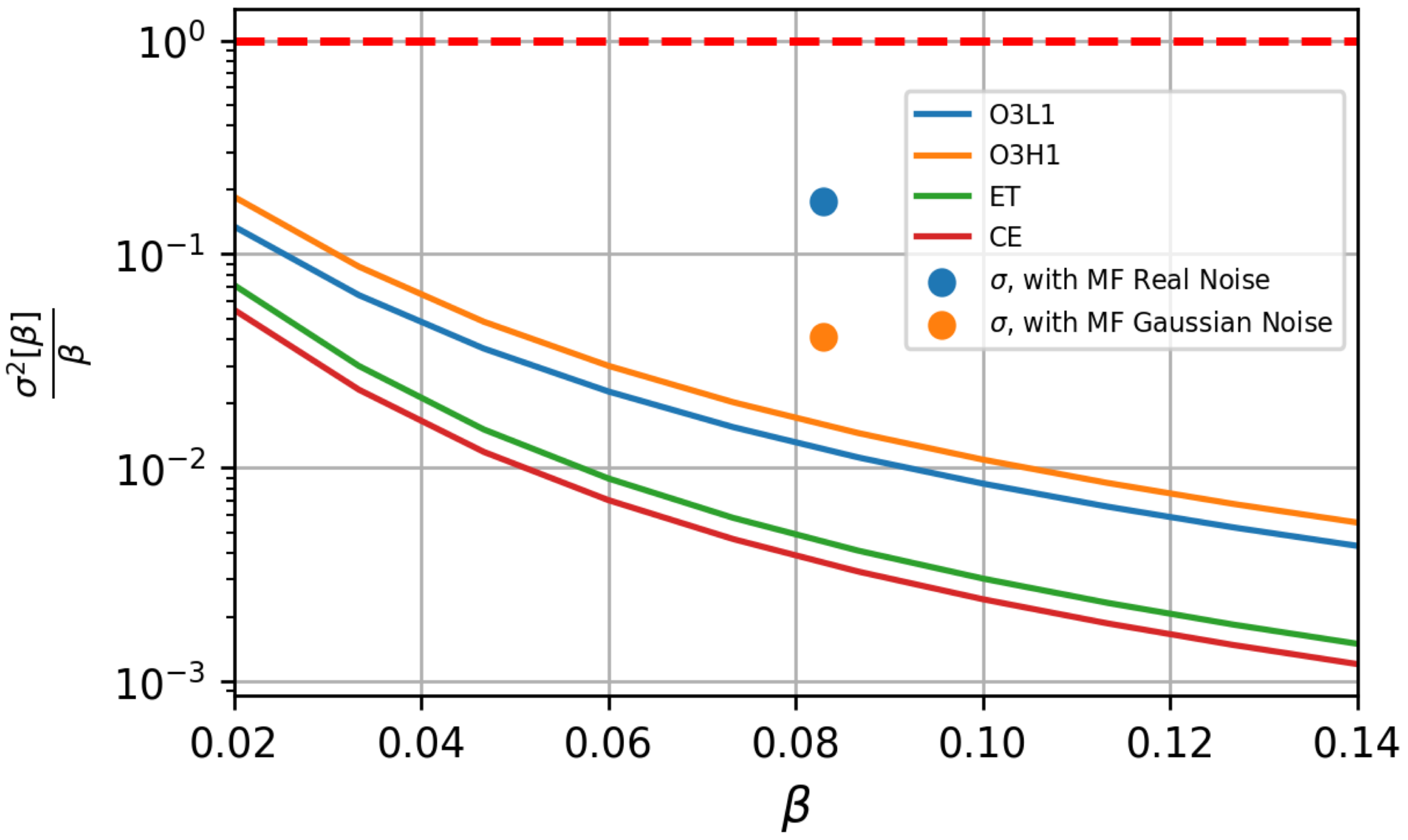
$$\sigma^2(\hat{\theta}) = \frac{C_1^2(\hat{\theta})}{SNR} + \frac{C_2^2(\hat{\theta})}{SNR} + \dots$$

The ratio between the covariance and the rotation parameter, give us the relative error of

$$Error = \frac{\sqrt{\sigma_1^2[\beta] + \sigma_2^2[\beta]}}{\beta}$$

In the plot, we show the relative error considering simulated data from ET, CE detectors and real data in O3 LIGO run.

Comparison of theoretical estimates with the minimum error of parameter estimation derived from asymptotic expansion. The relative error is around 10%.



Vitale. V. et.al. 2007

Conclusions:

- Using the catalogue of Richers, S. et.al. 2017 ($0.02 < \beta < 0.14$), we find and analytical expression that allows to approximate a gravitational wave form for the core bounce phase of rapidly rotation CCSN,

- Using Matched filter technique, we add Gaussian noise and real LIGO O3 noise in the numerical waveform to estimate the β parameter.

- Using CRLB we calculated the error of the β parameter, and we find that its is around 10% which are a very good approximation.

- **We are using now the more recent and improved GW signals from Abdikamalov et. al. 2025. Watch Emmanuel Avila's presentation, July 25, 3:50 PM.**

Thank you