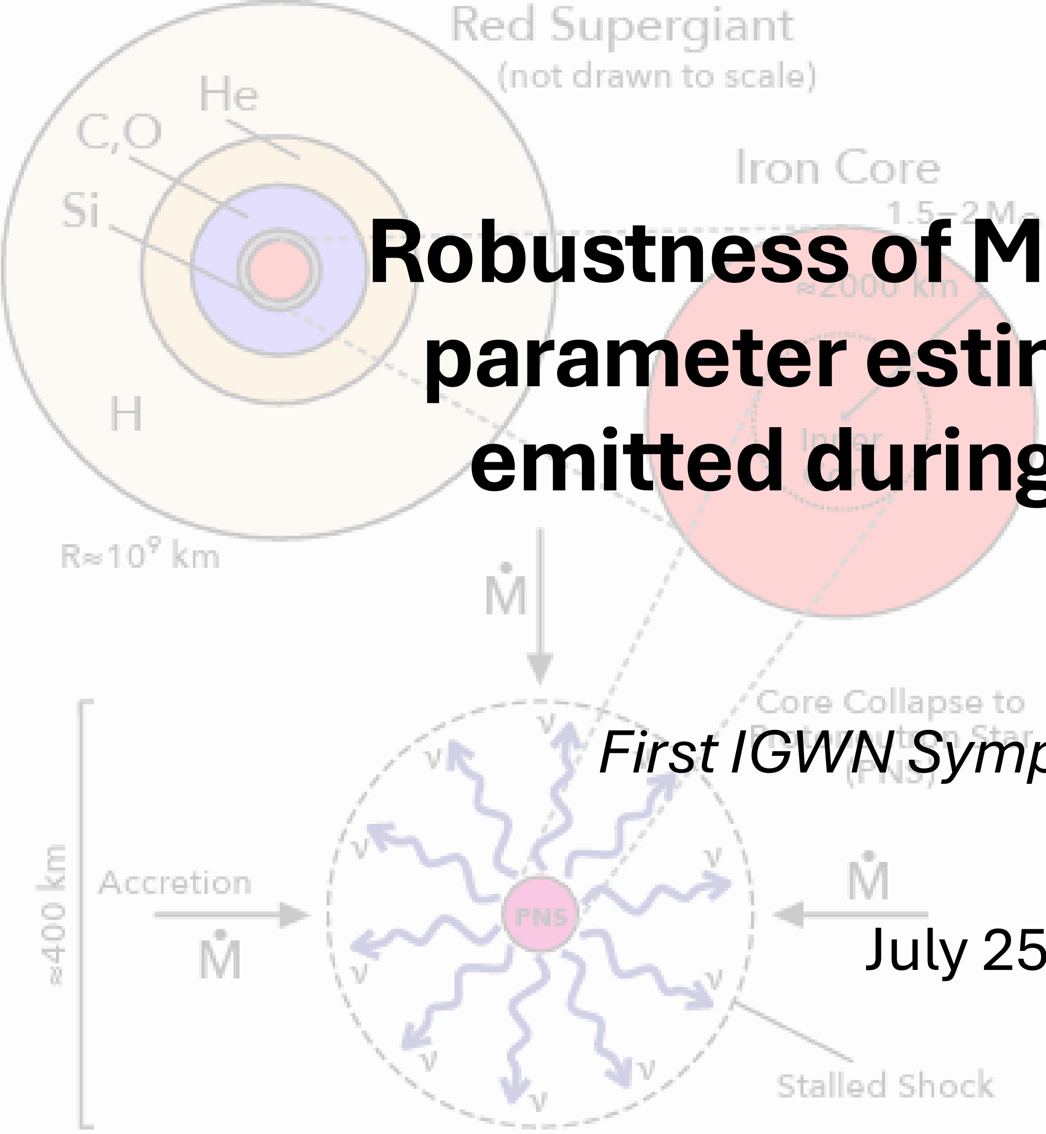

Robustness of Markov Chain – Monte Carlo in parameter estimation of gravitational waves emitted during Core-Bounce phase of Core Collapse Supernovae.

First IGWN Symposium on CCSN GW Theory and Detection

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Image Credit: (Ott,2016)

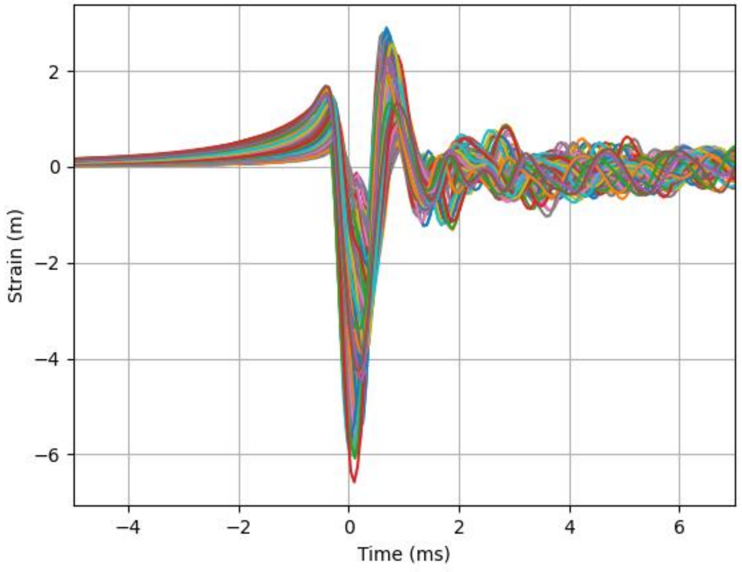
Outline

1. Motivation.
2. Core Bounce Signal Analytical Model (CBS).
3. Parameter Estimation (PE) with Gaussian colored Noise.
4. PE with O3 noise.
5. Prior sensitivity with Gaussian colored Noise.
6. Relative probability a variation in the number of parameters in the Analytical Model.

Motivation

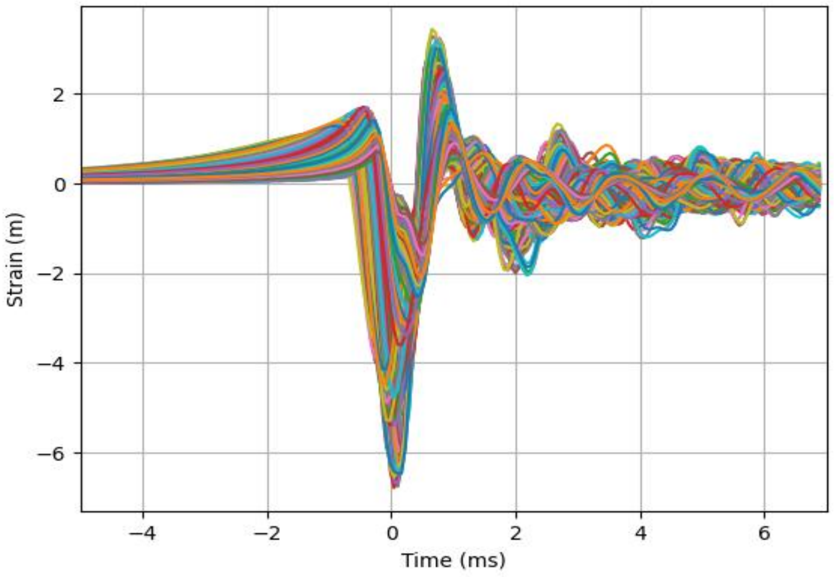
According to (Richers, 2017) and (Abylkairov, 2025) the Core Bounce phase of the GW produced by a RR CCSN can be templatable.

Richers Catalog



126 selected signals
and with a lowpass filter.

Abylkairov Catalog



452 Numerical Relativity
Signals without filter

In this study we use a phenomenological analytical model to perform parameter estimation for the parameter:

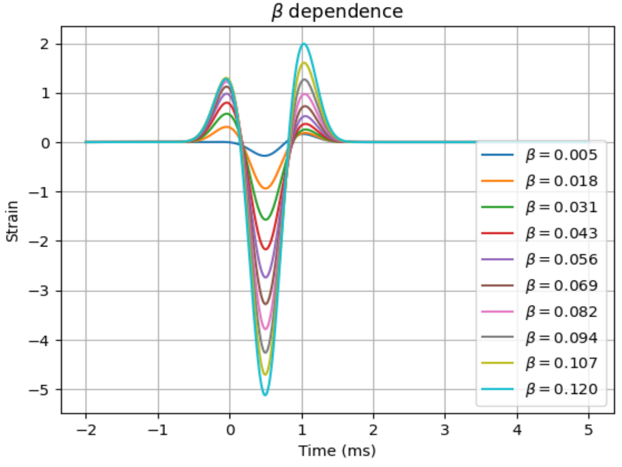
$$\beta = \frac{T}{|W|}$$

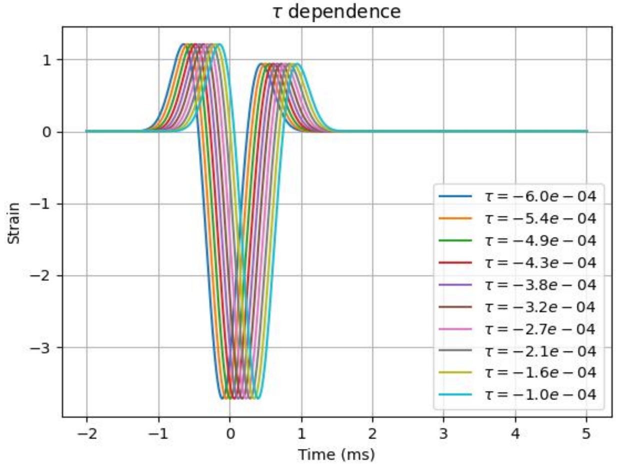
Core-Bounce Model

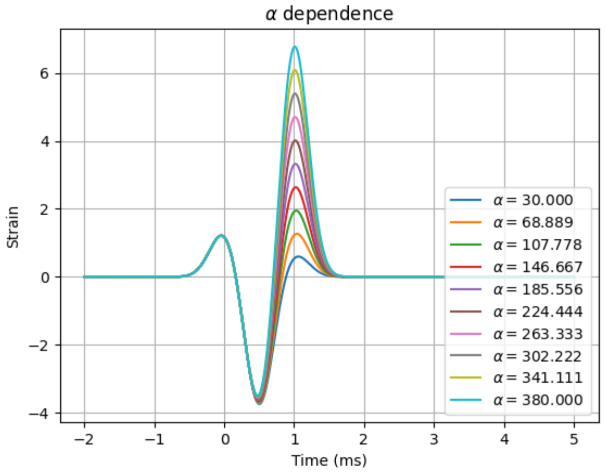
This model was proposed in (Villegas, 2024) with a matched filtering frequentist approach for PE, and keeping parameter '**s**' fixed.

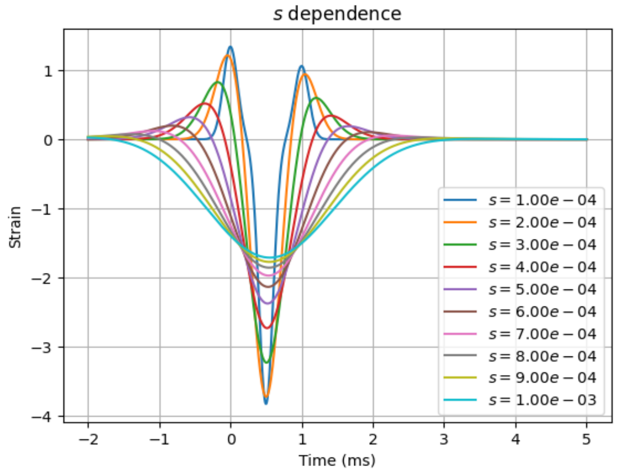
$$h_+(t;\beta,\tau,\alpha,s) = \sum_{i=1}^{2} h_i(\beta)e^{(t-\mu_i(\tau))^2/2s^2} + h_3(\alpha,\beta)e^{(t-\mu_3(\tau))^2/2s^2}$$

$$\begin{cases} \beta & : T/|W|, \\ \tau & : \text{Time of bounce}, \\ \alpha & : \text{Amplitude of third peak}, \\ s & : \text{Time scale of the bounce}. \end{cases}$$









Bayesian Inference with Markov Chain-Monte Carlo

$$d(t) = h(t;\theta) + n(t)$$

Data time series = signal + noise

Likelihood Function (Assuming Gaussianity)

$$\mathcal{L}(d|h(\theta)) \propto e^{-\langle d-h(\theta)|d-h(\theta)\rangle}$$

$$p(\theta|d) = \frac{\mathcal{L}(d|h(\theta))\pi(\theta)}{Z(d)}$$

Prior Distribution

Posterior Probability density Functions (PDF)

Marginalized Evidence of the observation

$$Z(d) = \int_{\Theta} \mathcal{L}(d|h(\theta))\pi(\theta)\, d\theta$$

Evaluation Metrics

Bayes Factor

$$B_N^M = \frac{Z_M}{Z_N}$$

Relative probability of occurrence between two hypothesis (models) that explain data.

Fitting Factor

$$FF = \frac{\langle h_i | h_j \rangle}{\sqrt{\langle h_i | h_i \rangle \langle h_j | h_j \rangle}}$$

Model complexity (Occam Factor)

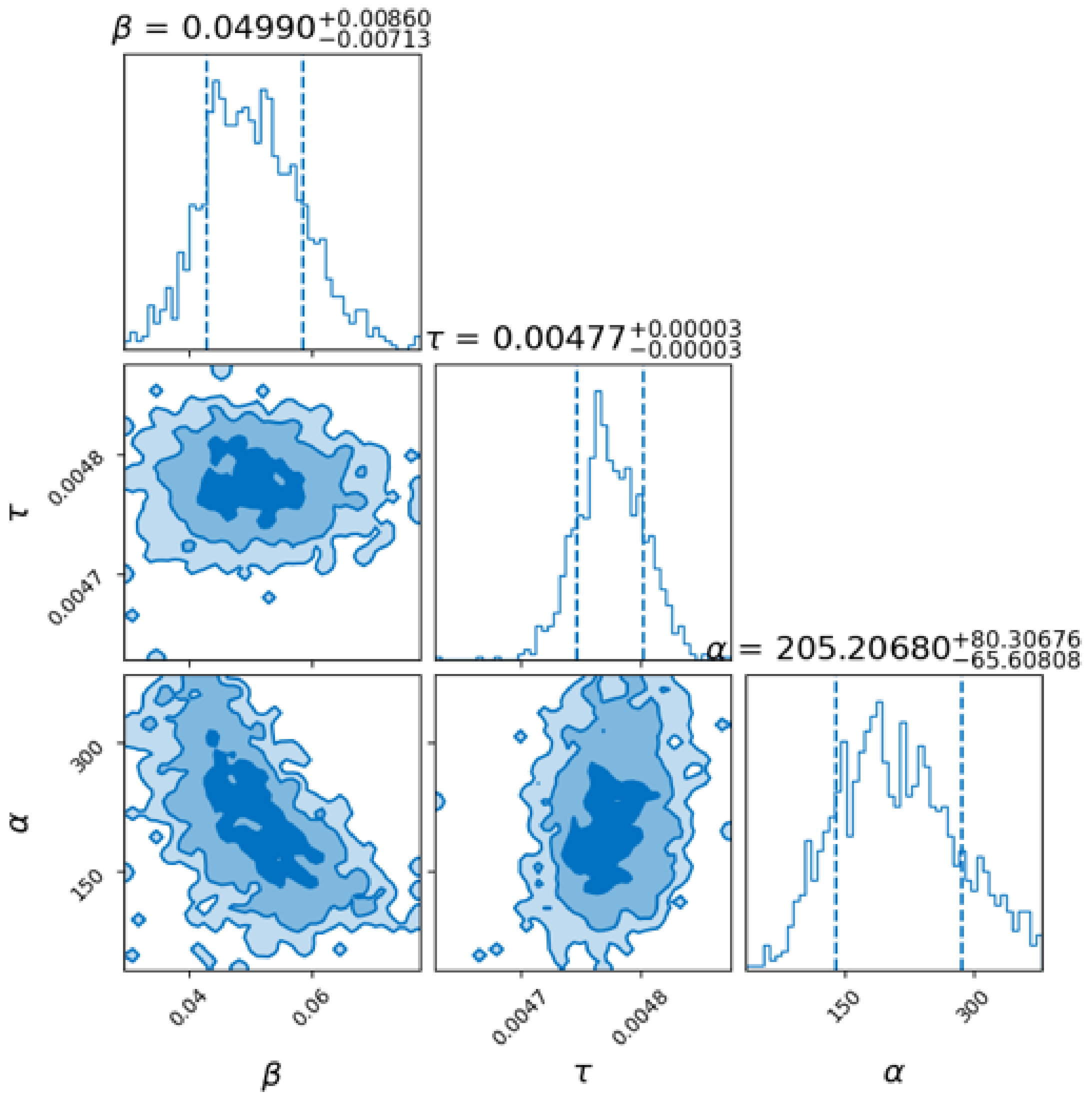
$$\mathcal{O} = \frac{\sigma_{p(\theta|D)}}{\sigma_{\pi(\theta)}}$$

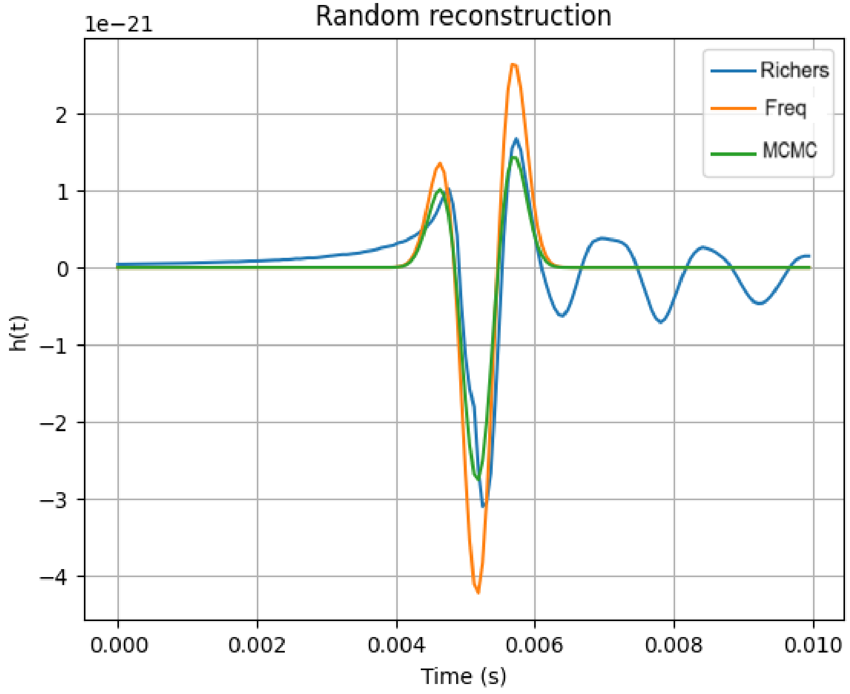
$\sigma_{p(\theta|D)}$ Standard deviation for posterior PDF.

$\sigma_{\pi(\theta)}$ Standard deviation for prior PDF.

Ratio of the spanned volume of priors and posterior PDF *(Mackay, 2003)*.

PE Richers Catalog in LIGO Gaussian Colored Noise (LGCL)

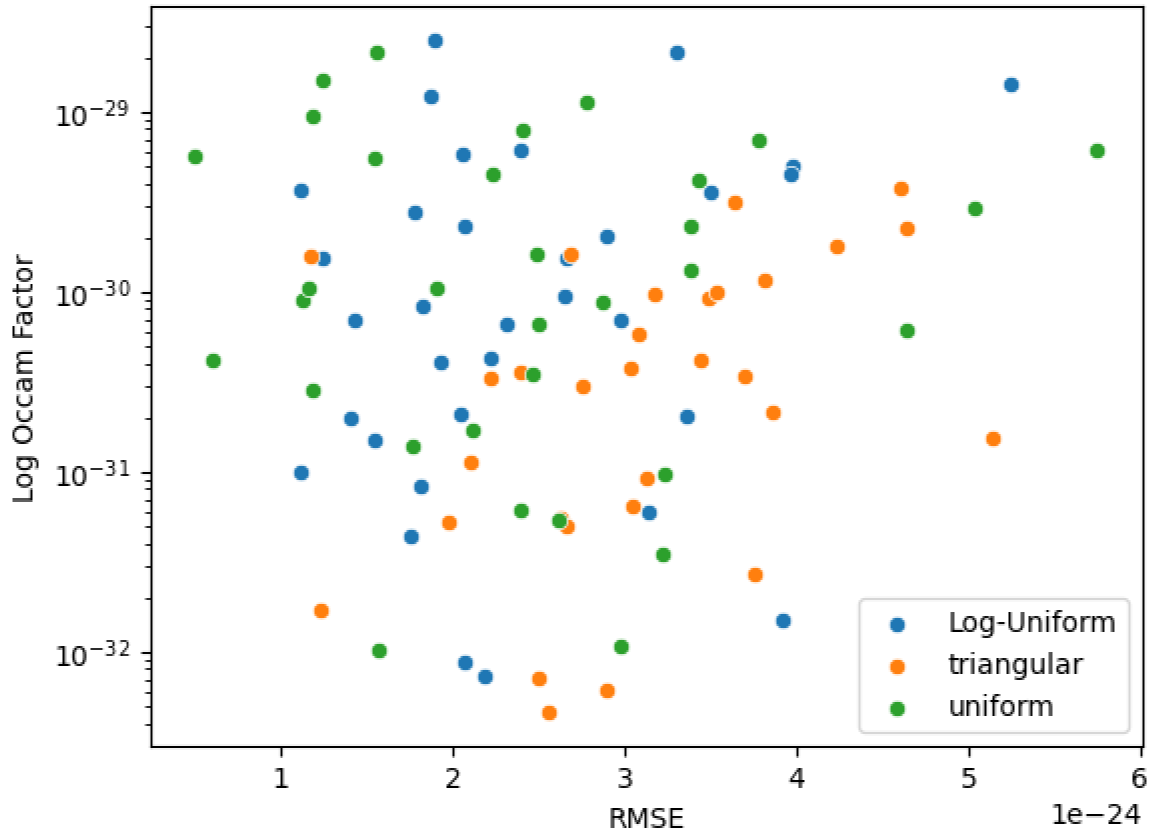


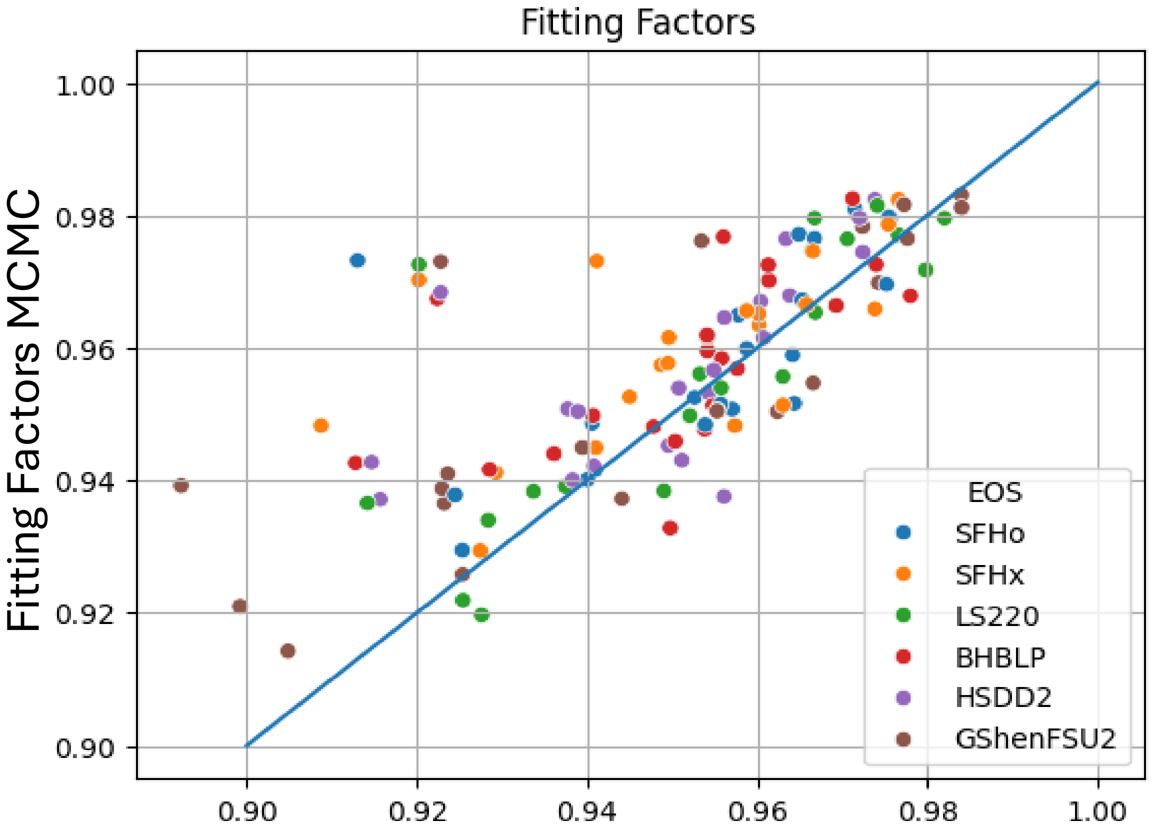


Rotational Profile [km]	Ω[rad s^{-1}]	EOS	No. of profiles
A1 (300)	4.0 - 12.0	SFHo,	12
A2 (467)	4.0 - 7.0	SFHx,	24
A3 (634)	3.0 - 7.0	LS220,	48
A4 (1268)	2.0 - 6.0	BHBLP,	18
A5 (10,000)	2.0 - 4.0	HSDD2, GShenFSU2.1	24

Random realization at 30 kpc for the signal with name A300w5.0_SFHx

Results PE Richers Catalog LGCN





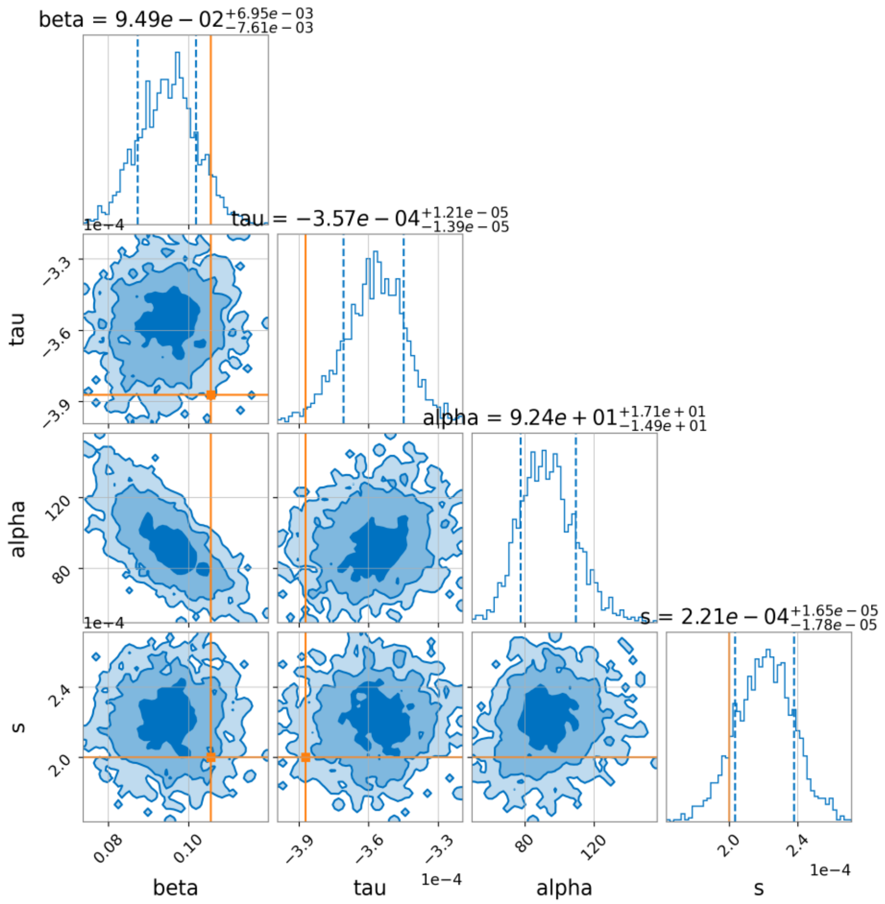
Fitting Factors Frequentist Matched Filtering

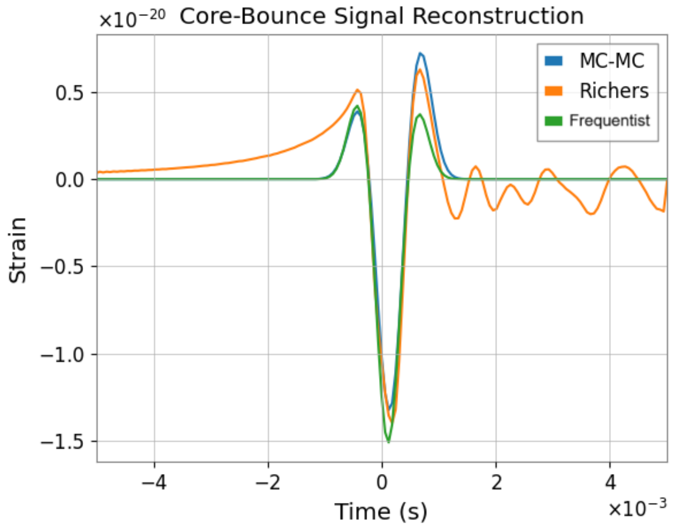
Model complexity is below one, so it suggests model overfitting

68.3% Of the reconstructed signals with MC-MC showed a fitting factor greater that the ones reported in the Model (FF).

Results PE O3 noise

Whitened O3 noise at 5kpc for highly rotating signal and 10 kpc for medium and slow regimes.

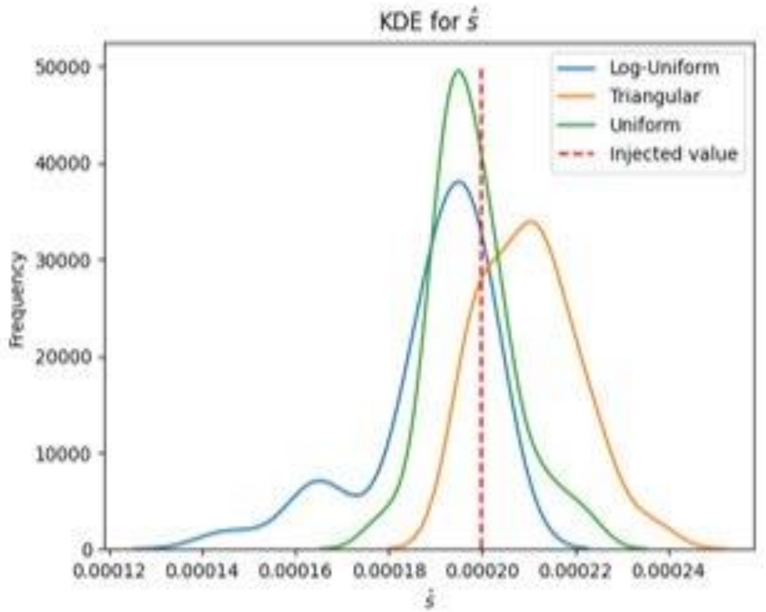


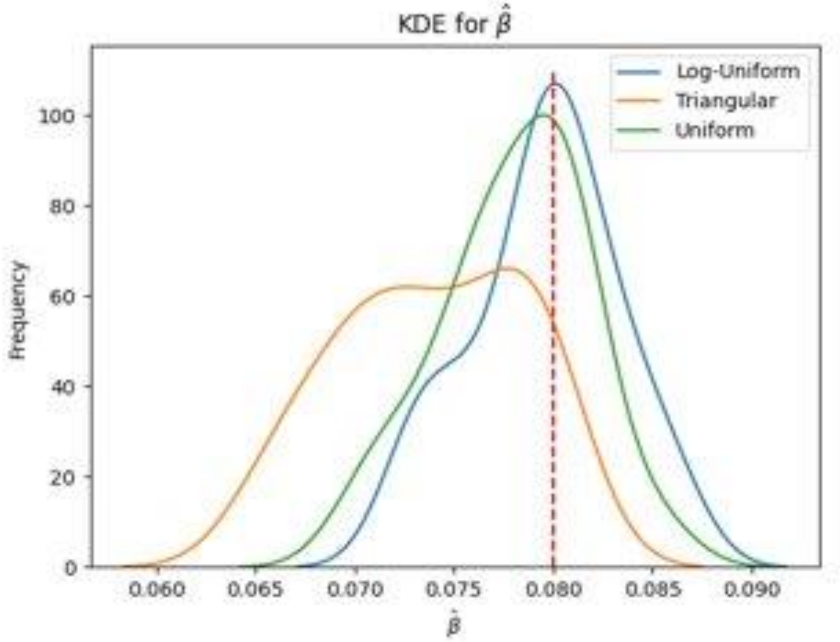


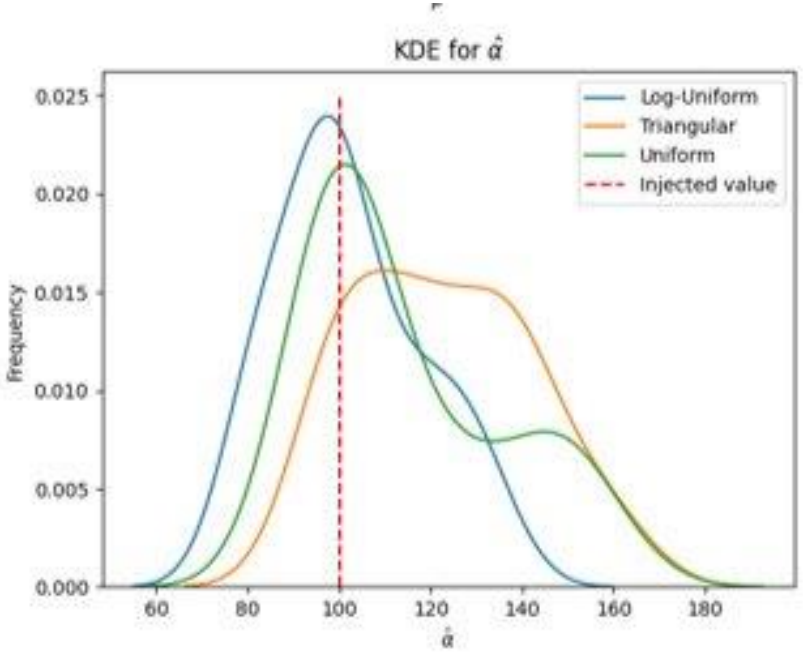
Frequentist β	MCMC β	σ_β
0.106	0.095	0.0073
0.083	0.092	0.034
0.021	0.0597	0.041

For slowly and medium rotating signals there is overestimation and higher uncertainties.

Prior Sensitivity in LGCN



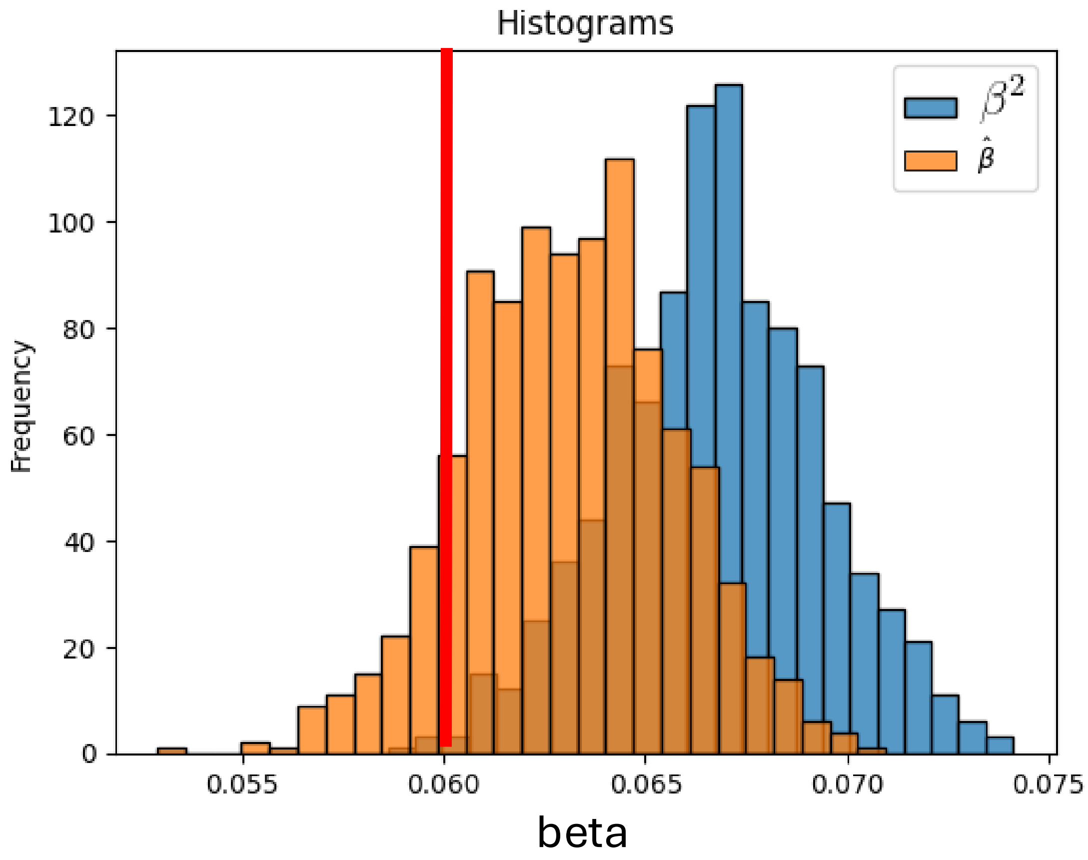


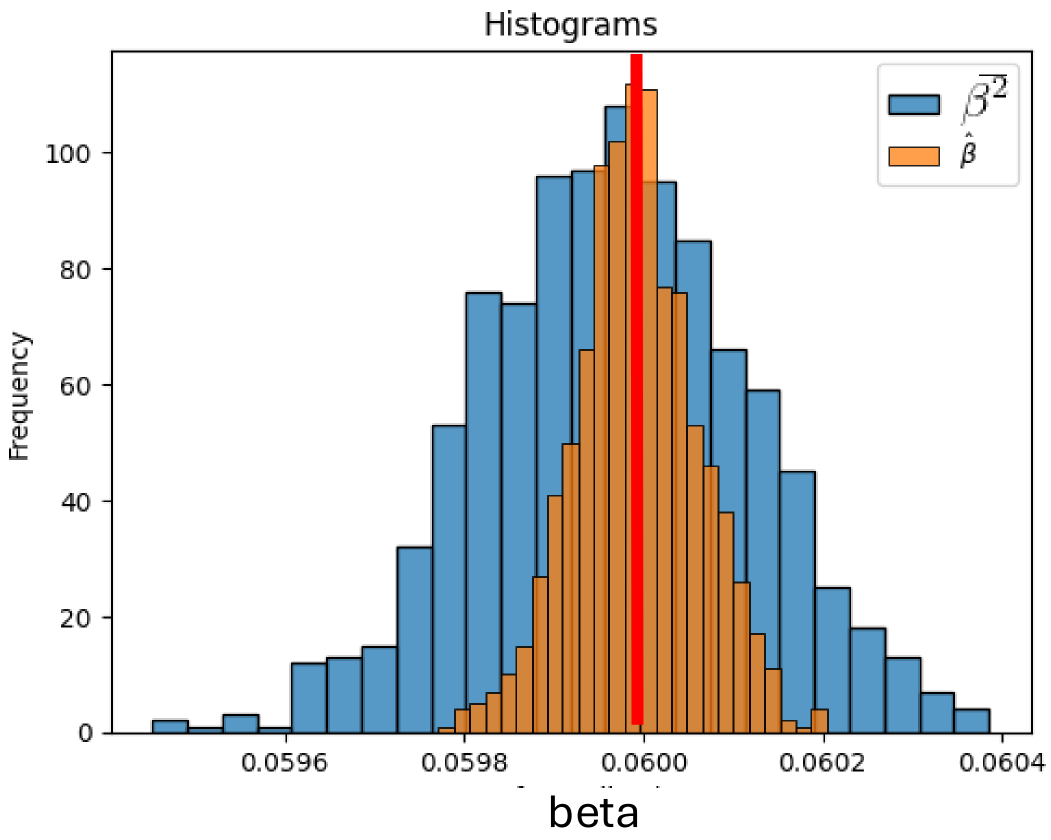


Prior	$\beta\ (10^{-2})$	$\hat{\beta}\ (10^{-2})$	$\sigma_\beta\ (10^{-3})$
Uniform	2.10	2.02	4.58
	8.90	8.99	5.71
Log-Uniform	2.10	1.86	4.41
	8.90	8.63	5.92
Triangular	2.10	2.45	4.56
	8.90	8.86	5.69

37 injections at 10 kpc.
And individual cases for different values of beta.

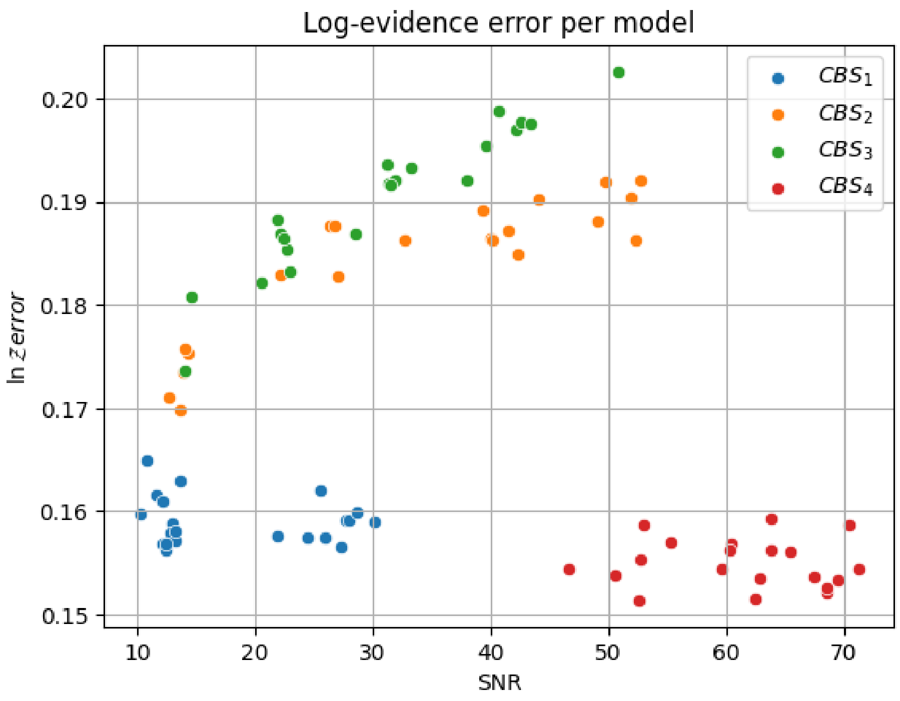
Priors in $\beta \sim U(0.005, 0.12)$ and $\beta^2 \sim U(2.5 \times 10^{-5}, 1.44 \times 10^{-2})$





Overestimation for 10 kpc (left) and high uncertainty in beta squared for 1 kpc (right). Using Gaussian colored noise based on LIGO PSD.

Model Comparison



Natural Log of Marginalized Evidence. Shows that 4-parametric model and 1-parameter model stand out in LGCN.

B_2^1	B_3^1	B_4^1	B_3^2	B_4^2	B_4^3
127.60	55.32	1.44	-72.28	-126.16	-53.89

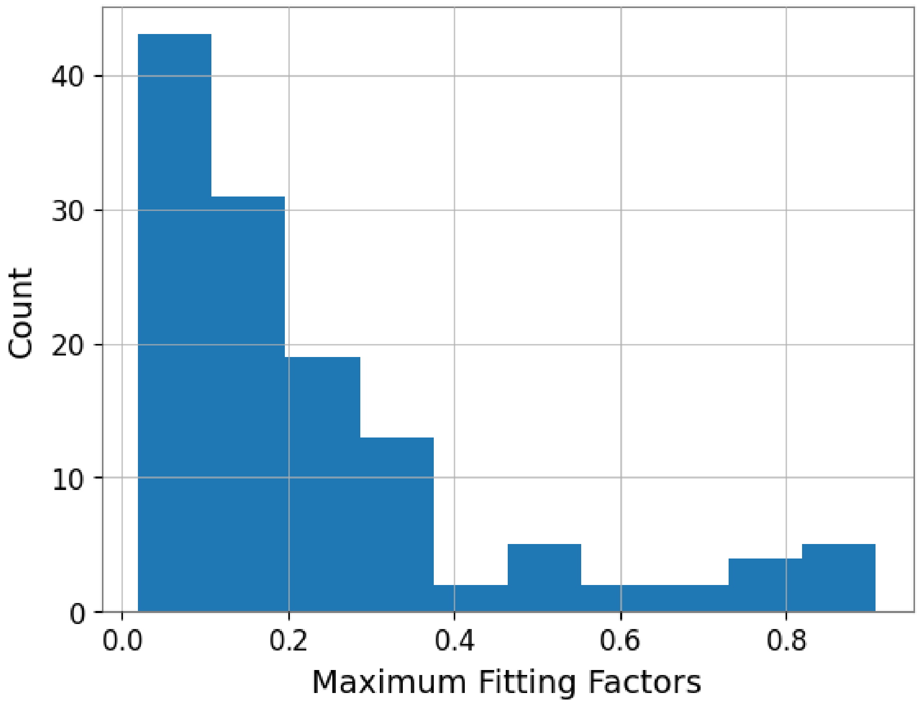
Log base 10 of Bayes Factors for models which differ in number of parameters 'x' :

Core Bounce Signal (CBSx).
CBS1 → Beta
CBS2 → Beta, tau
CBS3 → Beta, tau, alpha
CBS4 → Beta, tau, alpha, s

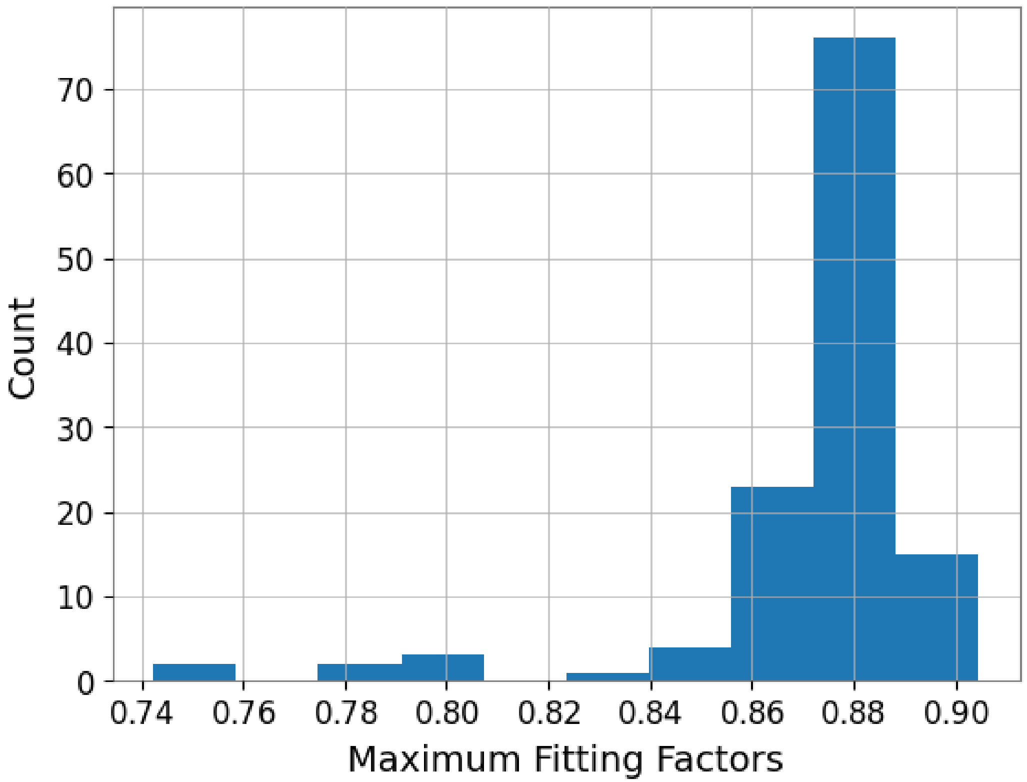
According to Jeffreys scale (Jeffreys, 1998), the analytical model with 1 free parameter and 4 free parameters are preferred.

Fitting Factors for a 4-parameter model

Using O3 noise at 10 kpc



126 values for beta were taken with an average fitting factor of 23.30% and median of 16.18%



625 points (beta,tau,alpha,s) were taken with an average fitting factor of 87.21% and median of 87.8%

Conclusions

- Bayesian Evidence and Fitting Factors suggests that a 1 parameter and 4-parameter model are more likely to explain data.
- Bayesian inference allows to refine the analytical model to find a better fit to numerical simulations.
- It seems to be a prior sensitivity introducing some bias and uncertainty in beta squared.
- Using real O3 noise requires further data pre-processing to enhance PE results for larger distances.

Prospects

- Perform PE and Fitting Factor calculation in Abylkairov Catalog.
- Use O3 noise and a network of detectors, including two polarizations of the metric perturbation.
- Improve the analytical model with physics-informed parameters.

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