Core-collapse Supernova Simulation with Subgrid Modeling of Fast Neutrino Flavor Conversion with Boltzmann Radiation Hydrodynamics Code

arXiv:2506.07017

Accepted to PRD

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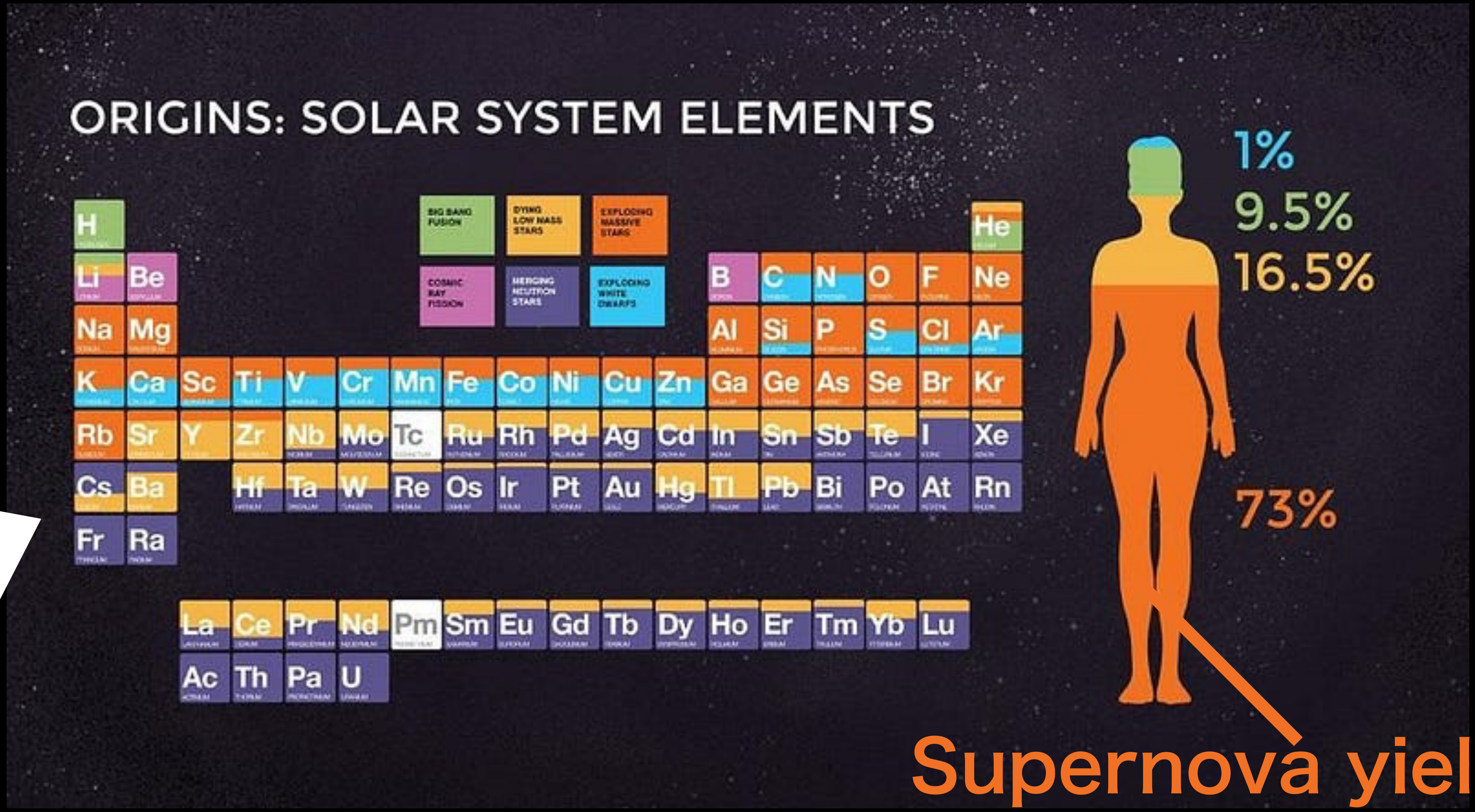
Core-collapse Supernovae (CCSNe)

- Energetic explosion at the end of stellar evolution.
- Plays central role for the evolution of the universe.



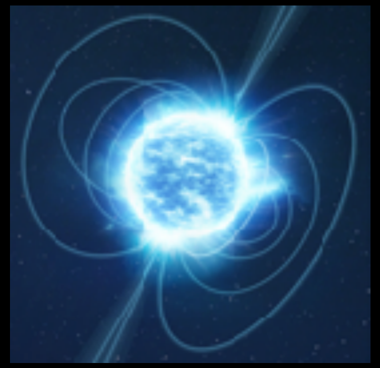
Explosive nucleosynthesis

Supernova yield

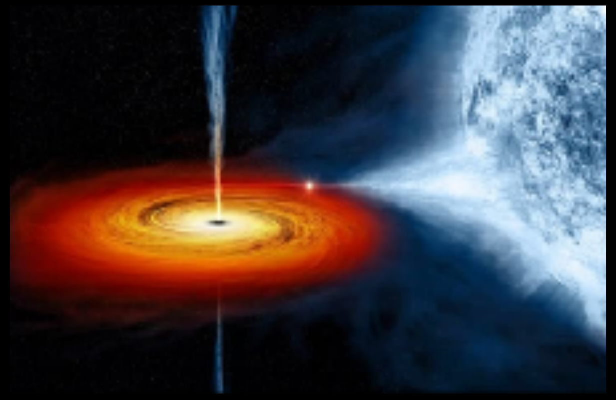
Supernova explosion

Formation of compact stars

Neutron stars



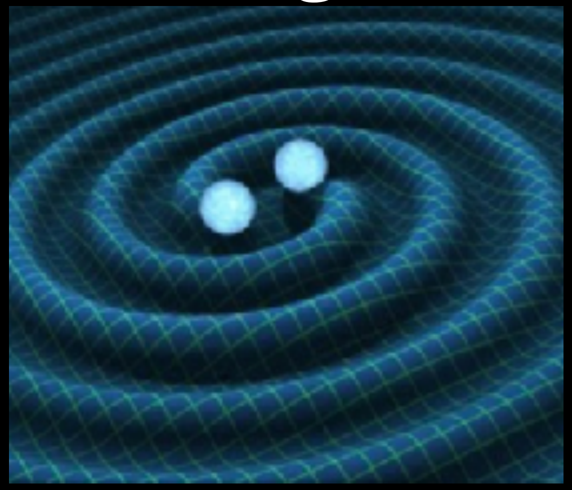
Black holes



High energy astrophysical phenomena

GRB

Compact binary mergers



Scenario of CCSN
Massive Star
Gravitational Collapse
Bounce
Explosion
H
CO
He
Si
Fe
ONeMg
Fe core formed due to nucleosynthesis
When Fe core grows too massive, it collapses
Core suddenly gets stiffened when the strong interaction take place
Neutron star or black hole
Shock wave reach outer envelope
Neutrino heating mechanism
Accretion
Photodissociation
Protoneutron star
shock
Shock stalls
p
n
ν_e
$\bar{\nu}_e$
Neutrinos heats up the matter behind the shock
Shock revival


Neutrinos inside CCSN
Free streaming
Intermediate: nontrivial
thermal eq. (Fermi-Dirac)
momentum: isotropic
Phase space distribution function $f(x^\mu, p^i)$
Boltzmann equation
$$p^\alpha \frac{\partial f}{\partial x^\alpha} - \Gamma^i_{\alpha\beta} p^\alpha p^\beta \frac{\partial f}{\partial p^i} = \left[\frac{\delta f}{\delta t}\right]_{\mathrm{coll}}$$

Truncated Moment Method

Distribution Function

$$f(r, \theta, \phi, \epsilon, \theta_\nu, \phi_\nu)$$

Boltzmann Equation

$$\frac{\partial f}{\partial t} + p^i \frac{\partial f}{\partial x^i} + \dot{p}^i \frac{\partial f}{\partial p^i} = C$$

Instead of Boltzmann transport, truncated moment method is often used.

Angular moment in momentum space

Moment eqs. (depend on higher moments)

0th $$\frac{\partial E}{\partial t} = L_1(E, M_1^i, M_2^{ij})$$

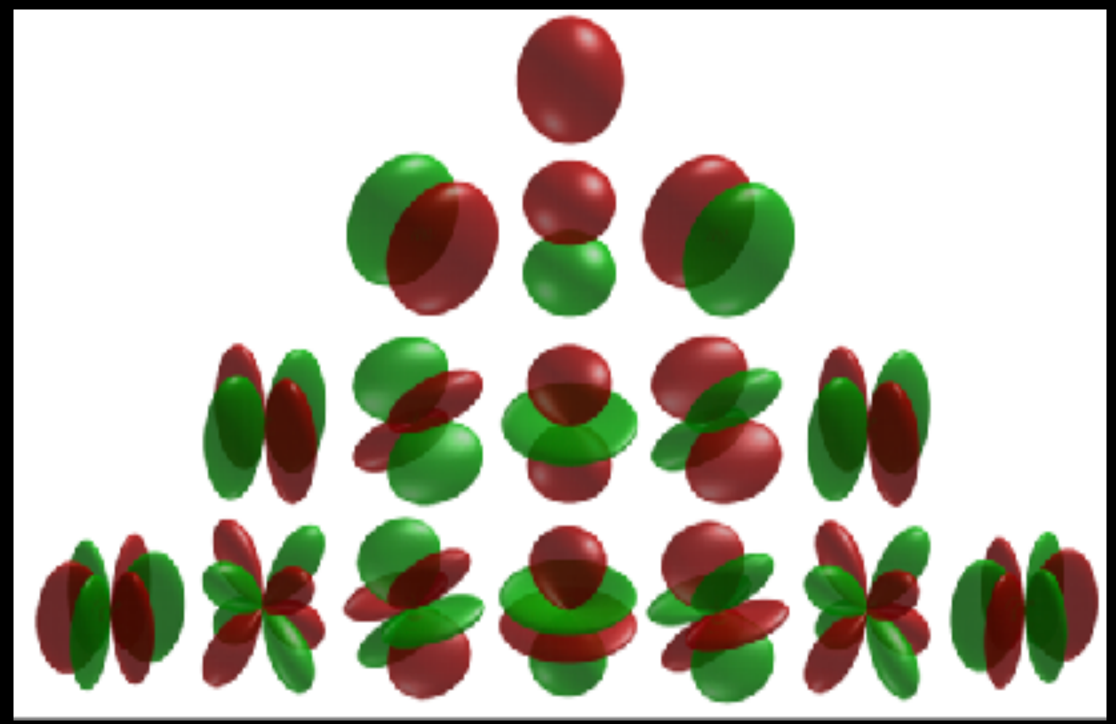
1st $$\frac{\partial M_1^i}{\partial t} = L_2(E, M_1^i, M_2^{ij})$$

Truncation

2nd $$\frac{\partial M_2^{ij}}{\partial t} = L_2(E, M_1^i, M_2^{ij}, M_3^{ijk})$$

⋮

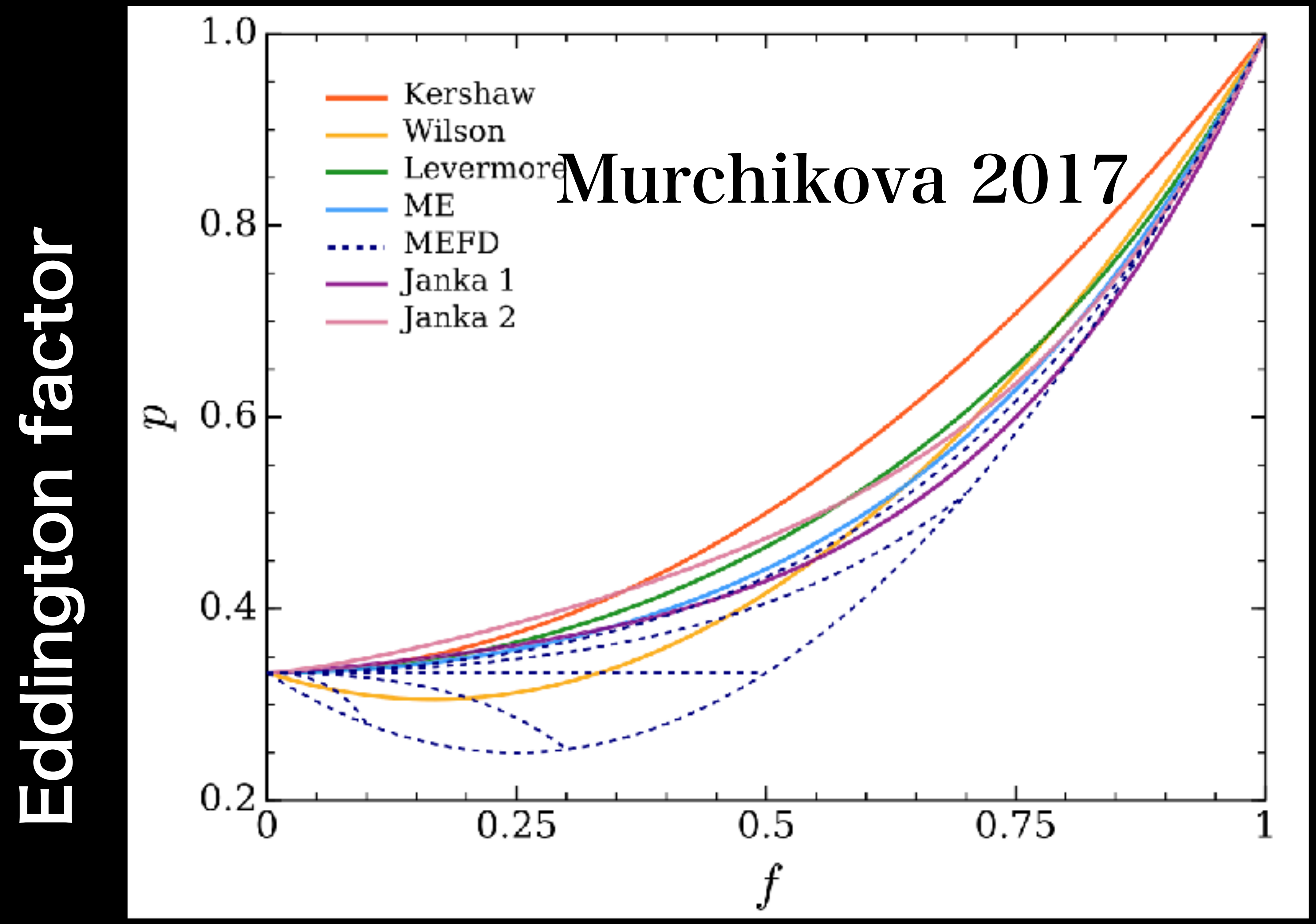
Part of momentum space information is lost



Analytical Closure

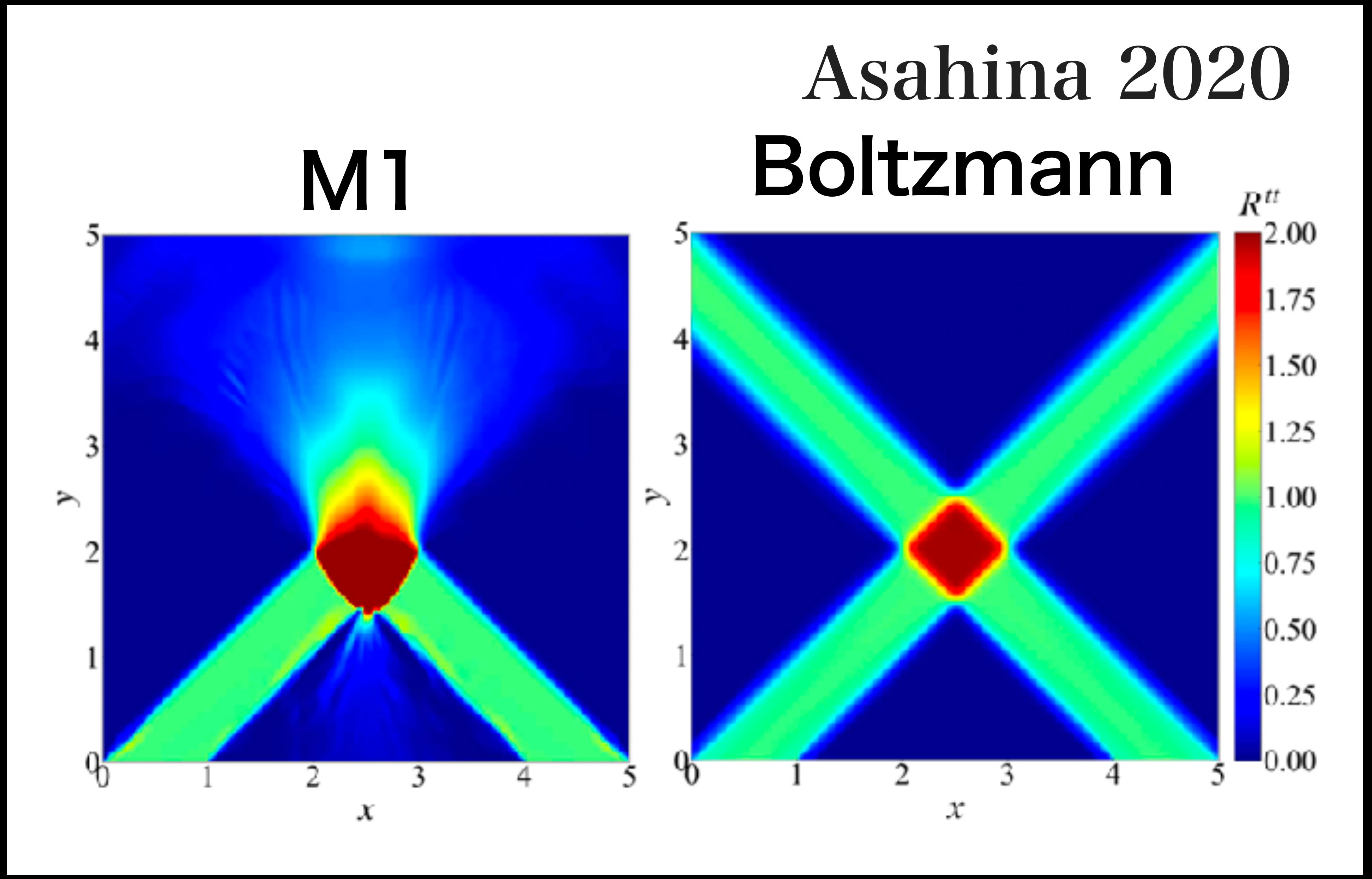
Assume closure relation to calculate 2nd moments only from 0th and 1st moments

$$P^{ij}_{M1} = \frac{3p-1}{2} P^{ij}_{thin} + \frac{3(1-p)}{2} P^{ij}_{thick}$$



Flux factor (function of 0th and 1st moment)

Moment method fails to solve ray crossing test



Boltzmann Radiation-hydro Simulation Project

Sumiyoshi 2012

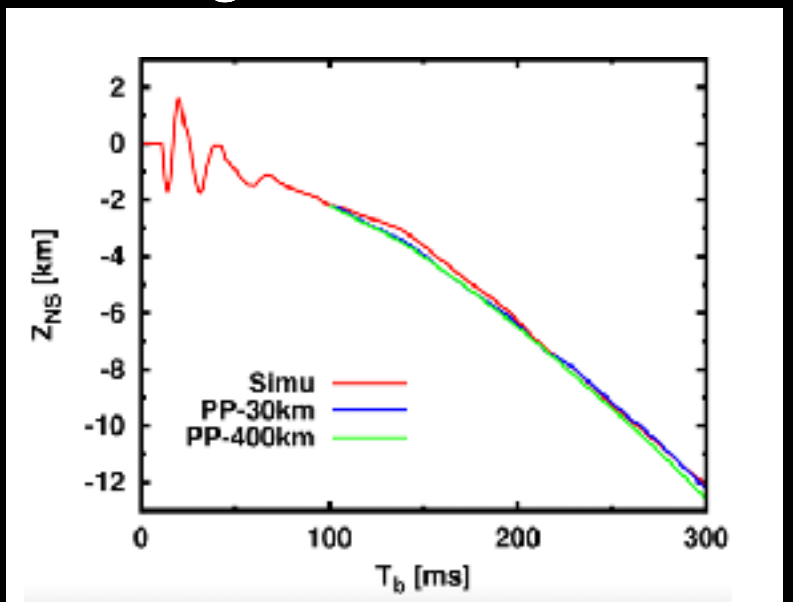
Nagakura 2014
Nagakura 2017
Nagakura 2019

RA+ 2021
RA+ 2023

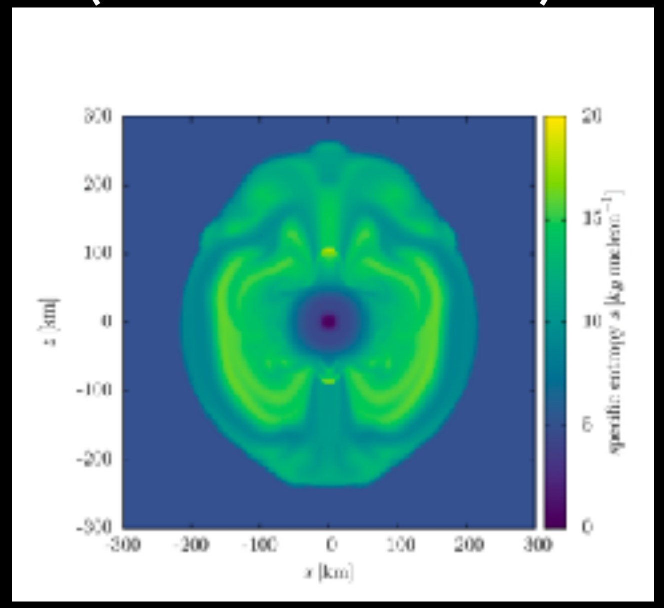
Harada in prep.

Boltzmann solver → **SR Boltzmann + Newtonian hydro** → **GR Boltzmann + GR hydro + 1D metric** → **GR Boltzmann + GR hydro + Numerical Relativity**

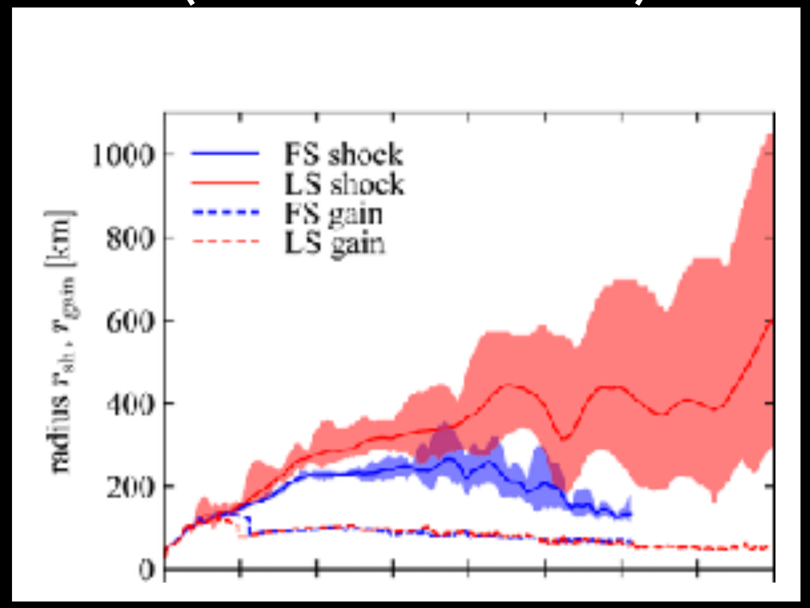
PNS kick
(Nagakura 2019)



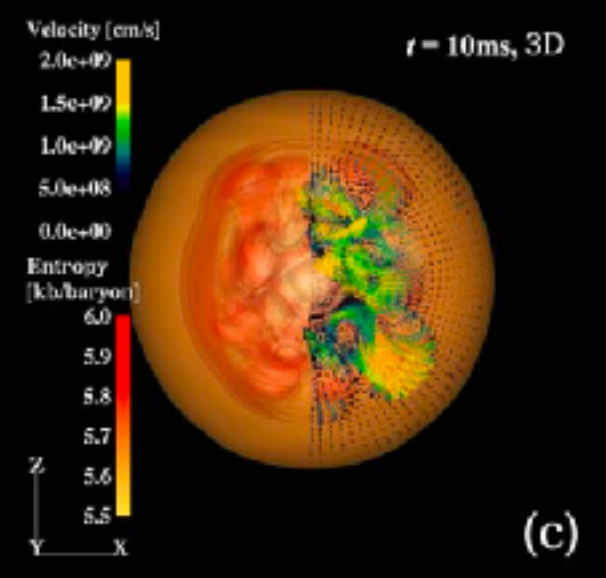
Rotating progenitor
(Harada 2019)



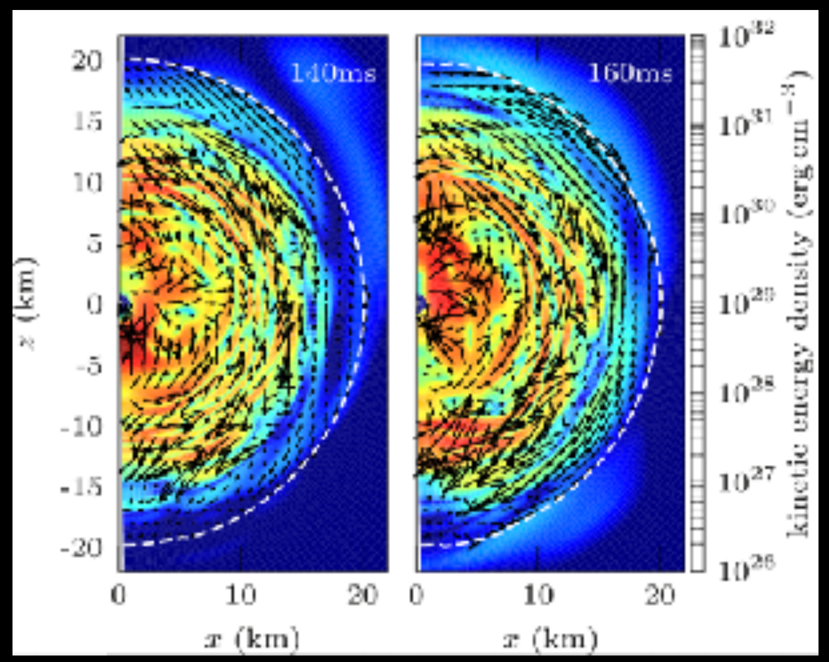
EOS dependence
(Harada 2020)



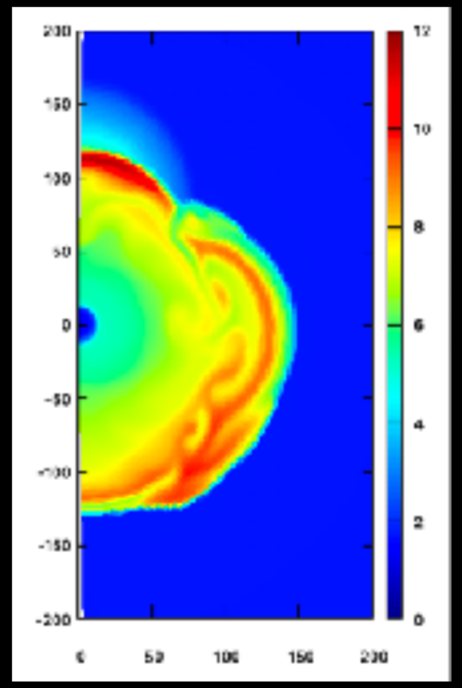
3D simulation
(Iwakami 2020)



PNS convection
(RA+2023)



GR CCSN simulation
(RA+ in prep.)



GR Boltzmann Neutrino Radiation Hydrodynamics Code

Boltzmann & hydrodynamics equations are solved together to simulate CCSN

Boltzmann equation

$$\frac{1}{\sqrt{-g}}\frac{\partial}{\partial x^\mu}\bigg|_{q_i}\left[\left(e^\mu_{(0)}+\sum_{i=1}^{3} l_i e^\mu_{(i)}\right)\sqrt{-g}f\right]-\frac{1}{\epsilon^2}\frac{\partial}{\partial\epsilon}\left(\epsilon^3 f\omega_{(0)}\right)$$
$$+\frac{1}{\sin\theta_\nu}\frac{\partial}{\partial\theta_\nu}\left(\sin\theta_\nu f\omega_{(\theta_\nu)}\right)-\frac{1}{\sin^2\theta_\nu}\frac{\partial}{\partial\phi_\nu}\left(f\omega_{(\phi_\nu)}\right)=S_{\rm rad}$$

Neutrino-matter interactions

Emission/Absorption

$$e^- + p \leftrightarrow \nu_e + n$$
$$e^+ + n \leftrightarrow \bar{\nu}_e + p$$
$$e^- + A \leftrightarrow \nu_e + A'$$

Scattering

$$\nu + N \leftrightarrow \nu + N$$
$$\nu + A \leftrightarrow \nu + A$$
$$\nu + e^- \leftrightarrow \nu + e^-$$

Pair

$$e^- + e^+ \leftrightarrow \nu + \bar{\nu}$$
$$N + N \leftrightarrow N + N + \nu + \bar{\nu}$$

Hydrodynamics equation

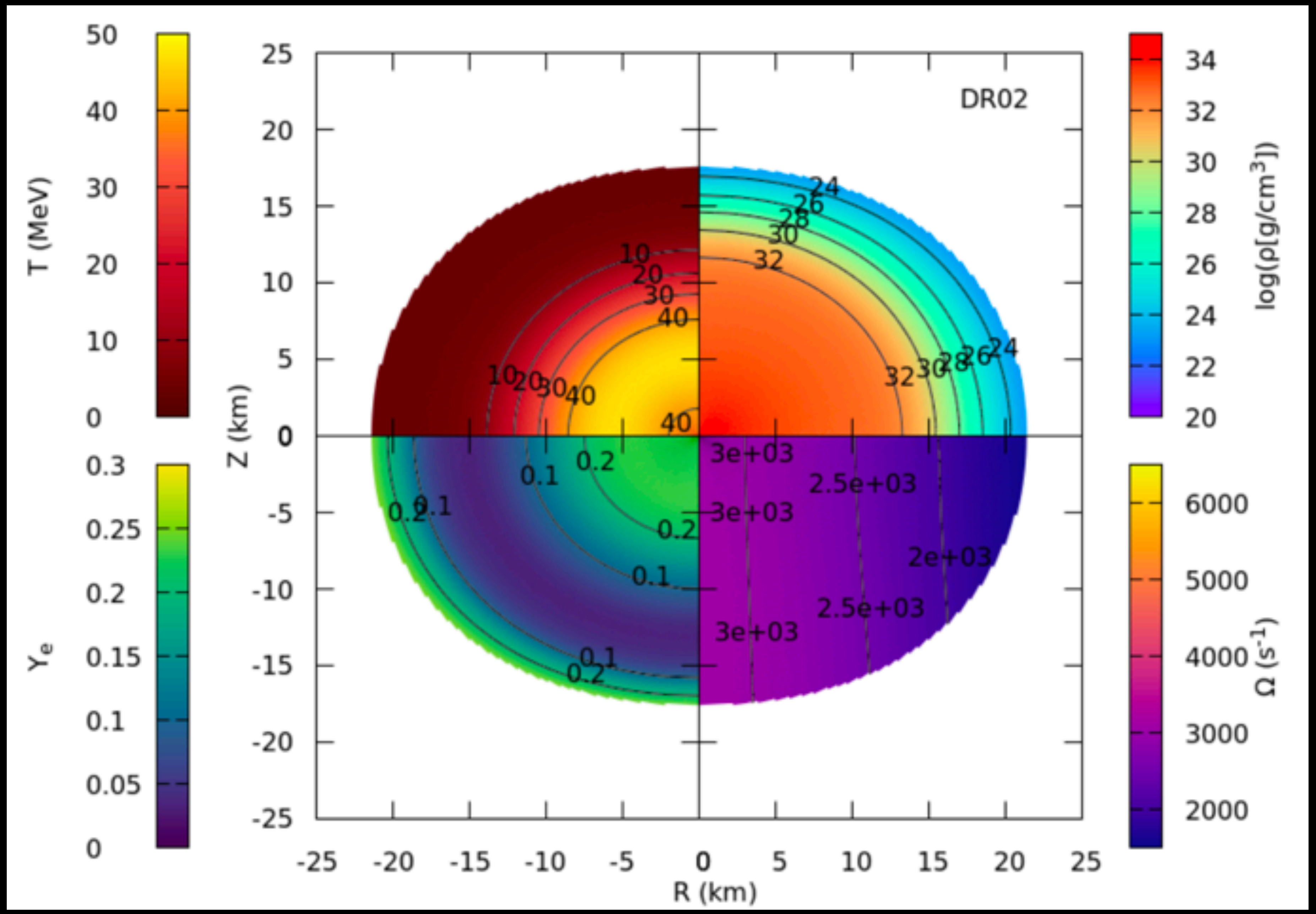
$$\partial_t\rho_* + \partial_j(\rho_* v^j) = 0$$
$$\partial_t S_i + \partial_j(S_i v^j + \alpha\sqrt{\gamma}P\delta_i{}^j)$$
$$= -S_0\partial_i\alpha + S_j\partial_i\beta^j - \frac{1}{2}\alpha\sqrt{\gamma}S_{jk}\partial_i\gamma^{jk} - \alpha\sqrt{\gamma}G_i$$
$$\partial_t(S_0-\rho_*) + \partial_k((S_0-\rho_*)v^k + \sqrt{\gamma}P(v^k+\beta^k))$$
$$= \alpha\sqrt{\gamma}S^{ij}K_{ij} - S_i D^i\alpha + \alpha\sqrt{\gamma}n^\mu G_\mu$$

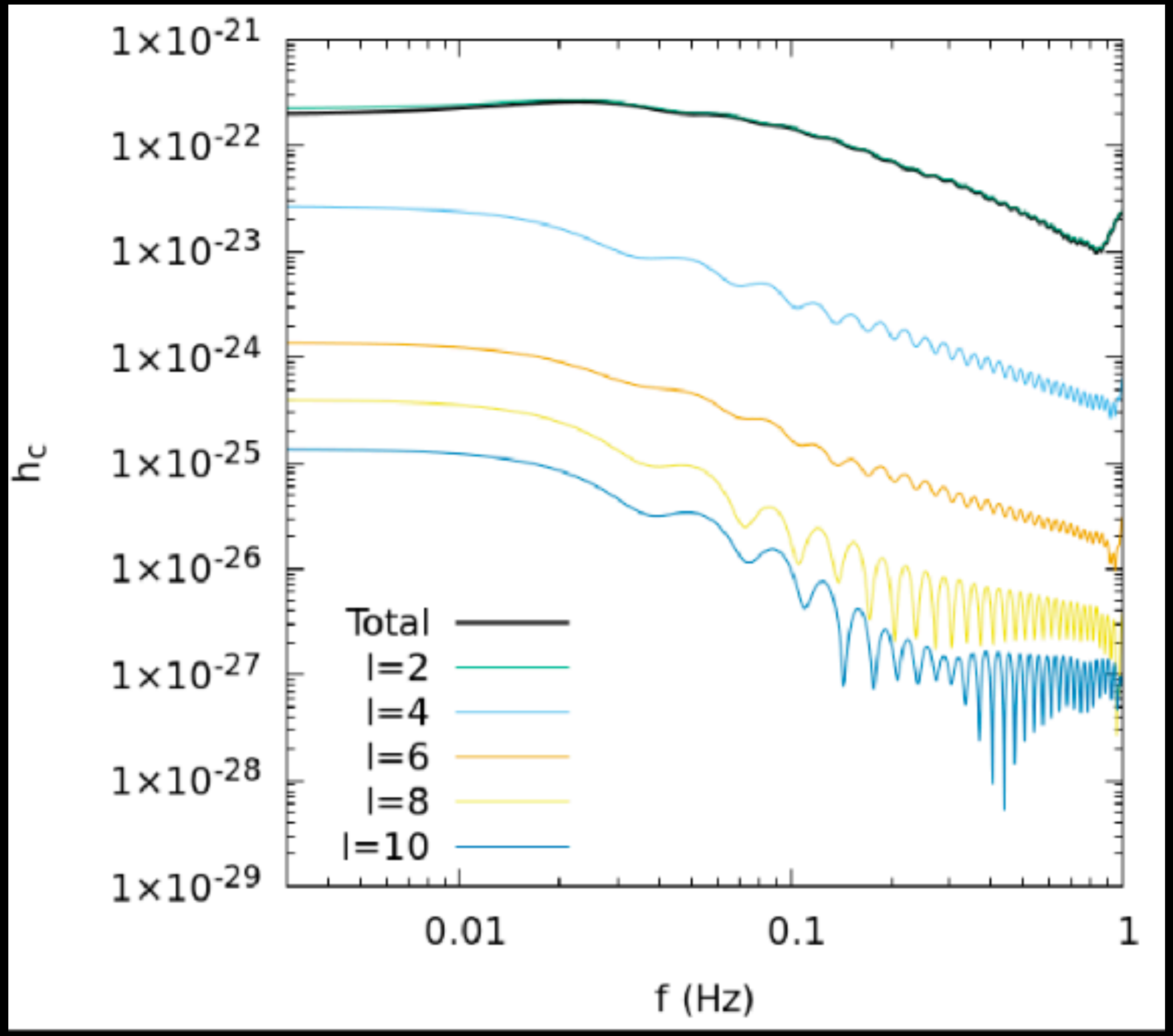
Spacetime metric

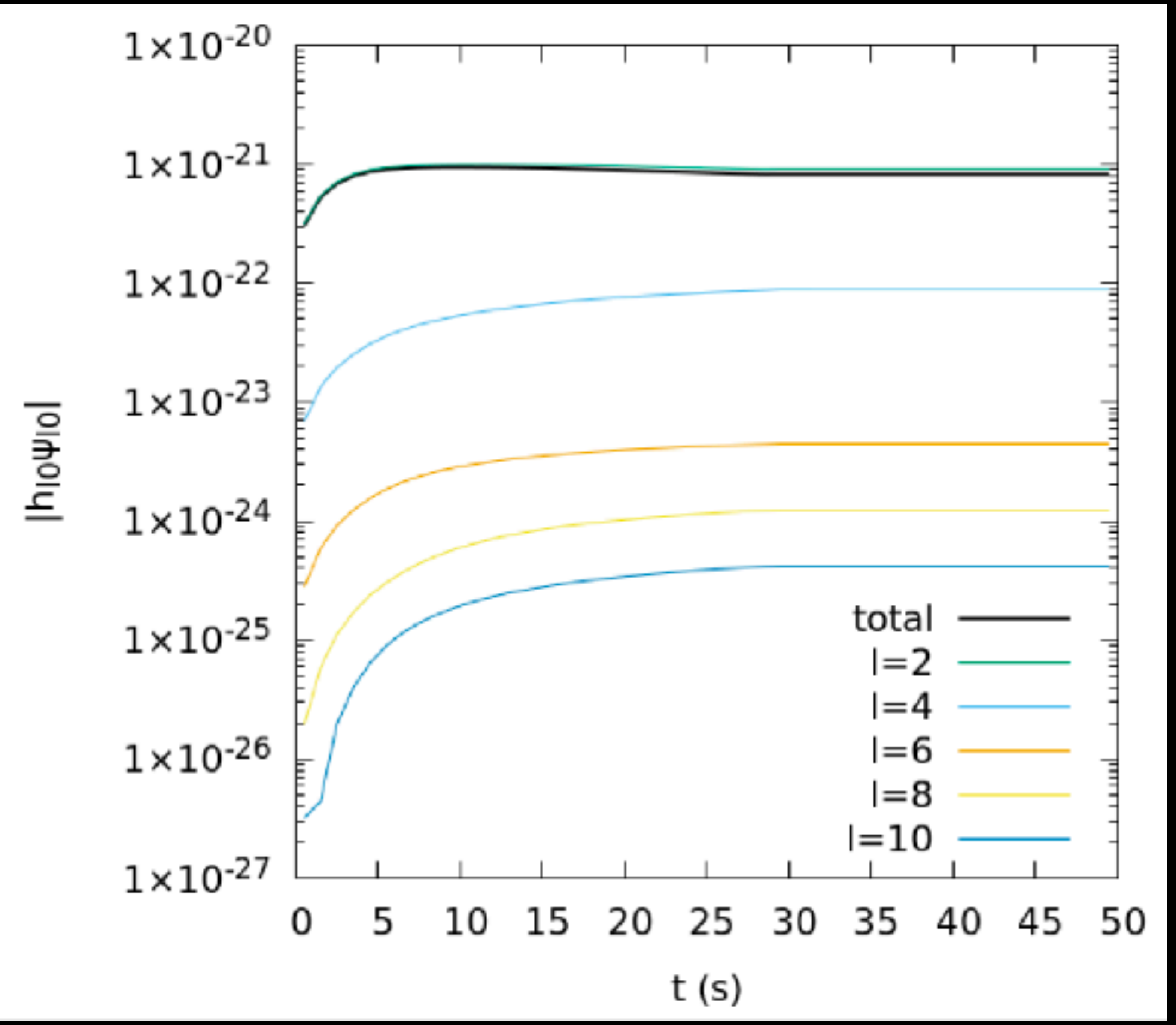
$$g_{\mu\nu} = {\rm diag}\left[-e^{2\Phi(t,r)}, \left(1-2m(t,r)/r\right)^{-1}, r^2, r^2\sin^2\theta\right]$$
$$\frac{\partial m}{\partial r} = 4\pi r^2(\rho h W^2 - P)$$
$$\frac{\partial\Phi}{\partial r} = \left(1-\frac{2m(t,r)}{r}\right)^{-1}\left(\frac{m(t,r)}{r^2}+4\pi r(\rho h v^2 + P)\right)$$

Memory GW from rotating PNS

Barrio, RA+ arXiv:2507.04784







Neutrino Oscillation

$$i v^{\mu}\partial_{\mu}\rho = \left[\underbrace{\frac{m_1^2+m_2^2}{4E}\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix} + \frac{m_2^2-m_1^2}{4E}\begin{pmatrix}-\cos 2\theta_{12} & \sin 2\theta_{12}\\ \sin 2\theta_{12} & \cos 2\theta_{12}\end{pmatrix}}_{\text{Vacuum term}} + \underbrace{\sqrt{2}G_F n_e \begin{pmatrix}1 & 0\\ 0 & 0\end{pmatrix}}_{\text{Matter term}} + \underbrace{\sqrt{2}G_F v_{\mu}\int dP' \rho(x, P') v'^{\mu}}_{\text{Neutrino self-interaction}}, \rho \right]$$

Density

- Vacuum oscillation (interstellar region): periodic oscillation with time
- MSW resonance (e.g. solar surface): instant conversion
- Matter suppression (e.g. inside stars): no neutrino oscillation

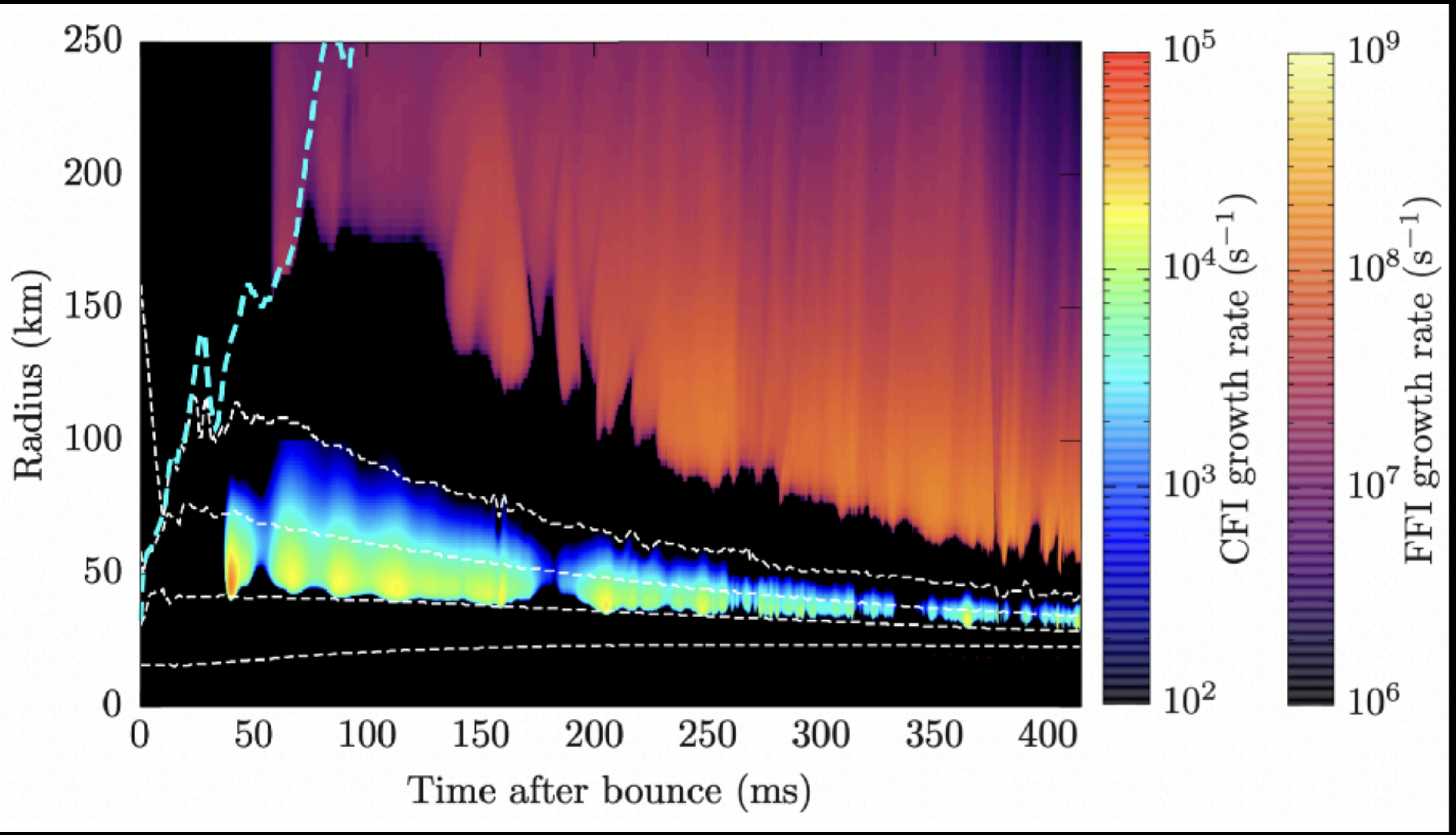
See also H. Nagakura's talk on Wed.

- **Collective oscillation (supernova core): nonlinear, can be very fast (~ns)**

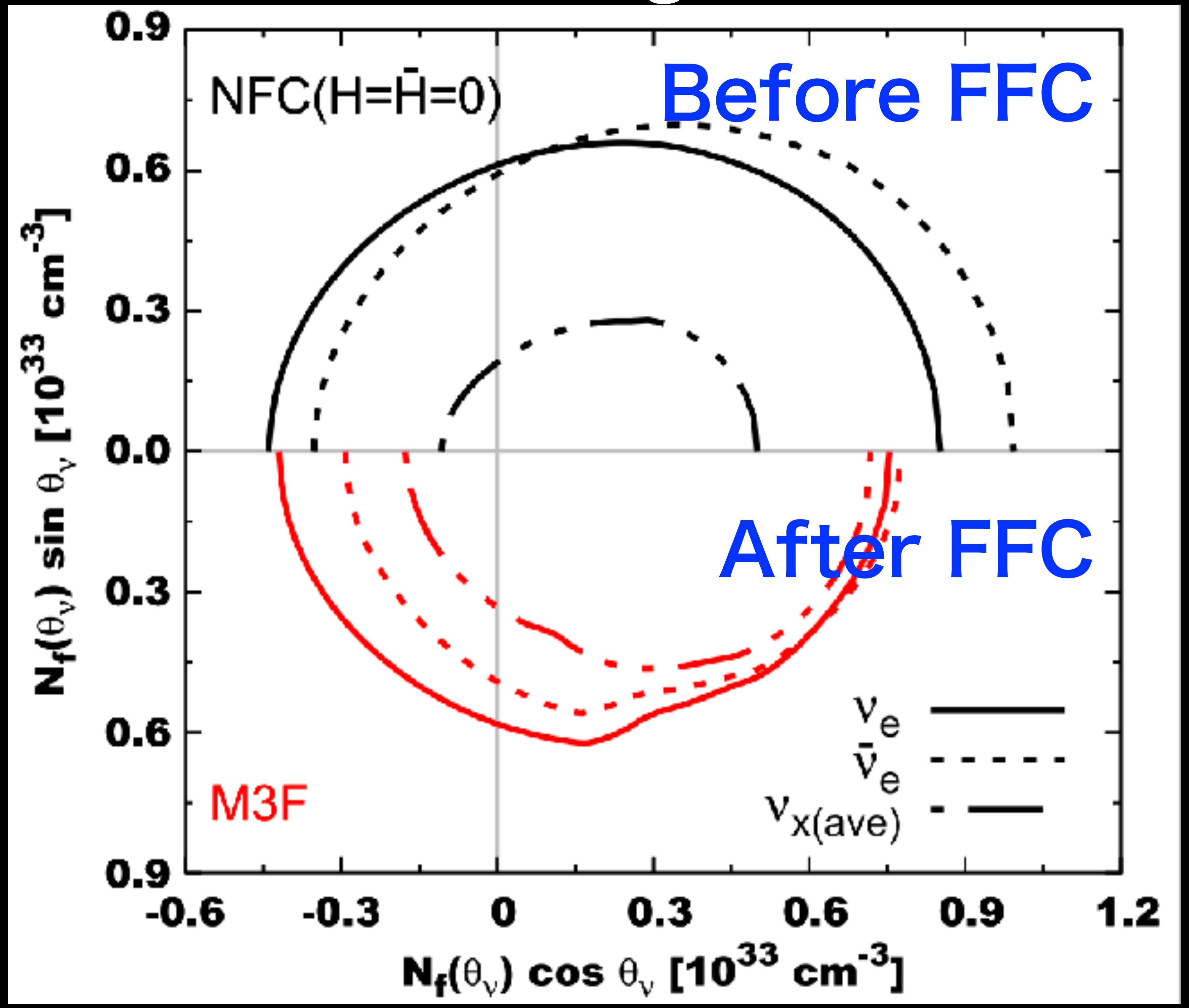
Fast Neutrino Flavor Conversion

- Fast flavor conversion (FFC) is one of collective oscillation modes, and getting great attention
- The conversion timescale can be ~ns, much shorter than the dynamical timescale.
- FFC is induced by angular crossing in momentum space

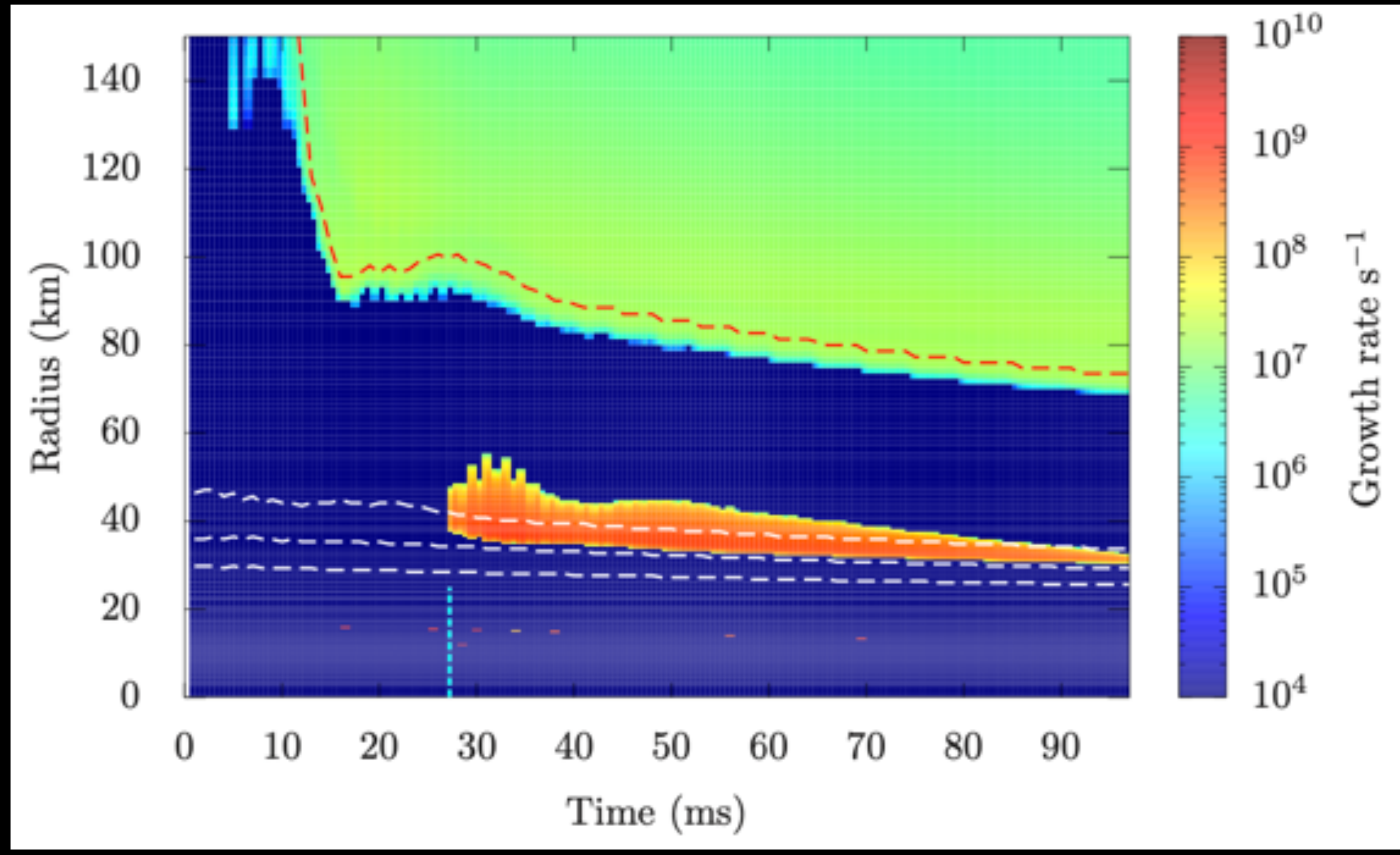
RA+2024



Nagakura 2023



Setup



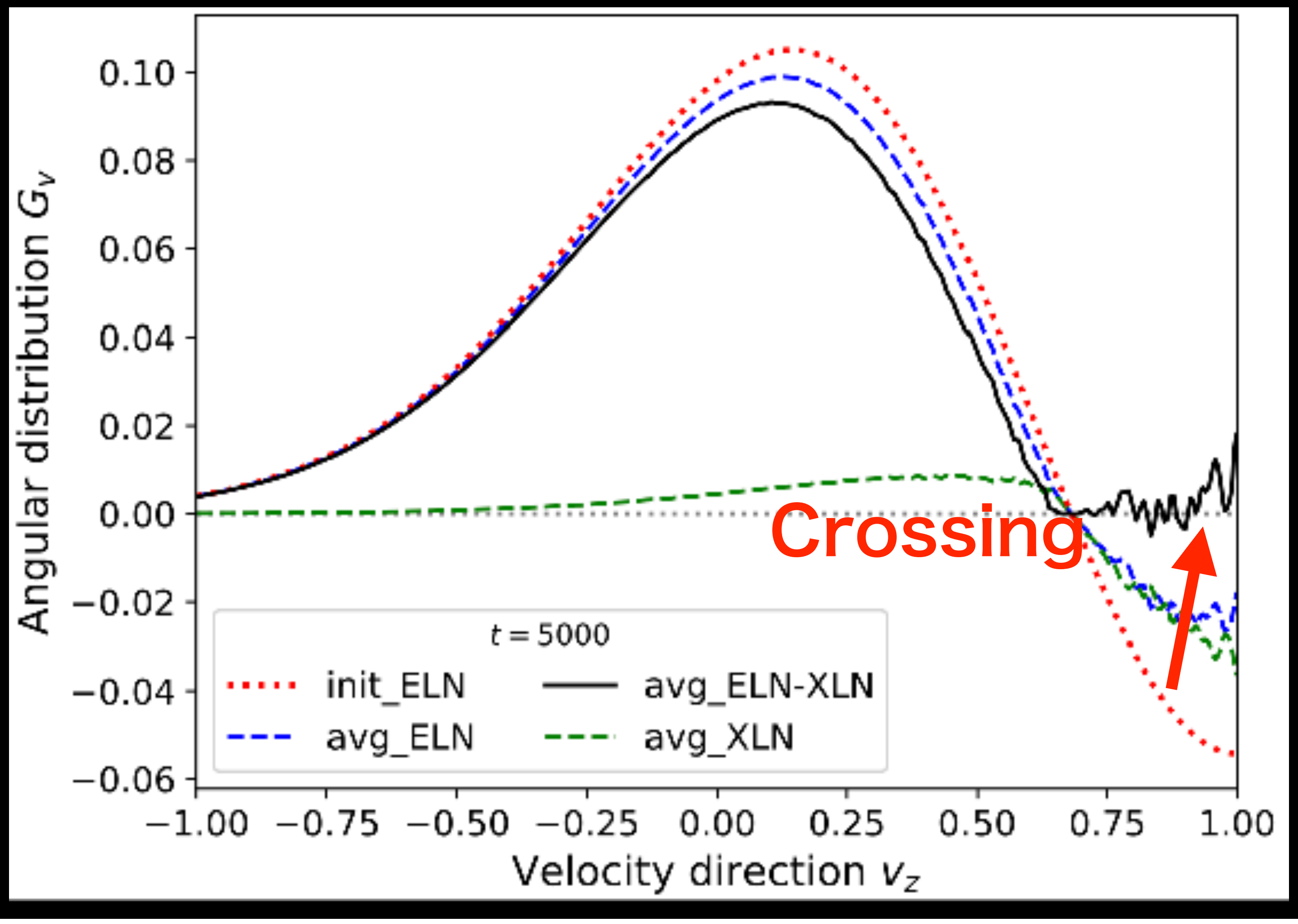
- Progenitor: $M_{\mathrm{ZAMS}} = 11.2 M_{\odot}$
- Nuclear matter: Furusawa-Togashi EOS
- 2D simulation => 1D relaxation run => 1D simulation w/ and w/o FFC

Comparison of Mixing Methods

4spBGK

Angle-dependent mixing method

Zaizen, Nagakura 2023



$$f_e^{\rm as} = \eta f_e + (1-\eta) f_x,\qquad f_x^{\rm as} = \frac{1-\eta}{2} f_e + \frac{1+\eta}{2} f_x,$$

$$\bar f_e^{\rm as} = \eta \bar f_e + (1-\eta) \bar f_x,\qquad \bar f_x^{\rm as} = \frac{1-\eta}{2} \bar f_e + \frac{1+\eta}{2} \bar f_x,$$

$B > A$

$$\eta = \begin{cases} 1/3 & (\Delta G < 0) \\ 1 - 2A/(3B) & (\Delta G \geq 0) \end{cases}$$

$B < A$

$$\eta = \begin{cases} 1/3 & (\Delta G > 0) \\ 1 - 2B/(3A) & (\Delta G \leq 0) \end{cases}$$

3spBGK

Impose 3-species assumption ($\nu_x = \bar\nu_x$) to 4spBGK model

$$f_e^{\rm as} = \eta f_e + (1-\eta) f_x,$$

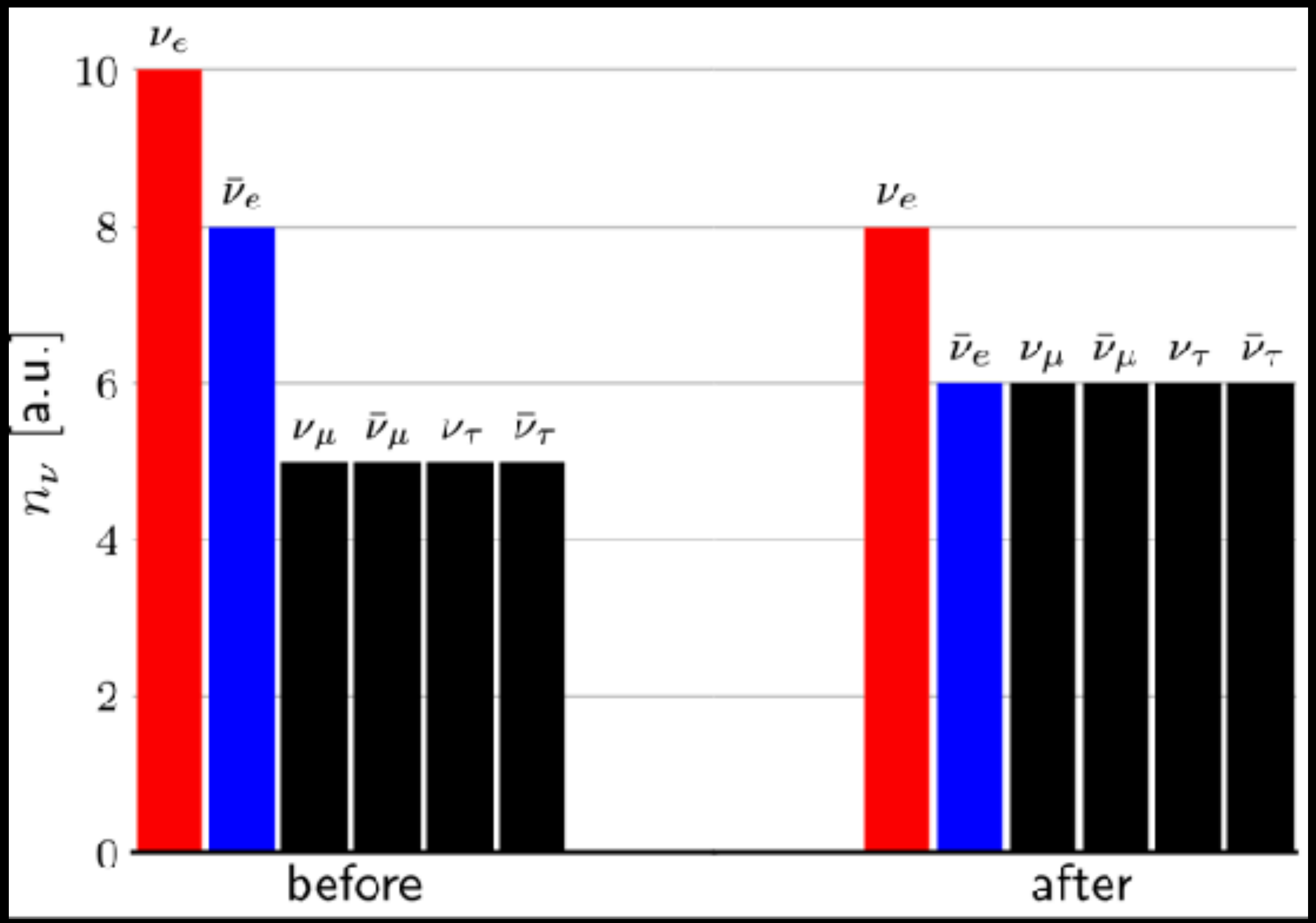
$$\bar f_e^{\rm as} = \eta \bar f_e + (1-\eta) f_x,$$

$$f_x^{\rm as} = \frac{1-\eta}{4} f_e + \frac{1-\eta}{4} \bar f_e + \frac{1+\eta}{2} f_x.$$

Lepton number violated!

3sp$\rho 11$

Simple equipartition with density threshold $\rho < 10^{11}$g/cc



$n_{\nu_e} > n_{\bar\nu_e}$

$$f_e^{\rm as} = f_e + \frac{2}{3}(f_x - \bar f_e),$$

$$\bar f_e^{\rm as} = \frac{\bar f_e + 2 f_x}{3},$$

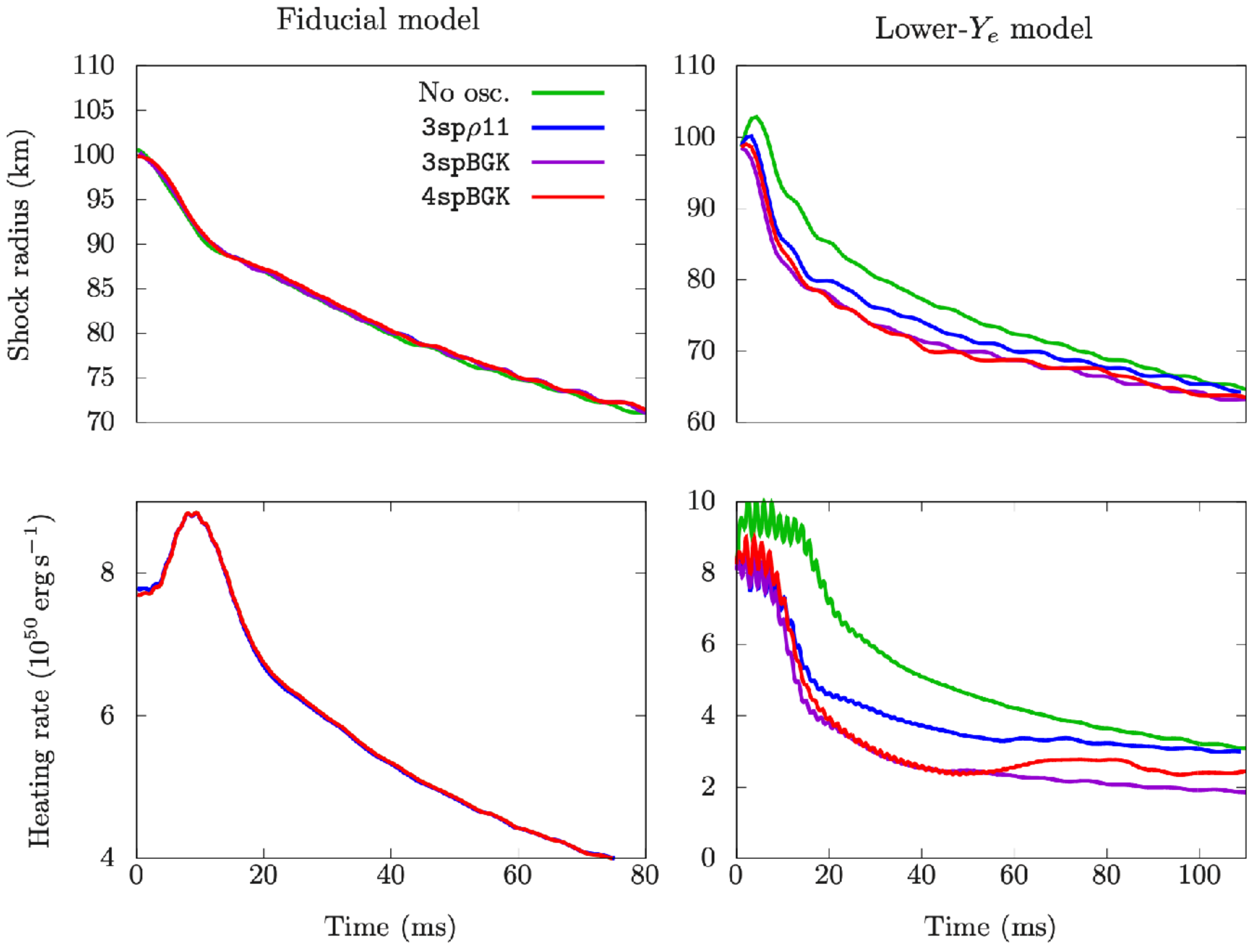
$$f_x^{\rm as} = \frac{\bar f_e + 2 f_x}{3}.$$

$n_{\nu_e} < n_{\bar\nu_e}$

$$f_e^{\rm as} = \frac{f_e + 2 f_x}{3},$$

$$\bar f_e^{\rm as} = \bar f_e + \frac{2}{3}(f_x - f_e),$$

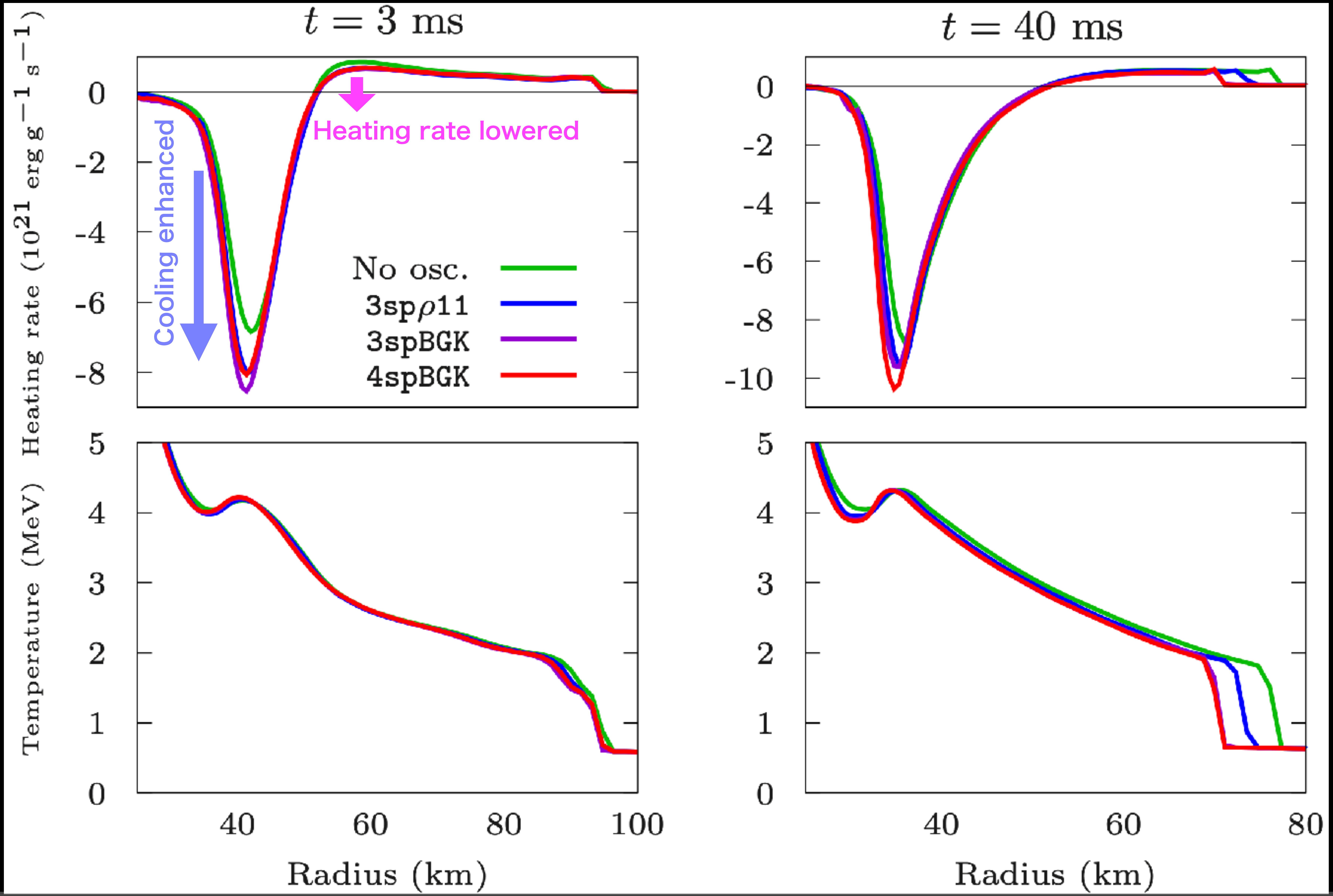
$$f_x^{\rm as} = \frac{f_e + 2 f_x}{3}.$$


Fiducial model
Lower-Y_e model
No osc.
3spρ11
3spBGK
4spBGK
Shock radius (km)
Heating rate (10^{50} erg s^{-1})
Time (ms)
Time (ms)

Flavor evolution at the appearance of FFI

Difference of energy spectra w.r.t no FFC model

Heating rate and temperature profiles



Summary

- We implemented BGK subgrid model to Boltzmann neutrino radiation hydrodynamics simulation and performed 1D CCSN simulations.
- For the models simulated, FFC can have negative effects onto CCSN dynamics.

Future Prospects

- Multi-dimensional CCSN simulation with FFC subgrid
- Progenitor and EOS dependences