Towards core-collapse supernovae asteroseismology including SASI

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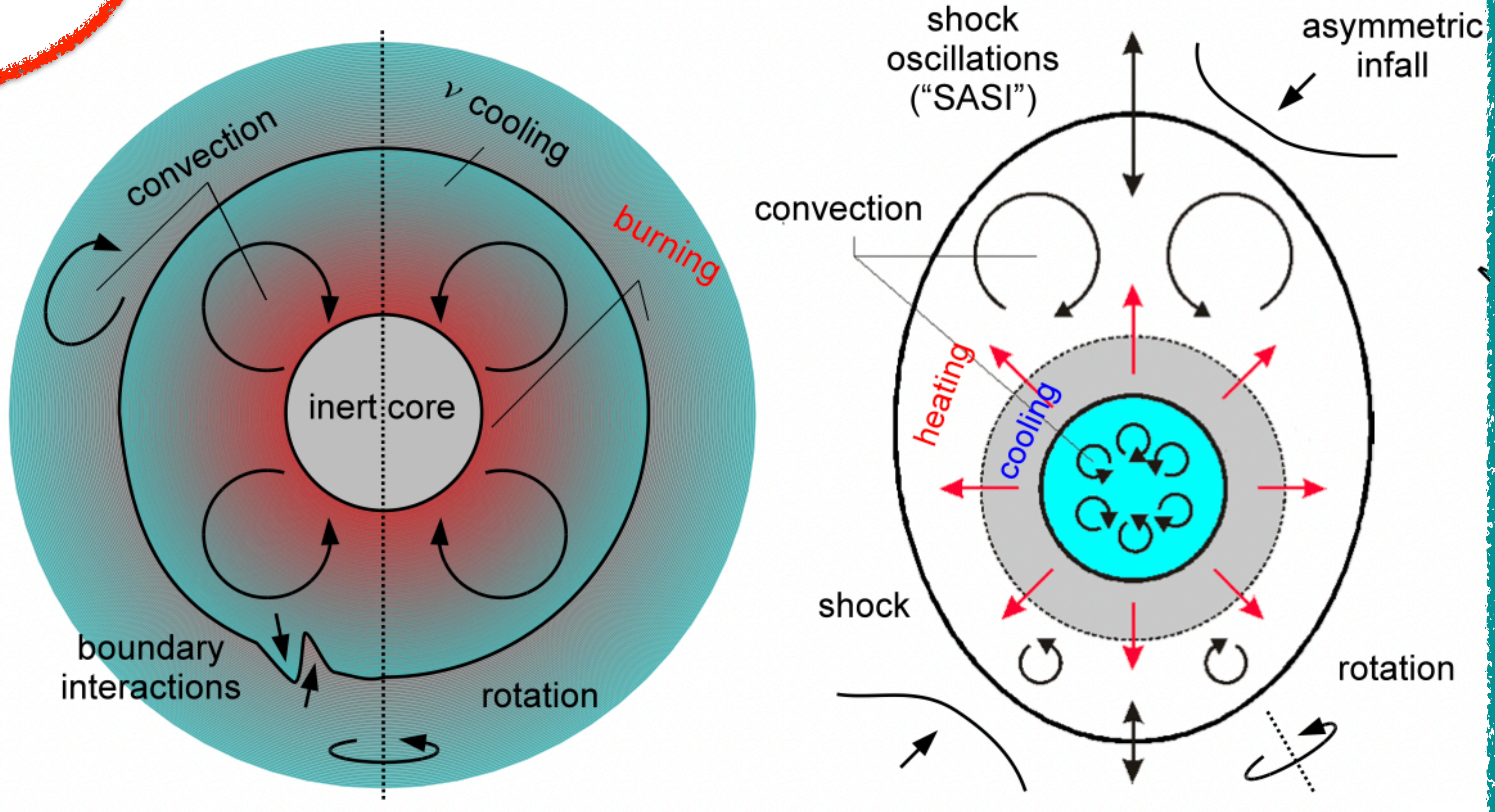
SN2025gw: 1st IGWN **Symposium**
July 21st 2025, Warsaw

Collaborators: R. Luna, P. Cerdá-Durán, A. Torres-Forné

Symposium

See Mueller's talk!

Anatomy of a CCSNe



a) C, Ne, O, and Si core and shell burning (years to hours)

b) Core collapse and shock revival (hundreds of milliseconds)

Müller, Living Reviews (2020)

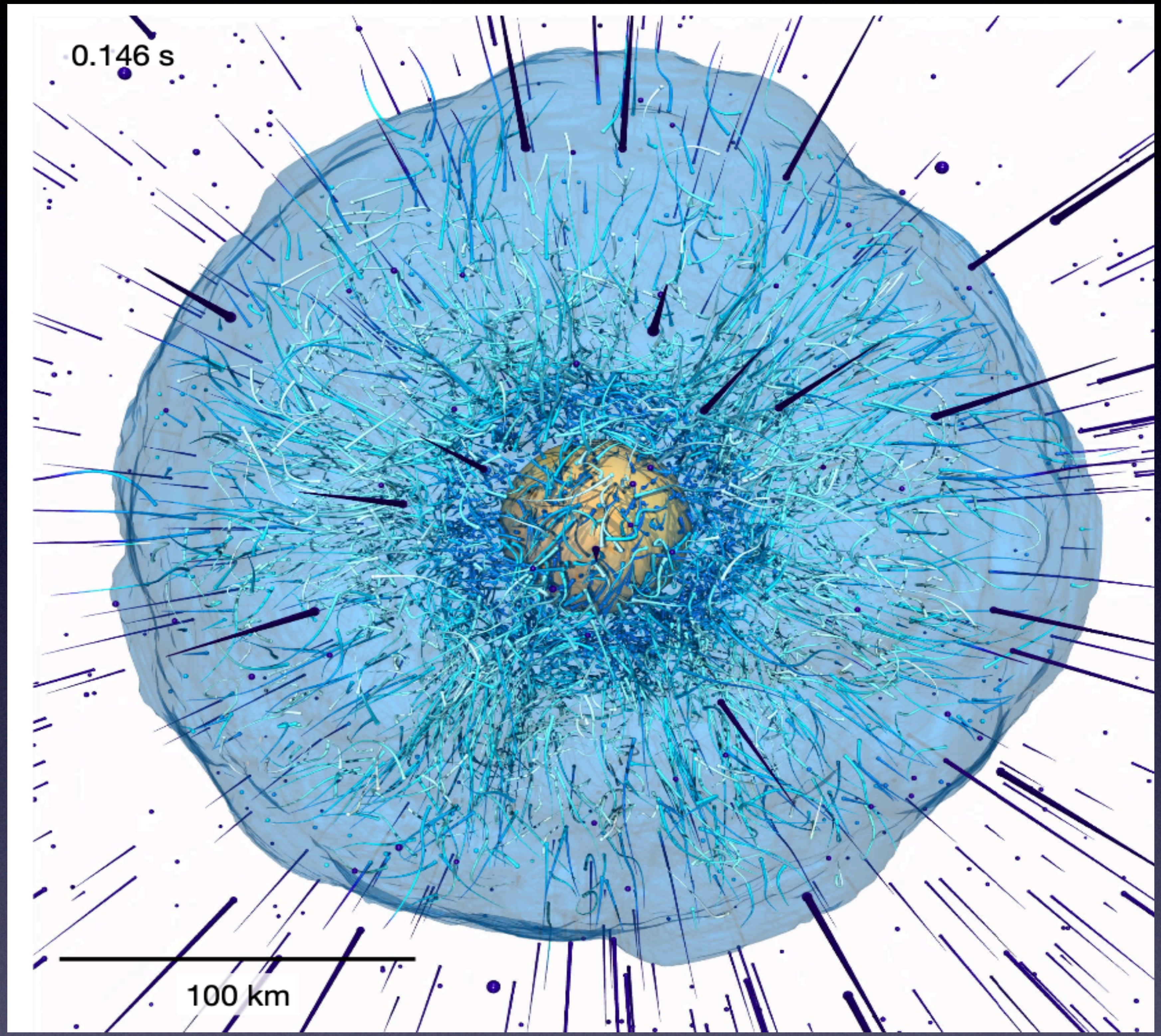
GW emission from CCSNe

Burrows et al (2021)

Proto-neutron star (PNS) phase (before explosion):

- Duration: 0.1 – 1 s
- PNS mass grows: $0.5\,M_\odot \rightarrow 1.2-2\,M_\odot$
- PNS shrinks: 30 km $\rightarrow$ 10 km
- PNS cools down

Convection, SASI



Reviews:

- Janka, Melson, and Summa, Ann. Rev. Nucl. Part. Sci. 66 341 (2016)
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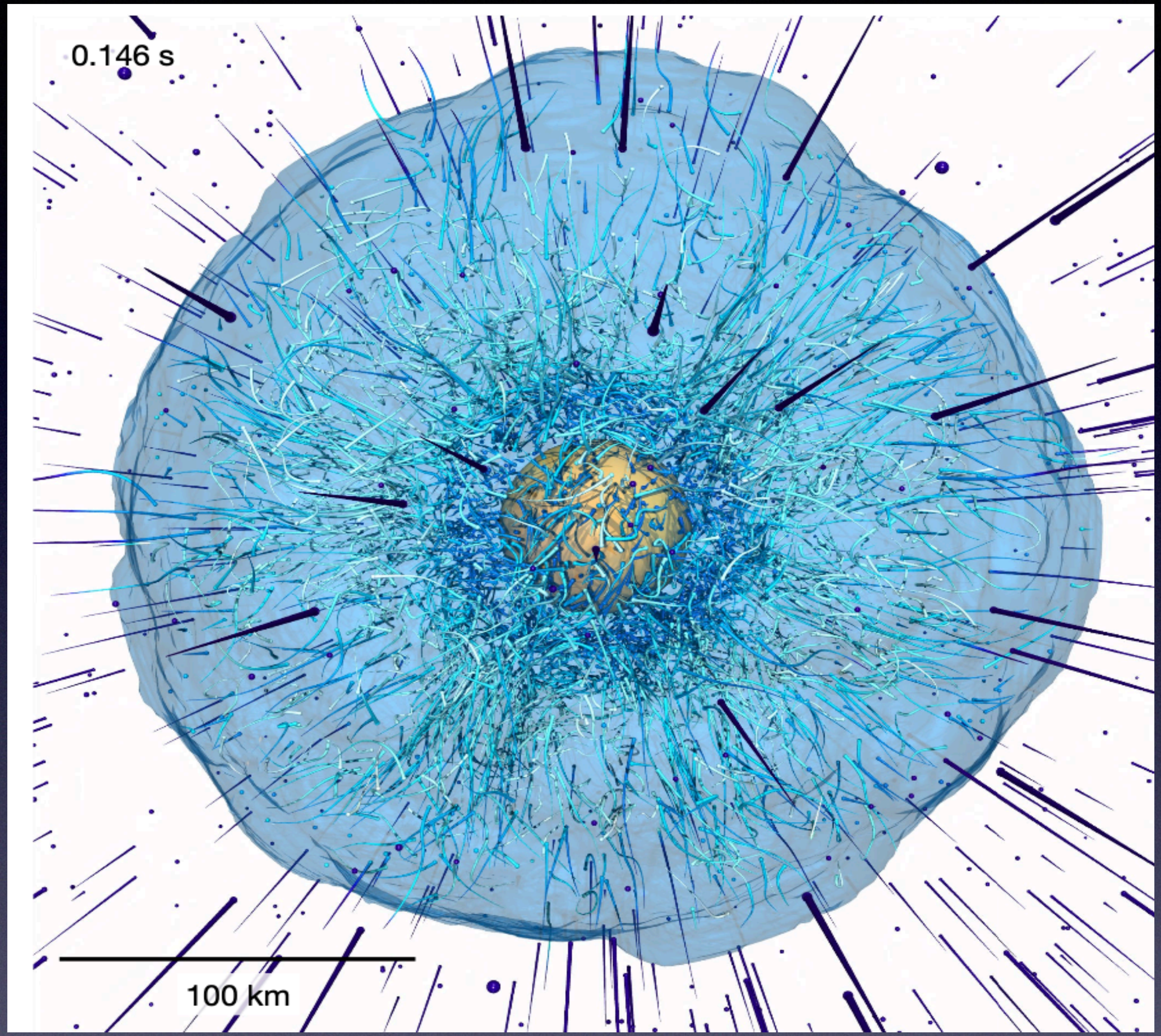

Convection, SASI

Excitation of PNS normal modes

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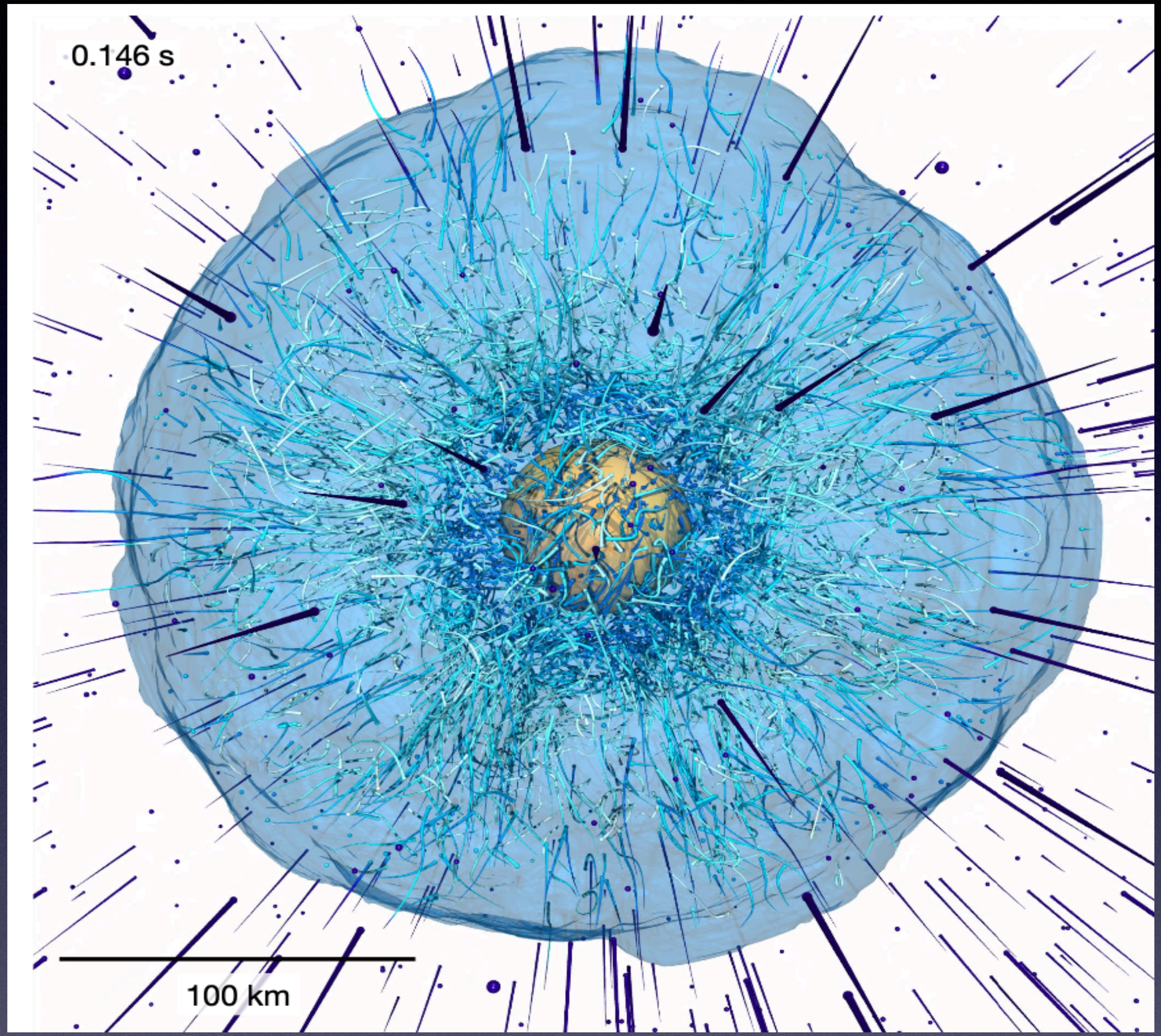

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Excitation of PNS normal modes

MAIN MECHANISM OF GW EMISSION!!

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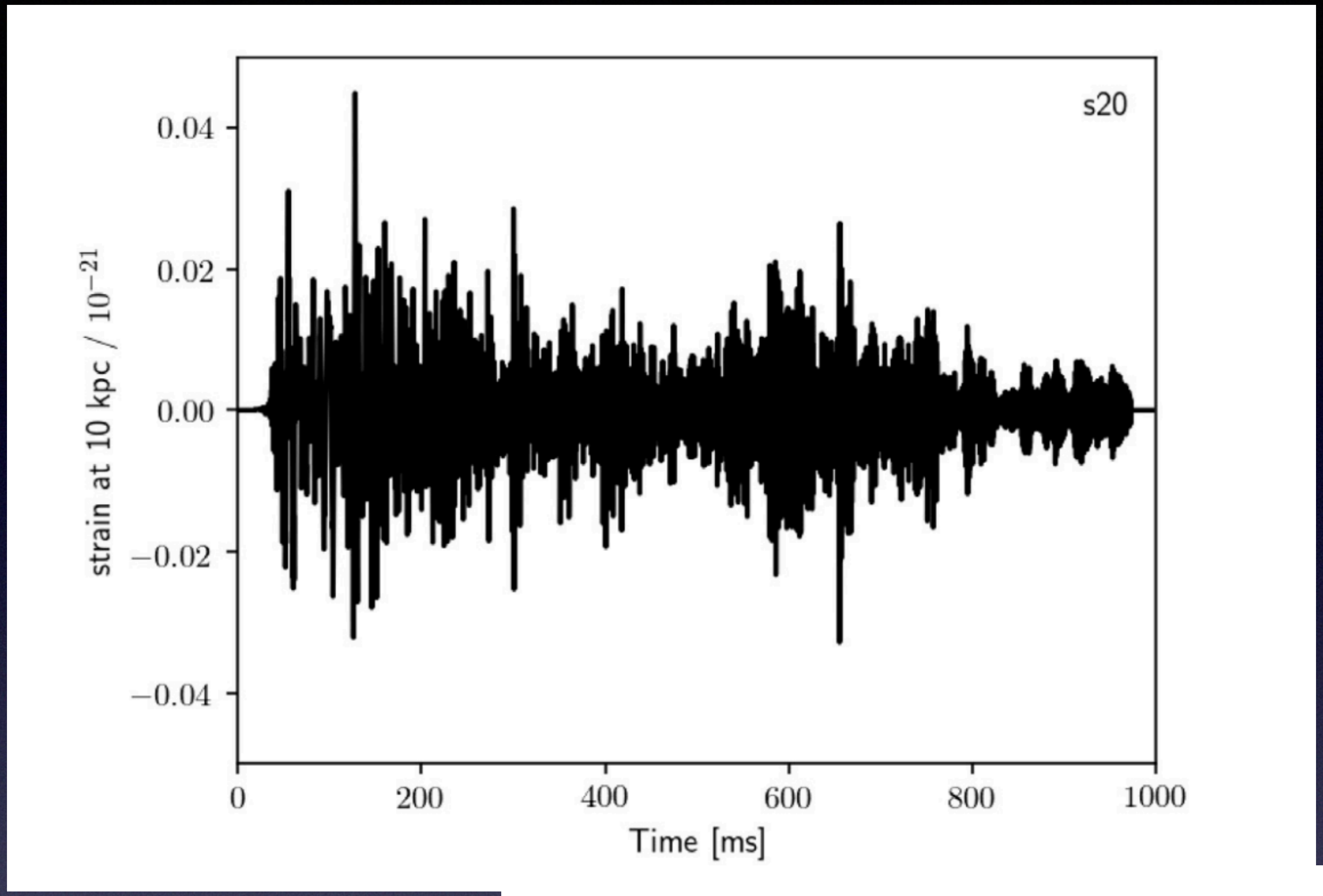
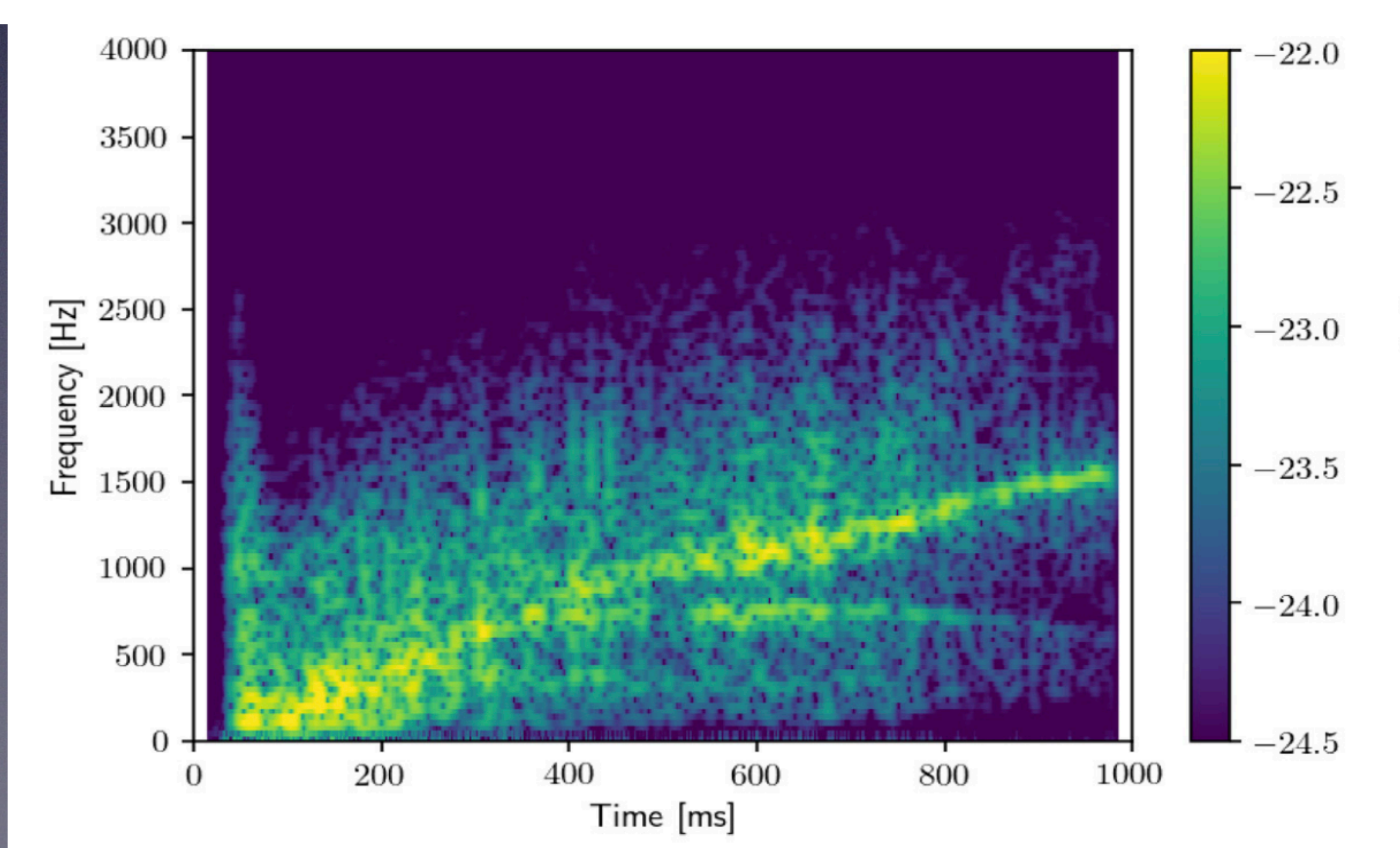
GW signal from CCSNe





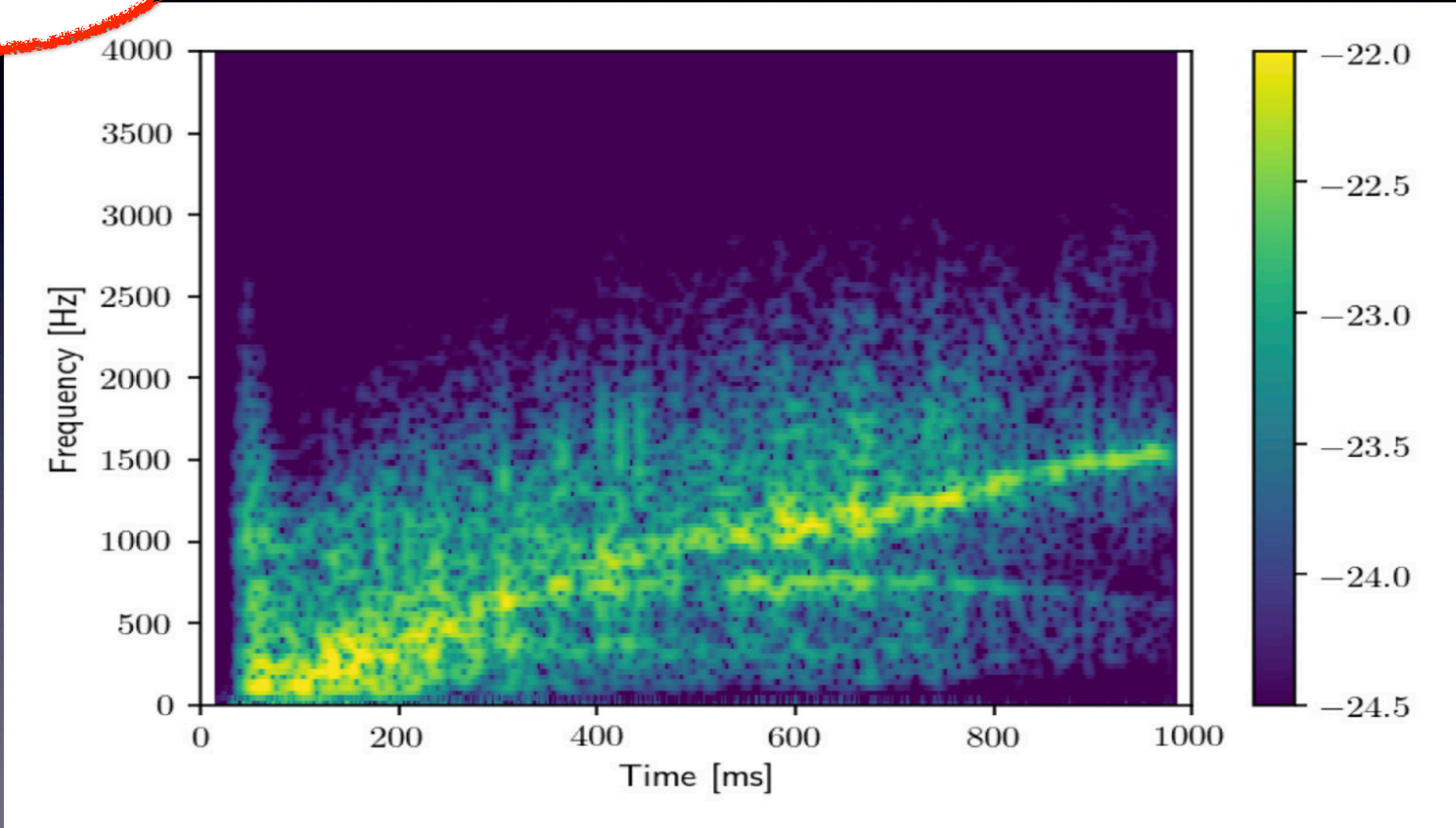
Highly stochastic!!

Spectrograms?

Torres-Forné et al. (2019)

Oscillation modes

Talk by Torres-Forné!



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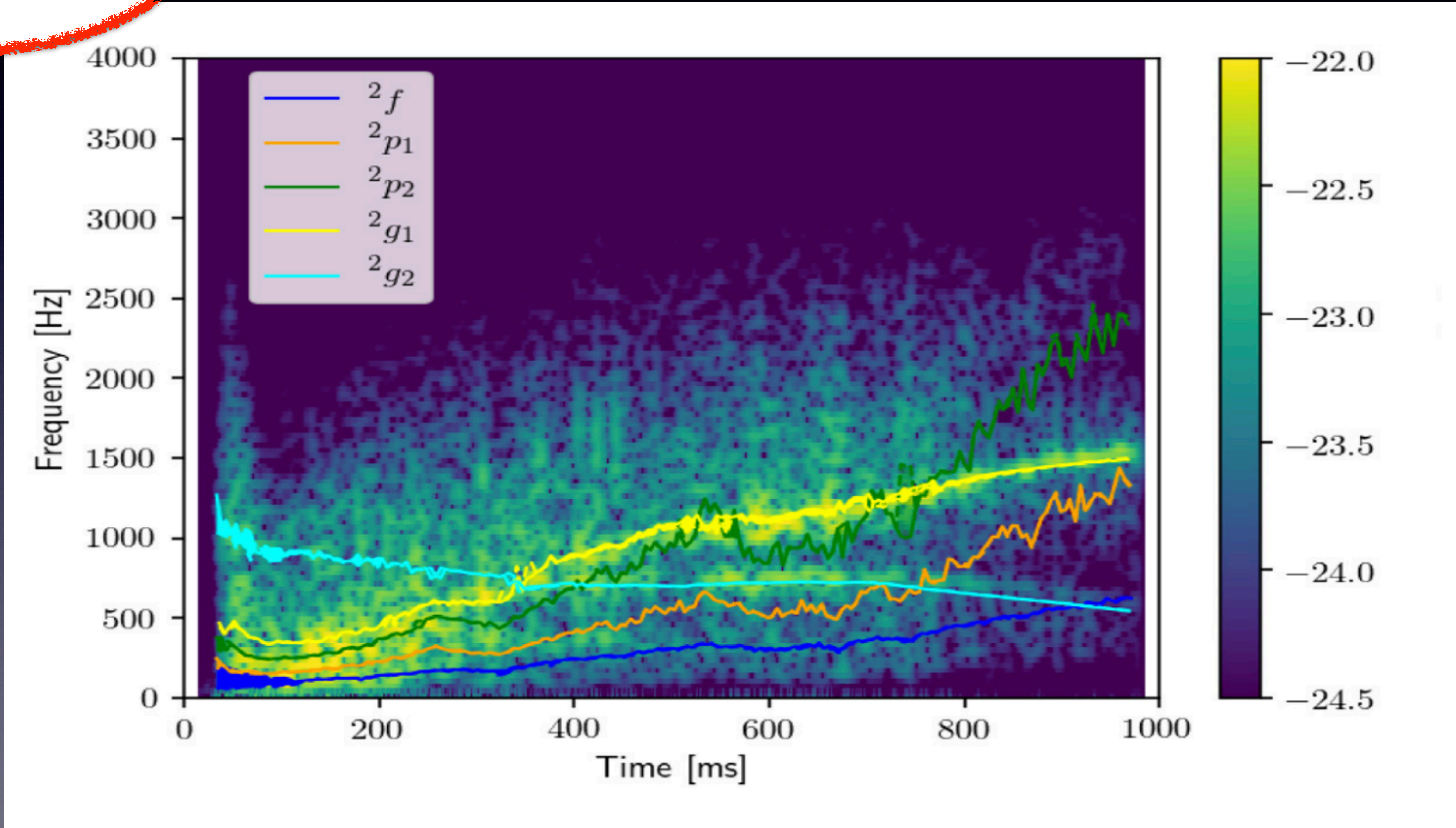
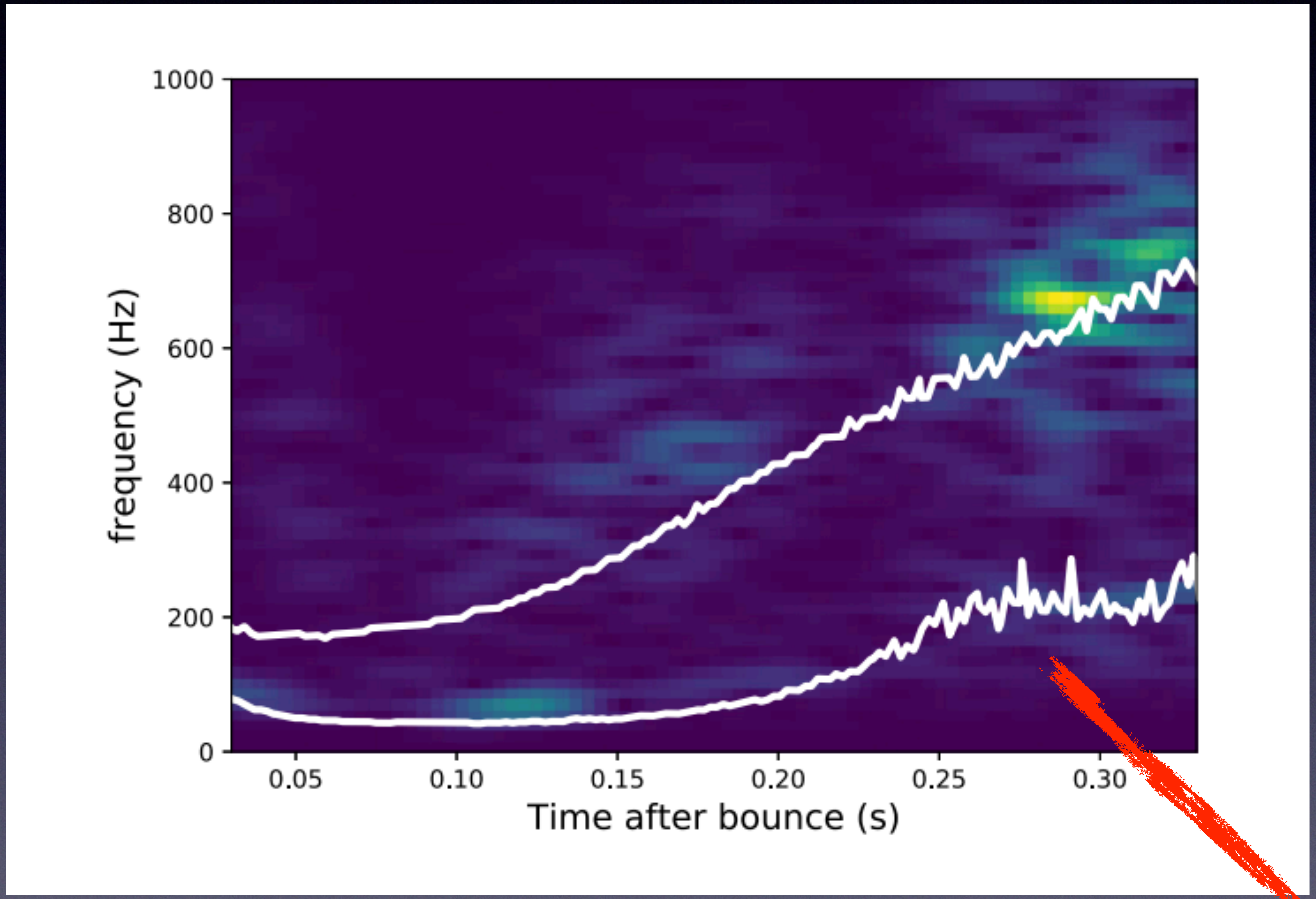


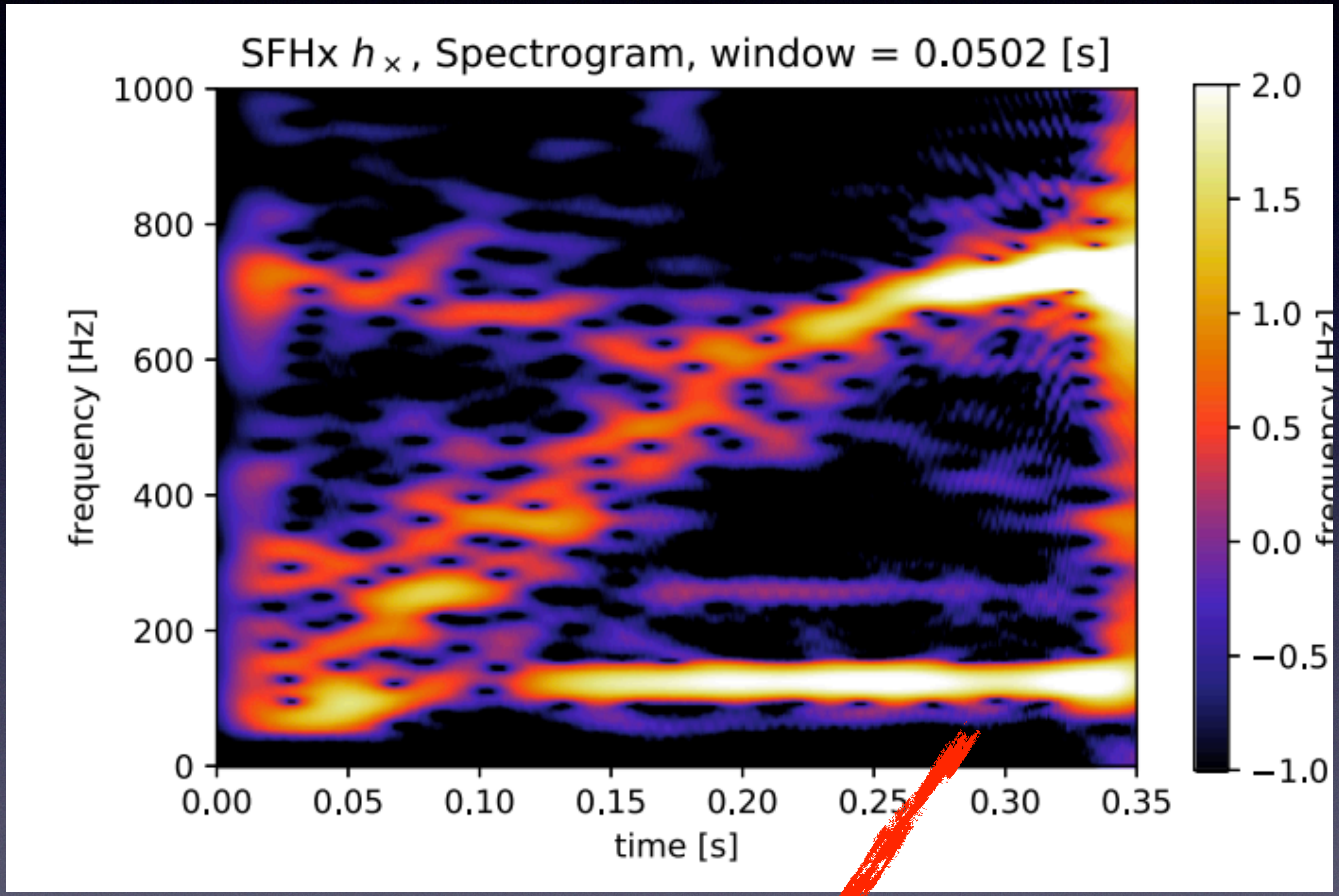
Torres-Forné et al. (2019)

BUT … there is more!

Powell et al. (2021)



Kawahara et al. (2018)



SASI

Standing Accretion Shock Instability

Asteroseismology

Recipe

- General Relativistic Hydrodynamic Equations
- Linear perturbation equations of a star in hydrostatic equilibrium
- Solve the eigenvalue problem
- Use data from CCSNe simulations
- Classify the modes
- Infer properties of the star (universal relations)

See Murphy's talk: Newtonian analysis!

See also talks by Mueller, Torres-Forné, Sotani!

Asteroseismology

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Eigenvalue Problem

The system of equations:

σ: eigenfrequencies

$$[\mathbf{A}] \cdot [\eta] = \sigma [\mathbf{B}] \cdot [\eta]$$

η: perturbed variables matrix

$A, \; B$: coefficient matrices

+ Rankine - Hugoniot Boundary Conditions at the shock

* Spectral methods
* Physics Informed Neural Networks (PINNs)

Spectral Methods

- Chebyshev collocation method
- Differentiation matrix D
- Multi-domain MN; M: # of domains, N: # of points
- Discretised system of S equations → Sx(MN+1) matrices

ADVANTAGES

Exponential convergence - fast!
Small grids
Discretised equations in matrix form
All modes (also complex!) are computed at once
Regularity of the solution is ensured

Cases of study

l=2

1. p- and g-modes in a sphere
   - Ideal fluid with $c_s^2, \Gamma_1 \rightarrow constant$ and $\upsilon \rightarrow 0$
   - Buoyant region
   - Varying Brunt–Vaisälä frequency $\mathcal{N}^2 \equiv \frac{\alpha^2}{\psi^4}\mathcal{G}_\alpha\mathcal{B}$

2. Plane Waves in an 1D flow

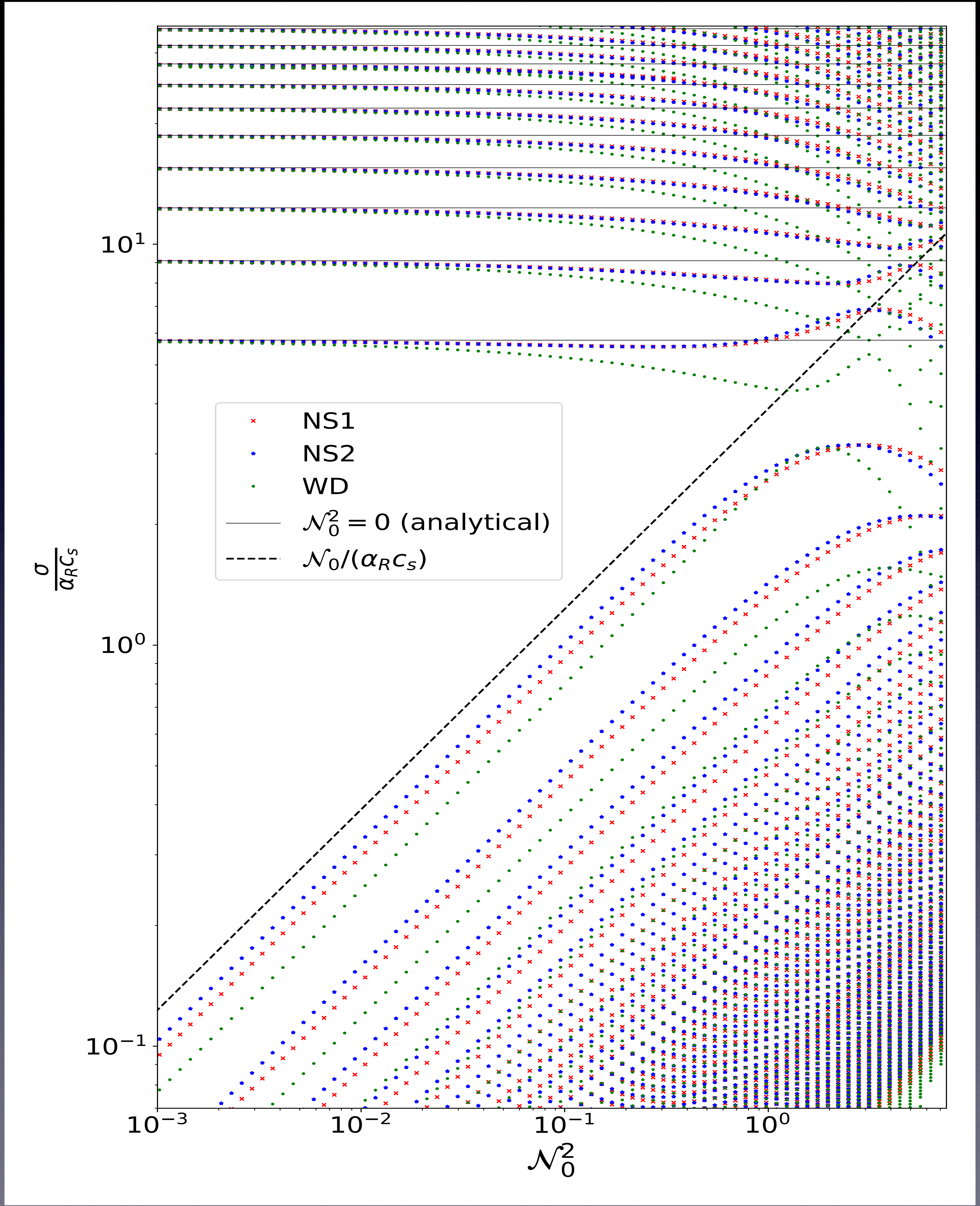
3. Accretion flow in a fluid sphere

- $N^2 = 0$
- Sound speed, pressure, density → constant
- Lapse → general
- Velocity < 0 and subsonic

Case I: Eigenfrequencies

Models

model	M ($M_\odot$)	R(km)	Γ_1	c_s^2	α_R
NS1	1.4	10	2	0.1	0.81
NS2	2.0	10	2	0.1	0.74
WD	1.2	$5 \cdot 10^3$	4/3	0.1	0.84

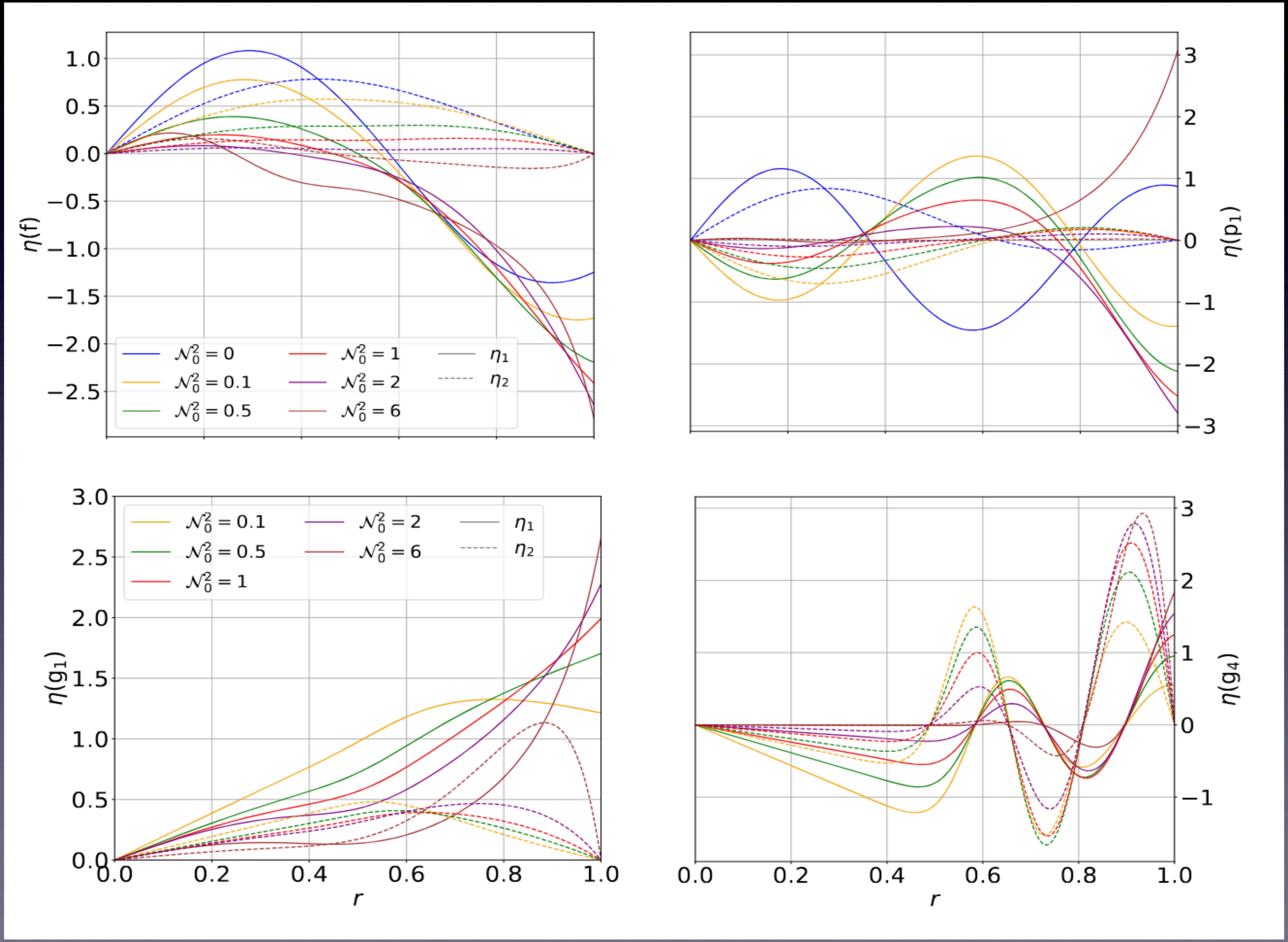


DT et al. (2025)
arXiv:2503.16317

Case I: Eigenfunctions

p-modes

g-modes



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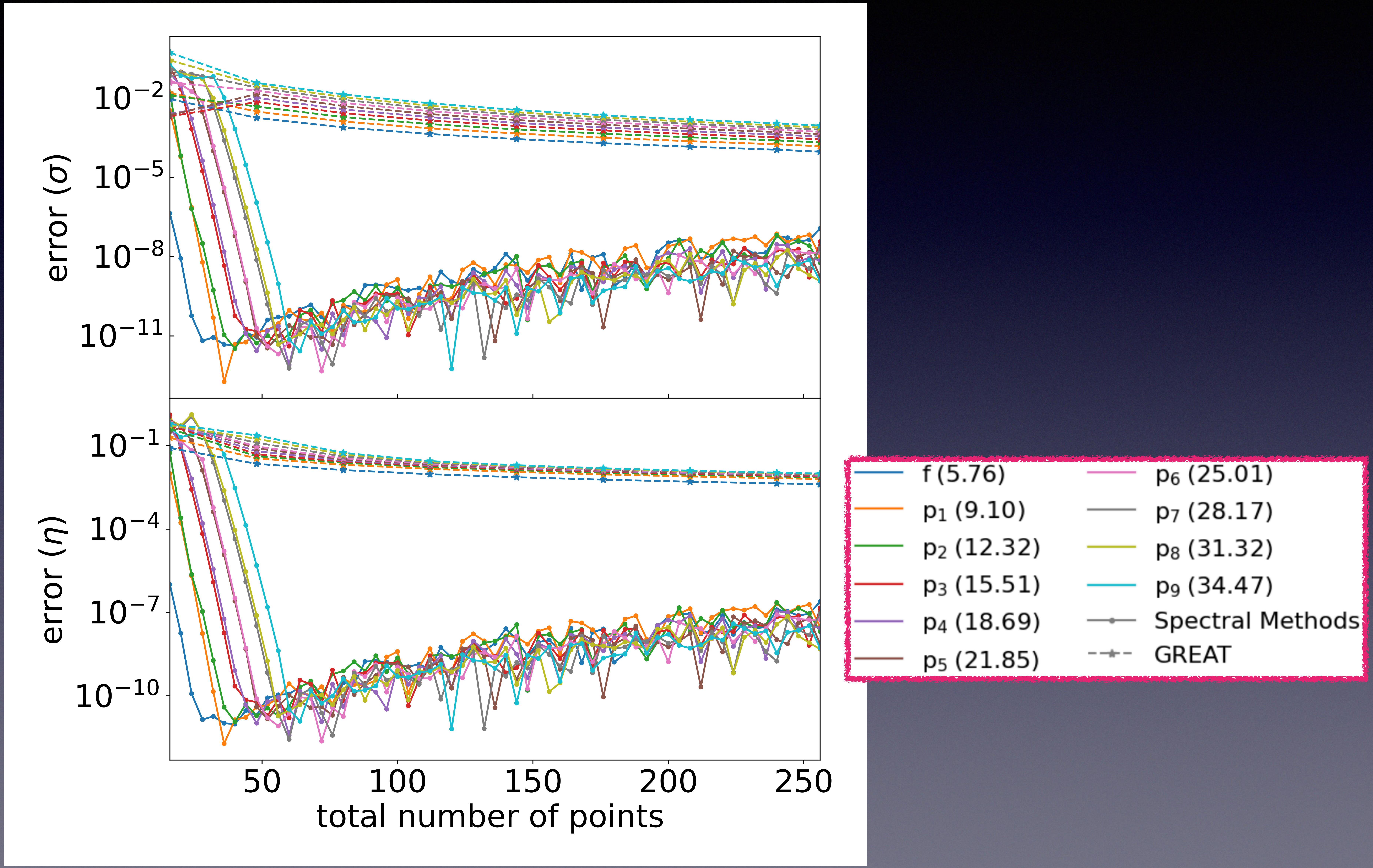
Case I: Convergence

- $N^2 = 0$
- Comparison to analytical solutions



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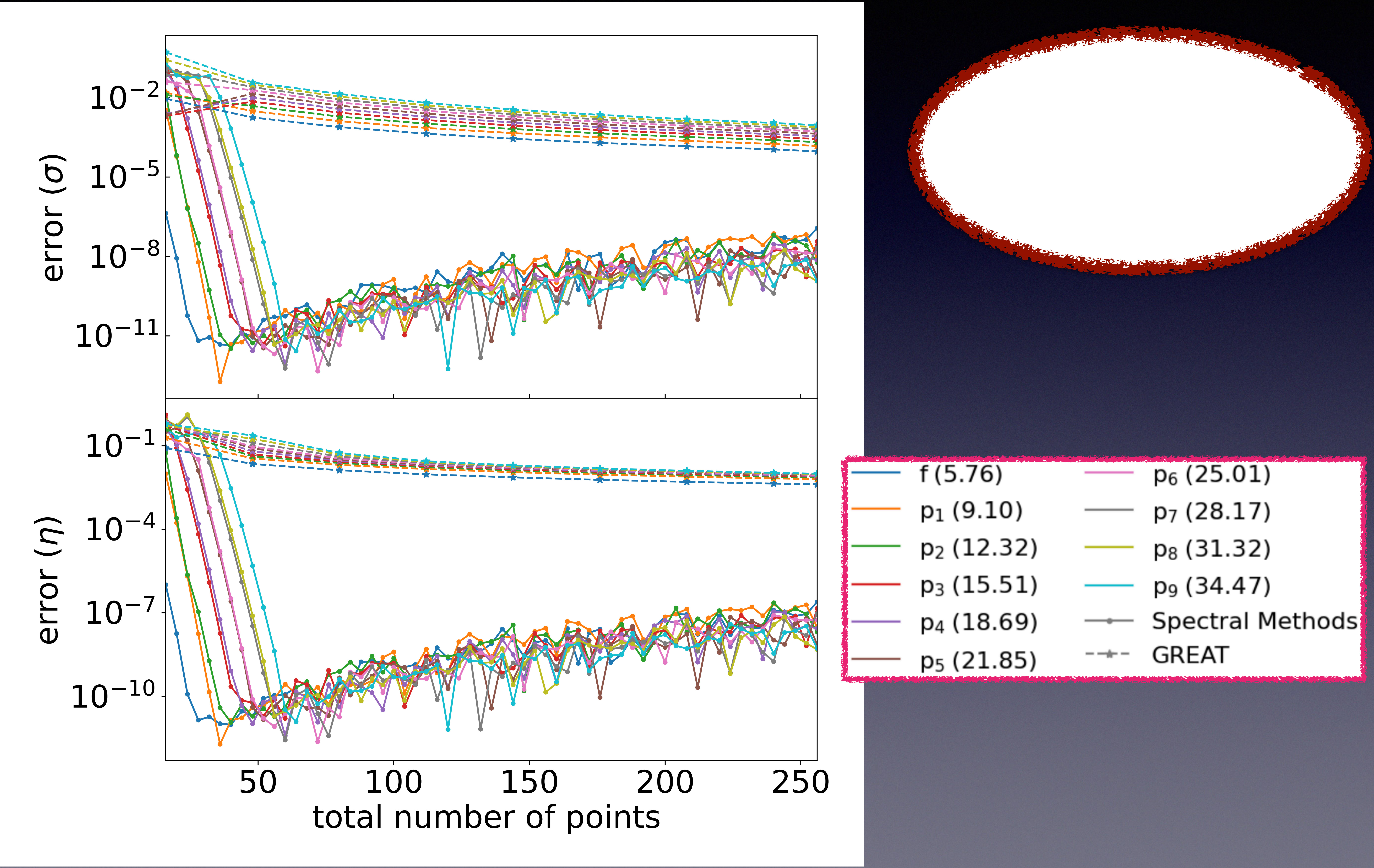

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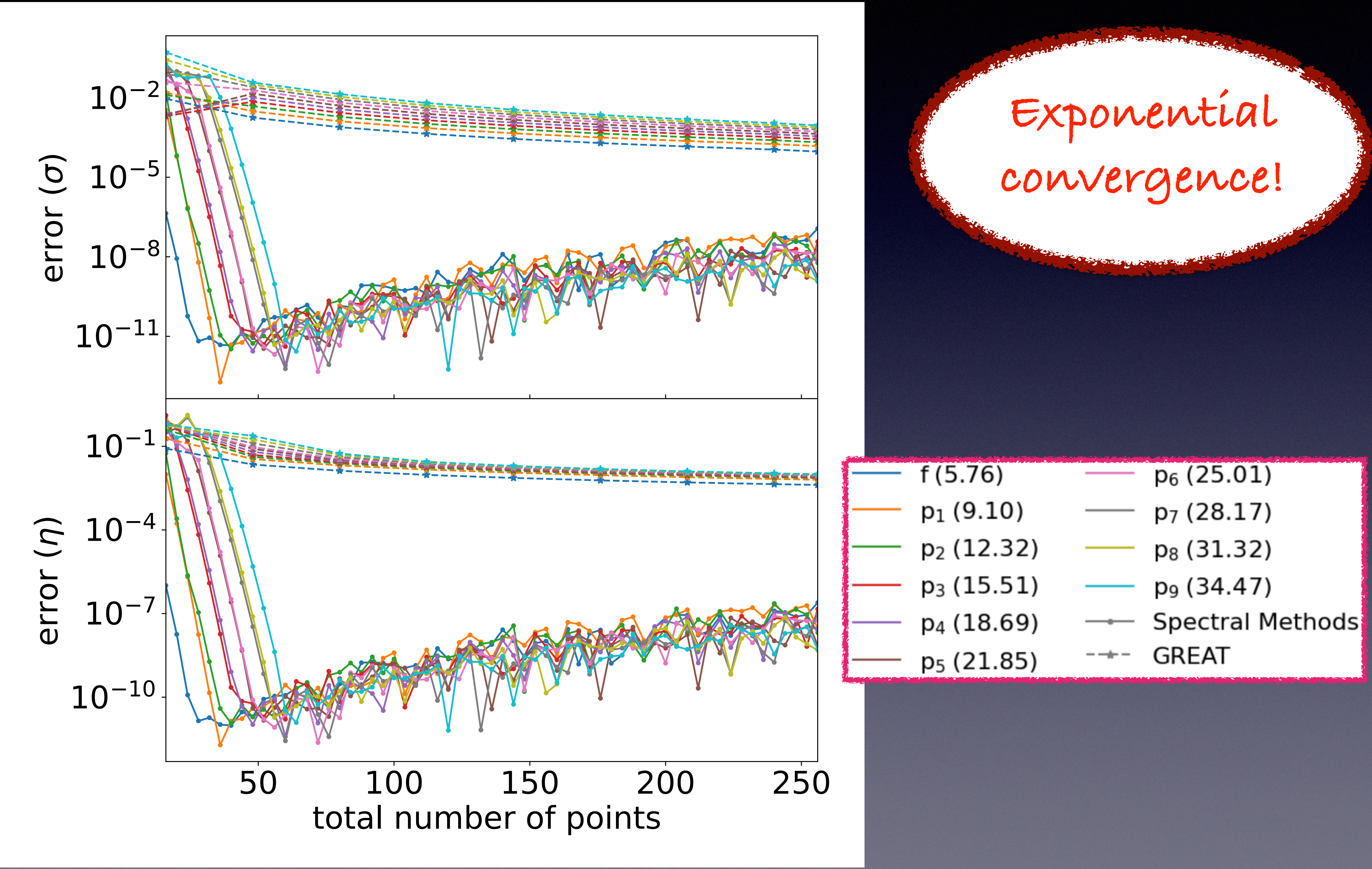
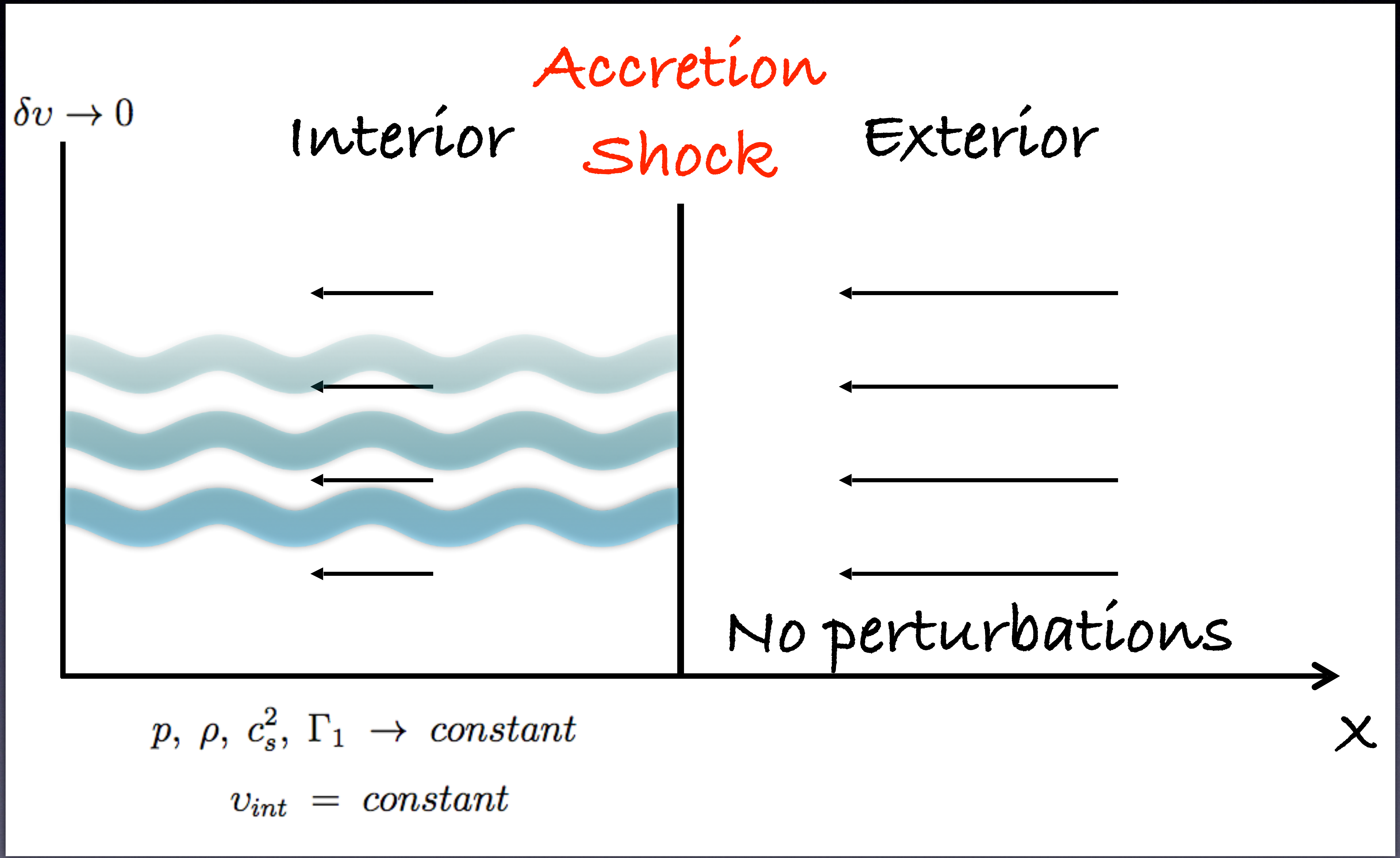


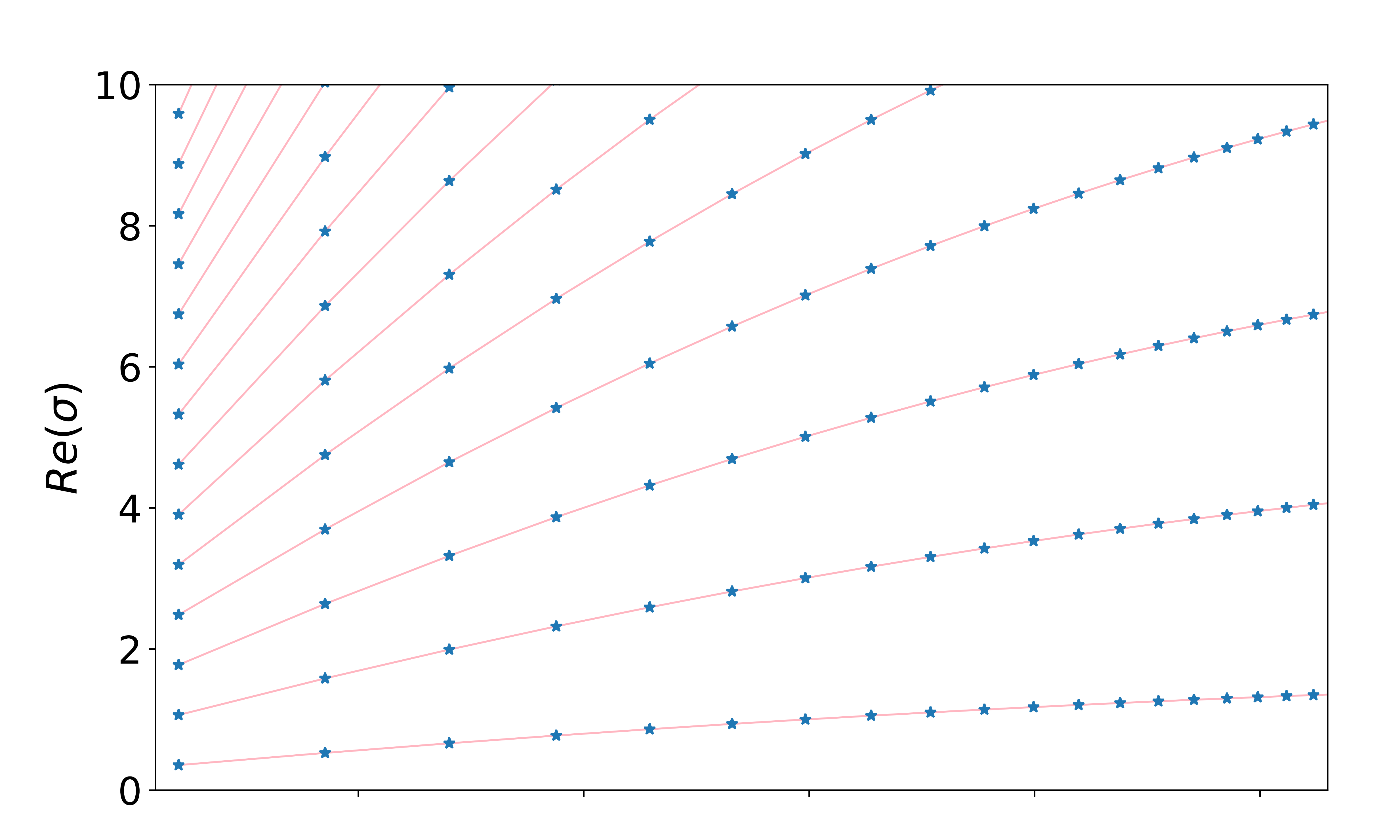
Exponential convergence!

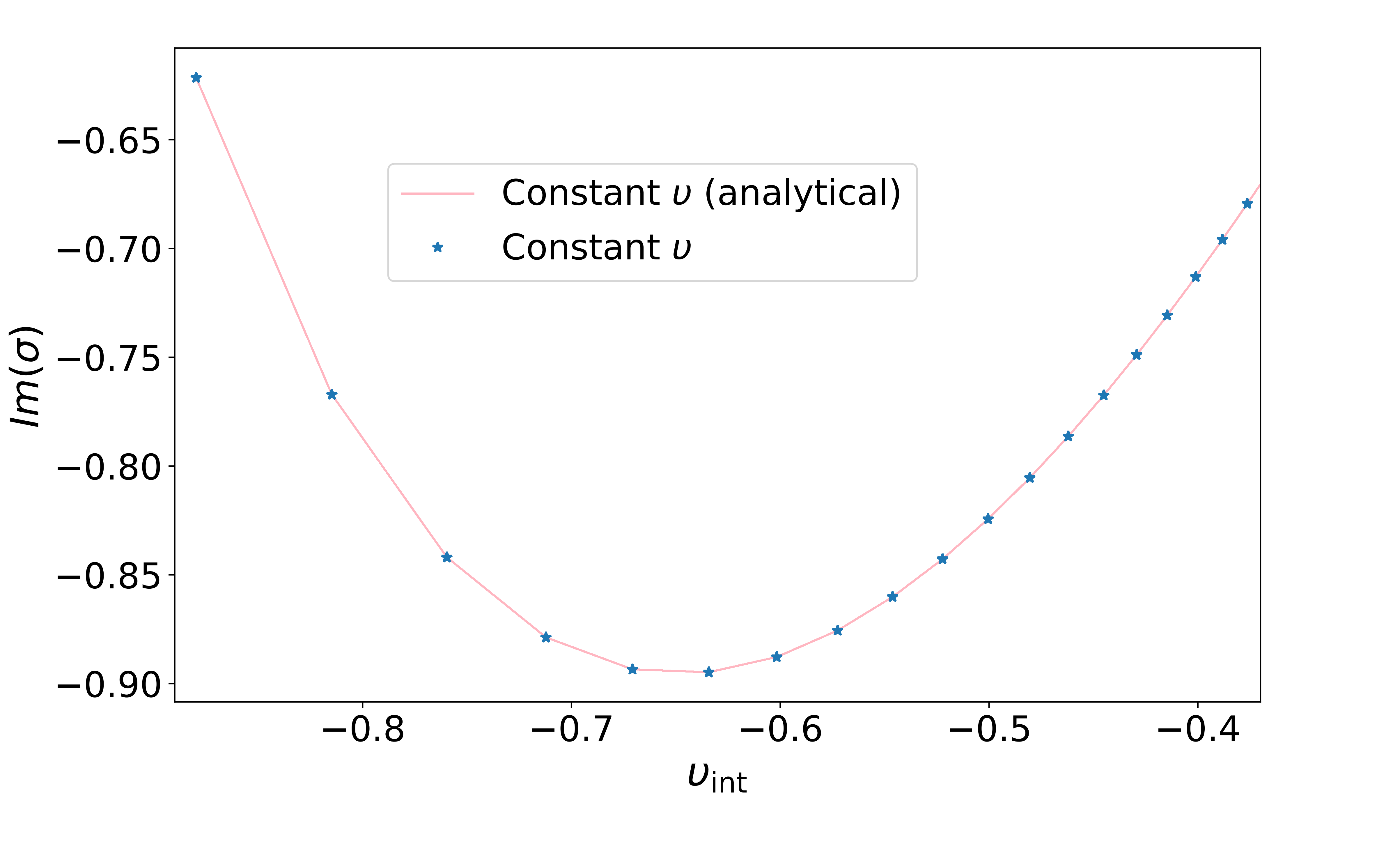
Case II: Plane Waves

Newtonian analysis



Case II: Eigenfrequencies





- ☆ Analytical and numerical solutions agree
- ☆ p-modes with decreasing frequency
- ☆ Frequencies have imaginary part
- ☆ Stable modes

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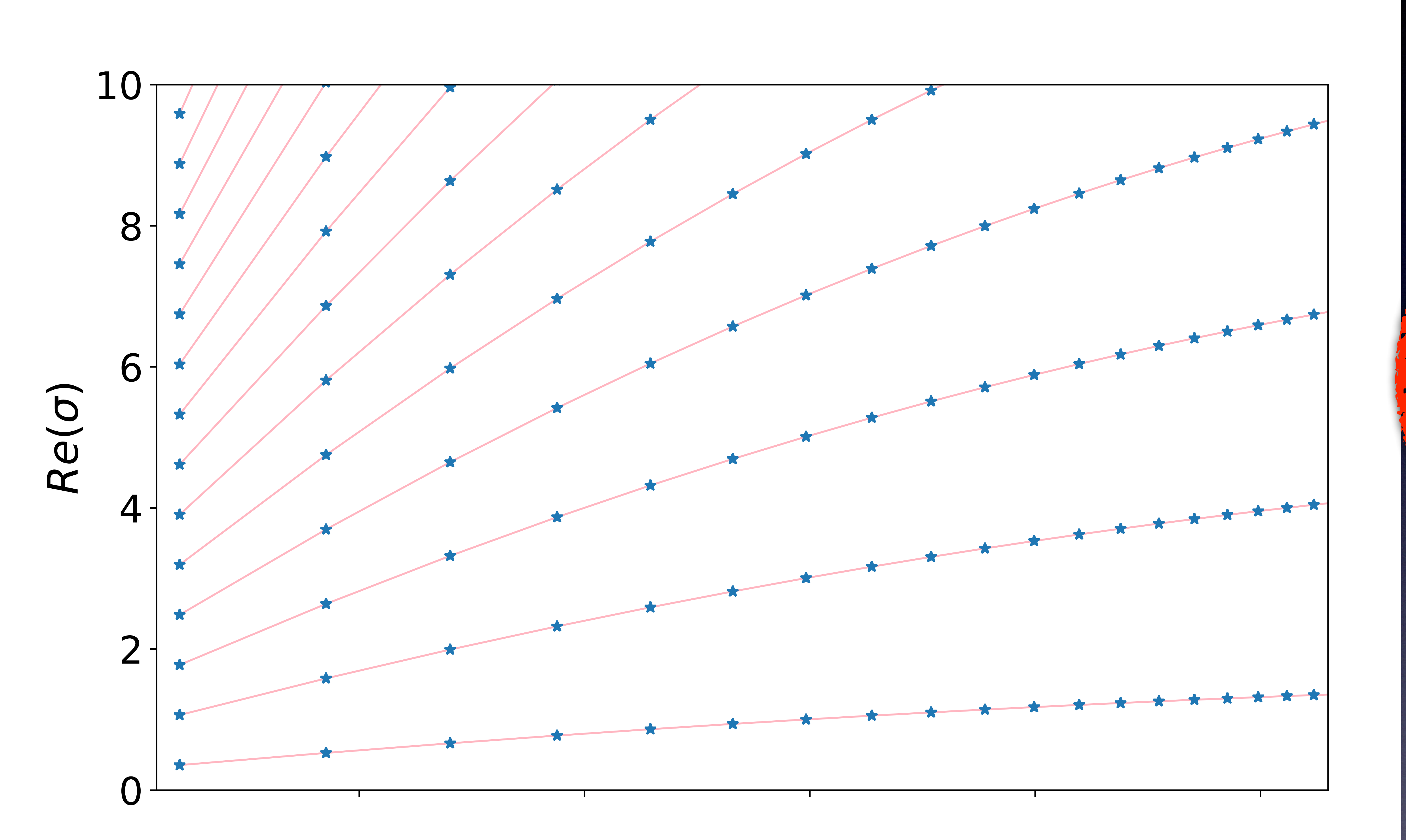
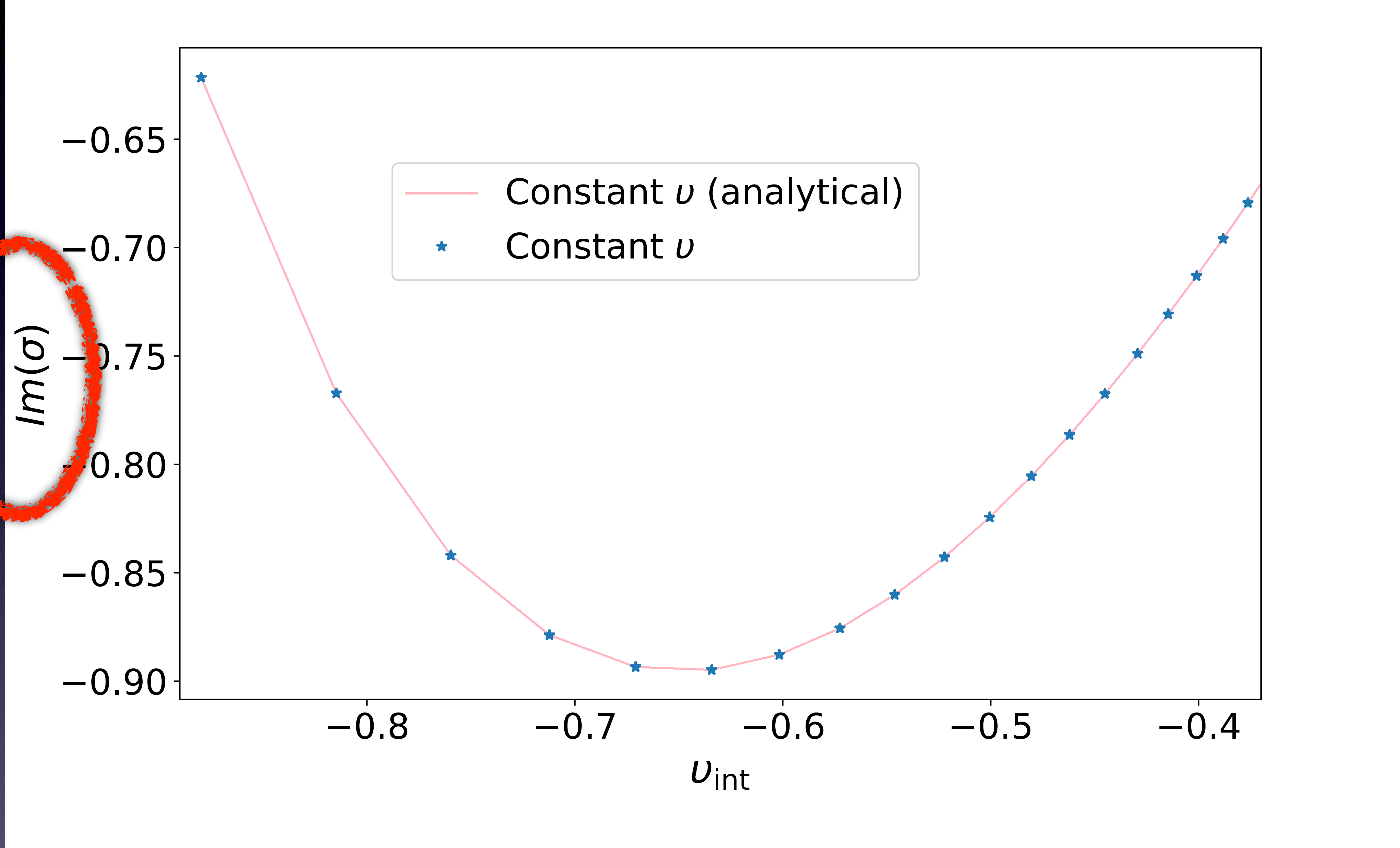
Case II: Eigenfrequencies





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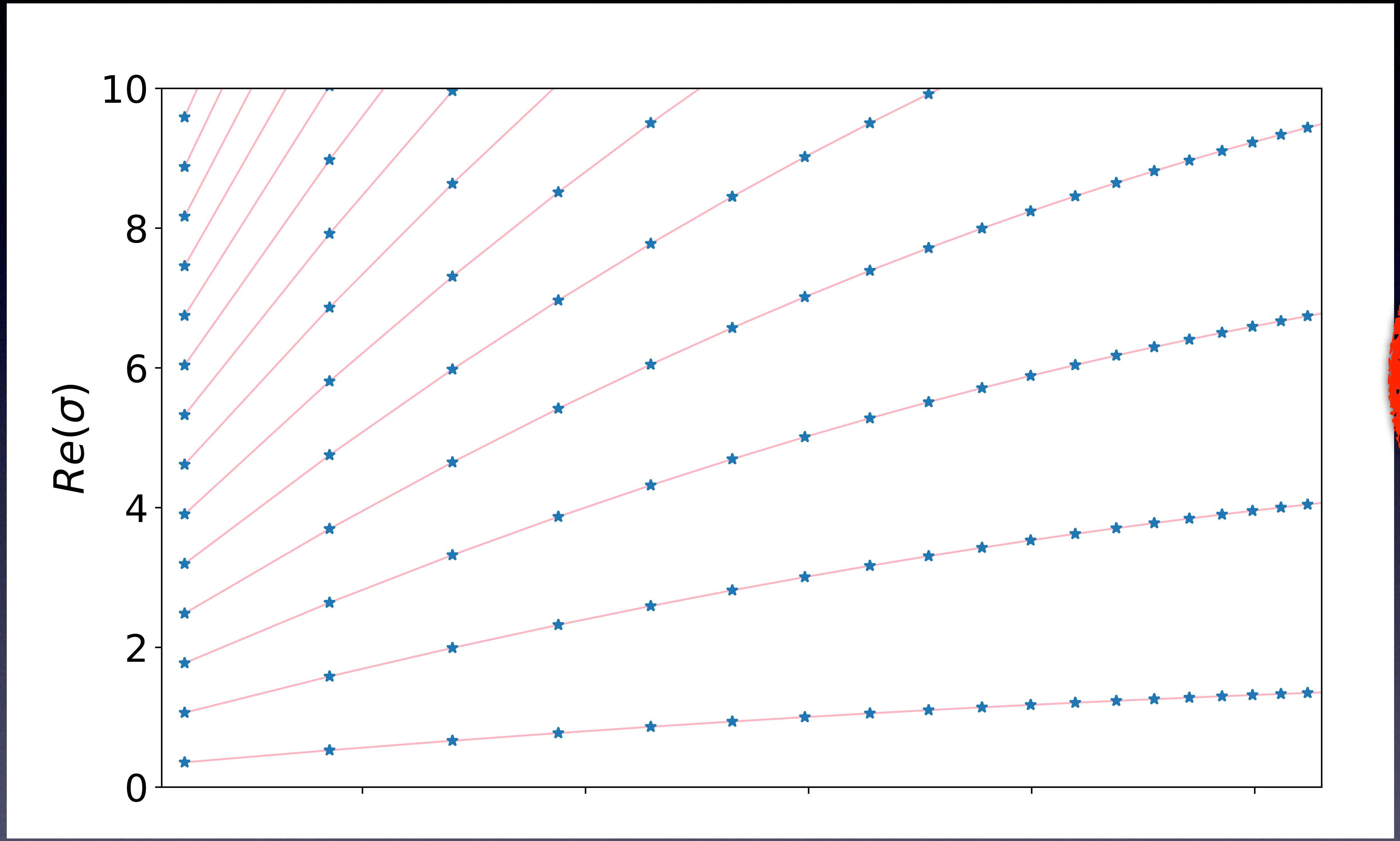

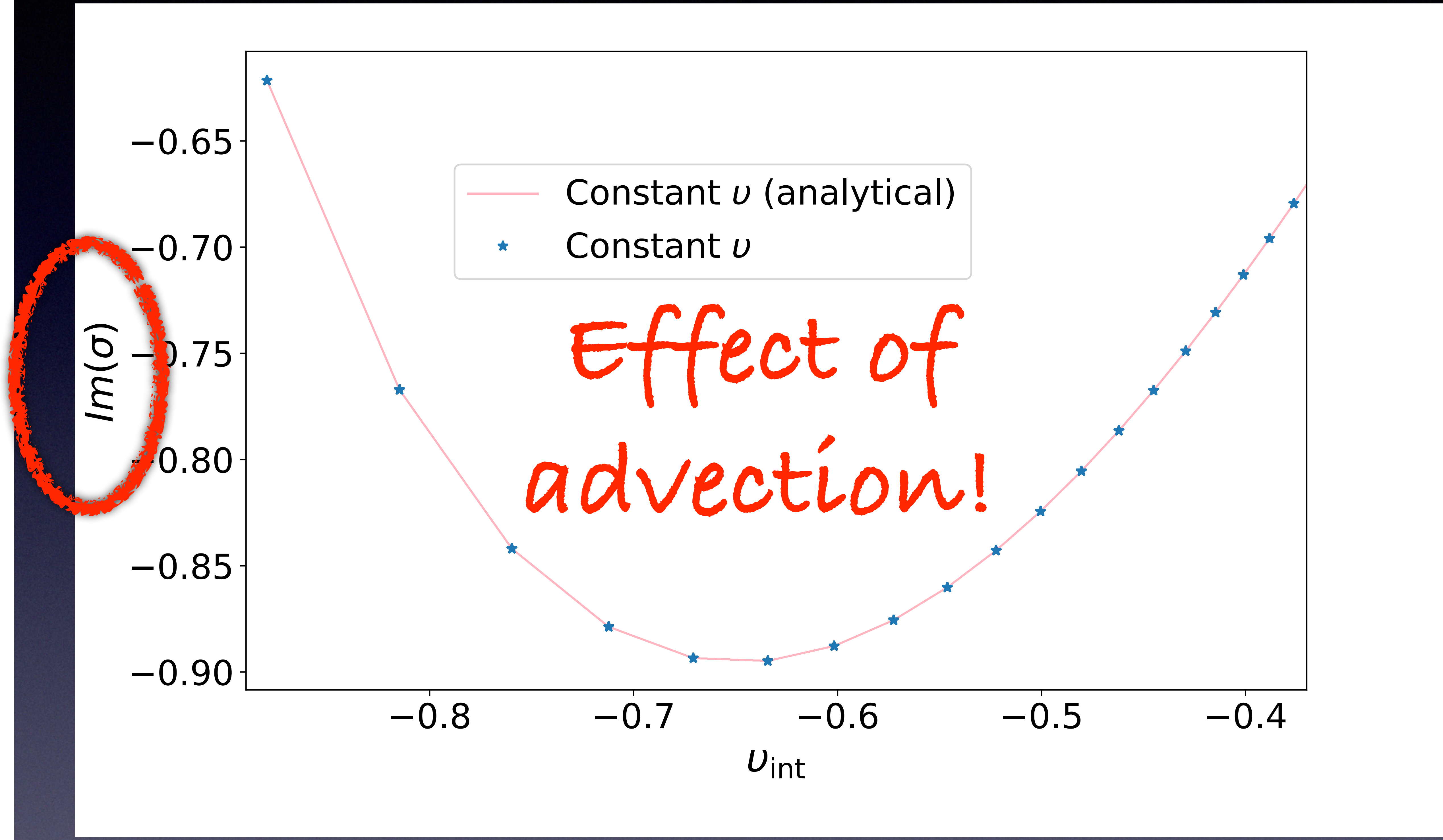

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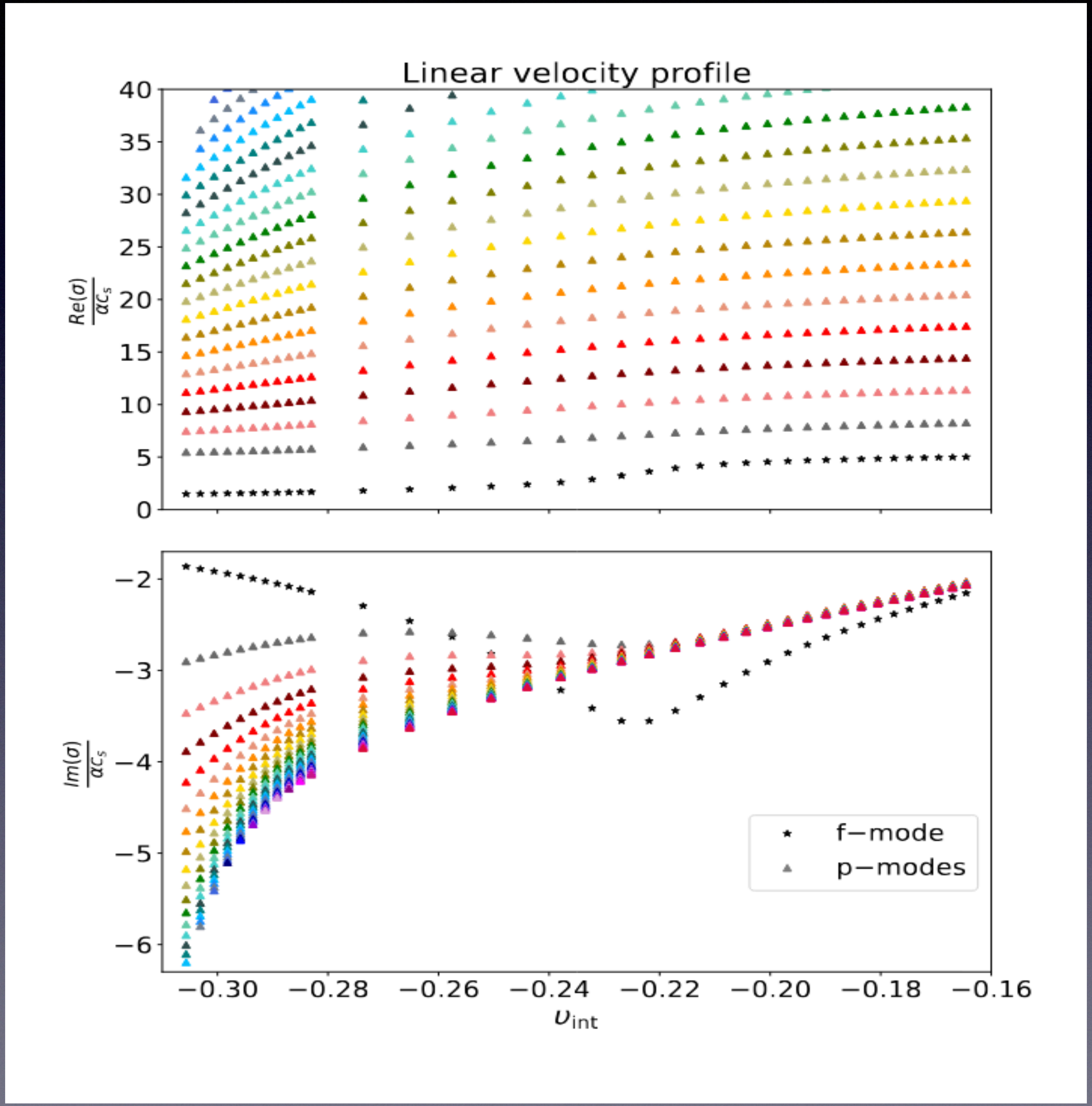
Case III: Eigenfrequencies

STABLE

Real part
of the frequency

Imaginary part
of the frequency



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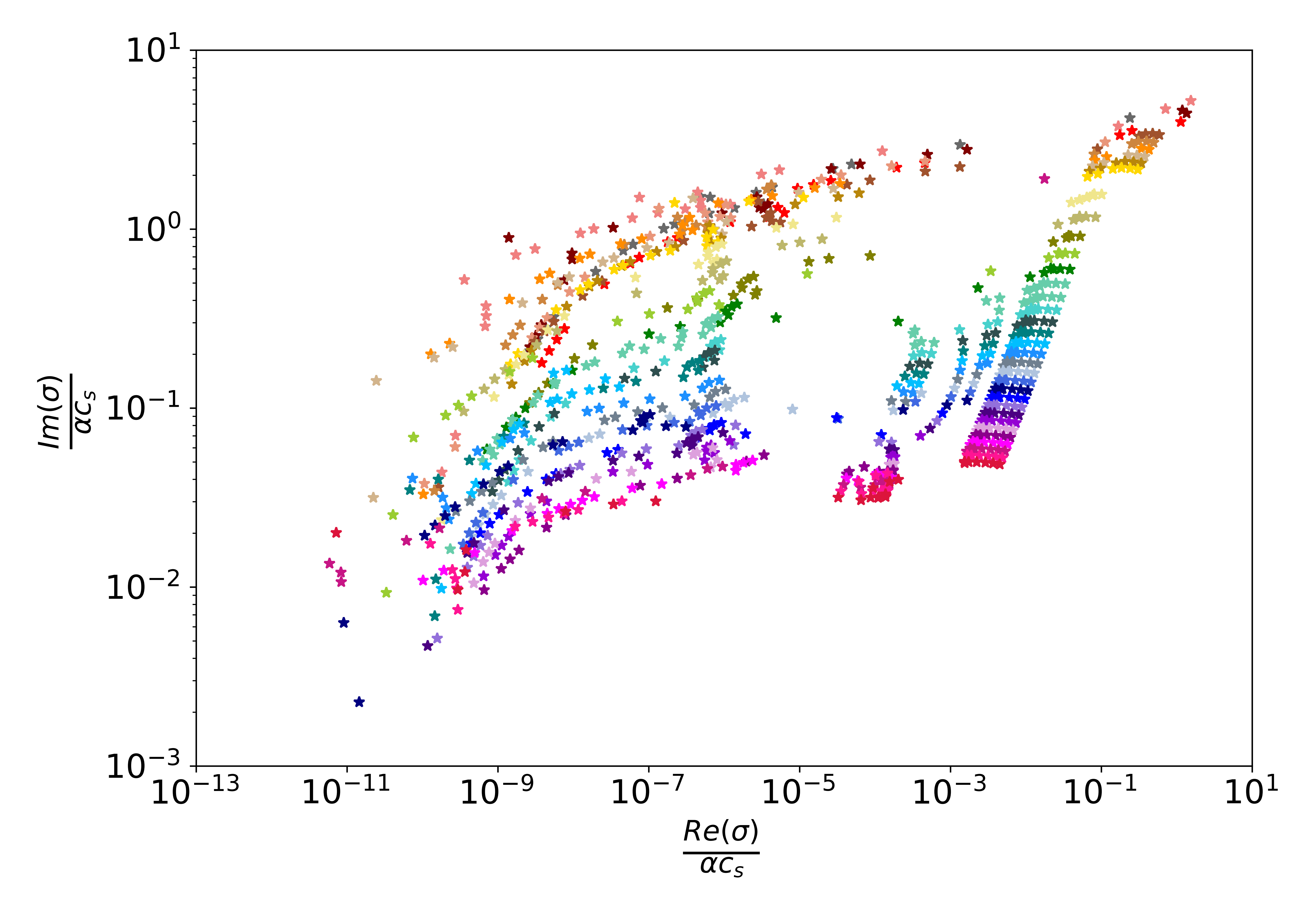
Case III: Eigenfrequencies

UNSTABLE

Colour scale: interior velocity



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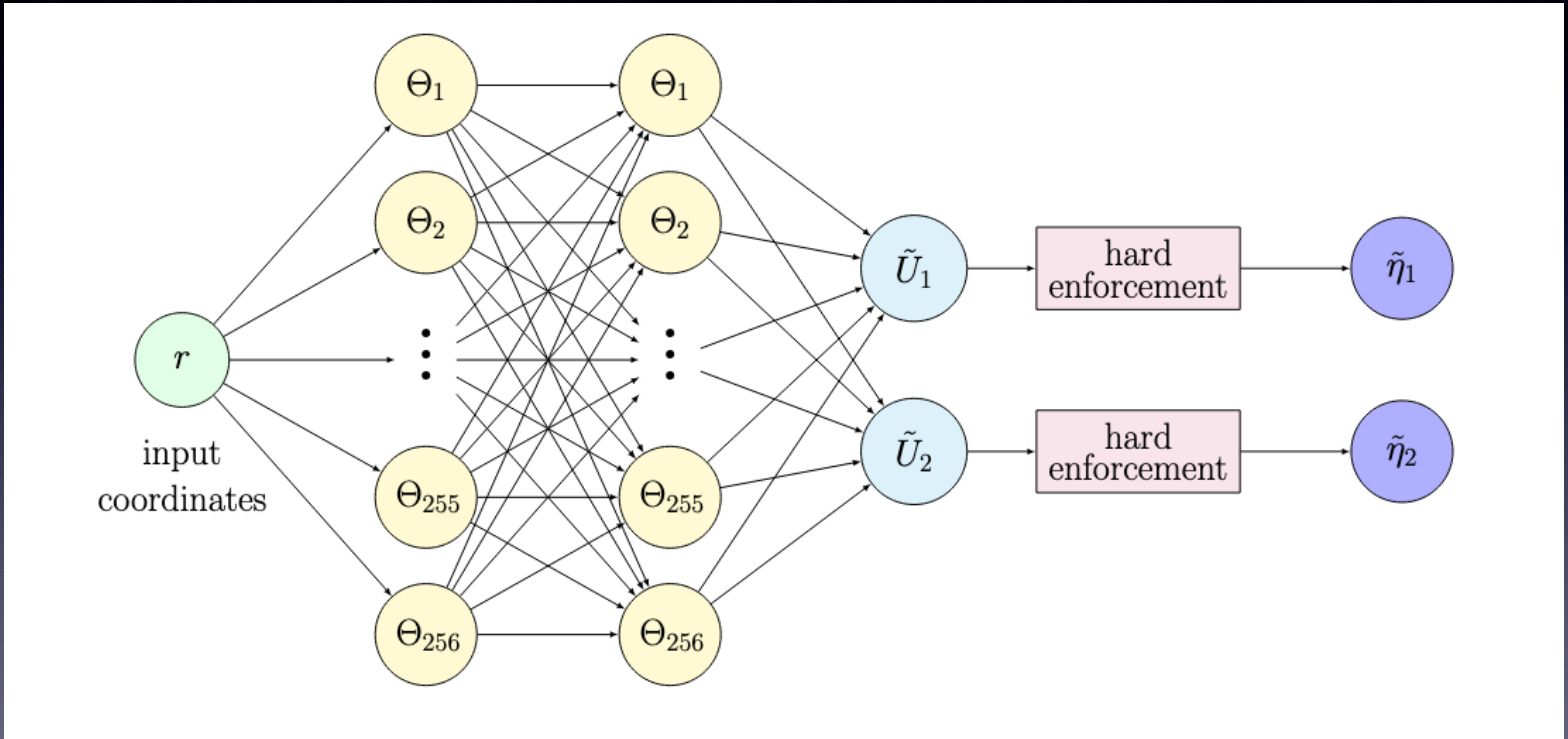
Implementing PINNs

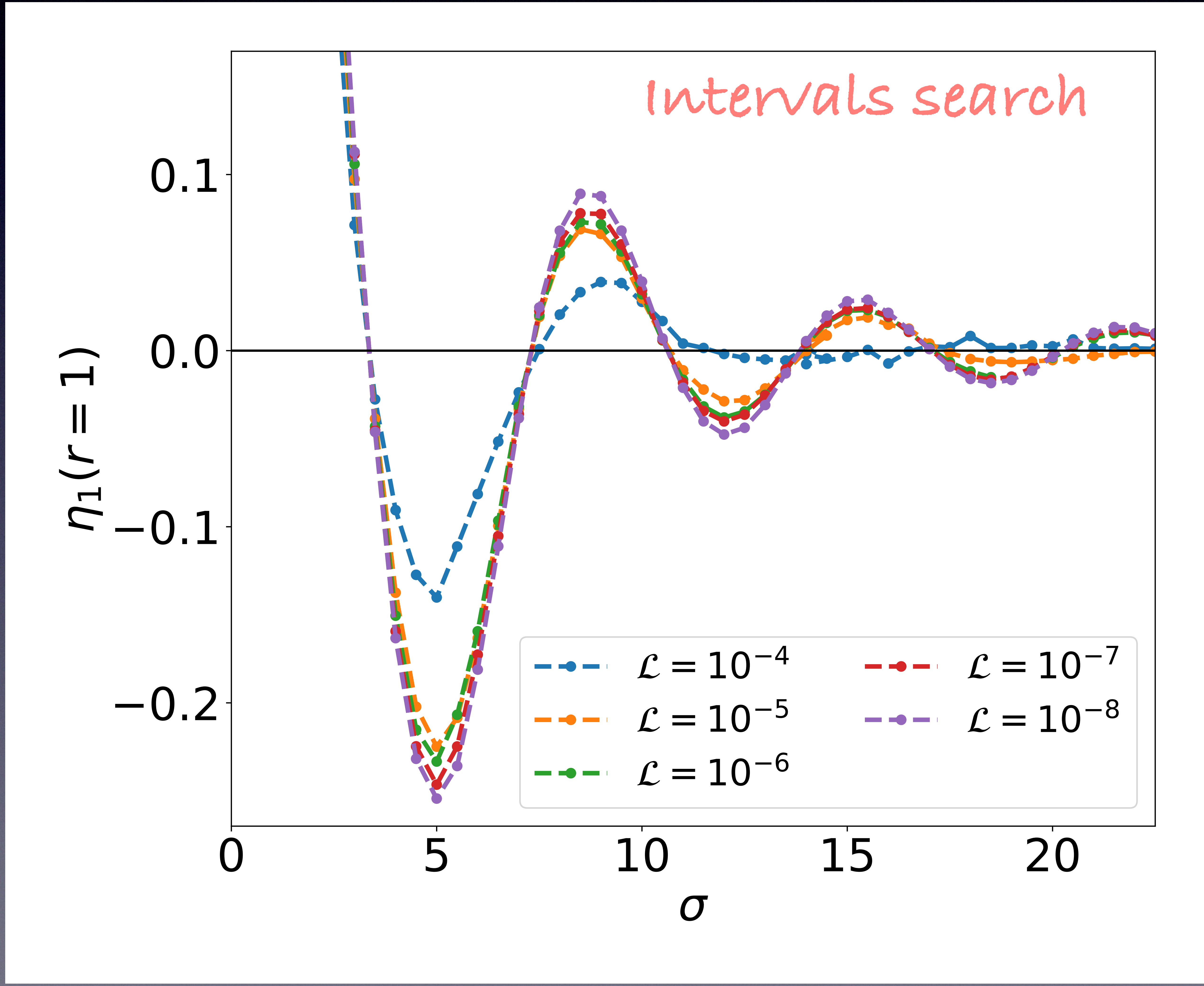


DT et al. (2025)
(arXiv:2504.12183)

PINNs I

- Static sphere of radius unity, density and sound speed
- $N^2 = 0$
- Analytical solutions

1. Find the intervals containing eigenvalues (s-PINN)

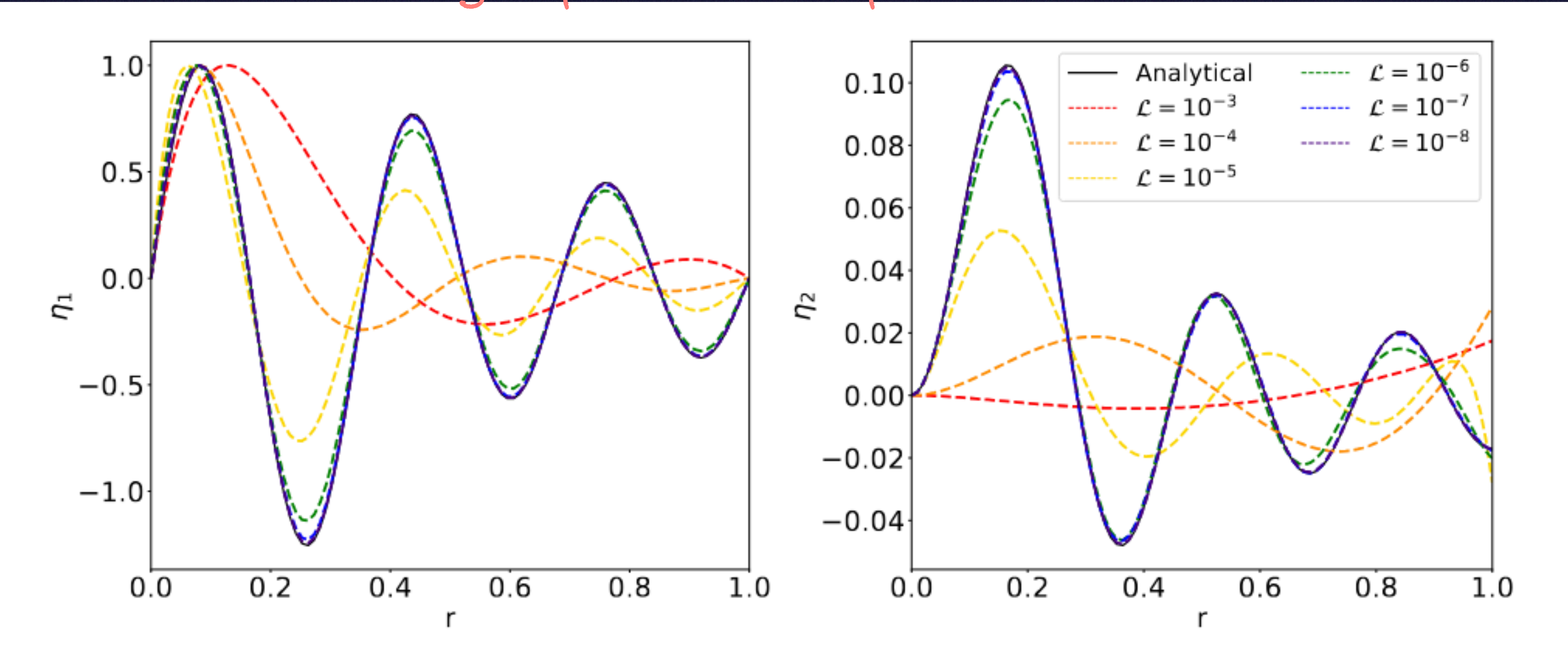


DT et al. (2025)
(arXiv:2504.12183)

PINNs II

2. Calculate the eigenvalues using:
   a. Bisection method (s-PINN) **vs**
   b. f-PINN: the frequency as a parameter

Eigenfunctions of the 5th mode



Summary

Asteroseismology of CCSNe is very important because it could allow us to infer properties of the PNS directly from GW observations

✓ Included for the first time accretion flow + standing accretion shock

✓ Introduced two eigenvalue solvers: Spectral Methods + PINNs

Current & Future work

- Realistic cases ⟶ data from CCSNe simulations (SASI?)
- Full GR cases (non-Cowling)
- Extension to BNS (A. Revilla-Peña, R. Bondarescu)


Thank you for your attention!
Dziękuję za uwagę!

System of equations

General Relativistic Euler equations:

$$\boxed{\frac{1}{\sqrt{\gamma}}\partial_t\left[\sqrt{\gamma}D\right]+\frac{1}{\sqrt{\gamma}}\partial_i\left[\sqrt{\gamma}Dv^{*i}\right]=0}$$

$$\boxed{\frac{1}{\sqrt{\gamma}}\partial_t\left[\sqrt{\gamma}S_j\right]+\frac{1}{\sqrt{\gamma}}\partial_i\left[\sqrt{\gamma}S_jv^{*i}\right]+\alpha\partial_j p=\frac{\alpha\rho h}{2}u^\mu u^\nu\partial_j g_{\mu\nu}}$$

Background configuration:

- Non-rotating spherically symmetric star
- In equilibrium
- But NOT static

Perturbations $q \to q + \delta q$

Decomposition of the perturbations:

$$\delta q = \delta\hat{q}(r) Y_{lm}(\theta,\phi) e^{-i\sigma t}$$

$$\xi^r = \eta_1 Y_{lm} e^{-i\sigma t},$$
$$\xi^\theta = \eta_2 \frac{1}{r} \partial_\theta Y_{lm} e^{-i\sigma t},$$
$$\xi^\phi = \eta_2 \frac{1}{r\sin^2\theta} \partial_\phi Y_{lm} e^{-i\sigma t},$$

Assumptions:

- Cowling approximation $\delta\hat{\alpha} = 0 = \delta\hat{\psi}$
- Adiabatic perturbations

$$\left.\frac{\partial p}{\partial \rho}\right|_{\text{adiabatic}} = h c_s^2 = \frac{p}{\rho}\Gamma_1$$

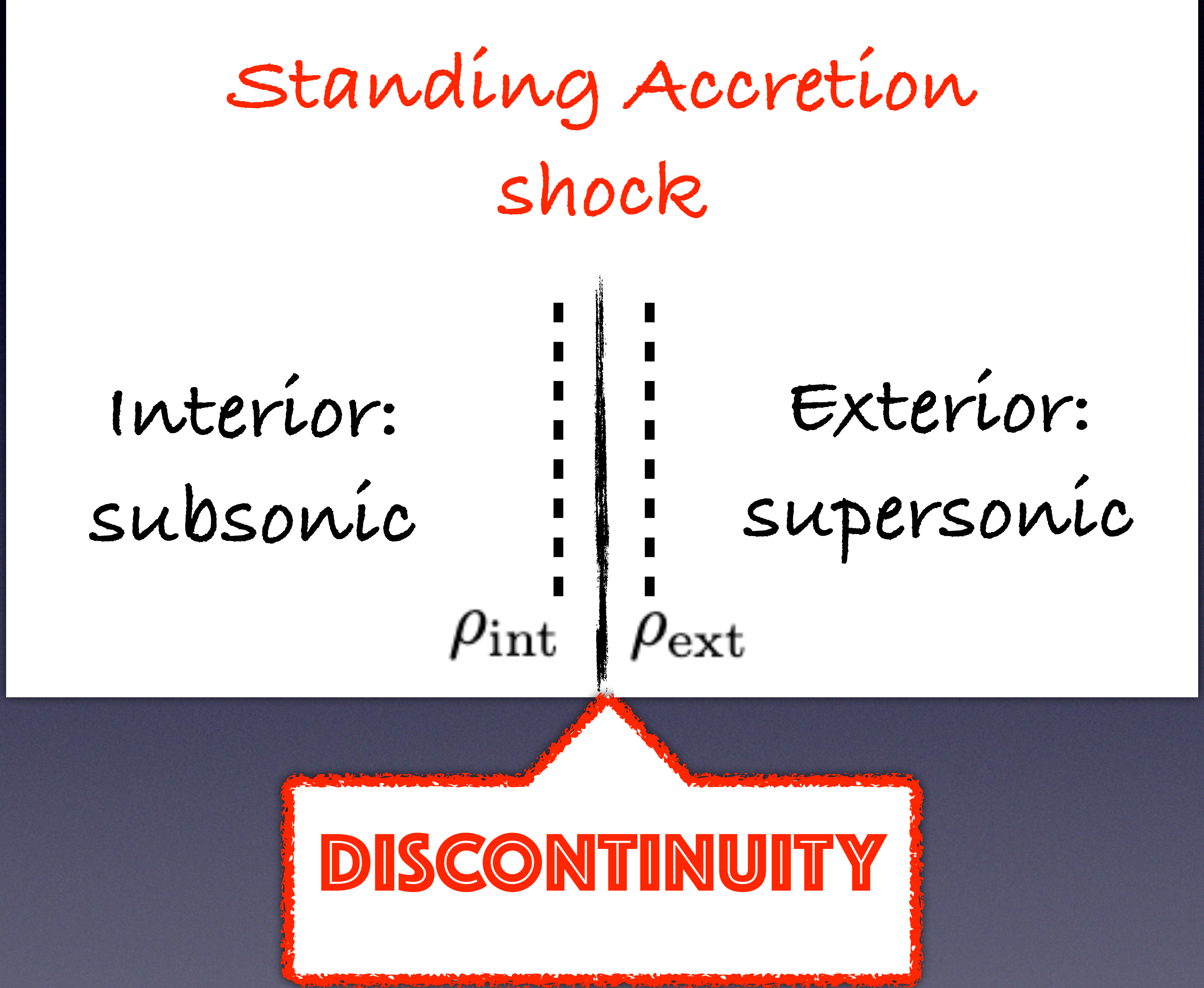
Rankine-Hugoniot conditions

General Relativistic

$$[[\rho u^{\mu}]]n_{\mu} = 0,$$

$$[[T^{\mu\nu}]]n_{\nu} = 0,$$

- Normal vector to the shock
- $[[F]] = F_{\rm int} - F_{\rm ext}$

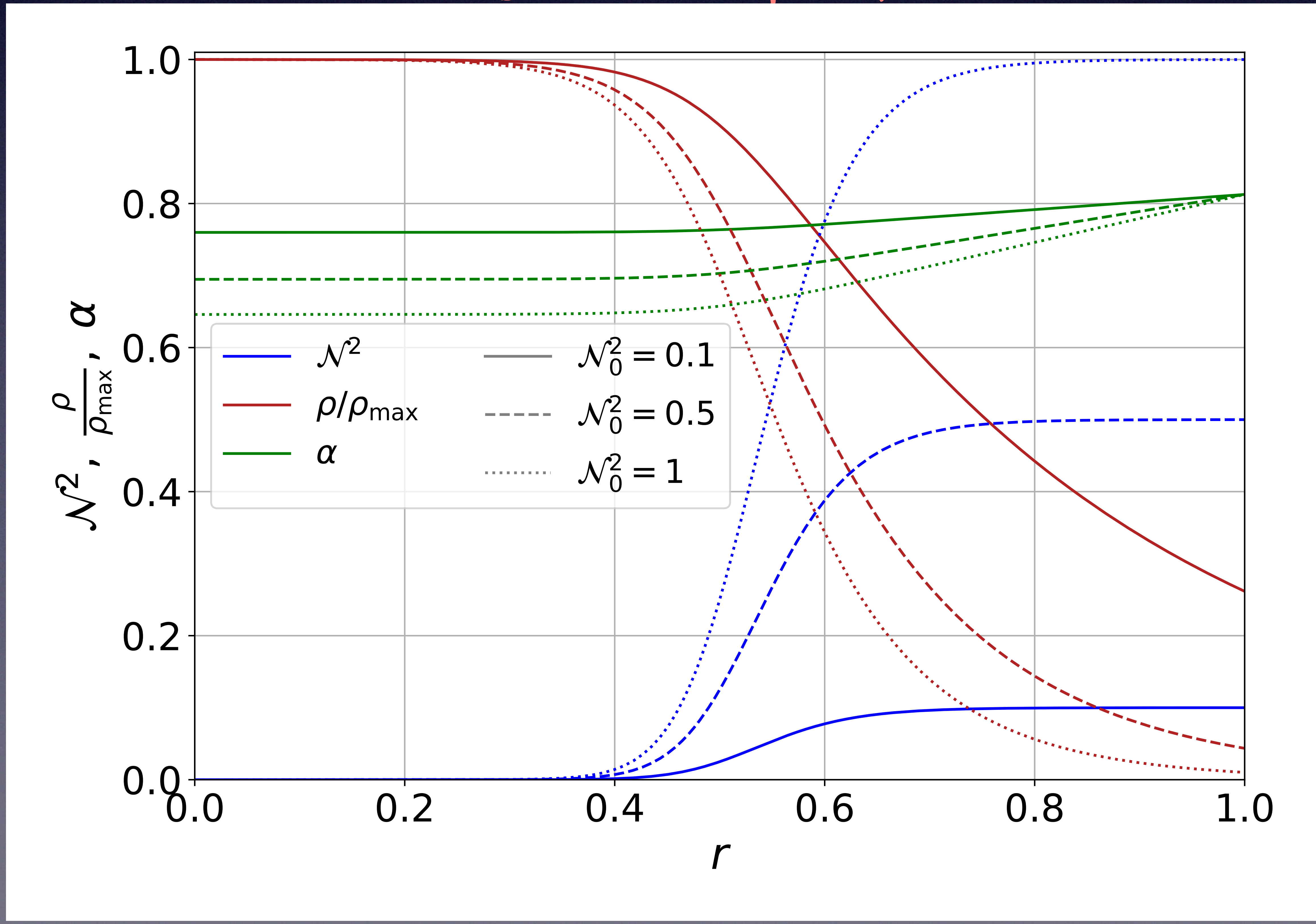


Ensure continuity of mass and conservation of energy-momentum fluxes

Case I: p- & g-modes

- Ideal fluid with $c_s^2,\ \Gamma_1 \ \rightarrow\ constant$ and $v\ \rightarrow\ 0$
- Buoyant region
- Varying Brunt-Vaisälä frequency $\mathcal{N}^2 \equiv \frac{\alpha^2}{\psi^4}\mathcal{G}_\alpha\mathcal{B}$

Background profiles



DT et al. (2025)
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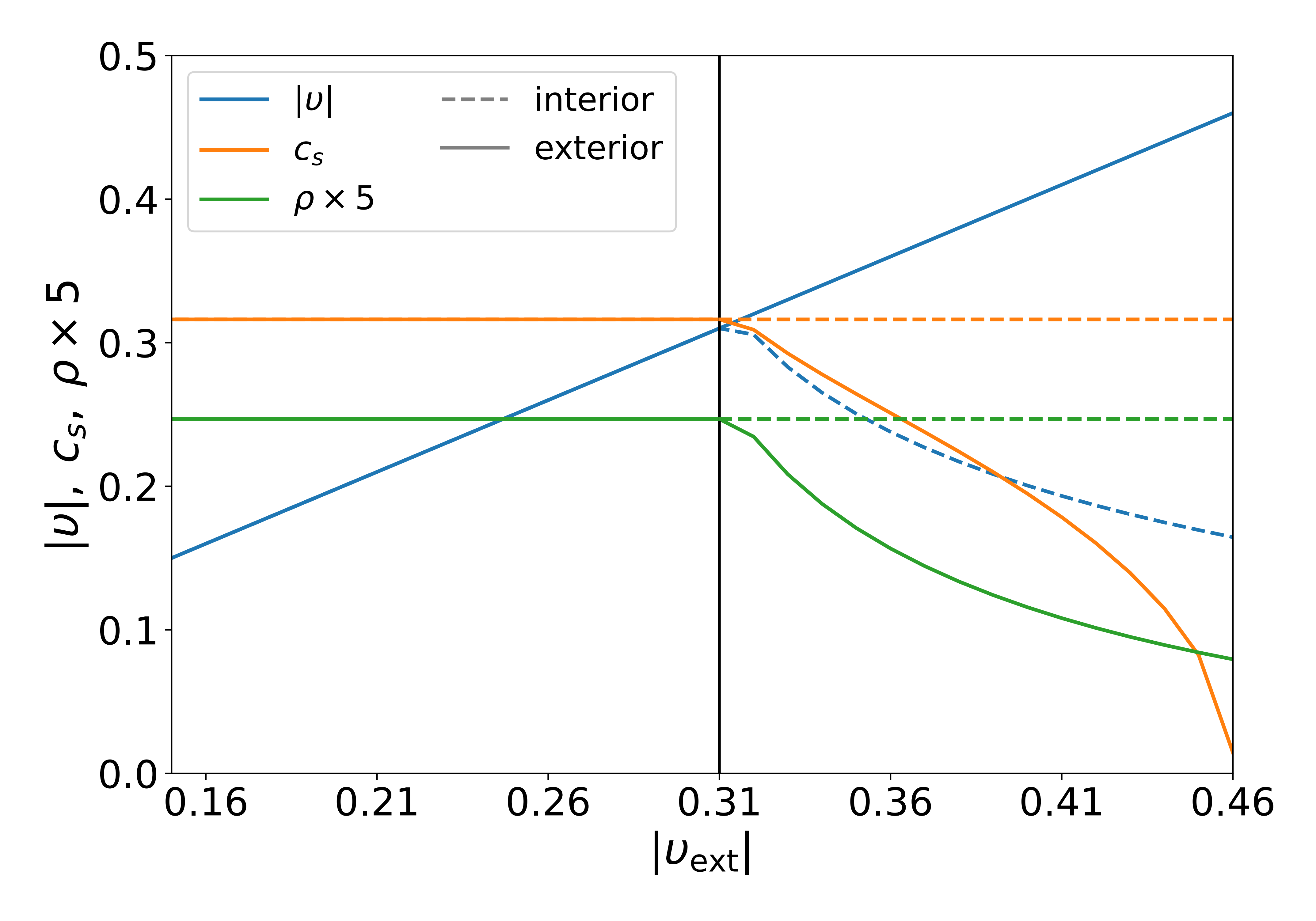
Case III: Advection

- NSI $\longrightarrow$ $N^2 = 0$
- Sound speed, pressure, density $\longrightarrow$ constant
- Lapse $\longrightarrow$ general
- Velocity < 0 and subsonic

Background shock quantities



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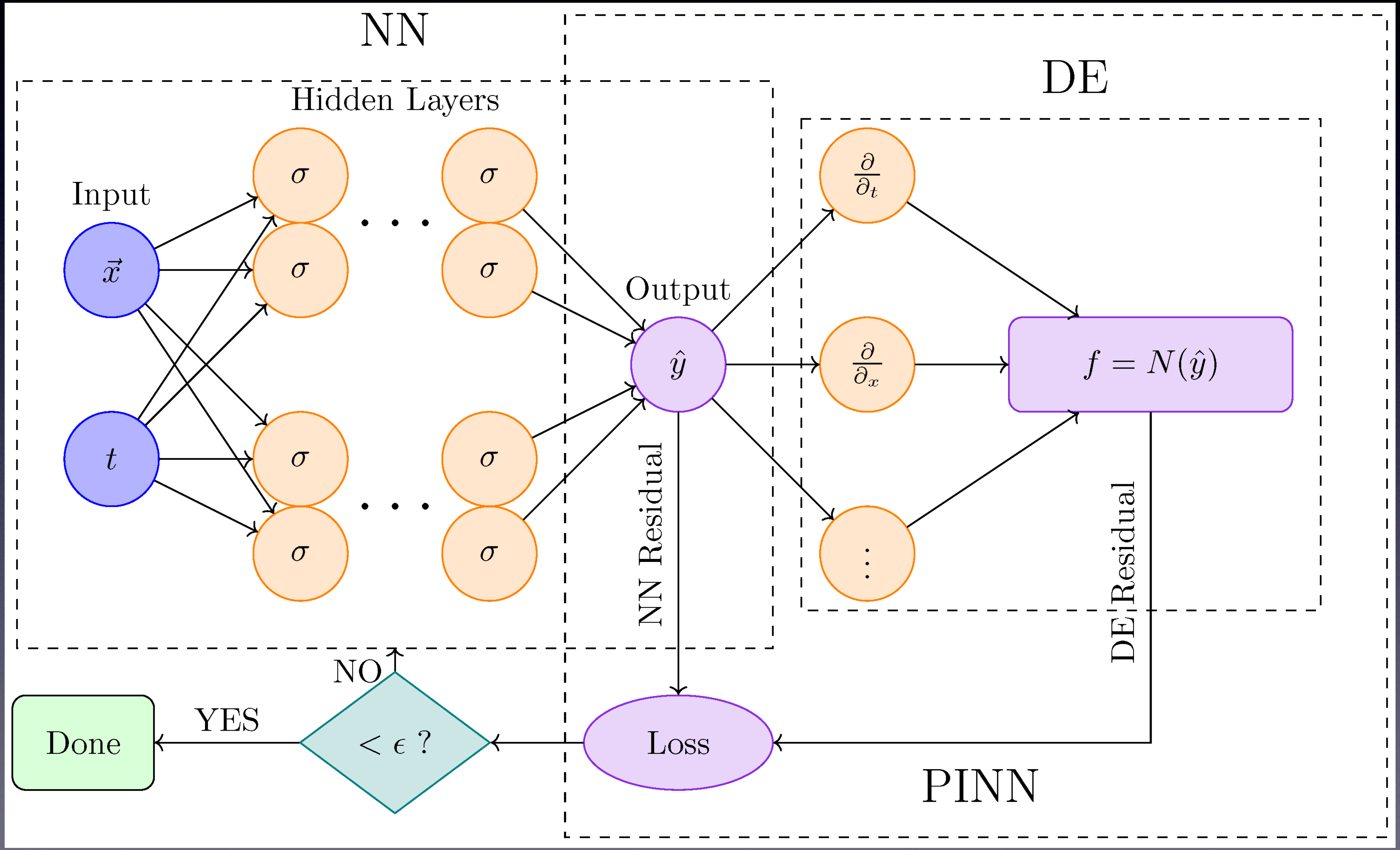
Physics Informed Neural Networks



ADVANTAGES

- Grid free
- Fast to code
- Easy implementation of initial & boundary conditions
- Flexible to add extra features
- Error control

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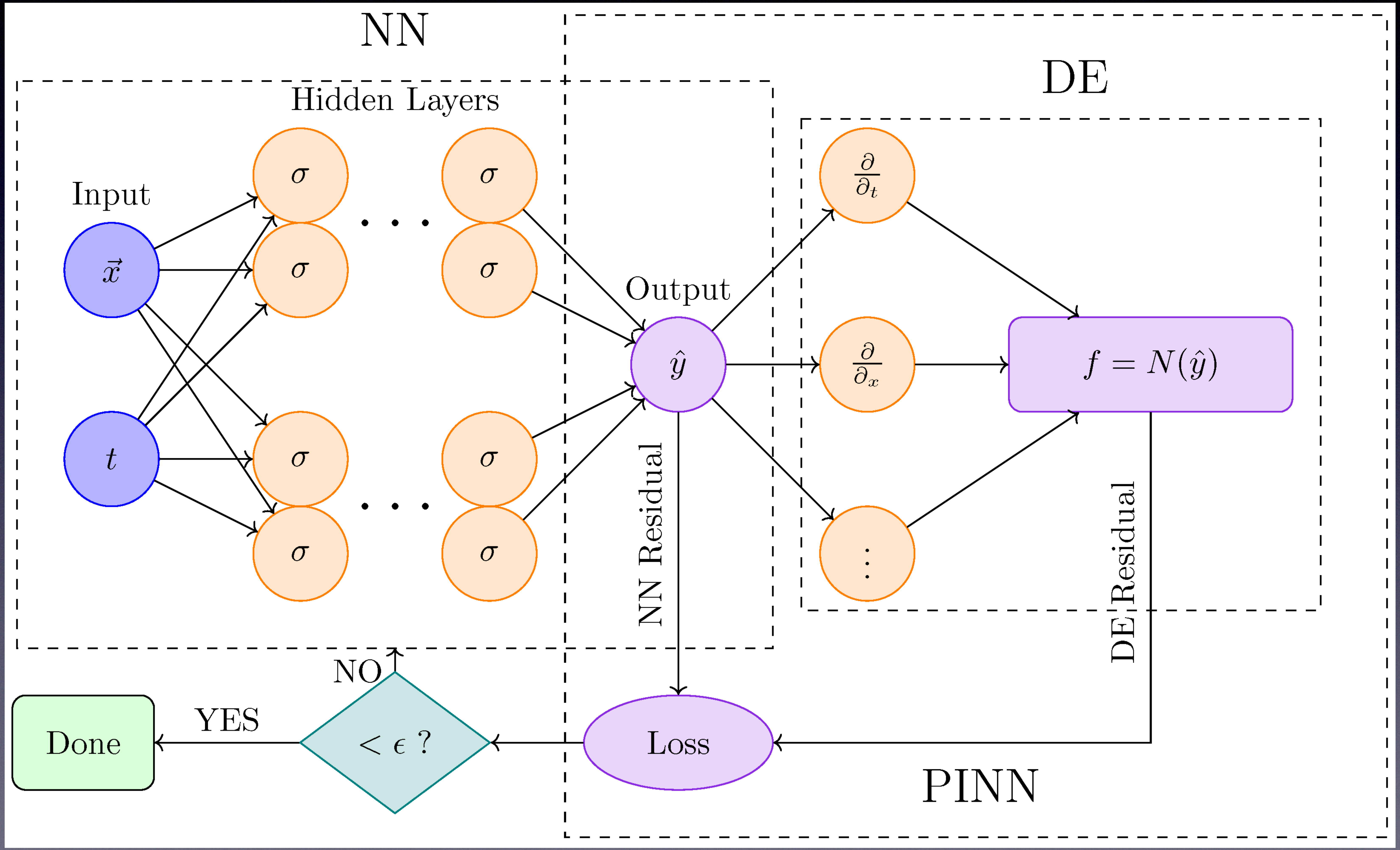

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Farea et al. (2014)

SMs
Newtonian simulation, model s20

Preliminary!