

A Horizontal Three-Higgs-Doublet Model

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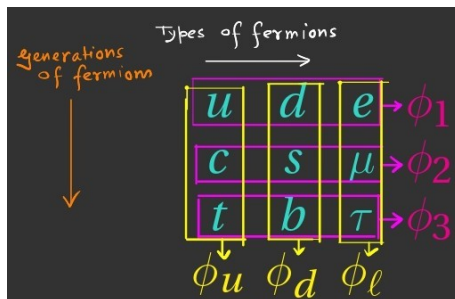
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Outline

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- Yukawa Sector
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Three-Higgs-Doublet Models (3HDMs)

- 3HDMs offer a unique way to approach the flavor puzzle.
Is it possible to assign masses to the massive fermions such that all quarks and leptons of a particular generation get the masses primarily from a particular doublet?
- Then, strong hierarchy among the VEVs of the three doublets
 \implies strong mass hierarchies between different generations of fermions
- This gives rise to a horizontal Yukawa structure as explained in the diagram



About FCNCs

- Unlike vertical (democratic) 3HDMs, natural flavor conservation (NFC) is absent here \implies scalar-mediated flavor changing neutral currents (FCNCs).
- In nHDMs, it is possible to work in the alignment limit, where a particular CP-even scalar becomes SM-like. This mitigates flavor constraints on that scalar.
- FCNCs mediated by the nonstandard scalars are still present.
- So nonstandard scalars must be much heavier than the electroweak scale to satisfy the constraints from flavor data.

We attempt to construct a 3HDM with the following properties:

- The charged fermions of a particular generation primarily receive their masses from a particular Higgs-doublet *i.e.* Choose appropriate Yukawa textures. Then, a strong VEV hierarchy, $v_1 \ll v_2 \ll v_3$ among the scalar doublets lead to the strong mass hierarchy among the different generations of fermions.
- All the FCNCs occur exclusively in the up-quark sector, with their effects being considerably suppressed by the off-diagonal elements of the CKM matrix. Similar to BGL models. This is what we call an up-type BGL 3HDM.
- No FCNC couplings in down-quark and charged lepton sector *i.e.* the neutral Higgs couplings in these sectors are flavor diagonal.
- The constraints arising from the flavor data are quite relaxed that sub-TeV scale non-standard scalars can successfully pass through the flavor constraints. Observable signatures in collider searches.
- It should be, at the very least, as economical as the NFC versions of 3HDMs in terms of the number of physical parameters. No compromise on the predictive power in comparison with NFC 3HDMs.

Implementation of the horizontal 3HDM

- Our construction is based on a 3HDM structure with a global $U(1)_1 \times U(1)_2$ horizontal symmetry.

$$U(1)_1 : (Q_L)_1 \rightarrow e^{-iq_1}(Q_L)_1, \quad (n_R)_1 \rightarrow e^{-2iq_1}(n_R)_1, \quad \phi_1 \rightarrow e^{iq_1}\phi_1, \quad (1a)$$

$$U(1)_2 : (Q_L)_2 \rightarrow e^{-iq_2}(Q_L)_2, \quad (n_R)_2 \rightarrow e^{-2iq_2}(n_R)_2, \quad \phi_2 \rightarrow e^{iq_2}\phi_2, \quad (1b)$$

$Q_L = (p_L \ n_L)^T$, the usual SM quark $SU(2)_L$ doublets

n_R , the down-type $SU(2)$ quark singlets

ϕ_i , the scalar doublets

The subscript refers to the different generations in the Lagrangian basis.

- All other quark and scalar fields transform trivially under the flavour symmetry.

Yukawa Sector

- The Yukawa Lagrangian,

$$\mathcal{L}_Y = - \sum_{k=1}^3 \left[\bar{Q}_L \Gamma_k \phi_k n_R + \bar{Q}_L \Delta_k \tilde{\phi}_k p_R \right] + \text{h.c.} , \quad (2)$$

p_R , the up-type quark $SU(2)_L$ singlets.

- The flavour indices (Γ_k and Δ_k are 3×3 matrices in the flavour space) are suppressed.
- The mass matrices for the down-type and up-type quarks are

$$M_n = \frac{1}{\sqrt{2}} \sum_{k=1}^3 v_k \Gamma_k , \quad M_p = \frac{1}{\sqrt{2}} \sum_{k=1}^3 v_k \Delta_k . \quad (3)$$

- The textures of Γ and Δ matrices that follow from the transformations of Eq. (1) are given below:

$$\Gamma_1 = \begin{pmatrix} y_1^d & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \quad \Gamma_2 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & y_2^d & 0 \\ 0 & 0 & 0 \end{pmatrix}; \quad \Gamma_3 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & y_3^d \end{pmatrix},$$

$$\Delta_1 = \begin{pmatrix} a_1 & b_1 & c_1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \quad \Delta_2 = \begin{pmatrix} 0 & 0 & 0 \\ a_2 & b_2 & c_2 \\ 0 & 0 & 0 \end{pmatrix}; \quad \Delta_3 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ a_3 & b_3 & c_3 \end{pmatrix}$$

- Γ_k 's and hence M_n are already diagonal \implies the mass of each generation in the down sector only receives contributions from the VEV of a dedicated ϕ_k .

$$\frac{1}{\sqrt{2}} \sum_{k=1}^3 v_k \Gamma_k = \frac{1}{\sqrt{2}} \text{diag} \left(v_1 y_1^d, v_2 y_2^d, v_3 y_3^d \right) = \text{diag} (m_d, m_s, m_b). \quad (5)$$

- Although not as obvious as the down sector, the same VEV hierarchy will lead to analogous effects in the up-quark sector, *i.e.*, the VEV of a particular doublet will be the major source of mass for a particular generation of quark in the up-sector as well.

- The physical quark fields, $u \equiv (u \ c \ t)^T$ and $d \equiv (d \ s \ b)^T$, are

$$n_L = U_L d_L, \quad n_R = U_R d_R, \quad p_L = V_L u_L, \quad p_R = V_R u_R, \quad (6)$$

- The diagonal mass matrices will be obtained as follows:

$$D_d = U_L^\dagger \cdot M_n \cdot U_R = \text{diag}(m_d, m_s, m_b), \quad (7a)$$

$$D_u = V_L^\dagger \cdot M_p \cdot V_R = \text{diag}(m_u, m_c, m_t), \quad (7b)$$

and the CKM matrix will be given by $V = V_L^\dagger U_L$.

- Since M_n is directly diagonal, U_L and U_R are diagonal. We take U_L to be the identity matrix, whereas U_R provides the necessary rephasings such that the elements of D_d are real and positive.
- Therefore, the CKM mixing arises exclusively from the up-sector as follows:

$$V = V_L^\dagger U_L = V_L^\dagger. \quad (8)$$

Scalar Sector

- Now we take a detour to look at the scalar sector of the model in order to set up notations and to define the scalar basis in which we will further calculate the flavor constraints.
- Taking into account the field transformations of Eq. (1), the scalar potential can be written as:

$$\begin{aligned} V_{U(1) \times U(1)} = & m_{11}^2 \phi_1^\dagger \phi_1 + m_{22}^2 \phi_2^\dagger \phi_2 + m_{33}^2 \phi_3^\dagger \phi_3 \\ & - \left(m_{12}^2 \phi_1^\dagger \phi_2 + m_{13}^2 \phi_1^\dagger \phi_3 + m_{23}^2 \phi_2^\dagger \phi_3 + \text{h.c.} \right) \\ & + \lambda_1 (\phi_1^\dagger \phi_1)^2 + \lambda_2 (\phi_2^\dagger \phi_2)^2 + \lambda_3 (\phi_3^\dagger \phi_3)^2 + \lambda_4 (\phi_1^\dagger \phi_1) (\phi_2^\dagger \phi_2) \\ & + \lambda_5 (\phi_1^\dagger \phi_1) (\phi_3^\dagger \phi_3) + \lambda_6 (\phi_2^\dagger \phi_2) (\phi_3^\dagger \phi_3) + \lambda_7 (\phi_1^\dagger \phi_2) (\phi_2^\dagger \phi_1) \\ & + \lambda_8 (\phi_1^\dagger \phi_3) (\phi_3^\dagger \phi_1) + \lambda_9 (\phi_2^\dagger \phi_3) (\phi_3^\dagger \phi_2), \end{aligned} \quad (9)$$

where m_{12}^2 , m_{13}^2 , and m_{23}^2 break the global $U(1)_1 \times U(1)_2$ symmetry softly.

- We also assume that all the potential parameters (including VEVs) are real.

- The individual VEVs after the EWSB, are further parameterized as follows:

$$v_1 = v \cos \beta_1 \cos \beta_2, \quad v_2 = v \sin \beta_1 \cos \beta_2, \quad v_3 = v \sin \beta_2, \quad (10)$$

with $v = \sqrt{v_1^2 + v_2^2 + v_3^2} = 246$ GeV being the total electroweak VEV.

- For later convenience, we go to a Intermediate basis by rotating the fields in the Lagrangian basis as follows:

$$\begin{pmatrix} H_0 \\ R_1 \\ R_2 \end{pmatrix} = O_\beta \begin{pmatrix} h_1 \\ h_2 \\ h_3 \end{pmatrix}, \quad \begin{pmatrix} G^0 \\ A'_1 \\ A'_2 \end{pmatrix} = O_\beta \begin{pmatrix} z_1 \\ z_2 \\ z_3 \end{pmatrix}, \quad \begin{pmatrix} G^\pm \\ H'^{\pm}_1 \\ H'^{\pm}_2 \end{pmatrix} = O_\beta \begin{pmatrix} w_1^\pm \\ w_2^\pm \\ w_3^\pm \end{pmatrix}, \quad (11)$$

- Here,

$$O_\beta = \begin{pmatrix} \cos \beta_2 \cos \beta_1 & \cos \beta_2 \sin \beta_1 & \sin \beta_2 \\ -\sin \beta_1 & \cos \beta_1 & 0 \\ -\cos \beta_1 \sin \beta_2 & -\sin \beta_1 \sin \beta_2 & \cos \beta_2 \end{pmatrix}. \quad (12)$$

- In Eq. (11), G^0 and G^\pm stand for the neutral and charged Goldstone bosons respectively.

- The rest of the fields are not the physical mass eigenstates, in general.
- The rotations to the mass eigenstates are as follows,

$$\begin{pmatrix} G^\pm \\ H_1^\pm \\ H_2^\pm \end{pmatrix} = O_{\gamma_2} \begin{pmatrix} G^\pm \\ H_1'^\pm \\ H_2'^\pm \end{pmatrix}, \quad \begin{pmatrix} G^0 \\ A_1 \\ A_2 \end{pmatrix} = O_{\gamma_1} \begin{pmatrix} G^0 \\ A_1' \\ A_2' \end{pmatrix}, \quad \begin{pmatrix} h \\ H_1 \\ H_2 \end{pmatrix} = O_{\alpha_3} \begin{pmatrix} H_0 \\ R_1 \\ R_2 \end{pmatrix}, \quad (13)$$

with O_{γ_1} and O_{γ_2} are,

$$O_{\gamma_1} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \gamma_1 & -\sin \gamma_1 \\ 0 & \sin \gamma_1 & \cos \gamma_1 \end{pmatrix}, \quad O_{\gamma_2} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \gamma_2 & -\sin \gamma_2 \\ 0 & \sin \gamma_2 & \cos \gamma_2 \end{pmatrix} \quad (14)$$

- We will work in the above defined Higgs basis under the alignment limit where the scalar H^0 is also the mass eigenstate corresponding to the SM Higgs boson. In this basis, R_1 and R_2 are rotate by the following matrix to obtain the physical eigenstates H_1 and H_2 .

$$O_{\alpha_3} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \alpha_3 & -\sin \alpha_3 \\ 0 & \sin \alpha_3 & \cos \alpha_3 \end{pmatrix} \quad (15)$$

Returning to the Yukawa Sector

- To understand the structure of the FCNCs and calculate the flavor constraints, we will determine the couplings of the physical quarks to the scalar fields defined in Eq. (11).
- These scalars are in general not mass eigenstates but the couplings to the physical eigenstates can be obtained by suitable rotations originating from the scalar sector alone.
- The Yukawa couplings of the CP-even scalars are

$$\begin{aligned} -\mathcal{L}_Y^{\text{CP-even}} &= \frac{1}{\sqrt{2}} \sum_{k=1}^3 \left[\bar{n}_L \Gamma_k h_k n_R + \bar{p}_L \Delta_k h_k p_R \right] + \text{h.c.} \\ &= \bar{d}_L \left(\frac{1}{\sqrt{2}} \sum_{k=1}^3 \Gamma_k h_k \right) d_R + \bar{u}_L V_L^\dagger \left(\frac{1}{\sqrt{2}} \sum_{k=1}^3 \Delta_k h_k \right) V_R u_R + \text{h.c.} \end{aligned} \quad . \quad (16)$$

- In the intermediate basis defined in Eq. (11), we can decompose h_k as

$$h_k = (O_\beta)_{1k} H_0 + (O_\beta)_{2k} R_1 + (O_\beta)_{3k} R_2 . \quad (17)$$

- By virtue of being a SM-like Higgs, the couplings of H_0 simplify to SM couplings.

$$-\mathcal{L}_Y^{H^0} = \frac{H_0}{v} \left[\bar{d}_L D_d d_R + \bar{u}_L D_u u_R \right] + \text{h.c.} = \frac{H_0}{v} \left[\bar{d} D_d d + \bar{u} D_u u \right]. \quad (18)$$

- The couplings of R_1 and R_2 that mediate FCNCs can be read off from Eq. (16) as

$$-\mathcal{L}_Y^R = R_1 \left[\bar{d}_L N_1^d d_R + \bar{u}_L N_1^u u_R \right] + R_2 \left[\bar{d}_L N_2^d d_R + \bar{u}_L N_2^u u_R \right] + \text{h.c.} \quad (19)$$

where,

$$N_1^d = \frac{1}{\sqrt{2}} \sum_{k=1}^3 (O_\beta)_{2k} \Gamma_k, \quad N_1^u = V_L^\dagger \left(\frac{1}{\sqrt{2}} \sum_{k=1}^3 (O_\beta)_{2k} \Delta_k \right) V_R, \quad (20a)$$

$$N_2^d = \frac{1}{\sqrt{2}} \sum_{k=1}^3 (O_\beta)_{3k} \Gamma_k, \quad N_2^u = V_L^\dagger \left(\frac{1}{\sqrt{2}} \sum_{k=1}^3 (O_\beta)_{3k} \Delta_k \right) V_R. \quad (20b)$$

- Taking Eqs. (5) and (12) into account, N_1^d assumes a simple shape:

$$\begin{aligned}
N_1^d &= \text{diag} \left(\frac{m_d(O_\beta)_{21}}{v_1}, \frac{m_s(O_\beta)_{22}}{v_2}, \frac{m_b(O_\beta)_{23}}{v_3}, \right) \\
&= \text{diag} \left(-\frac{m_d}{v} \frac{t_{\beta_1} t_{\beta_2}}{s_{\beta_2}}, \frac{m_s}{v} \frac{t_{\beta_2}}{t_{\beta_1} s_{\beta_2}}, 0 \right),
\end{aligned} \tag{21}$$

where t_α , s_α and c_α denote $\tan \alpha$, $\sin \alpha$, and $\cos \alpha$, respectively.

- Similarly

$$\begin{aligned}
N_2^d &= \text{diag} \left(\frac{m_d(O_\beta)_{31}}{v_1}, \frac{m_s(O_\beta)_{32}}{v_2}, \frac{m_b(O_\beta)_{33}}{v_3}, \right) \\
&= \text{diag} \left(-\frac{m_d}{v} t_{\beta_2}, -\frac{m_s}{v} t_{\beta_2}, \frac{m_b}{v} \frac{1}{t_{\beta_2}} \right).
\end{aligned} \tag{22}$$

- We can write N_1^u and N_2^u as

$$N_1^u = -\sin \beta_1 F_1 + \cos \beta_1 F_2 \tag{23}$$

$$N_2^u = -\cos \beta_1 \sin \beta_2 F_1 - \sin \beta_1 \sin \beta_2 F_2 + \cos \beta_2 F_3 \tag{24}$$

where,

$$F_k = \frac{1}{\sqrt{2}} V_L^\dagger \cdot \Delta_k \cdot V_R = \frac{1}{v_k} V_L^\dagger (P_k M_p) V_R \tag{25}$$

- Now, we can write F_k as,

$$F_k = \frac{1}{v_k} V_L^\dagger (P_k M_p) V_R = \frac{1}{v_k} (V_L^\dagger P_k V_L) D_u \quad (26)$$

where, P_k is the projection matrix which has all elements zero except the (k,k) element.

- Now, we can expand,

$$(V_L^\dagger P_k V_L)_{ab} = V_{ak} V_{bk}^* \quad (27)$$

- Therefore

$$(F_k)_{ab} = \frac{1}{v_k} V_{ak} (V_{bk}^*) m_b \quad (28)$$

- Using this, we can write N_1^u and N_2^u as

$$(N_1^u)_{ab} = \left(\frac{-t_{\beta_1} t_{\beta_2}}{s_{\beta_2}} (V)_{a1} (V)_{b1}^* + \frac{t_{\beta_2}}{t_{\beta_2} s_{\beta_2}} (V)_{a2} (V)_{b2}^* \right) \frac{(D_u)_{bb}}{v} \quad (29)$$

$$(N_2^u)_{ab} = \left(-t_{\beta_2} (V)_{a1} (V)_{b1}^* - t_{\beta_2} (V)_{a2} (V)_{b2}^* + \frac{1}{t_{\beta_2}} (V)_{a3} (V)_{b3}^* \right) \frac{(D_u)_{bb}}{v} \quad (30)$$

- The pseudoscalar couplings follow similarly.

$$\begin{aligned}
-\mathcal{L}_Y^{\text{CP-odd}} &= \frac{1}{\sqrt{2}} \sum_{k=1}^3 \left[\bar{n}_L \Gamma_k (iz_k) n_R - \bar{p}_L \Delta_k (iz_k) p_R \right] + \text{h.c.} \\
&\supset i A'_1 \left[\bar{d} \left(N_1^d P_R - N_1^{d\dagger} P_L \right) d - \bar{u} \left(N_1^u P_R - N_1^{u\dagger} P_L \right) u \right] \\
&+ i A'_2 \left[\bar{d} \left(N_2^d P_R - N_2^{d\dagger} P_L \right) d - \bar{u} \left(N_2^u P_R - N_2^{u\dagger} P_L \right) u \right] \quad (31)
\end{aligned}$$

- Similarly, for the charged scalars.

$$\begin{aligned}
-\mathcal{L}_Y^{\text{charged}} &= \sum_{i=1}^3 \left[\bar{p}_L \Gamma_i \phi_i^+ n_R + \bar{p}_R \Delta_i^\dagger (-\phi_i^+) n_L \right] + \text{h.c.} \\
&\supset \sqrt{2} H_1'^+ \left[\bar{u} \left(V N_1^d P_R - N_1^{u\dagger} V P_L \right) d \right] \\
&+ \sqrt{2} H_2'^+ \left[\bar{u} \left(V N_2^d P_R - N_2^{u\dagger} V P_L \right) d \right] + \text{h.c.} \quad (32)
\end{aligned}$$

Constraints from Perturbativity

- The perturbativity bounds are obtained by imposing the following condition on the nonstandard Yukawa couplings

$$|(N_{1,2}^{u,d,\ell})_{ab}| \leq \sqrt{4\pi} \quad (33)$$

Additionally, to minimize the hierarchy in Yukawa parameters, we impose that $v_2 > v_1$ and $v_3 > v_2$

- Also we choose two benchmarks
 - ① $(\tan \beta_1 = 5, \tan \beta_2 = 25)$ implying $v_1 \approx 2$ GeV, $v_2 \approx 10$ GeV and $v_3 \approx 246$ GeV, which follows a strong hierarchy $v_1 \ll v_2 \ll v_3$.
 - ② $(\tan \beta_1 = 2, \tan \beta_2 = 1)$ implying $v_1 \approx 78$ GeV, $v_2 \approx 156$ GeV and $v_3 \approx 174$ GeV, which follows a mild hierarchy $v_1 < v_2 < v_3$
- The subsequent plot shows the regions allowed from these constraints

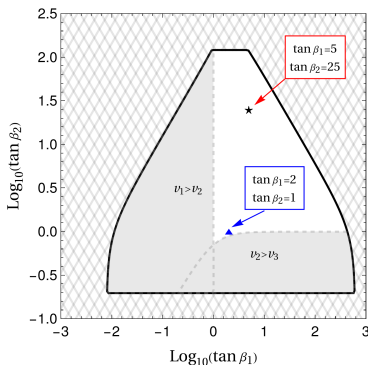


Figure: Allowed regions from the perturbativity constraints on the Yukawa couplings. The shaded areas inside the boundary on the left and on the bottom right denote the regions in which the VEV hierarchies invert: $v_1 > v_2$ (left) and $v_2 > v_3$ (bottom right).

Flavor Constraints

- We will analyze the flavor constraints in the alignment limit. In addition, to ensure automatic compatibility with the theoretical constraints from unitarity, boundedness from below and the ρ parameter, we impose the following relations:

$$m_{H_1} = m_{A_1} = m_{C_1} \equiv M_1, \quad m_{H_2} = m_{A_2} = m_{C_2} \equiv M_2, \quad \alpha_3 = \gamma_1 = \gamma_2 \equiv \gamma. \quad (34)$$

- We analyze the constraints arising from neutral meson oscillations, namely ΔM_K , ΔM_{B_s} , ΔM_{B_d} , and ΔM_D , as well as from $b \rightarrow s\gamma$ focusing on the leading order contributions.

- The constraints from D_0 - \bar{D}_0 mixing occurs at the tree-level due to the off-diagonal elements $(N_{1,2}^u)_{12}$ and $(N_{1,2}^u)_{21}$.
- N_1^u is particularly large for $\tan \beta_{1,2} \gg 1$. Consequently, it is preferable to selectively decouple one tier of nonstandard scalars, and arrange γ such that N_1^u couples predominantly to these decoupled scalars.
- Based on this strategy, we present benchmark scenarios that largely follow this assumption, with $M_1 \gg M_2$ along with $\gamma \approx 0$.
- After selectively decoupling the first tier of scalars by setting $\gamma = 0$ and M_1 to be large, we still need M_2 to be $\mathcal{O}(5\text{TeV})$ to accomodate a strong VEV hierarchy ($v_1 \ll v_2 \ll v_3$) given by the benchmark $\tan \beta_1 = 5$ and $\tan \beta_2 = 25$.
- On the other hand, if we work with a milder VEV hierarchy, $v_1 < v_2 < v_3$, reflected by the benchmark $\tan \beta_1 = 2$ and $\tan \beta_2 = 1$, then nonstandard masses below the TeV scale can be easily accommodated.
- These results are summarized in the following plots.

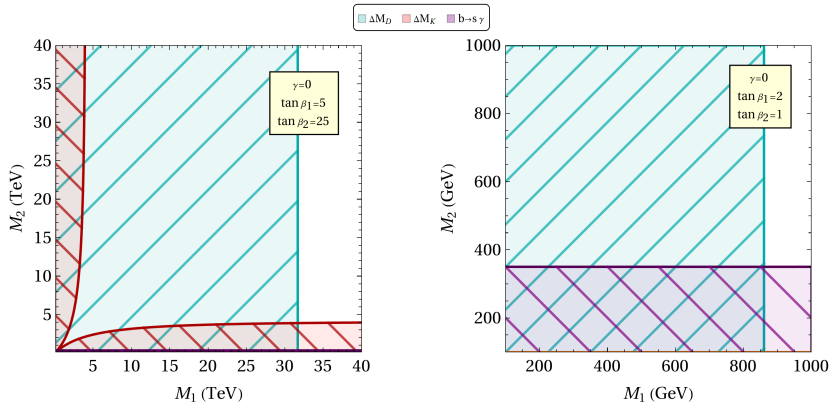
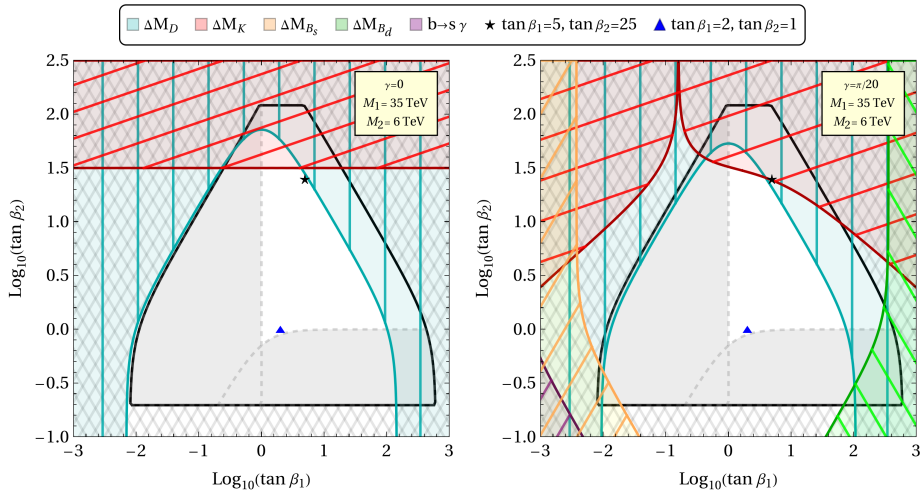


Figure: Constraints from flavor data for the case of $\gamma = 0$ with $\tan \beta_1 = 5$, $\tan \beta_2 = 25$ (**left**), and $\tan \beta_1 = 2$, $\tan \beta_2 = 1$ (**right**). The shaded region is forbidden by ΔM_D (cyan), ΔM_K (red) and $b \rightarrow s\gamma$ (purple). No additional constraints arise from $\Delta M_{B_{s,d}}$ for the displayed range of parameters. Please note that the ranges are different in the axes of the two plots.

- These results can also be cast in the $\tan\beta_1$ - $\tan\beta_2$ plane.
- For masses as light as $M_1 = 1$ TeV and $M_2 = 400$ GeV, the flavor constraints can be successfully satisfied with relatively mild VEV hierarchy $v_1 < v_2 < v_3$.
- However, if we insist on a stronger VEV hierarchy, $v_1 \ll v_2 \ll v_3$, then the flavor constraints will force the nonstandard masses to be much heavier than the electroweak scale.
- Additionally, we also see how changing $\gamma = 0$ significantly affects the allowed parameter space.



Conclusion

- We constructed a model that can successfully disentangle the primary sources of masses of different generations of charged fermions such that the k -th generation of the fermion gets its mass from the VEV of the k -th scalar doublet. The model also has the same number of parameters as the NFC 3HDM.
- However, The flavor constraints turned out to be very strong for the up-type version considered here. For the desired VEV hierarchy $v_1 \ll v_2 \ll v_3$, the constraints from neutral meson mixings forbid nonstandard masses below (5 TeV). To allow nonstandard scalars in the sub-TeV regime, we need to relax the aesthetic appeal of maintaining the hierarchy $v_1 \ll v_2 \ll v_3$, *i.e.*, $\tan \beta_{1,2} \gg 1$.
- This is because the strength of the tree-level FCNC interactions in the up-quark sector are proportional to the up-type masses.

Thank you