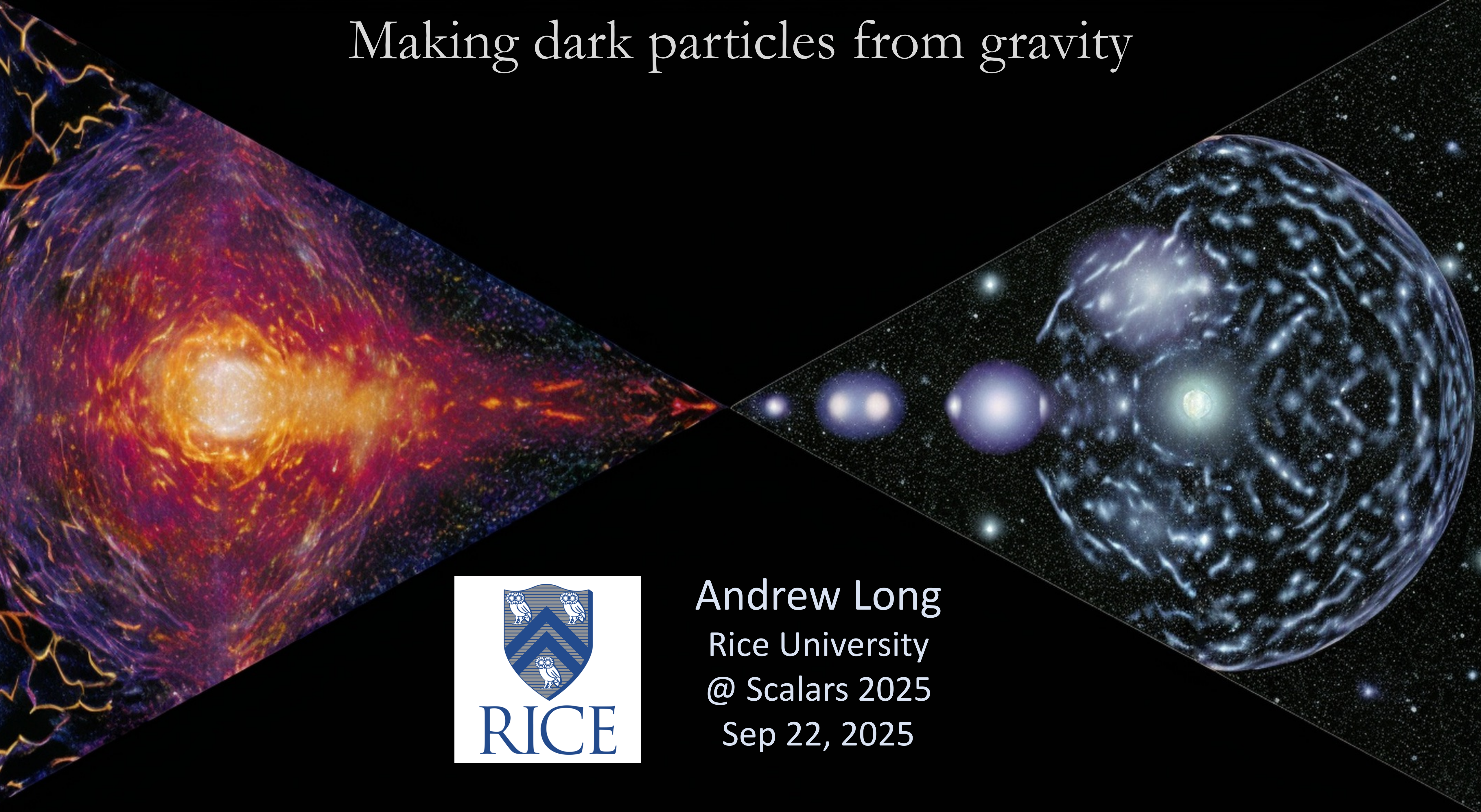
Making dark particles from gravity



Andrew Long
Rice University
@ Scalars 2025
Sep 22, 2025

Image credit: Chris Stabb

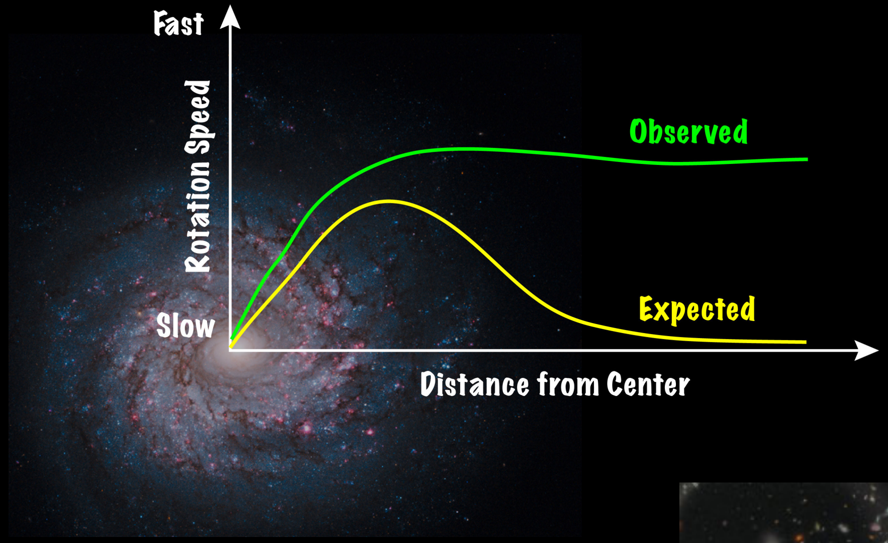
motivation

making dark matter from gravity

dark matter pulls on things

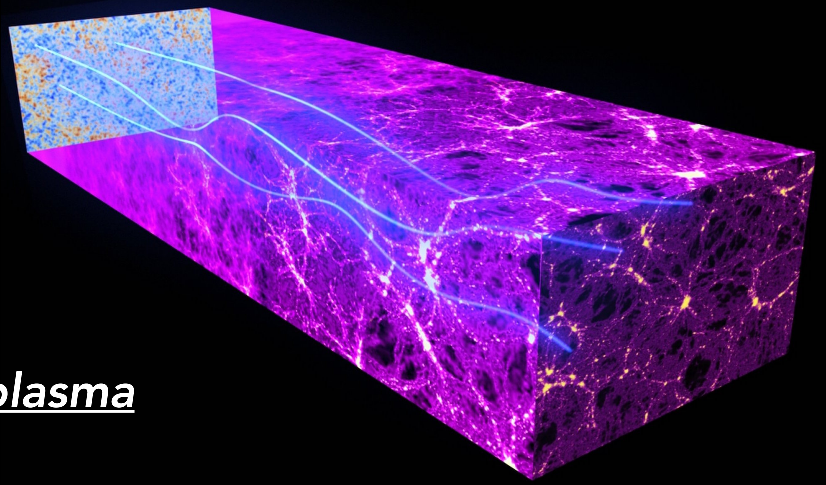
Dark matter pulls on stars in galaxies

(galactic rotation curves)



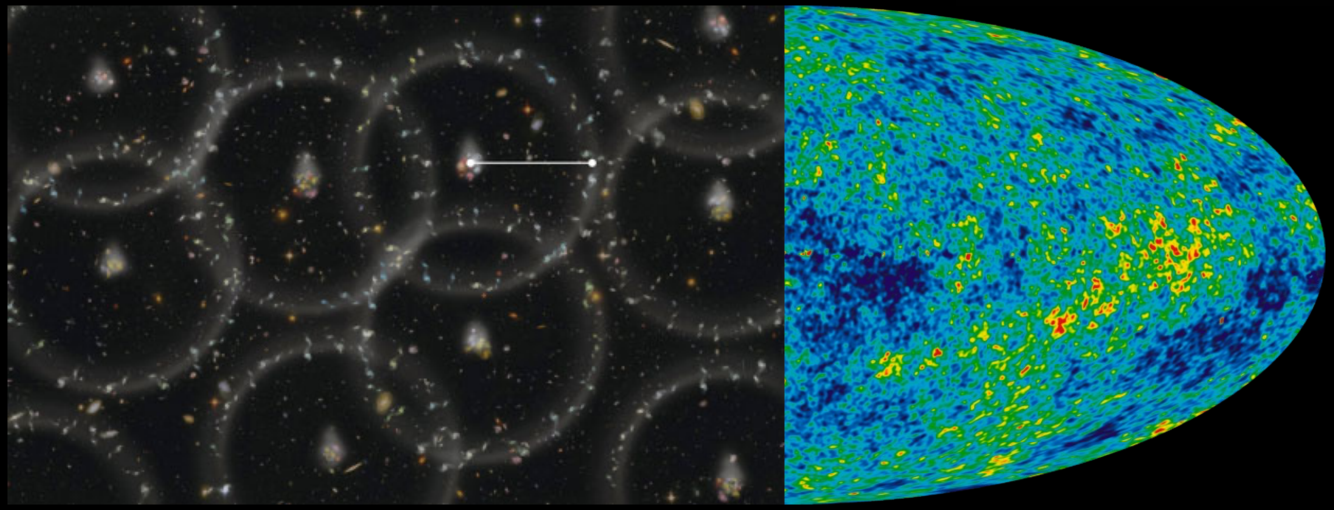
Dark matter pulls on light

(gravitational lensing)

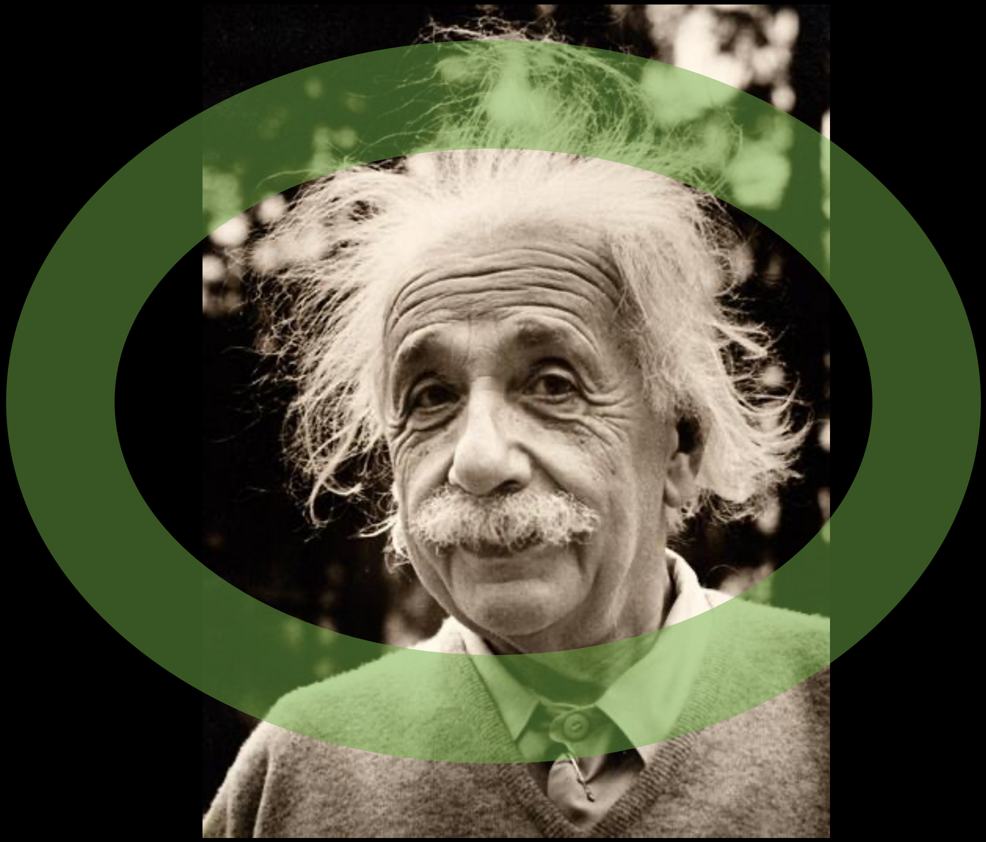


Dark matter pulled on e^-p^+ plasma

(CMB & large scale structure)



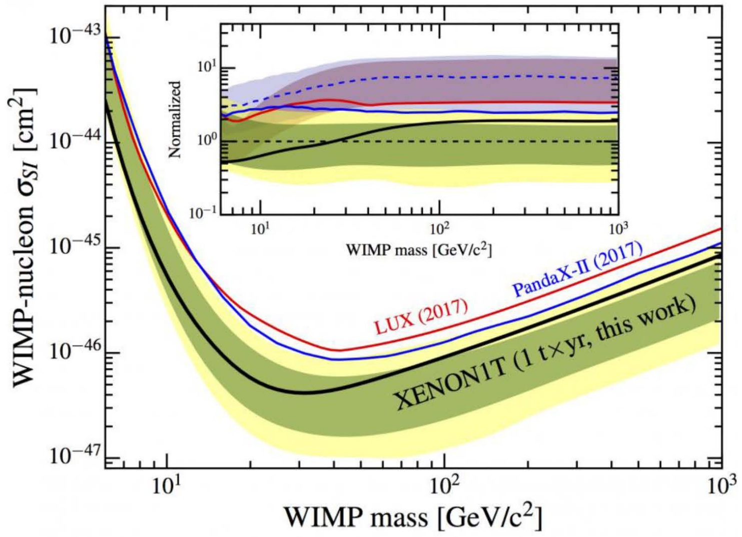
don't need a dark force



no evidence (yet) of dark matter bumping into things

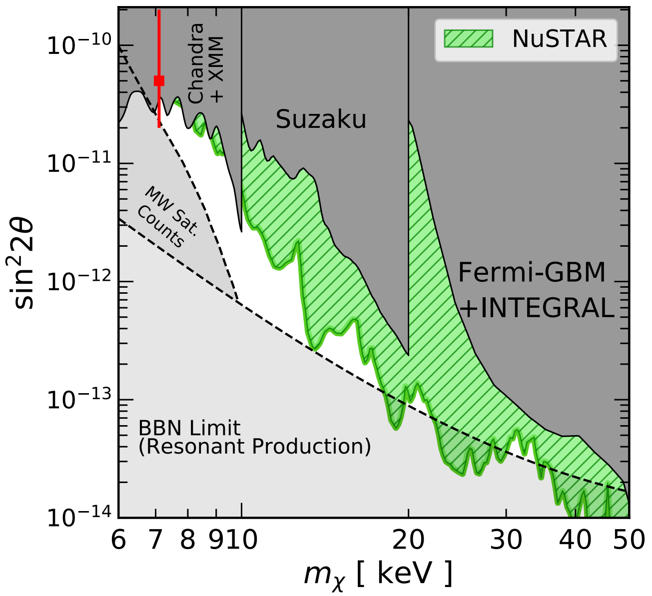
No dark matter bumping into things

(direct detection; 1805.12562)



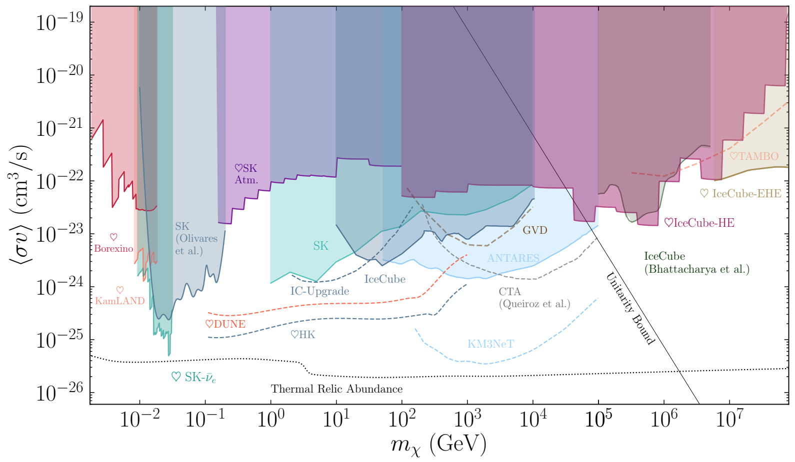
No dark matter decaying into things

(X-ray emission; 1908.09037)



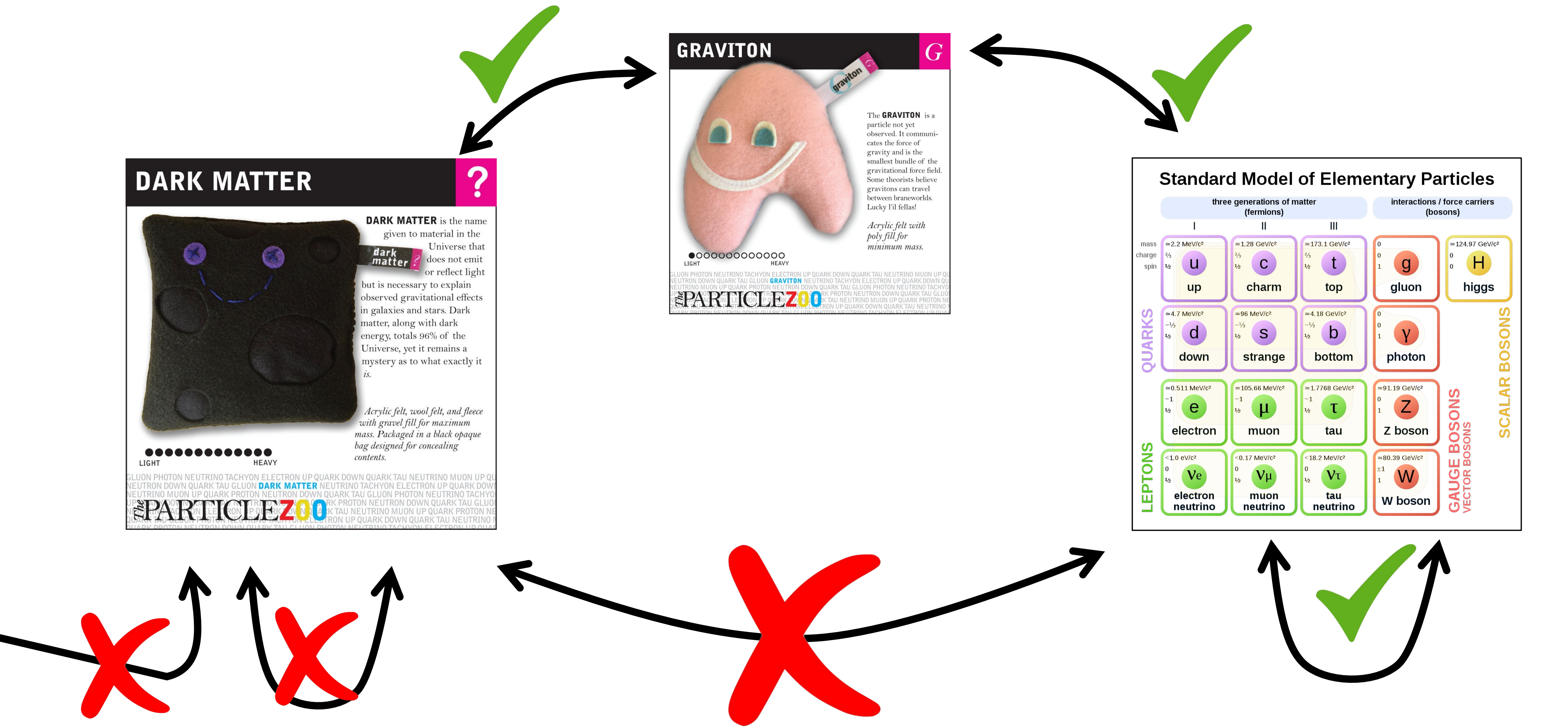
No dark matter bumping into itself

(annihilation to ν's; 1912.09486)



(notwithstanding hints of new physics, there's no overwhelming evidence)

the hypothesis:



the problem:

how do we use gravity
to make dark matter?

Can dark matter be created from gravity?

- Yes, via gravitational collapse into a PBH
- Yes, via Hawking radiation from an evaporating PBH
- Yes, via graviton-mediated thermal freeze in
- Yes, via graviton-mediated inflaton/moduli decay
- Yes, via graviton-mediated inflaton/moduli annihilation
- Yes, via inhomogeneous cosmological spacetimes
- Yes, via cosmological expansion in an FRW spacetime

collectively, we call these **gravitational particle production** (GPP),
& distinguish the last as **cosmological gravitational particle production** (CGPP)

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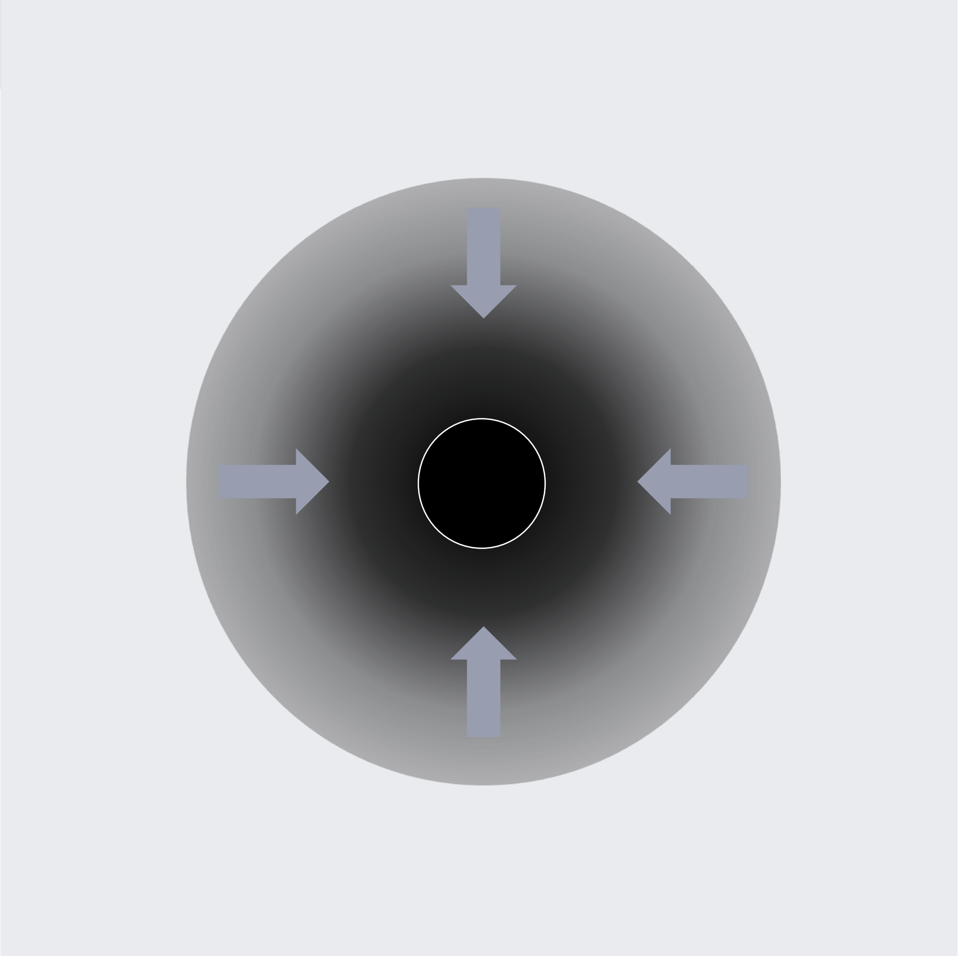

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Hawking (1971); Hawking & Carr (1974); + many others

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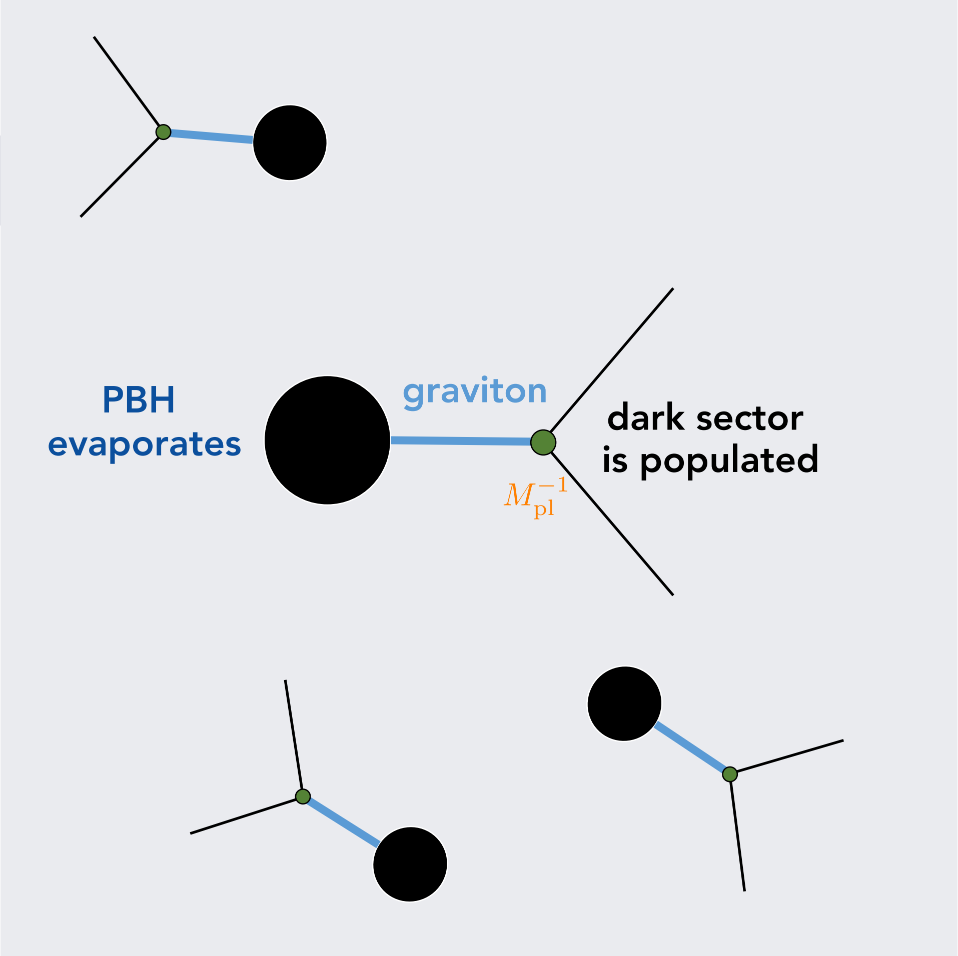

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Fujita, Kawasaki, Harigaya (2014); Lennon, March-Russel, Petrosian-Byrne, Tillim (2017); Morrison, Profumo, Yu (2018); Allaverdi, Dent, Osinski (2018); Hooper, Krnjaic, McDermott (2019)

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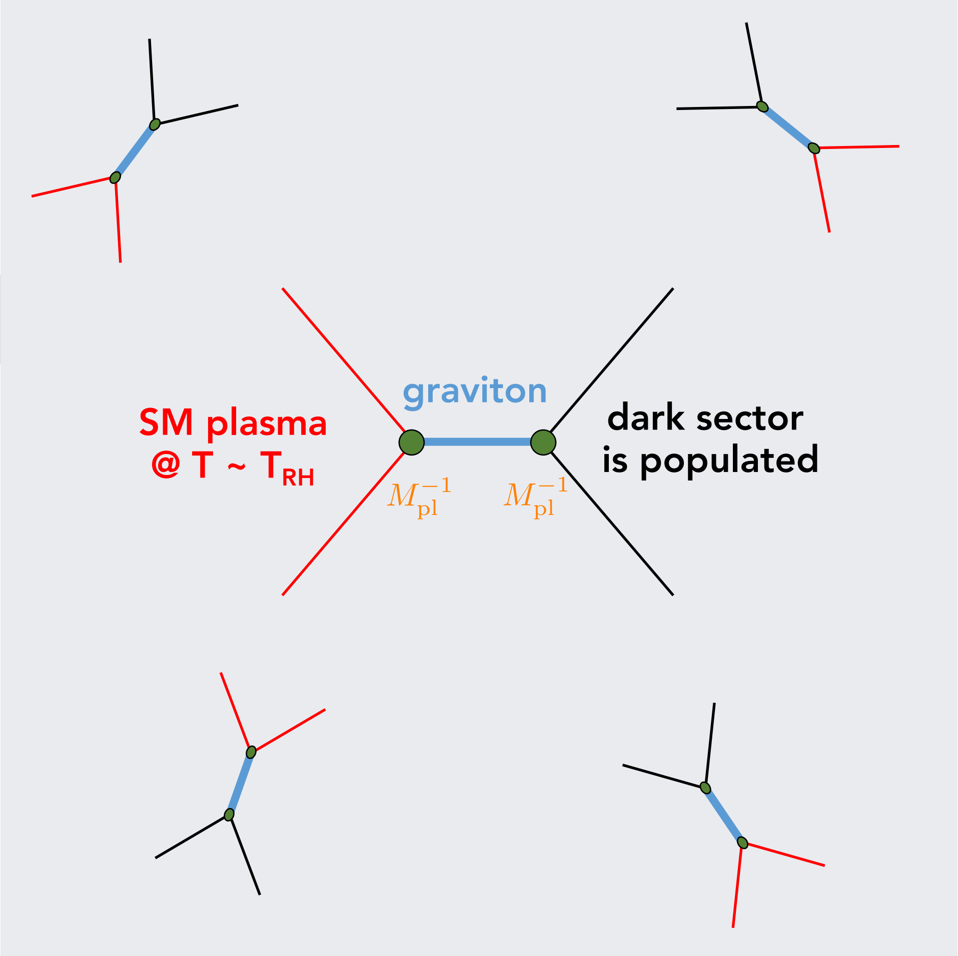

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Garny, Sandora, Sloth (2015); Chianese, Fu, King (2020,21); Redi, Tesi, Tillim (2020); Garcia, Kaneta, Mambrini, Olive, Verner (2021); Clery, Mambrini, Olive, Sherkin, Verner (2022); Barman & Bernal (2021)

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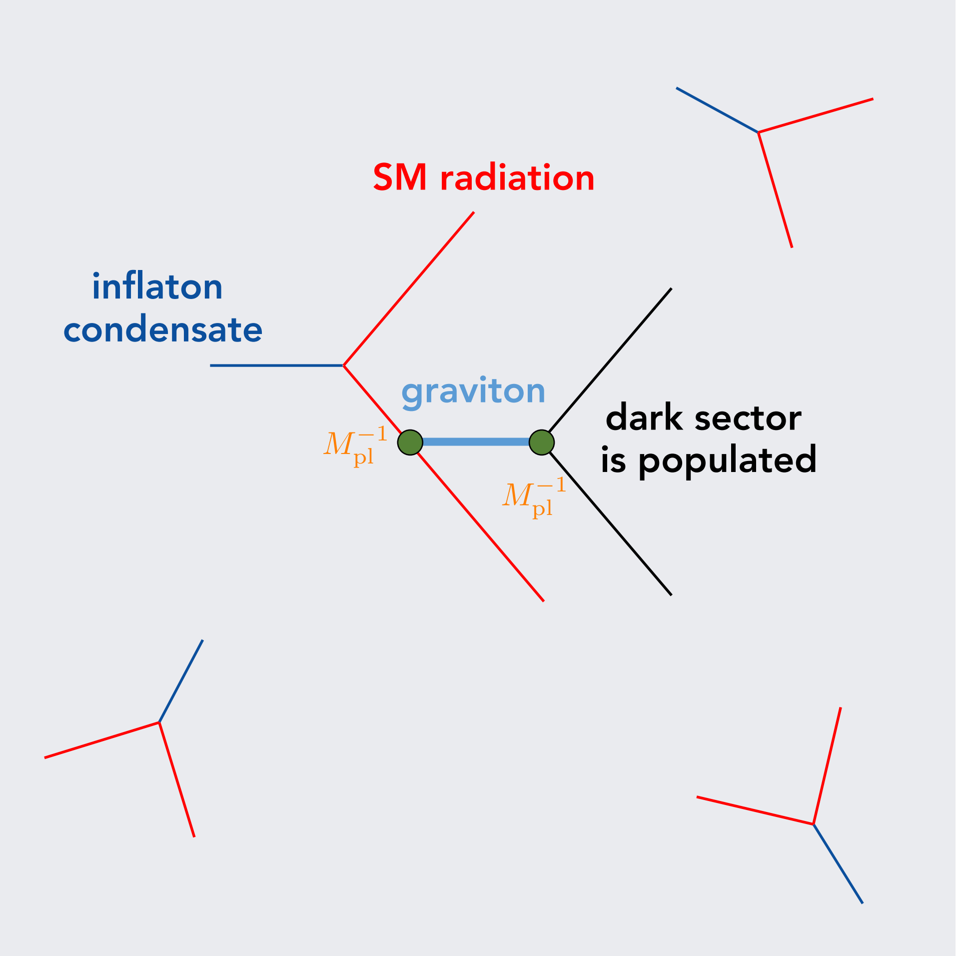

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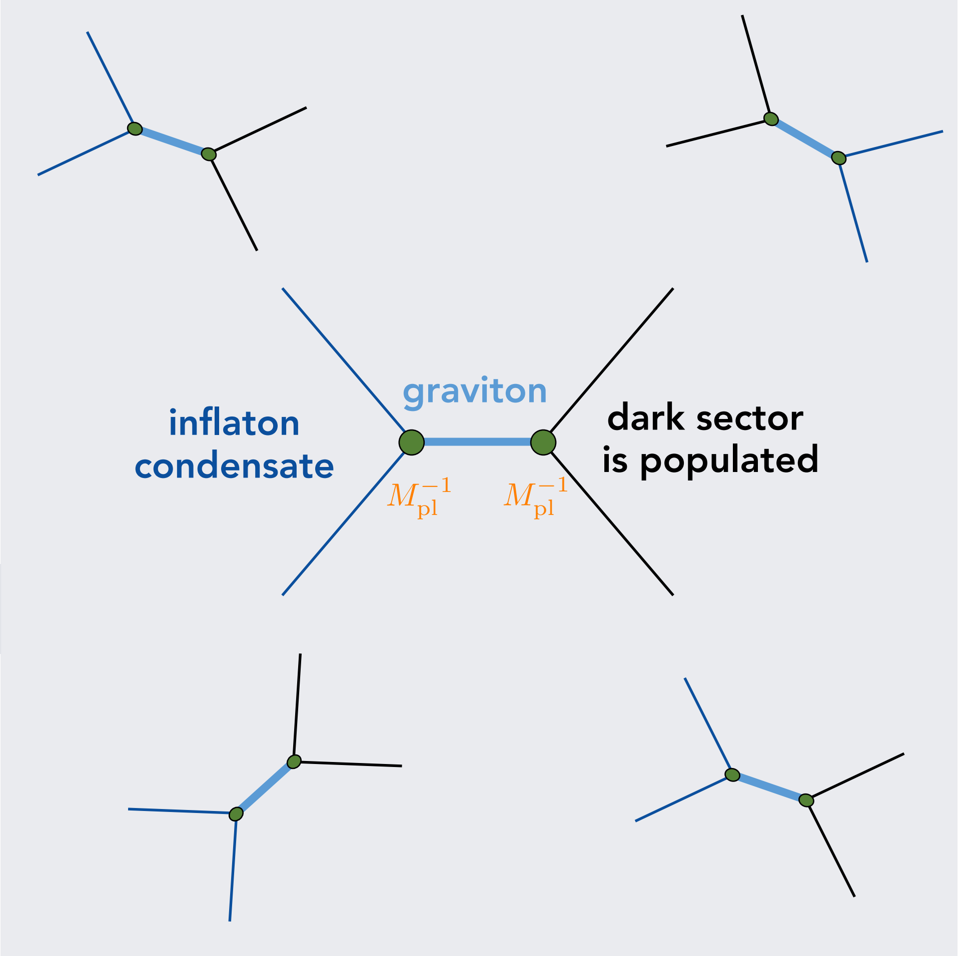

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Ema, Jinno, Mukaida, Nakayama (2016); Ema, Nakayama, Tang (2019a,b); Clery, Mambrini, Olive, Verner (2021); Barman & Bernal (2021)

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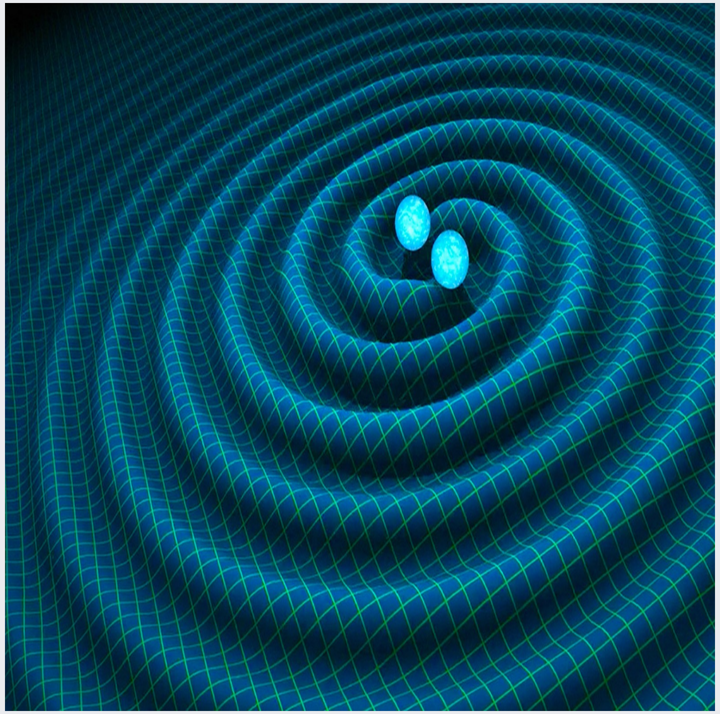

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Maleknejad & Kopp (2024a,b); Garani, Redi, Tesi (2024, 25)

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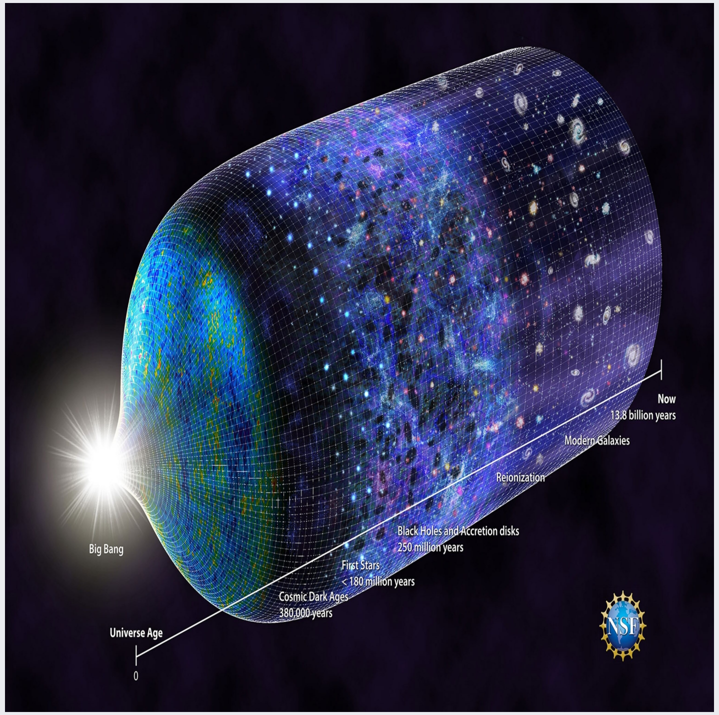

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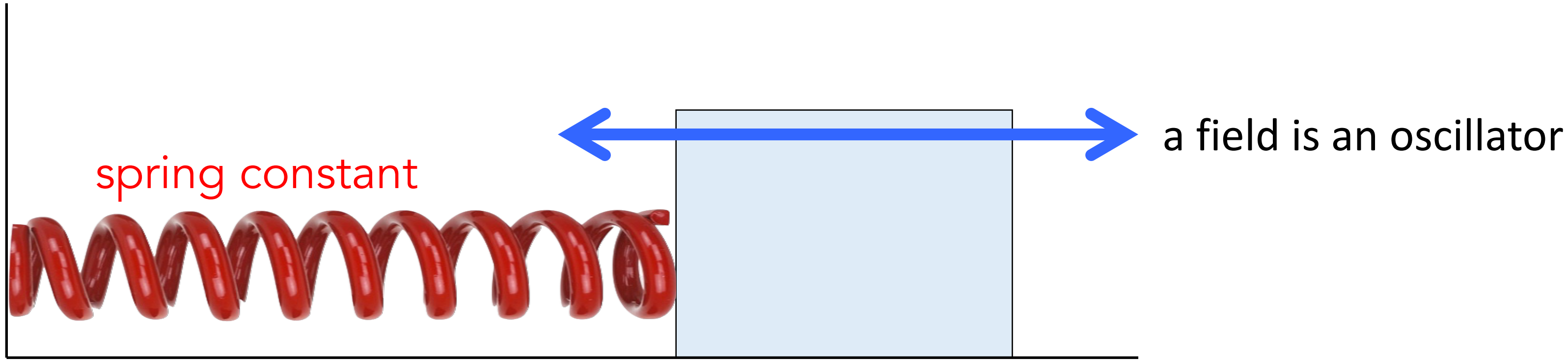
CGPP for any particle species: Parker (1969, 76); applications to dark matter: Kuzmin & Tkachev (1999); Chung, Kolb, Riotto (1999); Ahmed, Grzadkowski, Socha (2020); nice reviews: Ford (2021); Kolb & Long (2023) let's focus on:

cosmological gravitational particle production (CGPP)

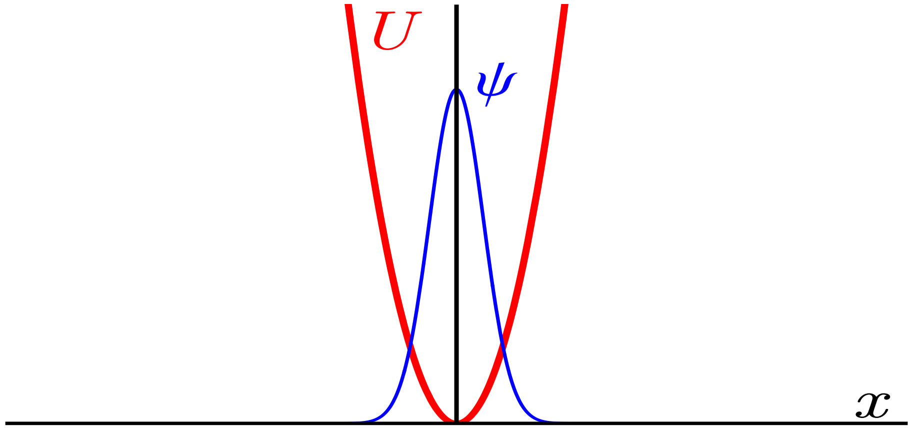
$a(t)$ → $\hat{H}(t)$ → $\langle \hat{N}_{\vec{k}} \rangle$

cosmological expansion — time-dependent Hamiltonian — particles produced

An analogy with 1D quantum mechanics

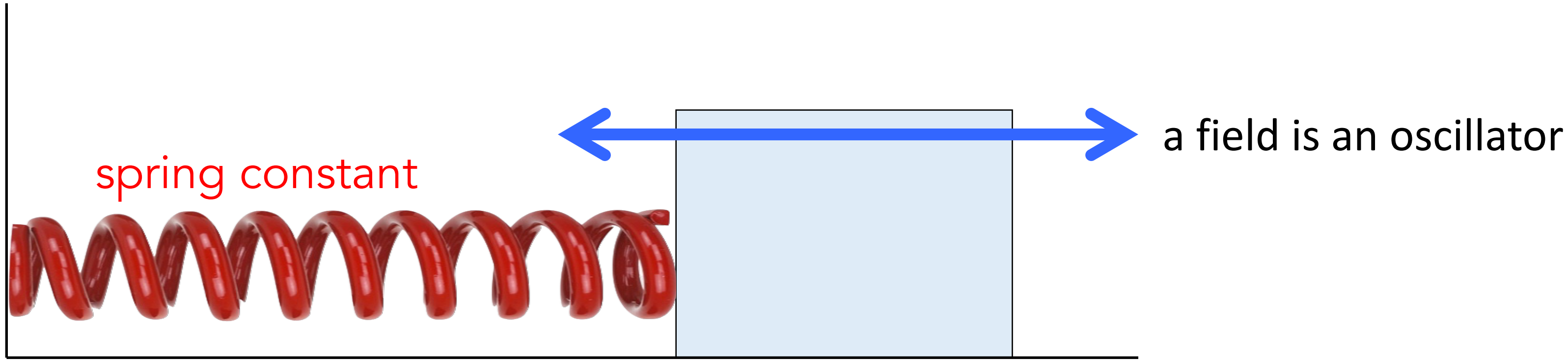


Spring constant is varied
slowly (adiabatically)



Spring constant is varied
abruptly (*non*-adiabatically)

An analogy with 1D quantum mechanics



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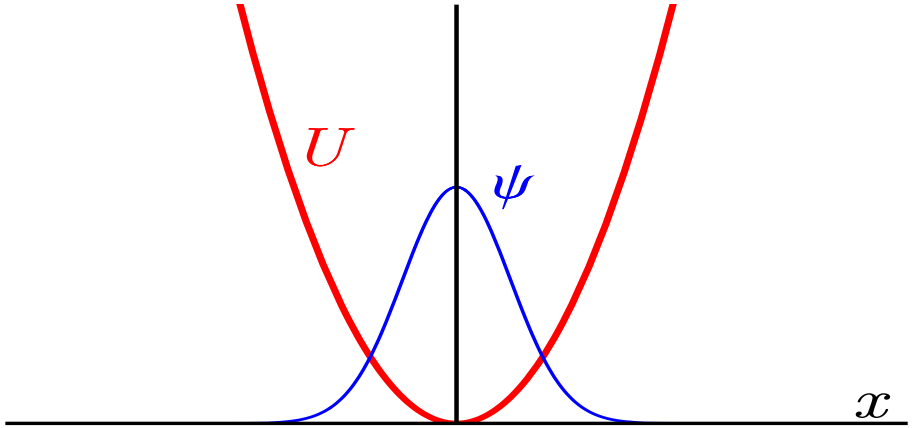

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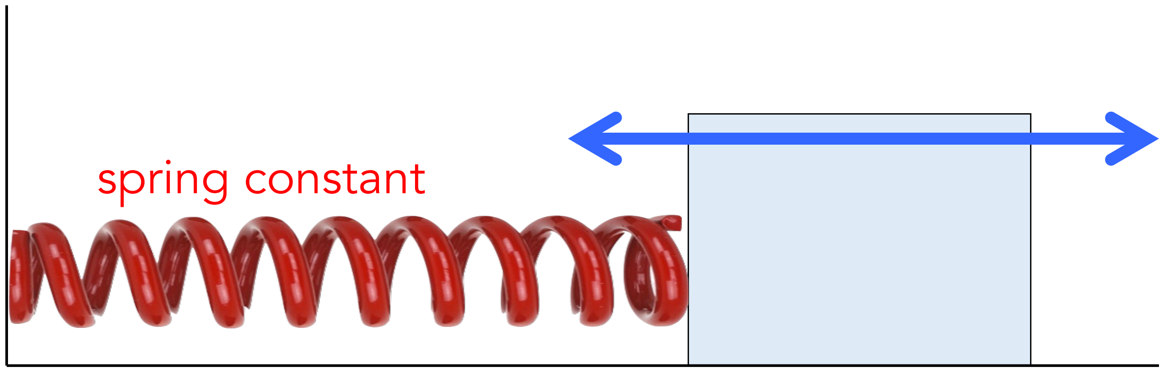

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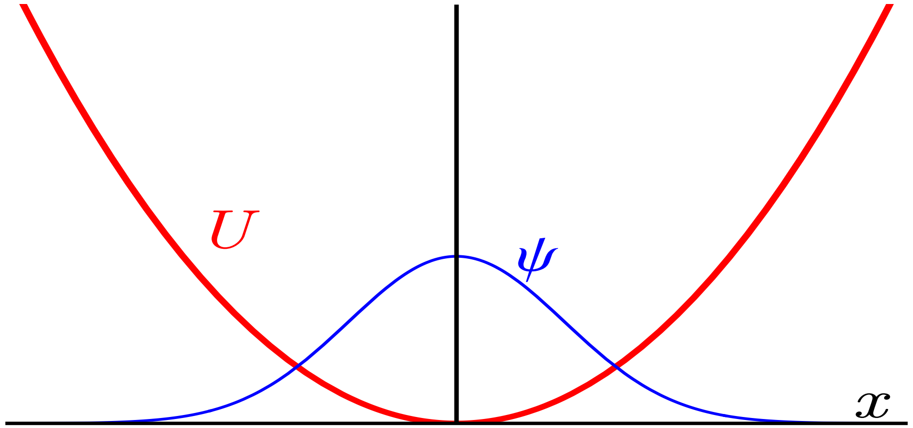

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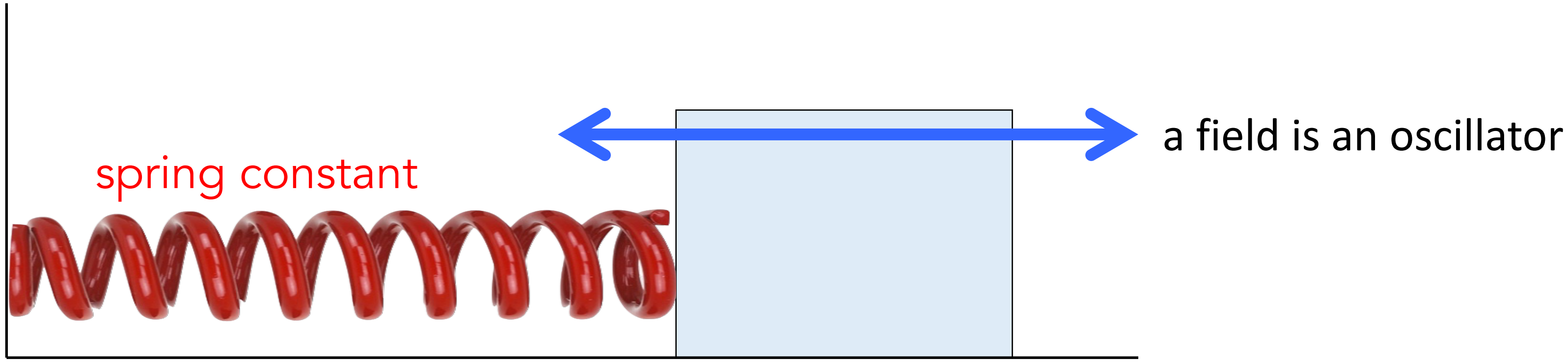

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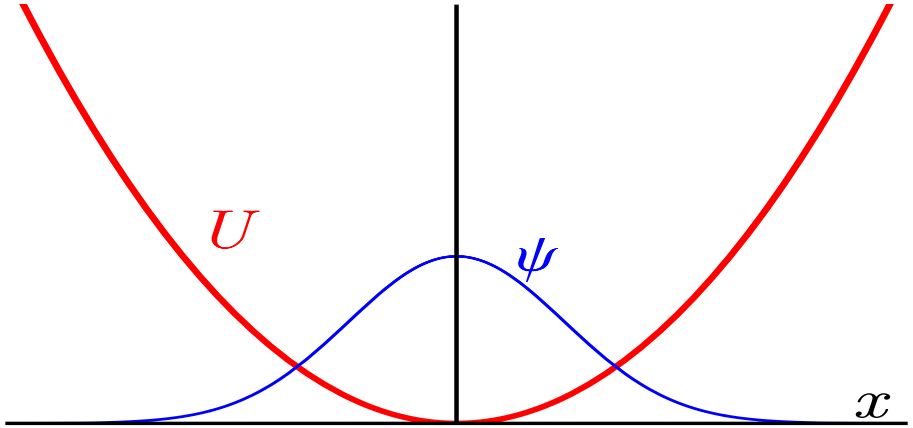

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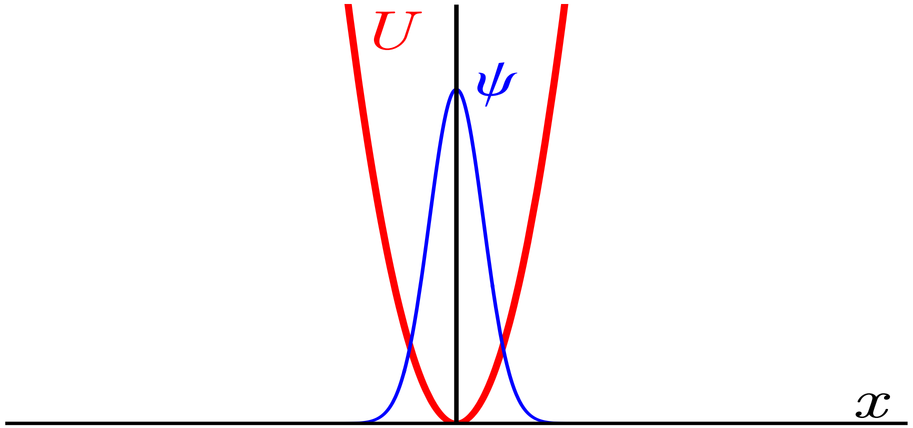

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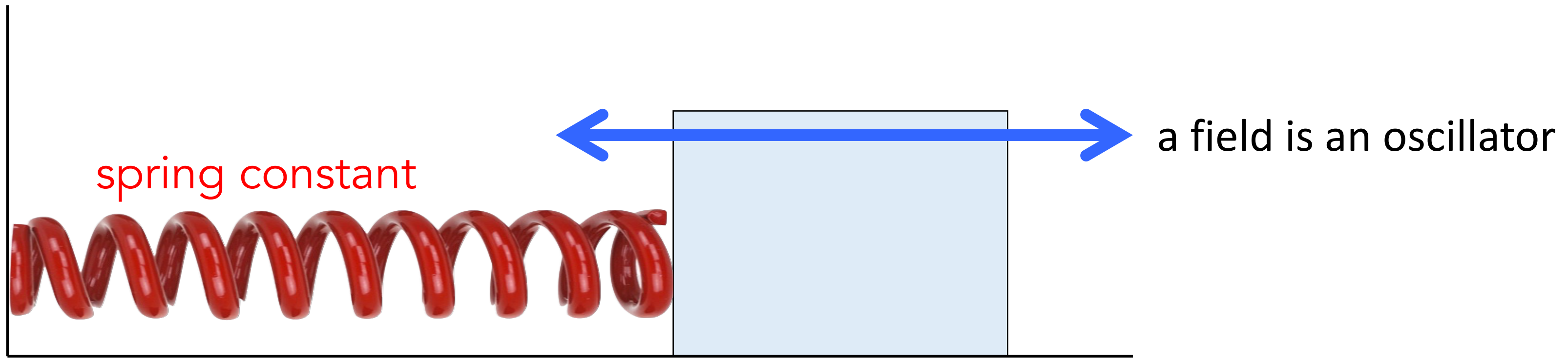

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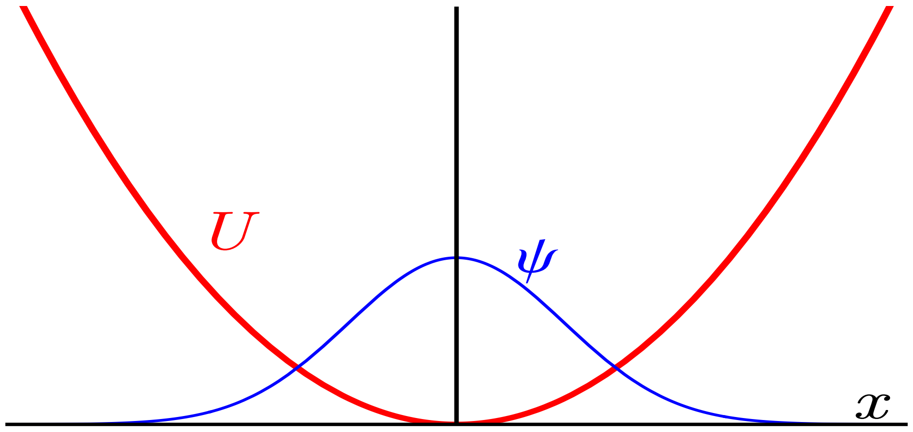

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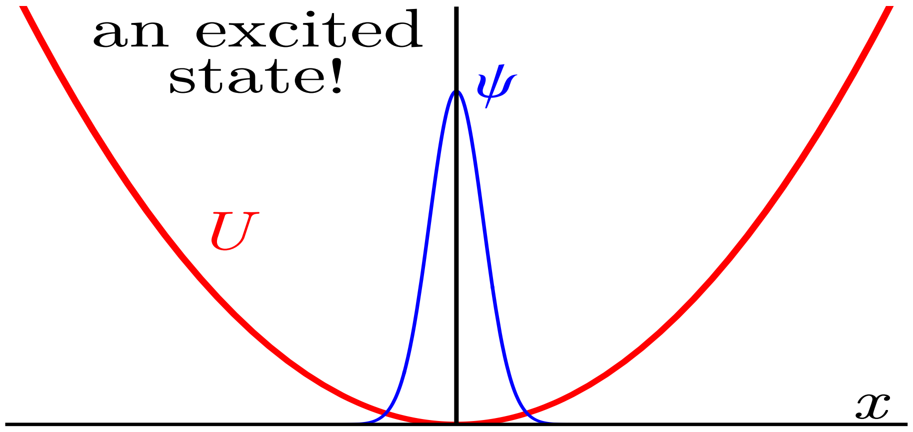

Getting a feel for the numbers

Consider CGPP occurring at the end of inflation

scale of inflation: $m_\phi \sim H_{\mathrm{inf}} \sim H_e$

scale of dark matter: m_χ

Dimensional analysis implies:

$$n_\chi(t_e) = H_e^3 \, f(m_\chi/m_\phi)$$

one particle per Hubble volume

(NB: t_e & H_e = "end" of inflation)

Lessons learned:

- faster expansion (big H_e) ➔ more particles,

so CGPP strongest during inflation / end of inflation

- mass ➔ need not be superheavy,

so CGPP threatens to overproduce stable relics

Relic abundance today:

$$\Omega_\chi h^2 \approx (0.1 f) \left(\frac{m_\chi}{10 \ \mathrm{GeV}} \right) \left(\frac{H_e}{10^{12} \ \mathrm{GeV}} \right)^{3/2}$$

assuming instant RH:

$$T_{\mathrm{RH}} \approx (8.4 \times 10^{14} \ \mathrm{GeV}) \left(\frac{H_e}{10^{12} \ \mathrm{GeV}} \right)^{1/2}$$

(NB: f contains additional m_χ dependence)

people are asking:

- **observables?** relic abundance, particle energy spectrum, power spectrum of inhomogeneities (i.e., isocurvature), non-Gaussianity, secondary grav waves, cosmic rays from late-time decay …
- **d.m. dependence?** dark matter mass & spin, nonminimal dark matter gravitational interaction, dark matter self-interaction …
- **inf. dependence?** model of inflation, i.e. connection with UV embedding, duration of inflation, model of reheating temp & EOS …

what should we call DM that only interacts gravitationally?

[Submitted on 14 Oct 1998]

WIMPZILLAS!

Edward W. Kolb, Daniel J. H. Chung, Antonio Riotto

Despicable dark relics: generated by gravity with unconstrained masses

Malcolm Fairbairn[1], Kimmo Kainulainen[2,4], Tommi Markkanen[3] and Sami Nurmi[2,4]

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Production of purely gravitational dark matter: the case of fermion and vector boson

Yohei Ema,[a,b] Kazunori Nakayama[c,d] and Yong Tang[c]

Completely dark matter from rapid-turn multifield inflation

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a 2018 play by Martin McDonagh

Can spin-0 dark matter arise from CGPP?

yes! … but isocurvature

Scalar Dark Matter CGPP

review: [Kolb, Long (2023)]
see talks by Marcos Garcia & Oleg Lebedev

inflationary quantum fluctuations

light scalar spectator ($m_\chi < H_{\rm inf}$)

$$\delta\chi \sim H_{\rm inf}/2\pi$$

stores energy efficiently

field is frozen outside the horizon

$$\rho_\chi \sim m_\chi^2 \chi^2 \sim a^0$$

cosmological energy fraction

easily makes up all the dark matter

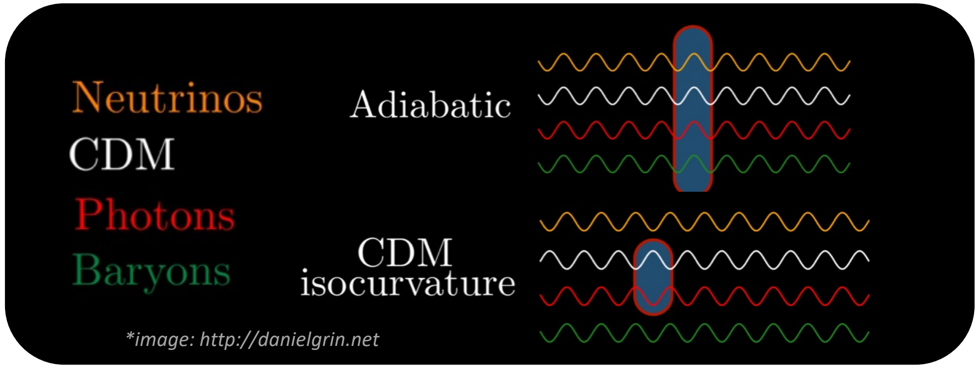
$$\frac{\Omega_\chi h^2}{0.1} \approx \begin{cases} \left(\frac{H_e}{10^{14}\ \mathrm{GeV}}\right)^2 \left(\frac{T_{\rm RH}}{10^2\ \mathrm{GeV}}\right) \log\left(\frac{k_\star^{(\rm late)}}{k_{\rm min}}\right) , & \text{late-RH (i.e., } H_{\rm RH} < m_\chi) \\ \left(\frac{H_e}{10^{14}\ \mathrm{GeV}}\right)^2 \left(\frac{m_\chi}{10^{-5}\ \mathrm{eV}}\right)^{1/2} \log\left(\frac{k_\star^{(\rm early)}}{k_{\rm min}}\right) , & \text{early-RH (i.e., } m_\chi < H_{\rm RH}) \end{cases}$$

assuming $w_{\rm RH}=0$

Scalar Dark Matter CGPP – isocurvature issues

inflaton & spectator fluctuations don't align in space

prediction: dark matter isocurvature



CMB observations consistent with completely adiabatic perturbations

Planck 2018 upper limit on isocurvature

$$\Delta^2_{\mathcal{S}}(k_{\rm cmb}) < 7.3 \times 10^{-11}$$

$$k_{\rm cmb} = 0.002\ {\rm Mpc}^{-1} a_0$$

$$k_{\rm cmb}/a_e H_e \approx e^{-50} \simeq 2 \times 10^{-22}$$

isocurvature is a problem for scalar CGPP

Scalar Dark Matter CGPP – evading isocurvature

1. Consider a heavy spectator instead: $m_\chi = O(1)\, H_{\mathrm{inf}}$
2. Consider a nonminimal coupling: $L_{\mathrm{int}} = \xi\, \chi^2 R$
3. Consider a long period of inflation: $N_{\mathrm{efold}} >> 60$
4. Consider a self-coupling: $L_{\mathrm{int}} = \lambda\, \chi^4$
5. Consider a coupling to the inflaton: $L_{\mathrm{int}} = g\, \phi^2\, \chi^2$

Scalar Dark Matter CGPP – evading isocurvature

[Ling & AL (2101.11621)]
see also: [Chung, Kolb, Riotto, Senatore (2004)]
[Garcia, Pierre, Verner (2023)]

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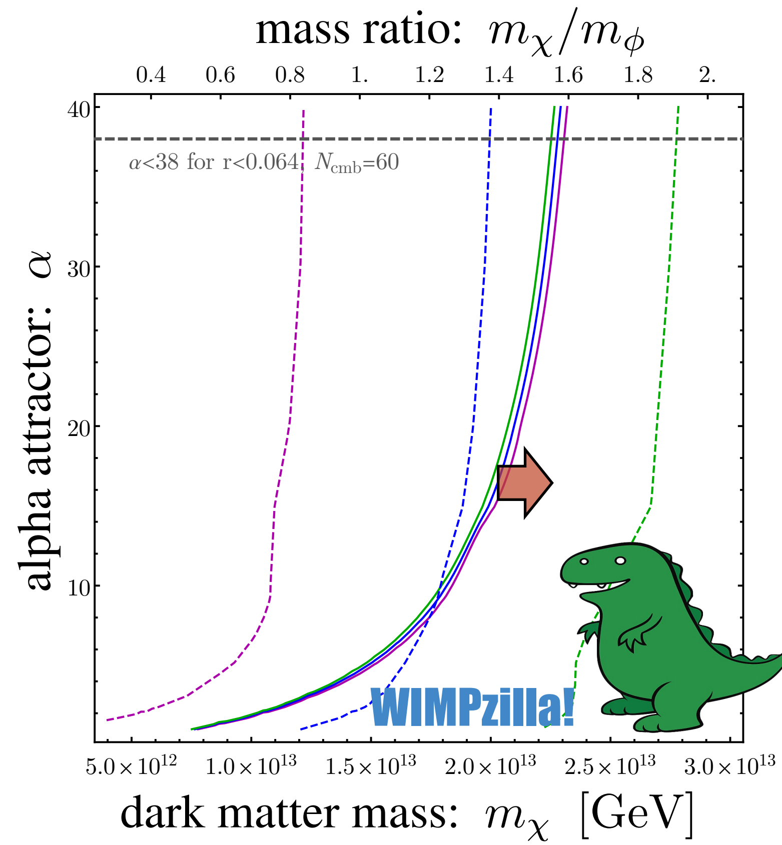

isocurvature avoidance

$$m_\chi \gtrsim (0.8 - 1.6)\, m_\phi$$

Scalar Dark Matter CGPP – evading isocurvature

[Kolb, AL, McDonough, & Payeur (2022)], [Garcia, Pierre, & Verner (2023)]
see also: [Markkanen, Rajantie, & Tenkanen (2018); Tenkanen (2019)]

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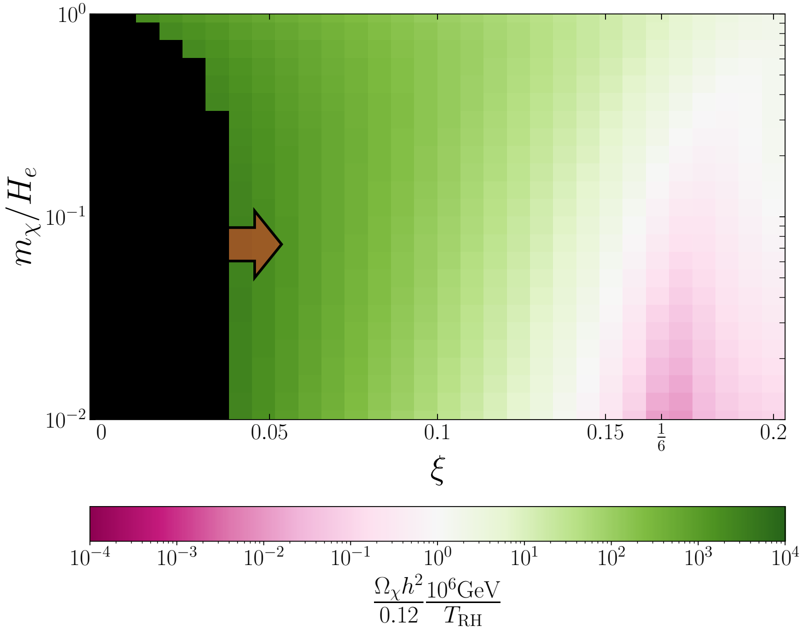


isocurvature avoidance

$$\xi \gtrsim 0.04$$

Scalar Dark Matter CGPP – signals of isocurvature

table: [CMB-S4 Science Book (2017)]
[Domenech, Passaglia, Renaux-Petel (2021)]
[Ebadi, Kumar, McCune, Tai, Wang (2023)]
[Domenech, Trankle (2025)]
figure: [Garcia & Verner (2025)]

it's not a bug, it's a feature!

future CMB observations will search for DM isocurvature

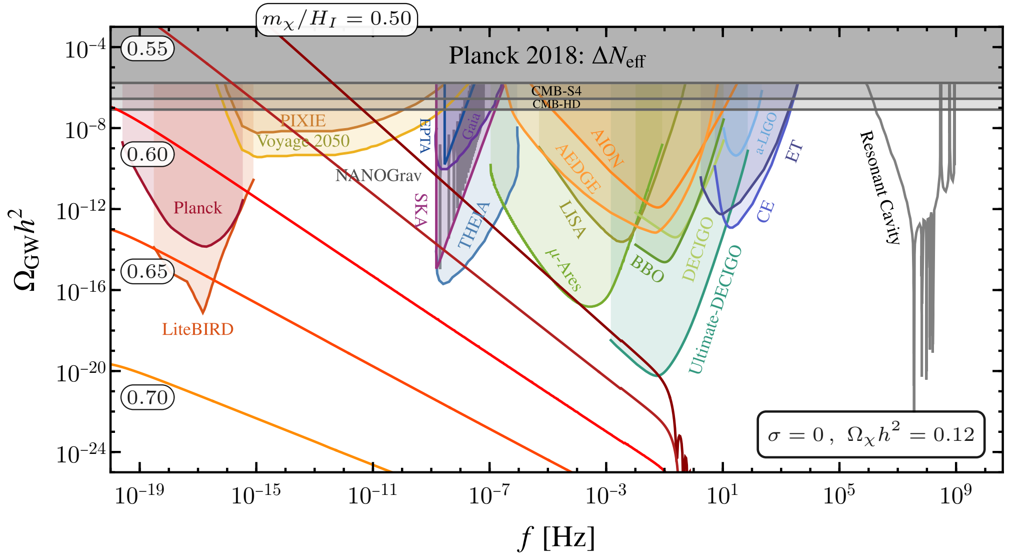
projected sensitivity of CMB-S4

Isocurvature scenario	*Planck*	CMB-S4
	$\Delta r_D / r_D^{\text{adi}}$	$\Delta r_D / r_D^{\text{adi}}$
$(b_{\text{by}}, c_{\text{before}}, L_{y_L})$	0.03	0.005
$(b_{\text{before}}, c_{\text{by}}, L_{y_L})$	0.01	0.004
$(b_{\text{by}}, c_{\text{after}}, L_{y_L})$	0.04	0.01
$(b_{\text{after}}, c_{\text{by}}, L_{y_L})$	0.008	0.002
$(b_{\text{by}}, c_{\text{by}}, L_{y_L})$	0.007	0.002
	$\Delta \chi_D / \chi^{\text{adi}}$	$\Delta \chi_D / \chi^{\text{adi}}$
$(b_{\text{after}}, c_{\text{after}}, L_{y_L})$	0.003	0.0004
	$\Delta \xi_{\text{lep}}^2$	$\Delta \xi_{\text{lep}}^2$
$(b_{\text{by}}, c_{\text{before}}, L_{\text{by}})$	0.02	0.002
$(b_{\text{before}}, c_{\text{by}}, L_{\text{by}})$	0.4	0.04
$(b_{\text{by}}, c_{\text{after}}, L_{\text{by}})$	0.3	0.04
$(b_{\text{after}}, c_{\text{by}}, L_{\text{by}})$	0.3	0.04
$(b_{\text{by}}, c_{\text{by}}, L_{\text{by}})$	0.3	0.04
$(b_{\text{after}}, c_{\text{after}}, L_{\text{by}})$	0.3	0.04

significant improvement expected

large isocurvature can source other signals

e.g., secondary gravitational waves



see talk by Marcos Garcia this afternoon

Can spin-1 dark matter arise from CGPP?

yes! … mostly L-polarization

Vector Dark Matter CGPP

[Graham, Mardon, Rajendran (2016]
figure: [Ahmed, Grzadkowski, Socha (2020)]
figure: [Kolb, AL (2020)]
[Gross, Karamitsos, Landini, Strumia (2020)]

cosmological energy fraction

easily makes up all the dark matter

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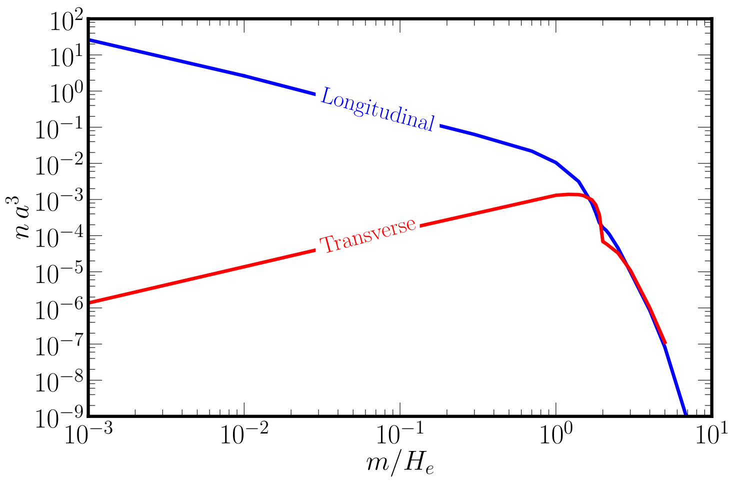
dependence on mass:

longitudinal polarization dominates



ultra-light VDM with a mass as low as 10 μeV could arise from CGPP

Vector Dark Matter CGPP – no isocurvature issues

[Graham, Mardon, Rajendran (2016)]
figure: [Ahmed, Grzadkowski, Socha (2020)]
[Kolb, AL (2020)]
[Gross, Karamitsos, Landini, Strumia (2020)]

energy depletes during inflation
after modes leave the horizon
b/c inverse metric in the mass term

$$\rho_A \sim m_A^2 \, g^{\mu\nu} A_\mu A_\nu \sim a^{-2}$$

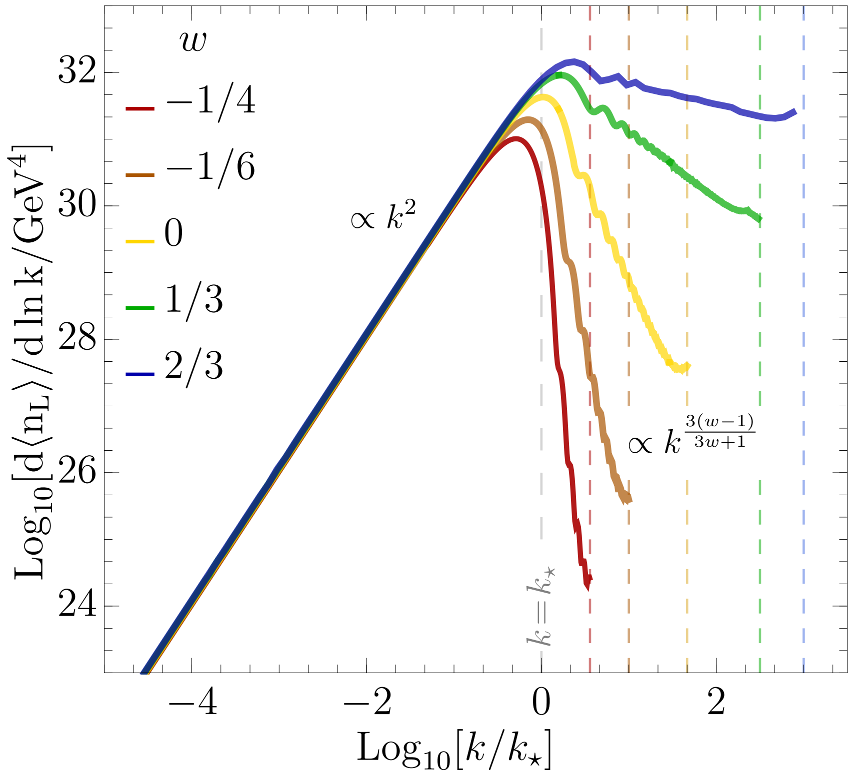
low-momentum modes
carry less energy
spectrum is automatically blue-tilted

$$d\rho / d \ln k \propto k^2$$

$$\Delta^2_{\text{iso}} \propto k^3$$

isocurvature is negligible at CMB scales!
prediction: small-scale isocurvature

$\frac{d\langle n_L(a_\star)\rangle}{d \ln k}(k)$ for $H_{rh} < m_X < H_I$



Can spin-2 dark matter arise from CGPP?

yes! … but ghosts

Tensor Dark Matter CGPP – earlier work

[Babichev et al (2016)]
[Alexander, Jenks, McDonough (2020)]
[Kolb, Ling, AL, Rosen (2023)]

Babichev et al
only considered thermal freeze in after inflation

Alexander et al
only considered maximal helicity in a dS background

our paper
first comprehensive study of CGPP for massive spin-2 particles in an inflationary background

Tensor Dark Matter CGPP – polarizations

Perform a scalar-vector-tensor (SVT) decomposition

$$\begin{aligned} v_{\mu\nu}(\eta, \boldsymbol{x}) &\sim \text{massive spin-2} \\ &\sim (\text{helicity } \lambda = \pm 2) \oplus (\text{helicity } \lambda = \pm 1) \oplus (\text{helicity } \lambda = 0) \end{aligned}$$

Tensor Dark Matter CGPP – polarizations

Perform a scalar-vector-tensor (SVT) decomposition

$$v_{\mu\nu}(\eta, \boldsymbol{x}) \sim \text{massive spin-2}$$
$$\sim (\text{helicity } \lambda = \pm 2) \oplus (\text{helicity } \lambda = \pm 1) \oplus (\text{helicity } \lambda = 0)$$

Tensor sector

$$\chi''_{k,\lambda}(\eta) + \omega_k^2(\eta)\,\chi_{k,\lambda}(\eta) = 0 \qquad \text{for } \lambda = \pm 2$$

$$\omega_k^2(\eta) = k^2 + a^2 m^2 - a''/a$$

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$$\omega_k^2(\eta) = k^2 + a^2 m^2 - f''/f \quad \text{where} \quad f = a^2 / \sqrt{k^2 + a^2 m^2}$$

Tensor Dark Matter CGPP – polarizations

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Vector sector

$$\chi''_{k,\lambda}(\eta) + \omega_k^2(\eta)\, \chi_{k,\lambda}(\eta) = 0 \qquad \text{for } \lambda = \pm 1$$

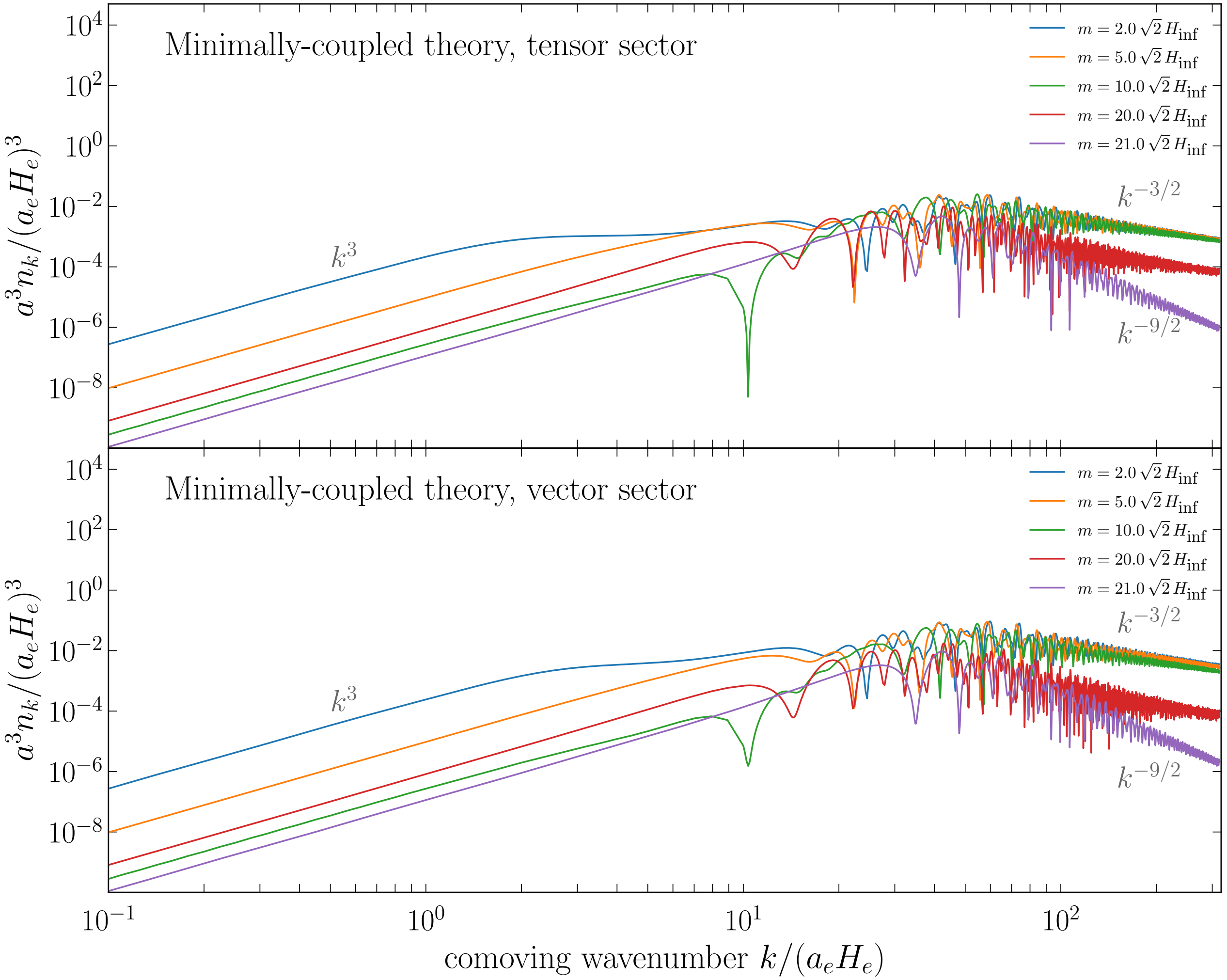
$$\omega_k^2(\eta) = k^2 + a^2 m^2 - f''/f \quad \text{where} \quad f = a^2/\sqrt{k^2 + a^2 m^2}$$

Scalar sector – it's complicated!

$$L_{S,\boldsymbol{k}} = K_\Pi\, |\tilde{\Pi}'|^2 + M_\Pi\, |\tilde{\Pi}|^2 + K_\mathcal{B}\, |\tilde{\mathcal{B}}'|^2 + M_\mathcal{B}\, |\tilde{\mathcal{B}}|^2 + \lambda_1\, \tilde{\Pi}^* \tilde{\mathcal{B}}' + \lambda_0\, \tilde{\Pi}^* \tilde{\mathcal{B}}$$

Tensor Dark Matter CGPP – numerical results

spectra: [Kolb, Ling, AL, Rosen (2302.04390)]
scattering: [Chung, Kolb, AL (1812.00211)]
interference: [Basso, Chung, Kolb, AL (2209.01713)]



Notable features:

1. Similar results for tensors & vectors
2. Low-k power law ~ k^3
3. High-k power law ~ $k^{-3/2}$ or $k^{-9/2}$
4. Wiggles!

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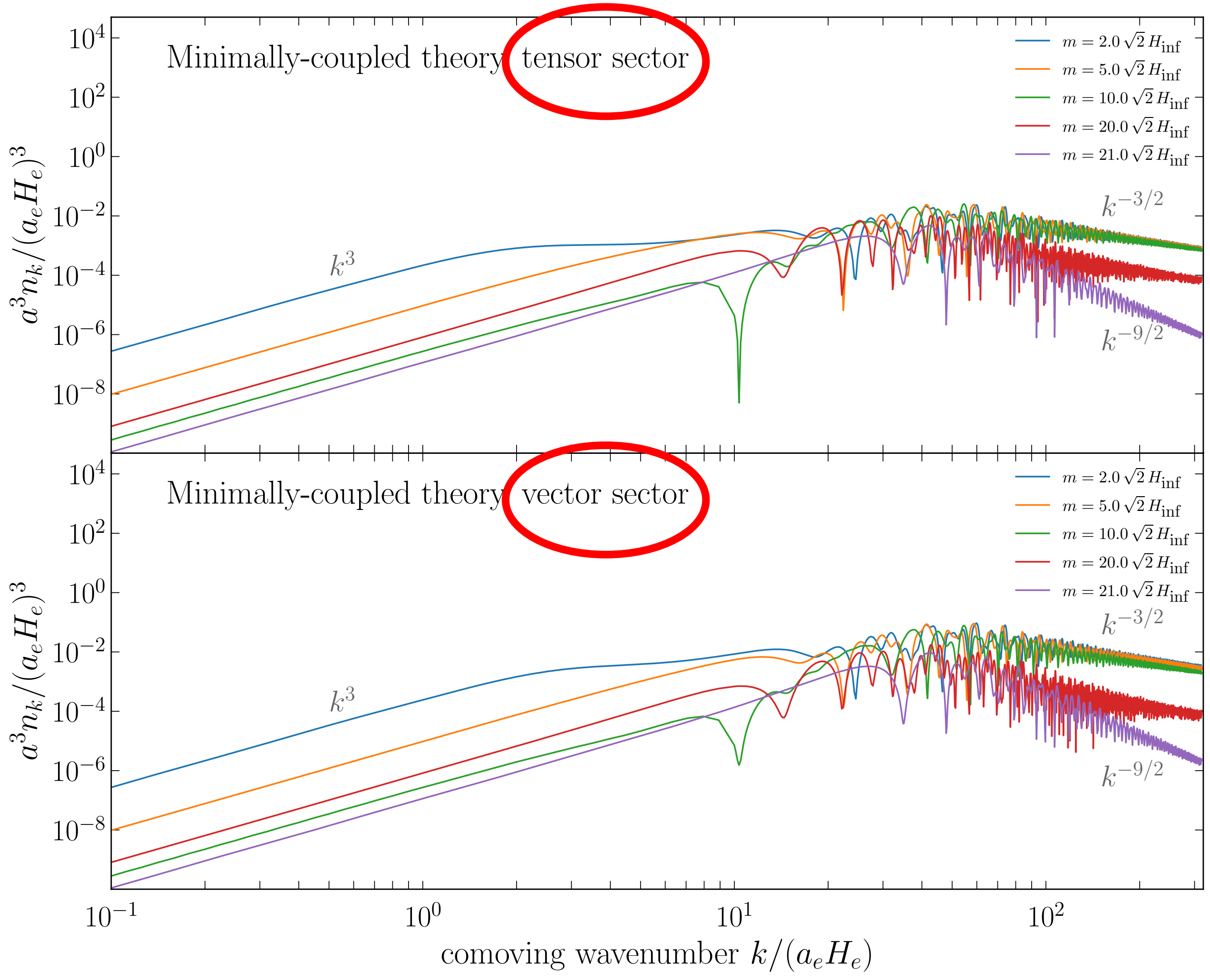

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$$\text{tensor sector:} \quad \omega_k^2(\eta) = k^2 + a^2 m^2 - a''/a$$
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equal for nonrelativistic modes

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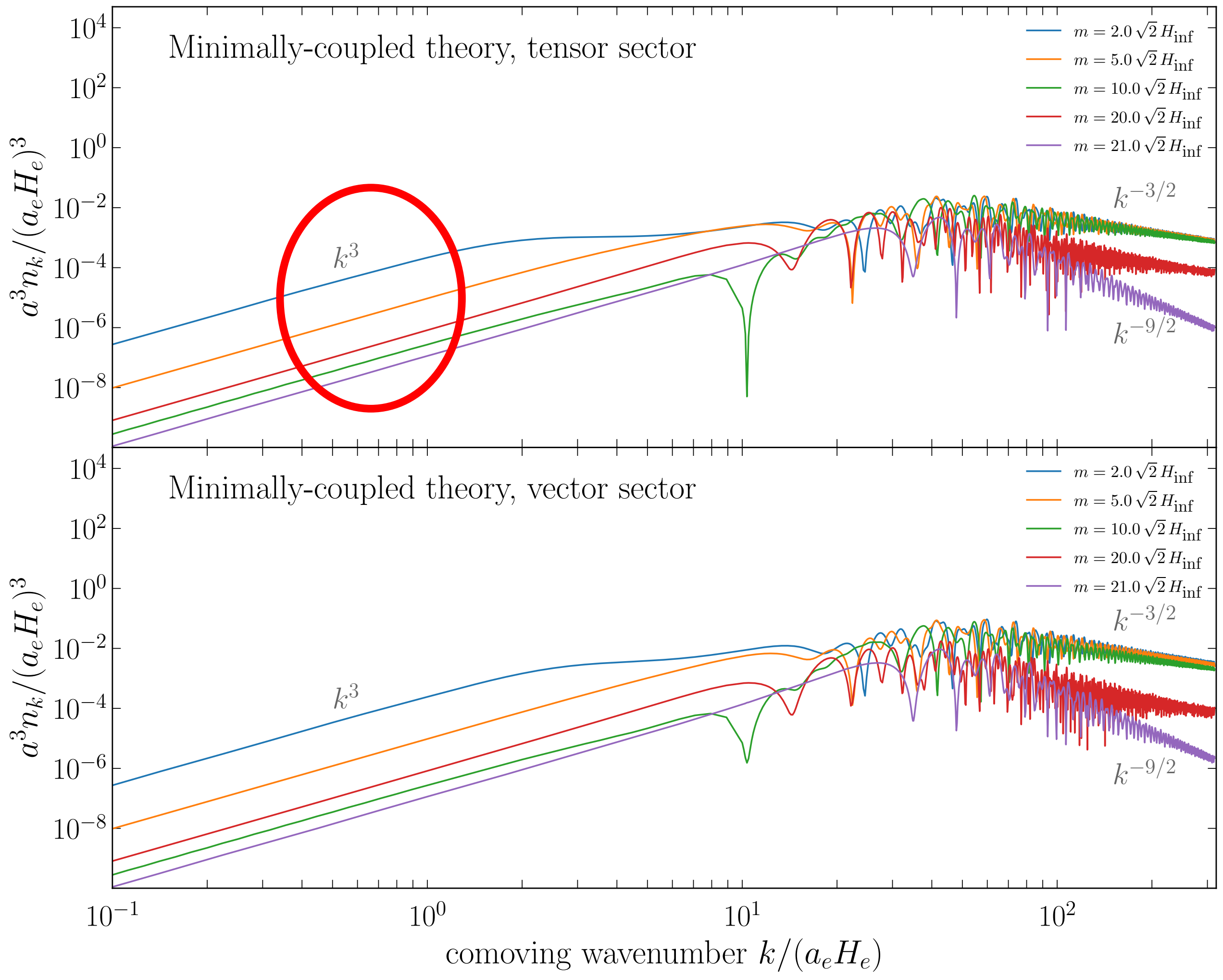

Notable features:

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4. Wiggles!

$$n_k \propto k^\nu \quad \text{with} \quad \nu = 3 - 2\left[\frac{9}{4} - \frac{m^2}{H_{\rm inf}^2}\right]^{1/2}$$

$$\mathrm{Re}[\nu] = 3 \quad \text{for } m > \tfrac{3}{2} H_{\rm inf}$$

low-k modes have familiar dS solution

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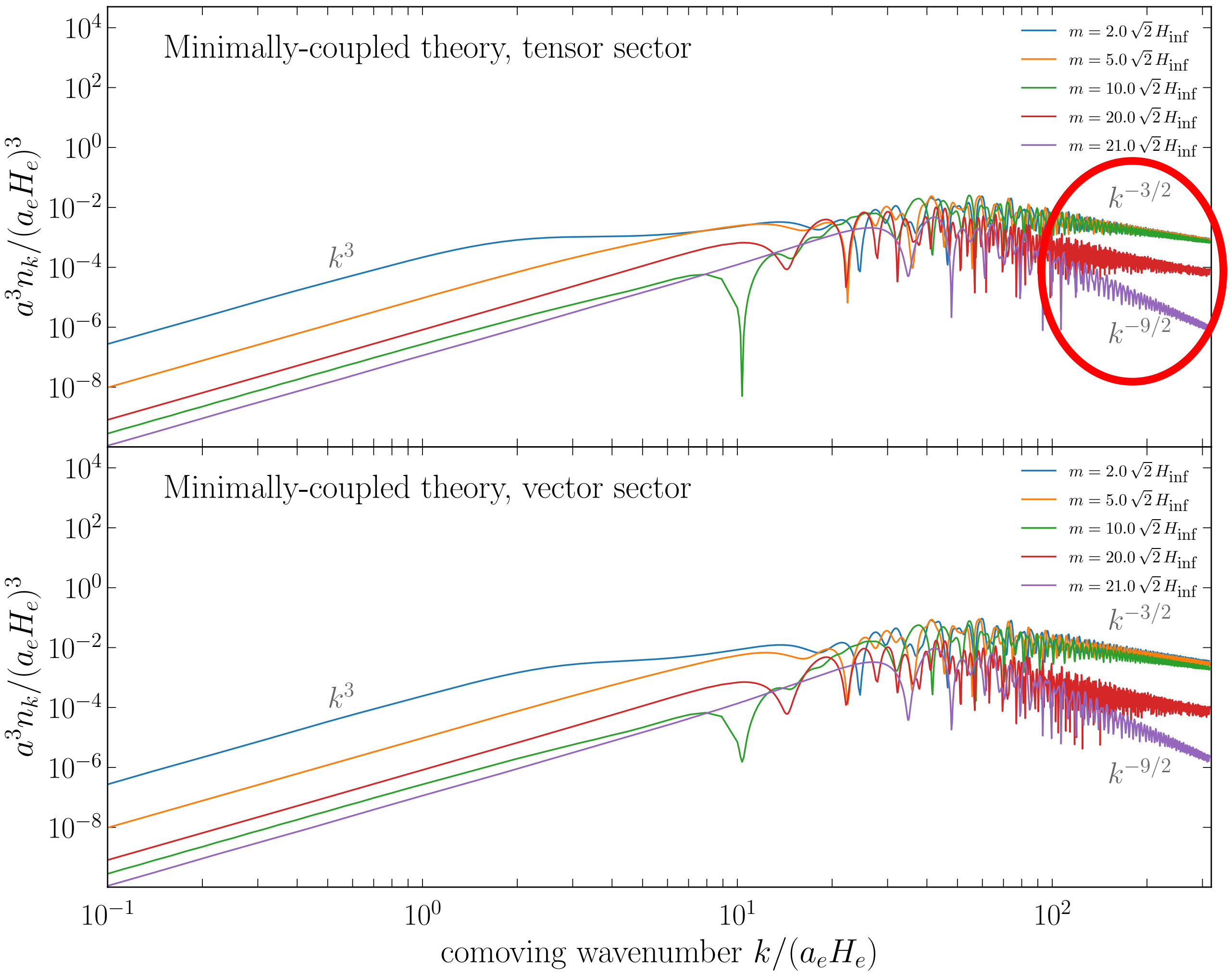

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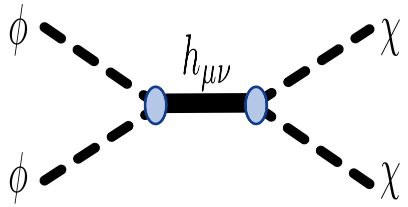

inflaton annihilation

$$\Gamma_{\phi\phi\to\chi\chi} \simeq \frac{C}{16\pi}\frac{\Phi^2}{M_P^4}m_\phi^4$$

$$\phi\phi \to \chi\chi \quad \Rightarrow \quad n_k \propto k^{-3/2}$$

$$\phi\phi\phi \to \chi\chi \quad \Rightarrow \quad n_k \propto k^{-9/2}$$

$$n\phi \to \chi\chi \quad \Rightarrow \quad n_k \propto k^{-3(2n-3)/2}$$

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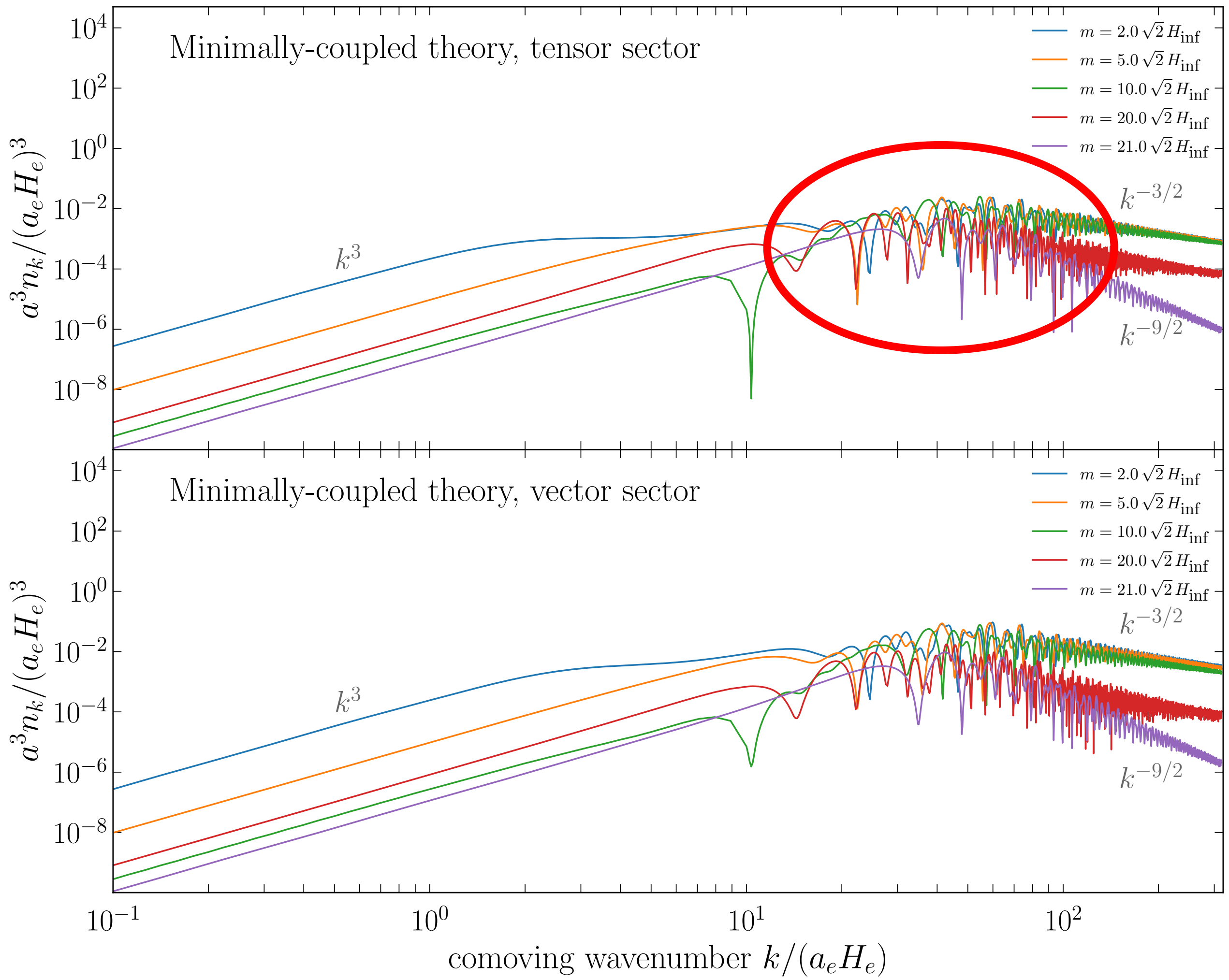

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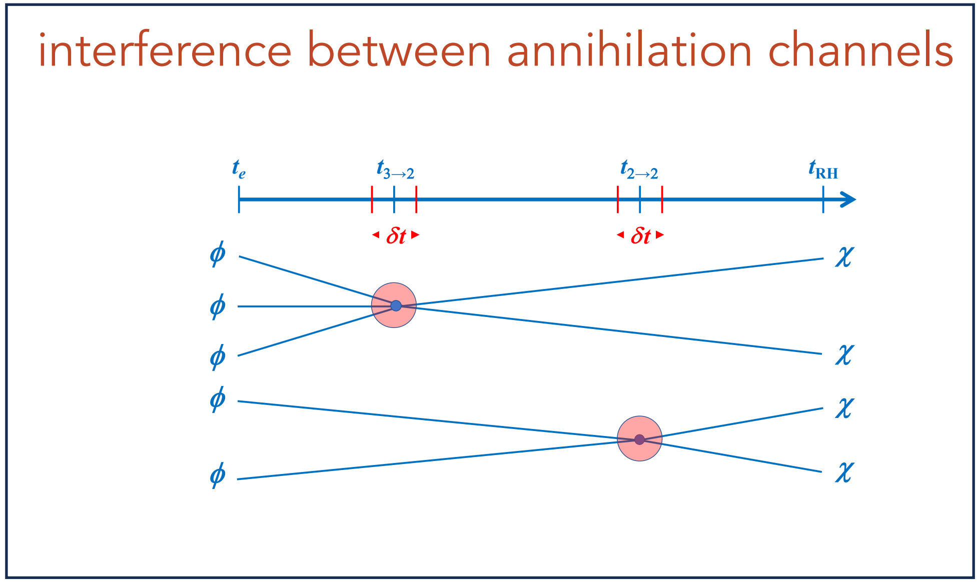


Tensor Dark Matter CGPP – longitudinal polarization

[Kolb, Ling, AL, Rosen (2302.04390)]

Scalar metric perturbations mix with scalar inflaton perturbation

$$L_S = \text{a messy function of } A,\, B,\, E,\, F,\, \text{and } \varphi_v$$

After imposing constraints (and a LOT of algebra) there are only two propagating degrees of freedom

$$L_{S,\boldsymbol{k}} = K_\Pi \, |\tilde{\Pi}'|^2 + M_\Pi \, |\tilde{\Pi}|^2 + K_{\mathcal{B}} \, |\tilde{\mathcal{B}}'|^2 + M_{\mathcal{B}} \, |\tilde{\mathcal{B}}|^2 + \lambda_1 \, \tilde{\Pi}^* \tilde{\mathcal{B}}' + \lambda_0 \, \tilde{\Pi}^* \tilde{\mathcal{B}}$$

$$K_\varphi = \frac{a^2}{2} \frac{H^2k^4 + 3a^2\left(m^2 - m_H^2\right)H^2k^2 + \frac{9}{4}a^4m^2\left(m^2 - m_H^2\right)H^2}{H^2k^4 + 3a^2\left(m^2 - m_H^2\right)H^2k^2 + \frac{3}{8}a^4m^2\left(6m^2H^2 - 4H^2m_H^2 - m_H^4\right)} \tag{3.17a}$$

$$M_\varphi = \frac{a^2}{2} \frac{c_{10}k^{10} + c_8k^8 + c_6k^6 + c_4k^4 + c_2k^2 + c_0}{\left[H^2k^4 + 3a^2\left(m^2 - m_H^2\right)H^2k^2 + \frac{3}{8}a^4m^2\left(6m^2H^2 - 4H^2m_H^2 - m_H^4\right)\right]^2} \tag{3.17b}$$

$$\begin{aligned}
c_{10} &= H^4 \\
c_8 &= \tfrac{1}{2}a^2H^2\Big[\left(12m^2H^2 + 8H^4 - 14H^2m_H^2 - m_H^4\right) + 4\tfrac{HV'(\bar\phi)\bar\phi'}{aM_P^2} + 2H^2V''(\bar\phi)\Big] \\
c_6 &= \tfrac{3}{8}a^4H^2\Big[\big(36m^4H^2 + 72m^2H^4 - 82m^2H^2m_H^2 - 64H^4m_H^2 \\
&\quad - 7m^2m_H^4 + 40H^2m_H^4 + 8m_H^6\big) \\
&\quad + 8\left(3m^2 - 4m_H^2\right)\tfrac{HV'(\bar\phi)\bar\phi'}{aM_P^2} \\
&\quad + 16\left(m^2 - m_H^2\right)H^2V''(\bar\phi)\Big] \\
c_4 &= \tfrac{3}{8}a^6\Big[4H^2\big(9m^6H^2 + 36m^4H^4 + 16m^2H^6 - 30m^4H^2m_H^2 - 76m^2H^4m_H^2 \\
&\quad - 3m^4m_H^4 + 31m^2H^2m_H^4 + 24H^4m_H^4 + 6m^2m_H^6 - 6H^2m_H^6 - 3m_H^8\big) \\
&\quad - 4m^2H^2\left(H^2 - m_H^2\right)\tfrac{V'(\bar\phi)^2}{M_P^2} \\
&\quad + \left(36m^4H^2 + 8m^2H^4 - 94m^2H^2m_H^2 + m^2m_H^4 + 48H^2m_H^4\right)\tfrac{HV'(\bar\phi)\bar\phi'}{aM_P^2} \\
&\quad + \left(36m^4H^2 - 58m^2H^2m_H^2 - m^2m_H^4 + 24H^2m_H^4\right)H^2V''(\bar\phi)\Big] \\
c_2 &= \tfrac{9}{32}a^8m^2\Big[H^2\big(18m^6H^2 + 120m^4H^4 + 128m^2H^6 - 78m^4H^2m_H^2 - 384m^2H^4m_H^2 \\
&\quad - 9m^4m_H^4 + 132m^2H^2m_H^4 + 128H^4m_H^4 + 23m^2m_H^6 - 32H^2m_H^6 - 16m_H^8\big) \\
&\quad - 8H^2\left(2m^2H^2 - 2m^2m_H^2 + m_H^4\right)\tfrac{V'(\bar\phi)^2}{M_P^2} \\
&\quad + 4\left(6m^4H^2 - 22m^2H^2m_H^2 + m^2m_H^4 + 14H^2m_H^4\right)\tfrac{HV'(\bar\phi)\bar\phi'}{aM_P^2} \\
&\quad + 4\left(m^2 - m_H^2\right)\left(12m^2H^2 - 10H^2m_H^2 - m_H^4\right)H^2V''(\bar\phi)\Big] \\
c_0 &= \tfrac{27}{32}a^{10}m^4\Big[-2H^2\left(2m^2H^2 - 2m^2m_H^2 + m_H^4\right)\tfrac{V'(\bar\phi)^2}{M_P^2} \\
&\quad - m^2\left(2H^2 - m_H^2\right)\left(4H^2 + m_H^2\right)\tfrac{HV'(\bar\phi)\bar\phi'}{aM_P^2} \\
&\quad + \left(m^2 - m_H^2\right)\left(6m^2H^2 - 4H^2m_H^2 - m_H^4\right)H^2V''(\bar\phi)\Big]
\end{aligned}$$

$$K_B = \frac{a^6m^2}{8} \frac{\left(8m^2H^2 - 6H^2m_H^2 - m^2m_H^2\right)k^4}{H^2k^4 + 3a^2\left(m^2 - m_H^2\right)H^2k^2 + \frac{3}{8}a^4m^2\left(6m^2H^2 - 4H^2m_H^2 - m_H^4\right)} \tag{3.17c}$$

$$M_B = \frac{a^6m^2}{8} \frac{c_{10}k^{10} + c_8k^8 + c_6k^6 + c_4k^4}{\left[H^2k^4 + 3a^2\left(m^2 - m_H^2\right)H^2k^2 + \frac{3}{8}a^4m^2\left(6m^2H^2 - 4H^2m_H^2 - m_H^4\right)\right]^2} \tag{3.17d}$$

$$\begin{aligned}
c_{10} &= H^2\left(8m^2H^2 - 8H^4 - 2H^2m_H^2 - m^2m_H^2\right) \\
c_8 &= a^2H^2\Big[\big(30m^4H^2 + 32m^2H^4 - 96H^6 - 3m^4m_H^2 - 56m^2H^2m_H^2 \\
&\quad + 48H^4m_H^2 + 5m^2m_H^4 + 6H^2m_H^4\big) \\
&\quad + \left(4m^2 - 24H^2\right)\tfrac{HV'(\bar\phi)\bar\phi'}{aM_P^2}\Big] \\
c_6 &= \tfrac{3}{8}a^4m^2\Big[\big(96m^4H^4 + 144m^2H^6 - 6m^4H^2m_H^2 - 252m^2H^4m_H^2 - 192H^6m_H^2 \\
&\quad + 8m^2H^2m_H^4 + 200H^4m_H^4 - 10H^2m_H^6 - m^2m_H^6\big) \\
&\quad + \left(8m^2m_H^2 - 16H^2m_H^2\right)\tfrac{HV'(\bar\phi)\bar\phi'}{aM_P^2}\Big] \\
c_4 &= \tfrac{3}{8}a^6m^4\Big[\big(36m^4H^4 - 48m^2H^6 + 64H^8 - 12m^2H^4m_H^2 - 32H^6m_H^2 \\
&\quad - 12m^2H^2m_H^4 + 4H^4m_H^4 + 12H^2m_H^6 - 3m^2m_H^6 + 2m_H^8\big) \\
&\quad - \left(24m^2H^2 - 16H^4 - 12m^2m_H^2 - 8H^2m_H^2 + 8m_H^4\right)\tfrac{HV'(\bar\phi)\bar\phi'}{aM_P^2}\Big]
\end{aligned}$$

$$L_2 = \frac{a^3m^2\bar\phi'}{2M_PH} \frac{H^2k^4 + \frac{3}{2}a^2\left(m^2 - m_H^2\right)H^2k^2}{H^2k^4 + 3a^2\left(m^2 - m_H^2\right)H^2k^2 + \frac{3}{8}a^4m^2\left(6m^2H^2 - 4H^2m_H^2 - m_H^4\right)} \tag{3.17e}$$

$$L_1 = -\frac{a^4m^2\bar\phi'}{M_P} \frac{\left(H^2 - \frac{1}{4}m_H^2 - \frac{1}{2}\frac{aHV'(\bar\phi)}{\bar\phi'}\right)k^4 - \frac{3}{2}a^2\left(m^2 - m_H^2\right)\left(H^2 + \frac{1}{4}m_H^2 + \frac{1}{2}\frac{aHV'(\bar\phi)}{\bar\phi'}\right)k^2}{H^2k^4 + 3a^2\left(m^2 - m_H^2\right)H^2k^2 + \frac{3}{8}a^4m^2\left(6m^2H^2 - 4H^2m_H^2 - m_H^4\right)} \tag{3.17f}$$

$$L_0 = \frac{a^3m^2\bar\phi'}{2M_PH} \frac{c_{10}k^{10} + c_8k^8 + c_6k^6 + c_4k^4 + c_2k^2}{\left[H^2k^4 + 3a^2\left(m^2 - m_H^2\right)H^2k^2 + \frac{3}{8}a^4m^2\left(6m^2H^2 - 4H^2m_H^2 - m_H^4\right)\right]^2} \tag{3.17g}$$

$$\begin{aligned}
c_{10} &= H^4 \qquad\qquad (3.17\text{h}) \\
c_8 &= \tfrac{1}{2}a^2H^4\Big[\left(9m^2 + 12H^2 - 13m_H^2\right) - 4\tfrac{aHV'(\bar\phi)}{\bar\phi'}\Big] \\
c_6 &= \tfrac{3}{8}a^4H^2\Big[\big(18m^4H^2 + 32m^2H^4 + 64H^6 - 48m^2H^2m_H^2 - 64H^4m_H^2 \\
&\quad + m^2m_H^4 + 28H^2m_H^4\big) \\
&\quad + 8\left(-4m^2H^2 + 4H^4 + m^2m_H^2\right)\tfrac{aHV'(\bar\phi)}{\bar\phi'}\Big] \\
c_4 &= \tfrac{3}{16}a^6m^2H^2\Big[\big(18m^4H^2 - 24m^2H^4 + 256H^6 - 54m^2H^2m_H^2 - 160H^4m_H^2 \\
&\quad + 9m^2m_H^4 + 60H^2m_H^4 - 7m_H^6\big) \\
&\quad + 4\left(-30m^2H^2 + 32H^4 + 12m^2m_H^2 + 4H^2m_H^2 - 7m_H^4\right)\tfrac{aHV'(\bar\phi)}{\bar\phi'}\Big] \\
c_2 &= \tfrac{9}{16}a^8m^4H^2\left(2H^2 - m_H^2\right)\Big[-\left(4H^2 + m_H^2\right)\left(3m^2 - 4H^2 - m_H^2\right) \\
&\quad + 4\left(-3m^2 + 2H^2 + 2m_H^2\right)\tfrac{aHV'(\bar\phi)}{\bar\phi'}\Big]
\end{aligned}$$

Tensor Dark Matter CGPP – longitudinal polarization

[Kolb, Ling, AL, Rosen (2302.04390)]

Scalar metric perturbations mix with scalar inflaton perturbation

$$L_S = \text{a messy function of } A,\ B,\ E,\ F, \text{ and } \varphi_v$$

After imposing constraints (and a LOT of algebra) there are only two propagating degrees of freedom

$$L_{S,\boldsymbol{k}} = K_\Pi\, |\tilde{\Pi}'|^2 + M_\Pi\, |\tilde{\Pi}|^2 + K_{\mathcal{B}}\, |\tilde{\mathcal{B}}'|^2 + M_{\mathcal{B}}\, |\tilde{\mathcal{B}}|^2 + \lambda_1\, \tilde{\Pi}^* \tilde{\mathcal{B}}' + \lambda_0\, \tilde{\Pi}^* \tilde{\mathcal{B}}$$

The second kinetic term coefficient is

$$K_{\mathcal{B}} = \frac{3a^6 m^2 (m^2 - m_H^2)}{4k^4 + 12a^2(m^2 - m_H^2)k^2 + 9a^4 m^2 (m^2 - m_H^2)}$$

and where we've defined: $m_H^2(\eta) = 2H(\eta)^2\left[1 - \epsilon(\eta)\right]$ where $\epsilon(\eta) = -\dot{H}/H^2$

Beware of ghosts

[Kolb, Ling, AL, Rosen (2302.04390)]

Higuchi (1986)
see also: Fasiello & Tolley (2013)

A wrong-sign kinetic term leads to dangerous ghosts!

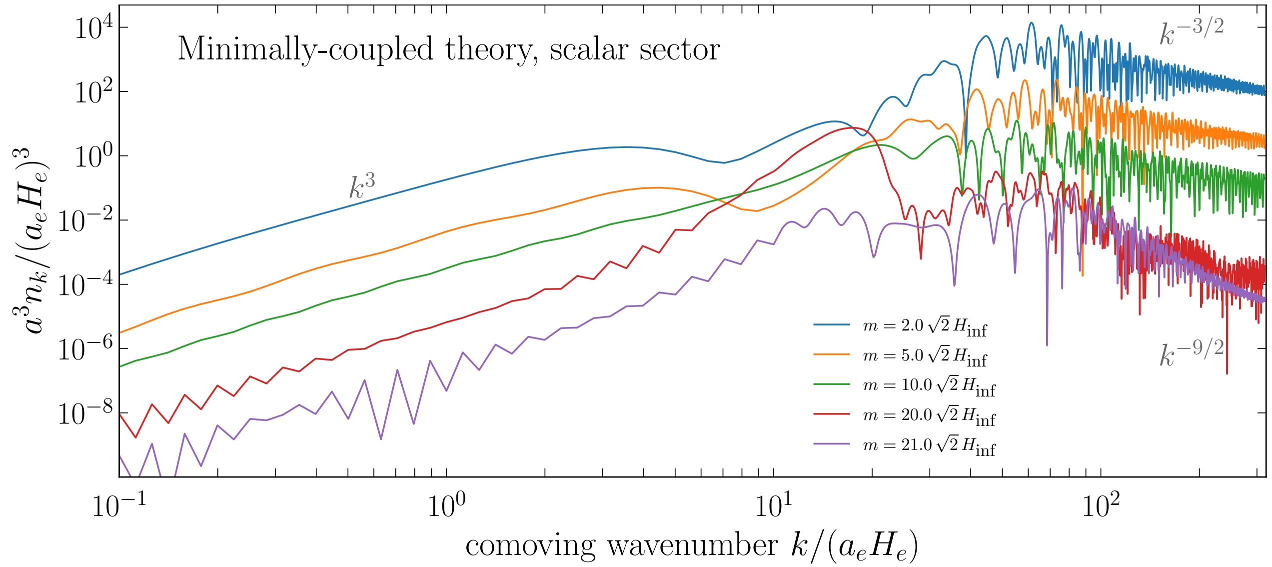
For massive spin-2 particles in FRW spacetime, ghost avoidance requires:

$$\boxed{m^2 > m_H^2(\eta)} = 2H(\eta)^2\left[1 - \epsilon(\eta)\right] \quad \text{where} \quad \epsilon(\eta) = -\dot{H}/H^2$$

- ➔ Generalizes the Higuchi bound (for dS) to FRW spacetime
- ➔ After inflation ε > 1 and any positive m^2 is ghost-free
- ➔ Implications for ultra-light spin-2 dark matter (e.g., time-dep mass)
- ➔ Implications for Kaluza-Klein (compact extra dimensions)
- ➔ Our numerical analysis focuses on $m^2 > 2\,H_{inf}^2$ to avoid the ghost

Tensor Dark Matter CGPP – longitudinal polarization

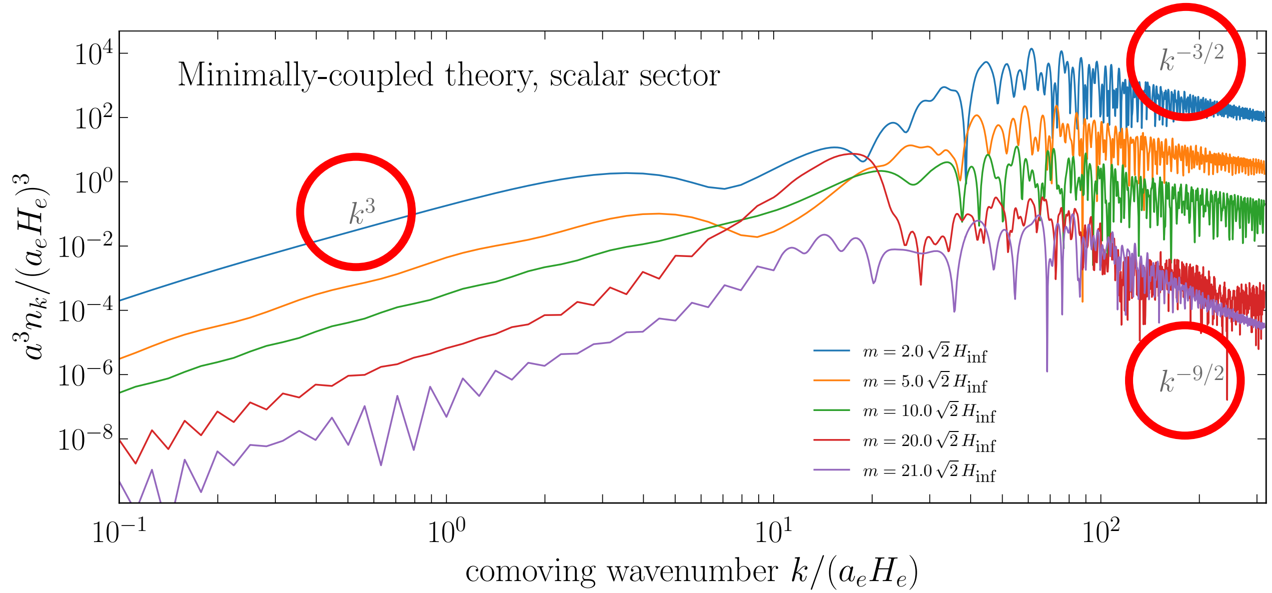
[Kolb, Ling, AL, Rosen (2302.04390)]



Notable features:

1. Same power laws & wiggles as T/V
2. Lowering mass raises amplitude

Tensor Dark Matter CGPP – longitudinal polarization



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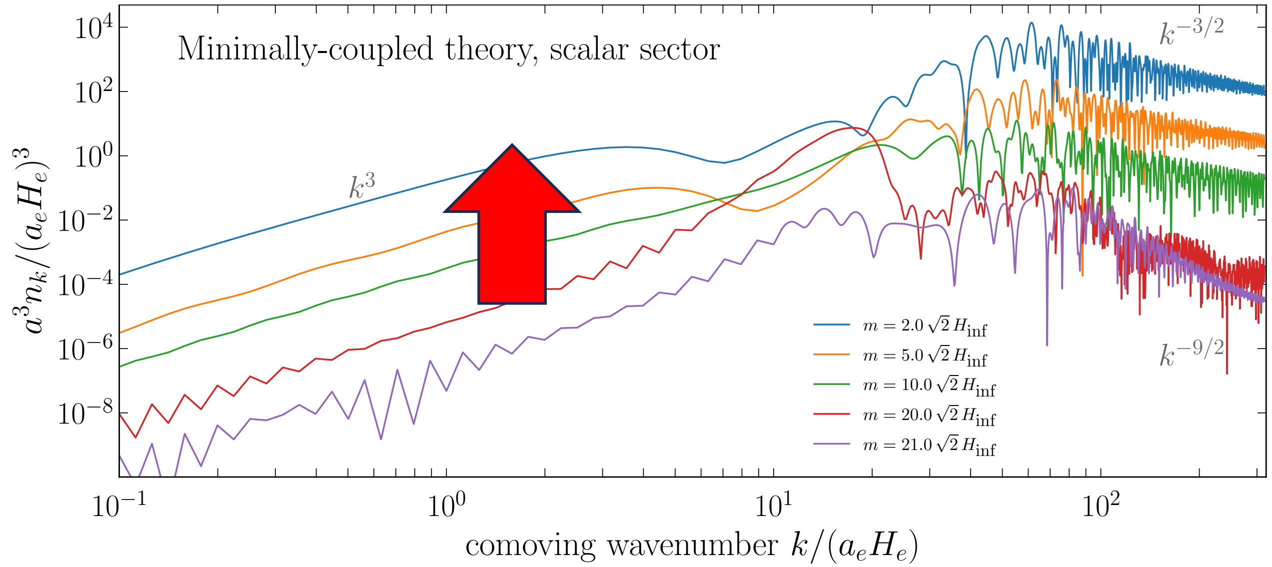

Notable features:

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Summary & Conclusions

Summary: Cosmological gravitational particle production (CGPP) arises when quantum fields `feel' the homogeneous expansion of the universe during inflation or at the end of inflation.

CGPP provides a simple explanation for the origin of dark matter (across wide mass & spin), and it leads to an unavoidable production of any (non-conformal) hidden-sector particles.

Conclusions:

- CGPP can explain the origin of dark matter even if it only interacts gravitationally.
- CGPP predicts dark matter isocurvature ➔ constrained @ CMB scales ➔ a target on small scales
- CGPP explains the origin of massive spin-2 dark matter ➔ they must be superheavy to avoid the Higuchi bound ghost instability

Tensor Dark Matter CGPP – summary results

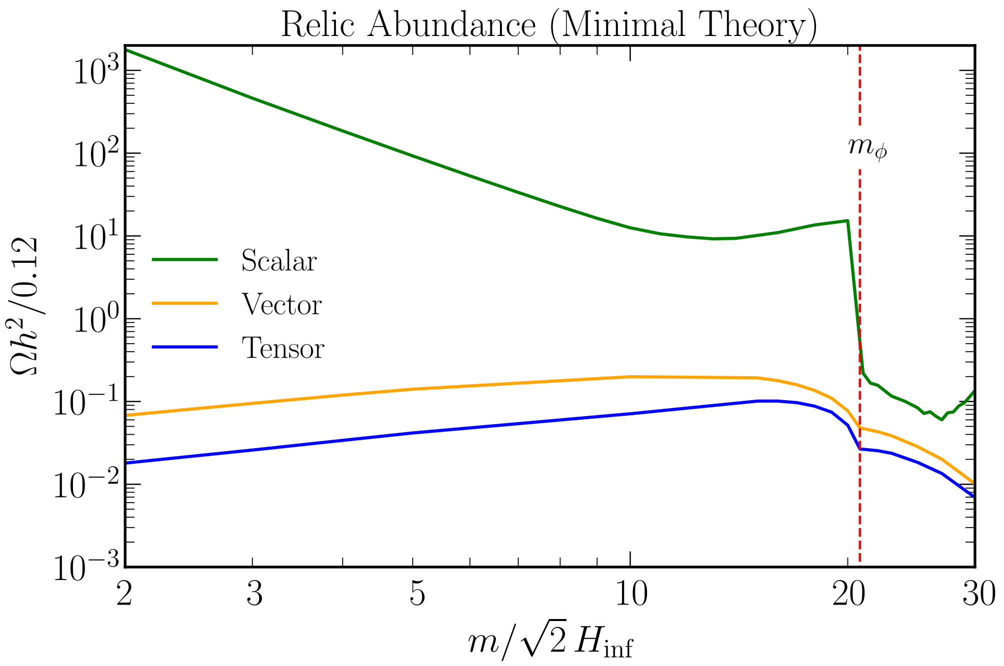
[Kolb, Ling, AL, Rosen (2302.04390)]

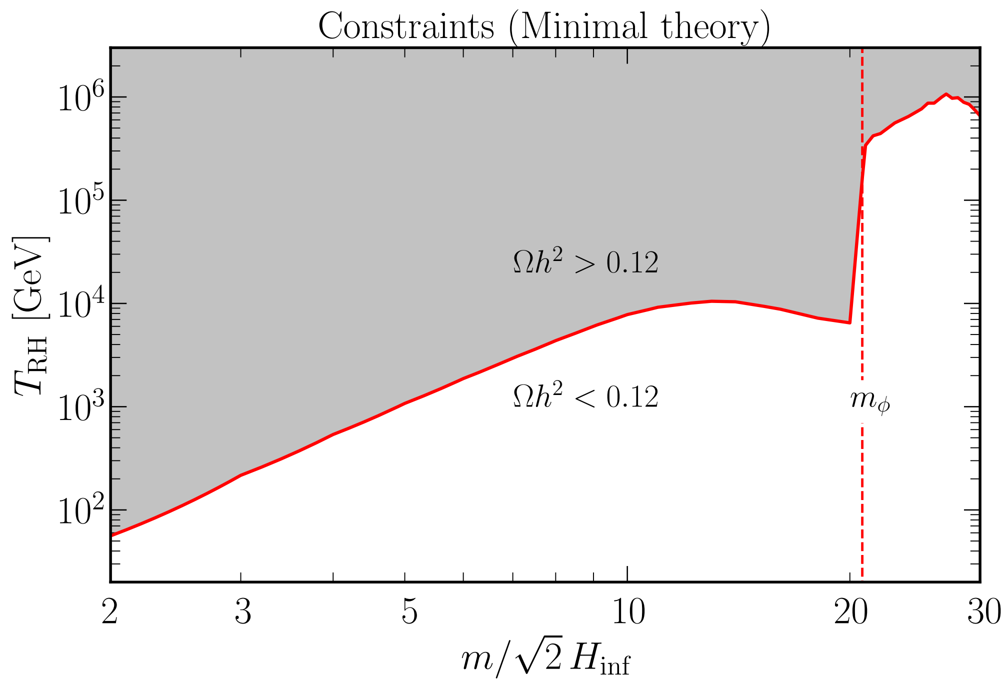
see also: Babichev et. al. (2016)

Assume: massive spin-2 particles are cosmologically long-lived

Relic abundance

$$\Omega h^2 \approx (0.114) \left(\frac{m}{10^{10}\ \mathrm{GeV}}\right) \left(\frac{H_e}{10^{10}\ \mathrm{GeV}}\right) \left(\frac{T_{\mathrm{RH}}}{10^{8}\ \mathrm{GeV}}\right) \left(\frac{a^3 n}{a_e^3 H_e^3}\right)$$





backup slides

scalar DM from CGPP

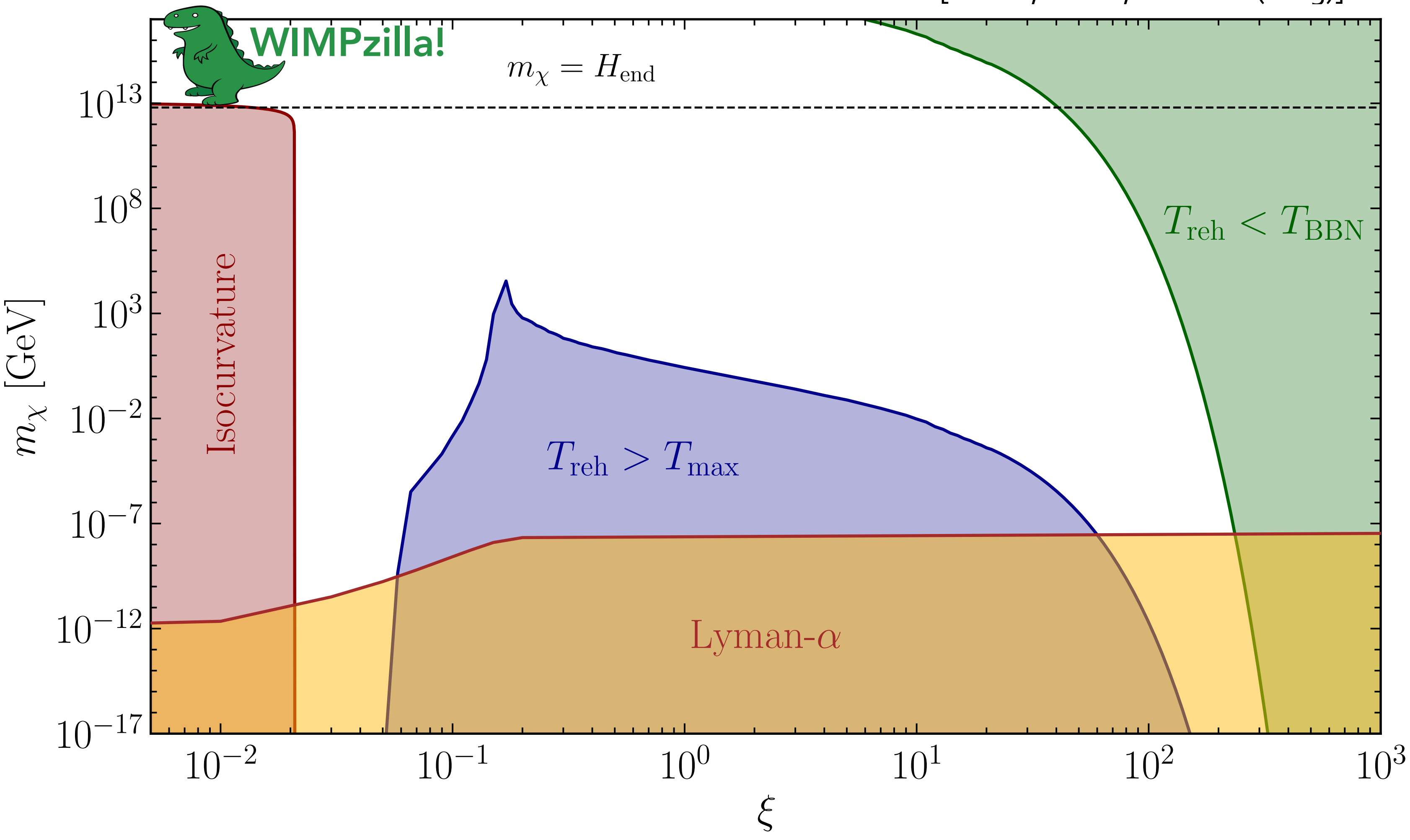
nonminimal coupling to gravity

Highlight: results for CGPP of spin-0 DM

$$\mathscr{L} \supset -\frac{1}{2}m_\chi^2\chi^2 + \frac{1}{2}\xi R\chi^2$$

see also: [Chung, Kolb, Riotto, & Senatore (2005)], [Ling & AL (2020)], [Kolb, AL, McDonough, & Payeur (2022)], [Lebedev, Solomko, & Yoon (2022)]

[Garcia, Pierre, & Verner (2023)]



ghost-free bigravity
details

General relativity

Covariant action for metric field $g_{\mu\nu}$

$$S[g_{\mu\nu}] = \int \mathrm{d}^4x \sqrt{-g} \left[\tfrac{1}{2} M_P^2 \, R[g] \right]$$

Linearize around Minkowski spacetime

$$g_{\mu\nu} = \eta_{\mu\nu} + \frac{2}{M_P} h_{\mu\nu}$$

$$S[h_{\mu\nu}] = \int \mathrm{d}^4x \left[-\tfrac{1}{2} \partial_\lambda h_{\mu\nu} \partial^\lambda h^{\mu\nu} + \partial_\mu h^{\nu\lambda} \partial_\nu h^\mu{}_\lambda - \partial_\mu h^{\mu\nu} \partial_\nu h + \tfrac{1}{2} \partial_\mu h \partial^\mu h + O(h^3) \right]$$

$$(h = \eta^{\mu\nu} h_{\mu\nu})$$

Counting degrees of freedom

$$h_{\mu\nu} \sim 16_{\text{components}} - 6_{\text{symmetric}} - 4_{\text{gauge}} - 4_{\text{constraint}} = 2_{\text{dof}}$$

$$(\delta h_{\mu\nu} = \partial_\mu \xi_\nu + \partial_\nu \xi_\mu) \quad (\text{transverse \& traceless})$$

➔ these are the two polarization modes of the massless graviton (x & + or h = +2, -2)

Adding a mass

Fierz & Pauli (1939)
see also the nice reviews by
Hinterbichler 1105.3735

Try to add mass terms

$$\delta S[h_{\mu\nu}] = \int \mathrm{d}^4x \left[-\tfrac{1}{2} m_1^2 h_{\mu\nu} h^{\mu\nu} - \tfrac{1}{2} m_2^2 h^\mu_\mu h^\nu_\nu \right]$$

A poor choice of these mass parameters leads to a theory with a ghost (in addition to a massive spin-2)

$$m^2_{\text{ghost}} = -\frac{1}{2} \frac{m_1^2 + 4m_2^2}{m_1^2 + m_2^2}$$

(Boulware-Deser ghost)

A clever choice of parameters avoids the ghost and yields a healthy theory of massive spin-2 field

$$S_{\text{FP}}[h_{\mu\nu}] = \int \mathrm{d}^4x \left[-\tfrac{1}{2}\partial_\lambda h_{\mu\nu} \partial^\lambda h^{\mu\nu} + \partial_\mu h^{\nu\lambda} \partial_\nu h^\mu_\lambda - \partial_\mu h^{\mu\nu} \partial_\nu h + \tfrac{1}{2} \partial_\mu h \partial^\mu h - \tfrac{1}{2} m^2 \left(h_{\mu\nu} h^{\mu\nu} - h^2 \right) \right]$$

(Fierz-Pauli action)

$$h_{\mu\nu} \sim 16_{\text{components}} - 6_{\text{symmetric}} - 1_{\text{gauge}} - 4_{\text{constraint}} = 5_{\text{dof}}$$

➔ the five polarization modes of a massive graviton (helicity = -2, -1, 0, +1, +2)

Going to FRW background – failed attempt

(Fierz-Pauli action)

$$S_{\rm FP}[h_{\mu\nu}] = \int \mathrm{d}^4x \left[-\tfrac{1}{2}\partial_\lambda h_{\mu\nu}\partial^\lambda h^{\mu\nu} + \partial_\mu h^{\nu\lambda}\partial_\nu h^\mu_{\ \lambda} - \partial_\mu h^{\mu\nu}\partial_\nu h + \tfrac{1}{2}\partial_\mu h \partial^\mu h - \tfrac{1}{2}m^2\left(h_{\mu\nu}h^{\mu\nu} - h^2\right)\right]$$

try promoting Minkowski derivatives
to FRW covariant derivatives

$$\nabla_\lambda h_{\mu\nu} = \partial_\lambda h_{\mu\nu} - \Gamma^\rho_{\lambda\mu} h_{\rho\nu} - \Gamma^\rho_{\lambda\nu} h_{\mu\rho}$$

$$S[h_{\mu\nu}] = \int \mathrm{d}^4x \left[-\tfrac{1}{2}\nabla_\lambda h_{\mu\nu}\nabla^\lambda h^{\mu\nu} + \nabla_\mu h^{\nu\lambda}\nabla_\nu h^\mu_{\ \lambda} - \nabla_\mu h^{\mu\nu}\nabla_\nu h + \tfrac{1}{2}\nabla_\mu h \nabla^\mu h - \tfrac{1}{2}m^2\left(h_{\mu\nu}h^{\mu\nu} - h^2\right)\right]$$

This procedure would re-introduce the Boulware-Deser ghost. Going to an FRW bkg without also introducing the matter sector is a violation of gauge symmetry. ($\delta h_{\mu\nu} = \partial_\mu \xi_\nu + \partial_\nu \xi_\mu$)

Successful attempt

Let's add a matter sector

$$S[g_{\mu\nu}] = \int \mathrm{d}^4x \sqrt{-g} \left[\tfrac{1}{2} M_P^2 R[g] - \tfrac{1}{2} g^{\mu\nu} \nabla_\mu \phi \nabla_\nu \phi - V(\phi) \right]$$

Linearize around an FRW background

$$g_{\mu\nu} = \bar{g}_{\mu\nu}^{(\mathrm{FRW})} + \frac{2}{M_P} u_{\mu\nu} \quad \text{and} \quad \phi = \bar{\phi}^{(\mathrm{FRW})} + \varphi_u$$

Resulting quadratic action

$$\begin{aligned}
\mathcal{L}^{(2)}_{\mathrm{massless}} &= \mathcal{L}^{(2)}_{uu} + \mathcal{L}^{(2)}_{u\varphi_u} + \mathcal{L}^{(2)}_{\varphi_u\varphi_u} \\
\mathcal{L}^{(2)}_{uu} &= -\tfrac{1}{2} \nabla_\lambda u_{\mu\nu} \nabla^\lambda u^{\mu\nu} + \nabla_\mu u^{\nu\lambda} \nabla_\nu u^\mu{}_\lambda - \nabla_\mu u^{\mu\nu} \nabla_\nu u + \tfrac{1}{2} \nabla_\mu u \nabla^\mu u \\
&\quad + \left(\bar{R}_{\mu\nu} - \tfrac{1}{M_P^2} \nabla_\mu \bar{\phi} \nabla_\nu \bar{\phi} \right) \left(u^{\mu\lambda} u_\lambda^\nu - \tfrac{1}{2} u^{\mu\nu} u \right) , \\
\mathcal{L}^{(2)}_{u\varphi_u} &= \tfrac{1}{M_P} \left[\left(\nabla_\mu \bar{\phi} \nabla_\nu \varphi_u + \nabla_\nu \bar{\phi} \nabla_\mu \varphi_u \right) \left(u^{\mu\nu} - \tfrac{1}{2} \bar{g}^{\mu\nu} u \right) - V'(\bar{\phi}) \varphi_u u \right] , \\
\mathcal{L}^{(2)}_{\varphi_u\varphi_u} &= -\tfrac{1}{2} \nabla_\mu \varphi_u \nabla^\mu \varphi_u - \tfrac{1}{2} V''(\bar{\phi}) \varphi_u^2 \, .
\end{aligned}$$

(massless spin-2 graviton + inflaton perturbation)

$$\begin{aligned}
\mathcal{L}^{(2)}_{\mathrm{massive}} &= \mathcal{L}^{(2)}_{vv} + \mathcal{L}^{(2)}_{v\varphi_v} + \mathcal{L}^{(2)}_{\varphi_v\varphi_v} \\
\mathcal{L}^{(2)}_{vv} &= -\tfrac{1}{2} \nabla_\lambda v_{\mu\nu} \nabla^\lambda v^{\mu\nu} + \nabla_\mu v^{\nu\lambda} \nabla_\nu v^\mu{}_\lambda - \nabla_\mu v^{\mu\nu} \nabla_\nu v + \tfrac{1}{2} \nabla_\mu v \nabla^\mu v \\
&\quad + \left(\bar{R}_{\mu\nu} - \tfrac{1}{M_P^2} \nabla_\mu \bar{\phi} \nabla_\nu \bar{\phi} \right) \left(v^{\mu\lambda} v_\lambda^\nu - \tfrac{1}{2} v^{\mu\nu} v \right) \\
&\quad - \tfrac{1}{2} m^2 \left(v^{\mu\nu} v_{\mu\nu} - v^2 \right) , \\
\mathcal{L}^{(2)}_{v\varphi_v} &= \tfrac{1}{M_P} \left[\left(\nabla_\mu \bar{\phi} \nabla_\nu \varphi_v + \nabla_\nu \bar{\phi} \nabla_\mu \varphi_v \right) \left(v^{\mu\nu} - \tfrac{1}{2} \bar{g}^{\mu\nu} v \right) - V'(\bar{\phi}) \varphi_v v \right] , \\
\mathcal{L}^{(2)}_{\varphi_v\varphi_v} &= -\tfrac{1}{2} \nabla_\mu \varphi_v \nabla^\mu \varphi_v - \tfrac{1}{2} V''(\bar{\phi}) \varphi_v^2 \, .
\end{aligned}$$

(massive spin-2 + inflaton perturbation)

Another approach: ghost-free bigravity

Hassan & Rosen (2012)

Field content: two metrics & two scalars

$$g_{\mu\nu}\ , \quad f_{\mu\nu}\ , \quad \phi_g\ , \quad \phi_f$$

A theory of bigravity with a minimal coupling to matter

$$S = \int \mathrm{d}^4x \, \Big[\tfrac{1}{2} M_g^2 \sqrt{-g}\, R[g] \; + \; \tfrac{1}{2} M_f^2 \sqrt{-f}\, R[f] \qquad \text{(metric kinetic terms)}$$

$$- \, m^2 M_*^2 \sqrt{-g}\, V(\mathbb{X};\, \beta_n) \qquad \text{(metric interactions)}$$

$$+ \sqrt{-g}\, \mathcal{L}_g(g, \phi_g) \; + \; \sqrt{-f}\, \mathcal{L}_f(f, \phi_f) \Big] \qquad \text{(coupling to matter)}$$

Matter-sector Lagrangians

$$\mathcal{L}_g(g, \phi_g) = -\tfrac{1}{2} g^{\mu\nu} \nabla_\mu \phi_g \nabla_\nu \phi_g - V_g(\phi_g)$$

$$\mathcal{L}_f(f, \phi_f) = -\tfrac{1}{2} f^{\mu\nu} \nabla_\mu \phi_f \nabla_\nu \phi_f - V_f(\phi_f)$$

$$\left(\begin{array}{c} M_*^{-2} = M_g^{-2} + M_f^{-2} \\ M_P^2 = M_g^2 + M_f^2 \end{array} \right)$$

Ghost-free bigravity

Hassan & Rosen (2012)

Field content: two metrics & two scalars

$$g_{\mu\nu}\ , \quad f_{\mu\nu}\ , \quad \phi_g\ , \quad \phi_f$$

A theory of bigravity with a minimal coupling to matter

$$S = \int \mathrm{d}^4x \Big[\tfrac{1}{2} M_g^2 \sqrt{-g}\, R[g] + \tfrac{1}{2} M_f^2 \sqrt{-f}\, R[f]$$ (metric kinetic terms)

$$- m^2 M_*^2 \sqrt{-g}\, V(\mathbb{X};\, \beta_n)$$ (metric interactions)

$$+ \sqrt{-g}\, \mathcal{L}_g(g, \phi_g) + \sqrt{-f}\, \mathcal{L}_f(f, \phi_f) \Big]$$ (coupling to matter)

Matter-sector Lagrangians

$$\mathcal{L}_g(g, \phi_g) = -\tfrac{1}{2} g^{\mu\nu} \nabla_\mu \phi_g \nabla_\nu \phi_g - V_g(\phi_g)$$

$$\mathcal{L}_f(f, \phi_f) = -\tfrac{1}{2} f^{\mu\nu} \nabla_\mu \phi_f \nabla_\nu \phi_f - V_f(\phi_f)$$

$$\left(\begin{array}{c} M_*^{-2} = M_g^{-2} + M_f^{-2} \\ M_P^2 = M_g^2 + M_f^2 \end{array} \right)$$

Proportional background & mirroring conditions

Backgrounds plus perturbations:

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + \tfrac{2}{M_g} h_{\mu\nu} \,, \quad f_{\mu\nu} = \bar{f}_{\mu\nu} + \tfrac{2}{M_f} k_{\mu\nu} \,, \quad \phi_g = \bar{\phi}_g + \varphi_g \,, \quad \text{and} \quad \phi_f = \bar{\phi}_f + \varphi_f$$

We seek solutions of the background equations of motion with

$$\bar{g}_{\mu\nu} = \bar{f}_{\mu\nu} = \text{FRW} \qquad \text{and} \qquad \frac{1}{M_g}\,\bar{\phi}_g = \frac{1}{M_f}\,\bar{\phi}_f \equiv \frac{1}{M_P}\,\bar{\phi}$$

The existence of such solutions places a constraint on the models:

$$\frac{1}{M_g^2}\,V_g(\bar{\phi}_g) = \frac{1}{M_f^2}\,V_f(\bar{\phi}_f) \equiv \frac{1}{M_P^2}\,V(\bar{\phi})$$

(mirroring condition)

Then the backgrounds obey the usual equations of motion (EOM) for an inflationary cosmology:

bkg. metric EOM: $\bar{R}_{\mu\nu} - \tfrac{1}{2}\bar{g}_{\mu\nu}\bar{R} + \Lambda\bar{g}_{\mu\nu} = \tfrac{1}{M_P^2}\,\bar{T}_{\mu\nu}$

bkg. inflaton EOM: $\Box\bar{\phi} - V'(\bar{\phi}) = 0$

Perturbations

Change variables

$$\frac{1}{M_*}\,u_{\mu\nu} = \frac{1}{M_f}h_{\mu\nu} + \frac{1}{M_g}\,k_{\mu\nu}\ , \qquad \frac{1}{M_*}\,v_{\mu\nu} = \frac{1}{M_g}\,h_{\mu\nu} - \frac{1}{M_f}\,k_{\mu\nu}$$

$$\frac{1}{M_*}\,\varphi_u = \frac{1}{M_f}\,\varphi_g + \frac{1}{M_g}\,\varphi_f\ , \qquad \frac{1}{M_*}\,\varphi_v = \frac{1}{M_g}\,\varphi_g - \frac{1}{M_f}\,\varphi_f$$

Quadratic action

$$S = \int \mathrm{d}^4x\,\sqrt{-\bar{g}}\,\left(\mathcal{L}^{(2)}_{\text{massless}} + \mathcal{L}^{(2)}_{\text{massive}} + \textbf{interactions}\right)$$

$$\begin{aligned}
\mathcal{L}^{(2)}_{\text{massless}} &= \mathcal{L}^{(2)}_{uu} + \mathcal{L}^{(2)}_{u\,\varphi_u} + \mathcal{L}^{(2)}_{\varphi_u\varphi_u} \\
\mathcal{L}^{(2)}_{uu} &= -\tfrac{1}{2}\nabla_\lambda u_{\mu\nu}\nabla^\lambda u^{\mu\nu} + \nabla_\mu u^{\nu\lambda}\nabla_\nu u^\mu{}_\lambda - \nabla_\mu u^{\mu\nu}\nabla_\nu u + \tfrac{1}{2}\nabla_\mu u\nabla^\mu u \\
&\quad + \left(\bar{R}_{\mu\nu} - \tfrac{1}{M_P^2}\nabla_\mu\bar{\phi}\nabla_\nu\bar{\phi}\right)\left(u^{\mu\lambda}u_\lambda^{\ \nu} - \tfrac{1}{2}u^{\mu\nu}u\right)\ , \\
\mathcal{L}^{(2)}_{u\,\varphi_u} &= \tfrac{1}{M_P}\left[\left(\nabla_\mu\bar{\phi}\nabla_\nu\varphi_u + \nabla_\nu\bar{\phi}\nabla_\mu\varphi_u\right)\left(u^{\mu\nu} - \tfrac{1}{2}\bar{g}^{\mu\nu}u\right) - V'(\bar{\phi})\varphi_u u\right]\ , \\
\mathcal{L}^{(2)}_{\varphi_u\varphi_u} &= -\tfrac{1}{2}\nabla_\mu\varphi_u\nabla^\mu\varphi_u - \tfrac{1}{2}V''(\bar{\phi})\varphi_u^2\ .
\end{aligned}$$

(massless spin-2 graviton + inflaton perturbation)

$$\begin{aligned}
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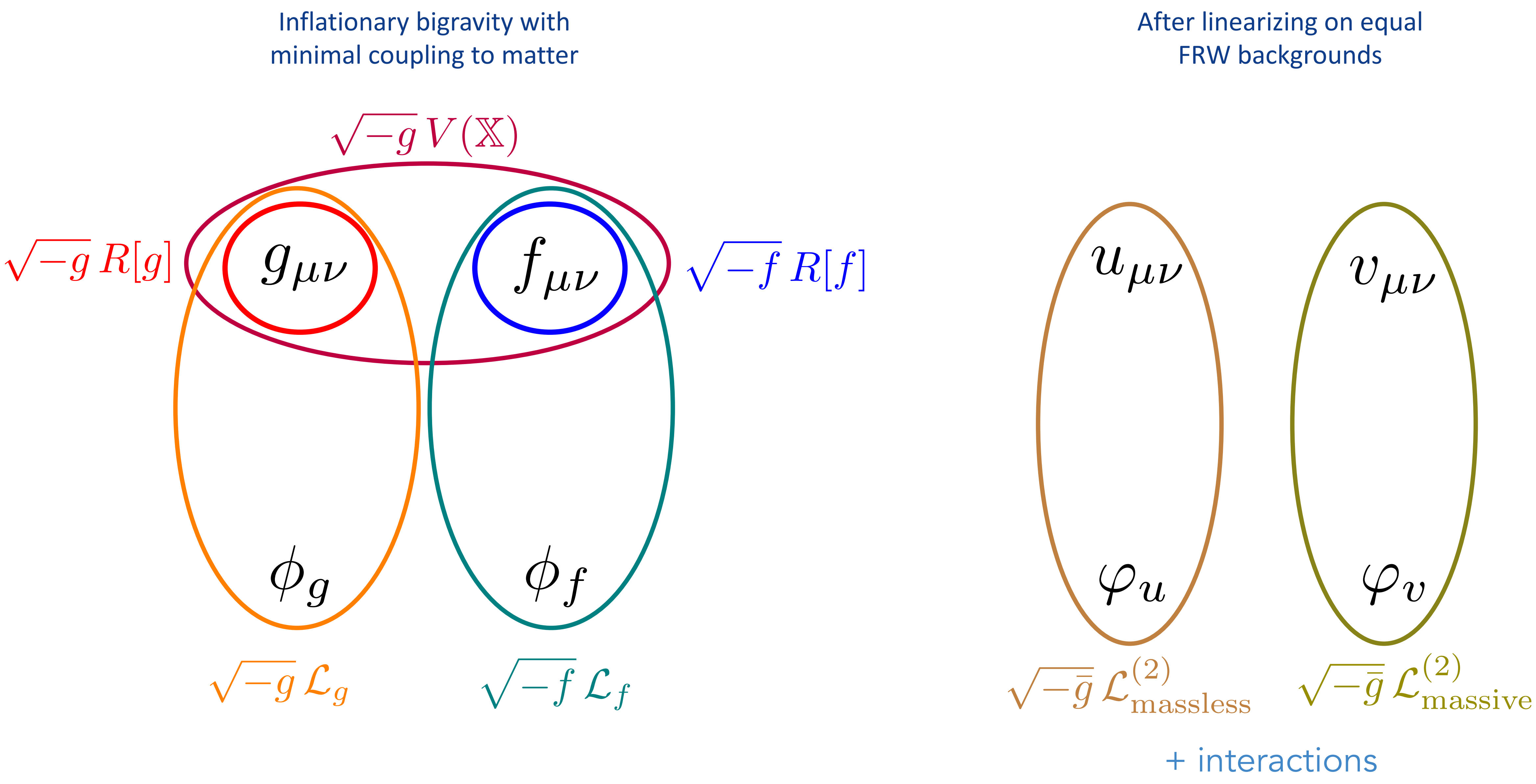
\mathcal{L}^{(2)}_{vv} &= -\tfrac{1}{2}\nabla_\lambda v_{\mu\nu}\nabla^\lambda v^{\mu\nu} + \nabla_\mu v^{\nu\lambda}\nabla_\nu v^\mu{}_\lambda - \nabla_\mu v^{\mu\nu}\nabla_\nu v + \tfrac{1}{2}\nabla_\mu v\nabla^\mu v \\
&\quad + \left(\bar{R}_{\mu\nu} - \tfrac{1}{M_P^2}\nabla_\mu\bar{\phi}\nabla_\nu\bar{\phi}\right)\left(v^{\mu\lambda}v_\lambda^{\ \nu} - \tfrac{1}{2}v^{\mu\nu}v\right) \\
&\quad - \tfrac{1}{2}m^2\left(v^{\mu\nu}v_{\mu\nu} - v^2\right)\ , \\
\mathcal{L}^{(2)}_{v\,\varphi_v} &= \tfrac{1}{M_P}\left[\left(\nabla_\mu\bar{\phi}\nabla_\nu\varphi_v + \nabla_\nu\bar{\phi}\nabla_\mu\varphi_v\right)\left(v^{\mu\nu} - \tfrac{1}{2}\bar{g}^{\mu\nu}v\right) - V'(\bar{\phi})\varphi_v v\right]\ , \\
\mathcal{L}^{(2)}_{\varphi_v\varphi_v} &= -\tfrac{1}{2}\nabla_\mu\varphi_v\nabla^\mu\varphi_v - \tfrac{1}{2}V''(\bar{\phi})\varphi_v^2\ .
\end{aligned}$$

(massive spin-2 + inflaton perturbation)



Inflationary bigravity with minimal coupling to matter
$\sqrt{-g}\,V(\mathbb{X})$
$\sqrt{-g}\,R[g]$
$g_{\mu\nu}$
$f_{\mu\nu}$
$\sqrt{-f}\,R[f]$
ϕ_g
ϕ_f
$\sqrt{-g}\,\mathcal{L}_g$
$\sqrt{-f}\,\mathcal{L}_f$
After linearizing on equal FRW backgrounds
$u_{\mu\nu}$
$v_{\mu\nu}$
φ_u
φ_v
$\sqrt{-\bar{g}}\,\mathcal{L}^{(2)}_{\rm massless}$
$\sqrt{-\bar{g}}\,\mathcal{L}^{(2)}_{\rm massive}$
+ interactions

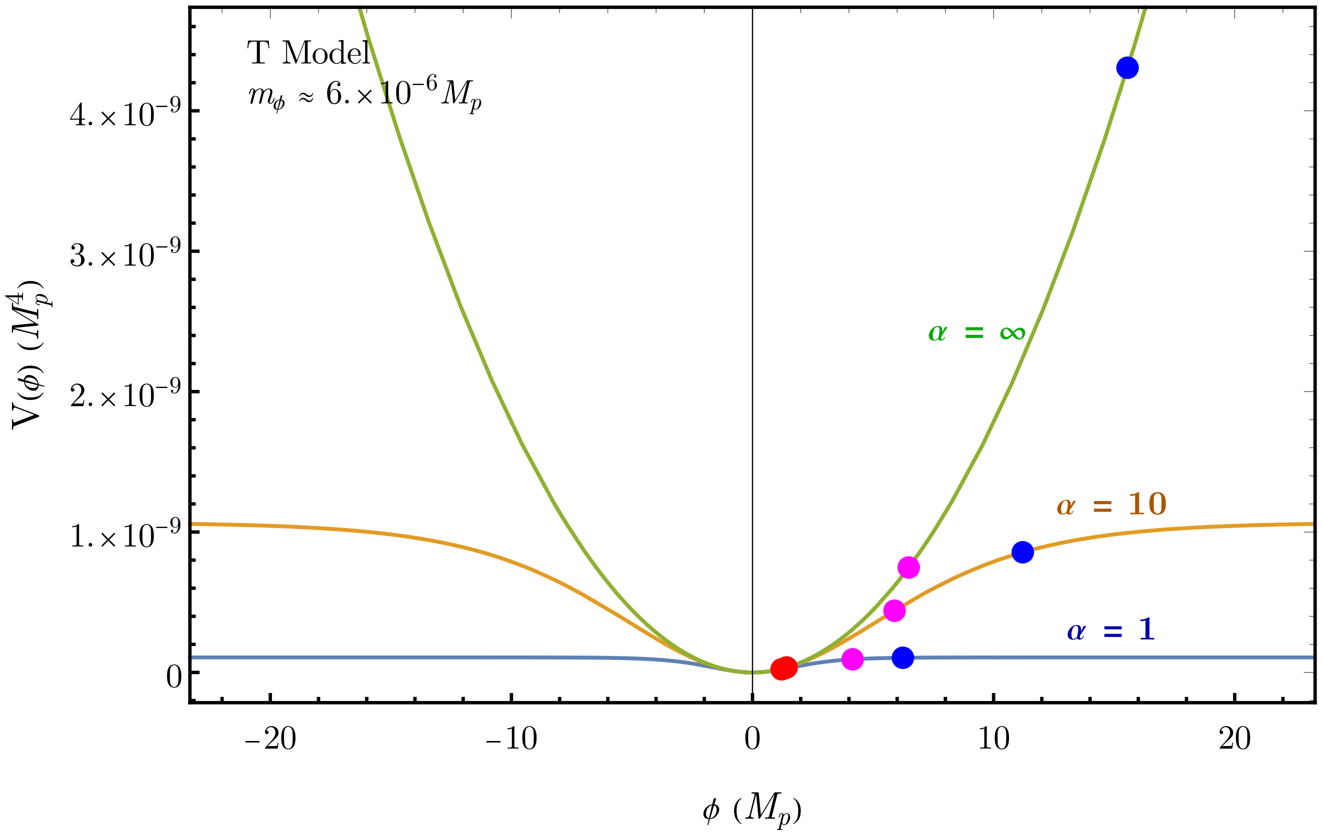
phenomenological considerations
in a concrete model

Example: alpha attractor

[Ling & AL (2101.11621)]

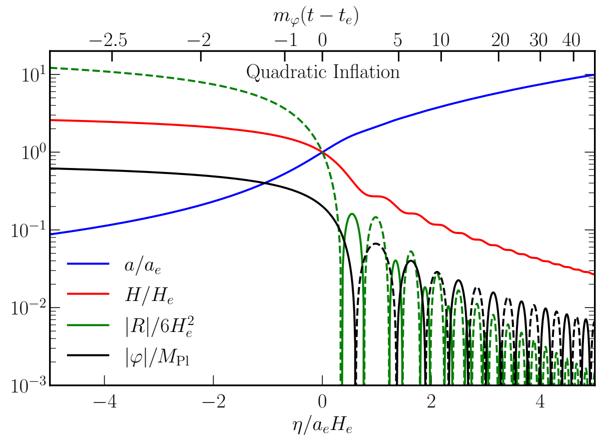
T-model
alpha attractor

$$V_T(\phi) = \alpha \mu^2 M_p^2 \tanh^2 \frac{\phi}{\sqrt{6\alpha} M_p}$$



$$\Rightarrow \begin{cases} \phi(t) \\ a(t) \end{cases}$$

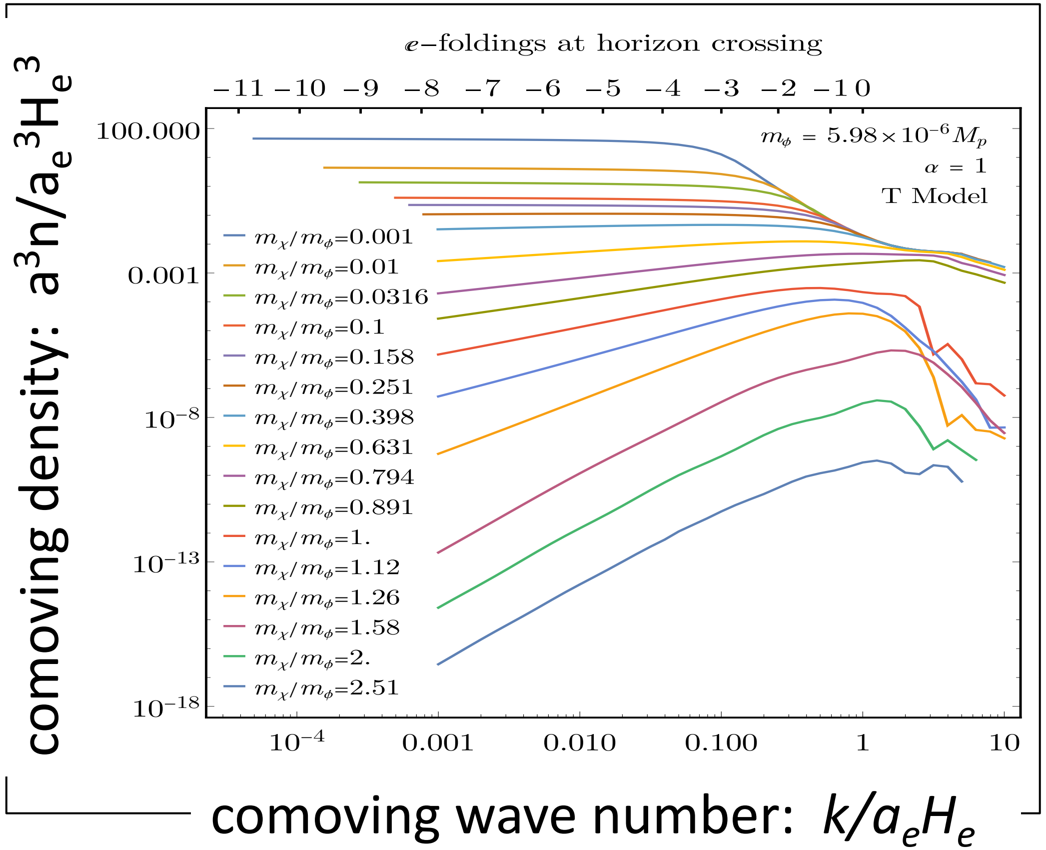
FRW background



Numerical results

[Ling & AL (2101.11621)]

comoving number density



Numerical results

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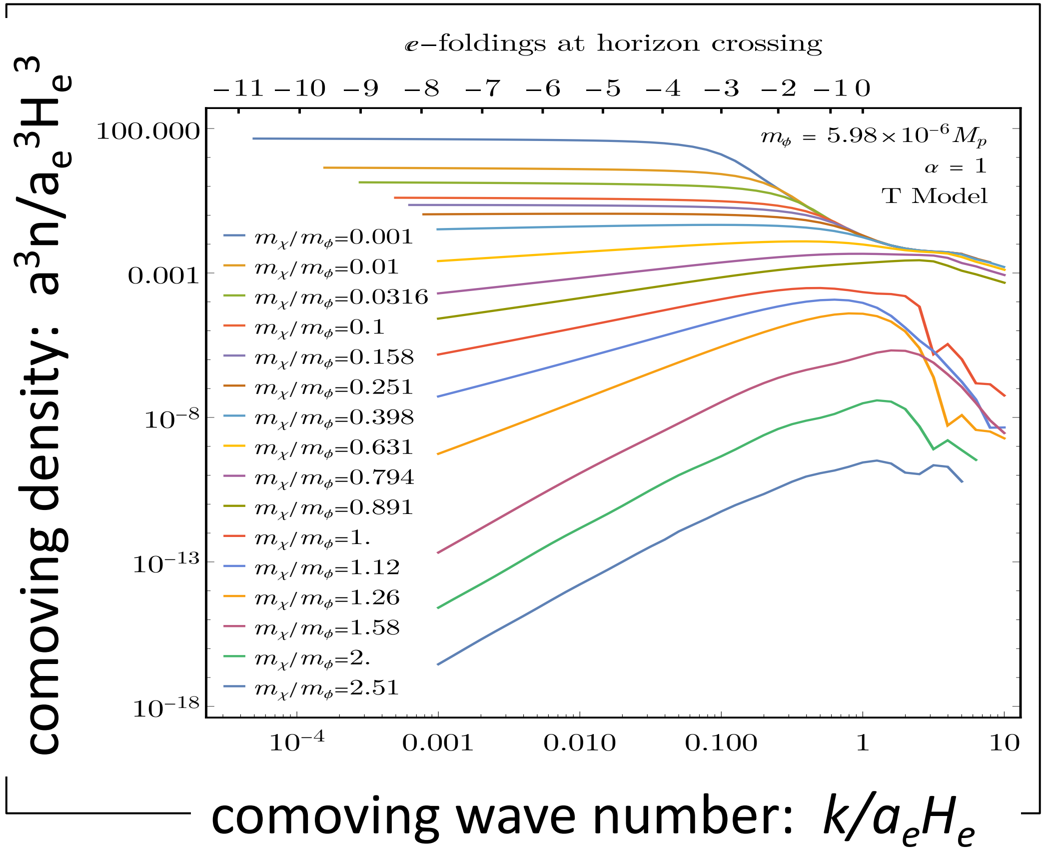
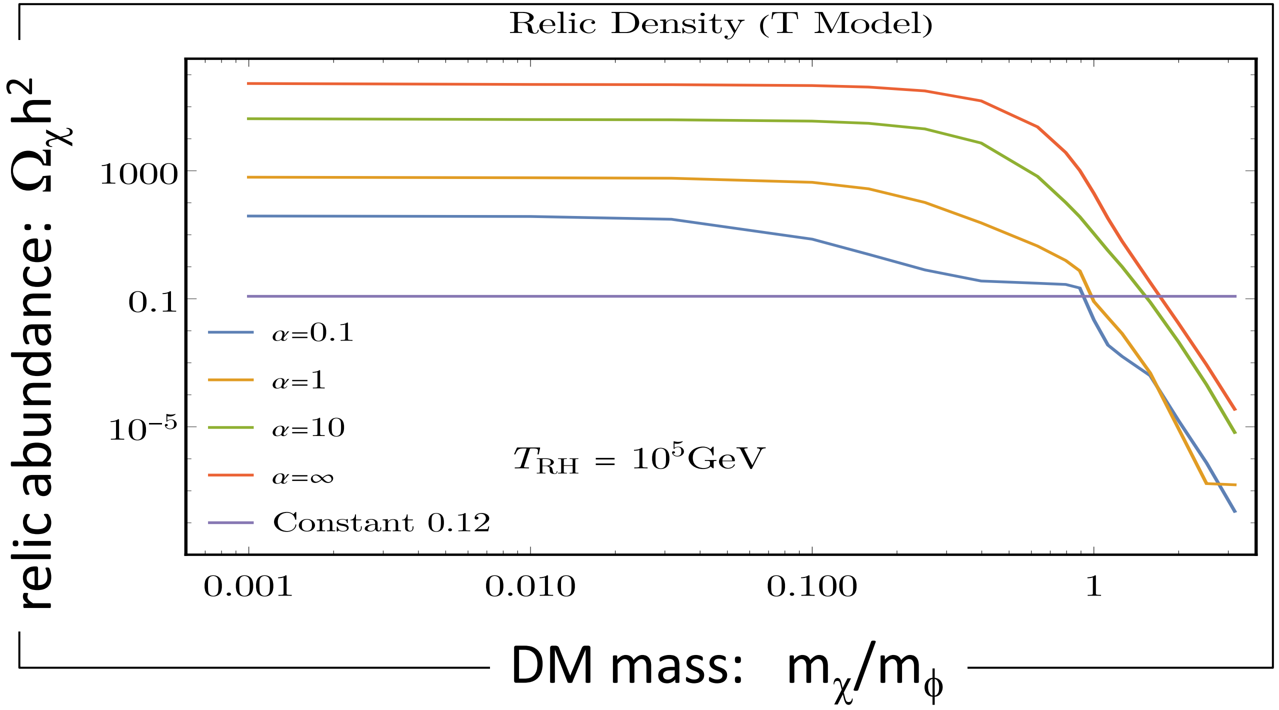


$$\Omega_\chi h^2 \simeq (0.114)\left(\frac{m_\chi}{10^{10}\ \mathrm{GeV}}\right)\left(\frac{H_e}{10^{10}\ \mathrm{GeV}}\right)\left(\frac{T_{\rm RH}}{10^{8}\ \mathrm{GeV}}\right)\left(\frac{a^3 n}{a_e^3 H_e^3}\right)$$

relic abundance

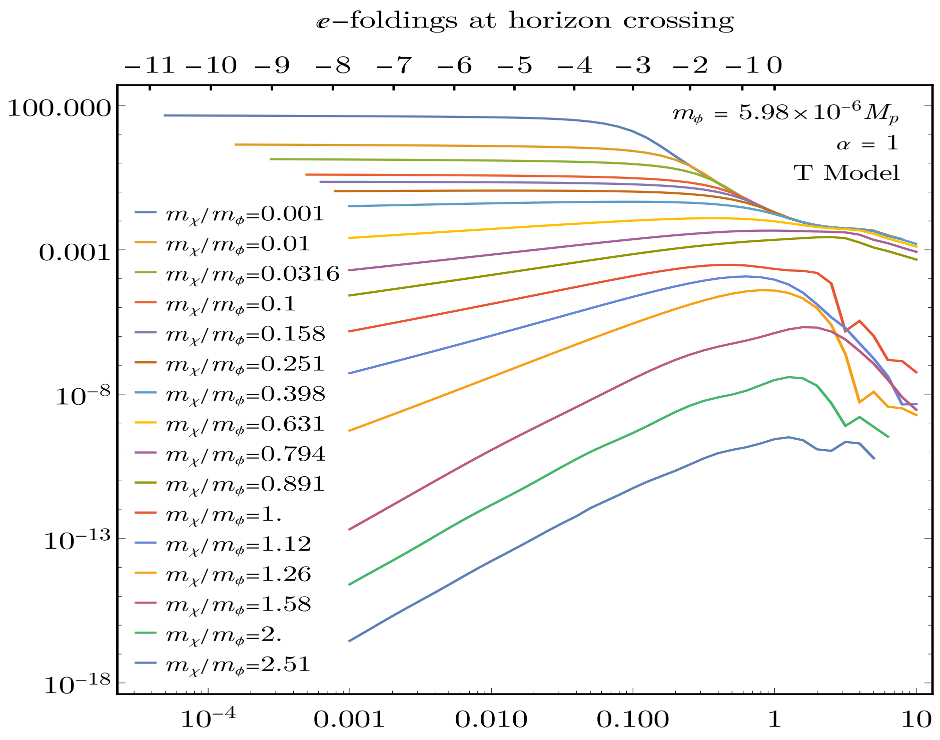


Numerical results

[Ling & AL (2101.11621)]

comoving number density

comoving density: $a^3 n / a_e^3 H_e^3$



comoving wave number: $k/a_e H_e$

$$k = a_e H_e \qquad \text{(Hubble-scale modes at the end of inflation)}$$

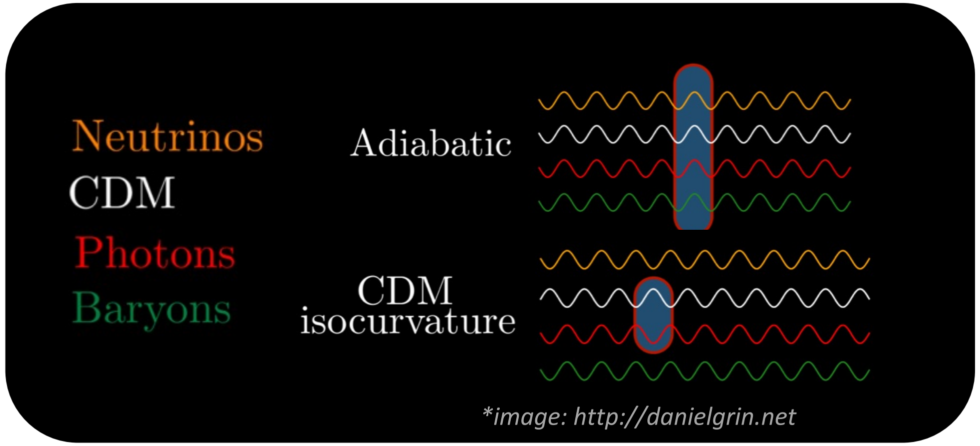
$$\lambda_{\text{phys,e}} = 2\pi a_e / k \simeq \left(10^{-29}\ \text{meters}\right)\left(\frac{H_e}{10^{14}\ \text{GeV}}\right)^{-1}$$

$$\lambda_{\text{phys,0}} = 2\pi a_0 / k \simeq \left(100\ \text{meters}\right)\left(\frac{H_e}{10^{14}\ \text{GeV}}\right)^{-1/3}\left(\frac{T_{\text{RH}}}{10^9\ \text{GeV}}\right)^{-1/3}$$

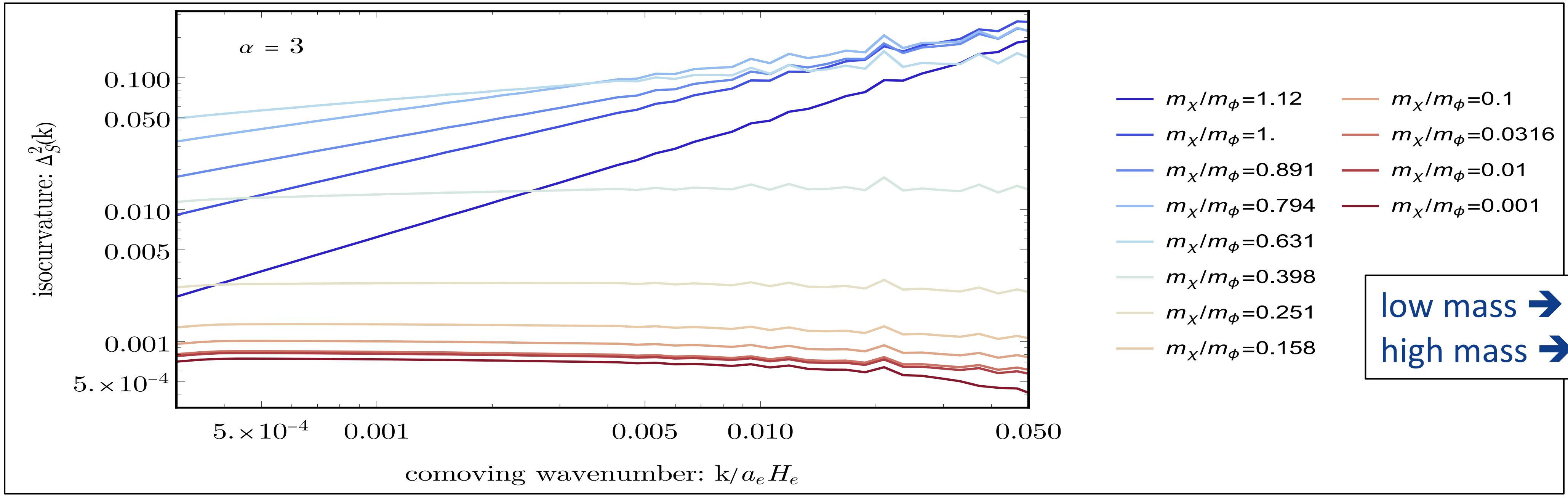
$$p_{\text{phys,0}} = \hbar k / a_0 \simeq \left(5 \times 10^{-18}\ \text{GeV}\right)\left(\frac{H_e}{10^{14}\ \text{GeV}}\right)^{1/3}\left(\frac{T_{\text{RH}}}{10^9\ \text{GeV}}\right)^{1/3}$$

CMB isocurvature

[Ling & AL (2101.11621)]



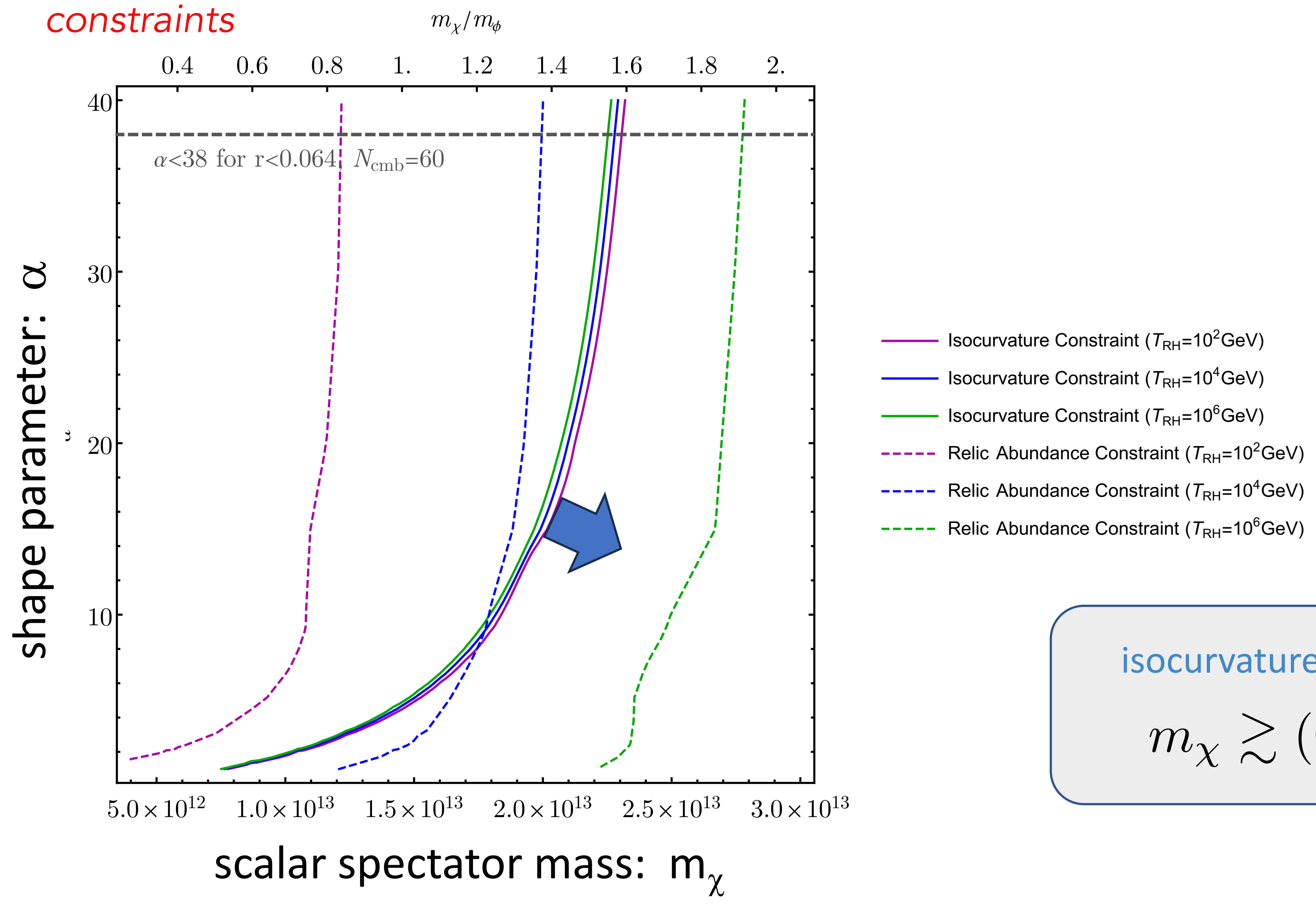
$$\Delta_S^2(k_{\mathrm{cmb}}) < 7.3 \times 10^{-11}$$
$$k_{\mathrm{cmb}} = 0.002\ \mathrm{Mpc}^{-1} a_0$$
$$k_{\mathrm{cmb}}/a_e H_e \approx e^{-50} \simeq 2 \times 10^{-22}$$



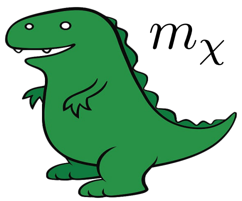
low mass ➔ red tilt
high mass ➔ blue tilt

Parameter space

[Ling & AL (2101.11621)]



WIMPzilla!



$m_\chi \gtrsim m_\phi$

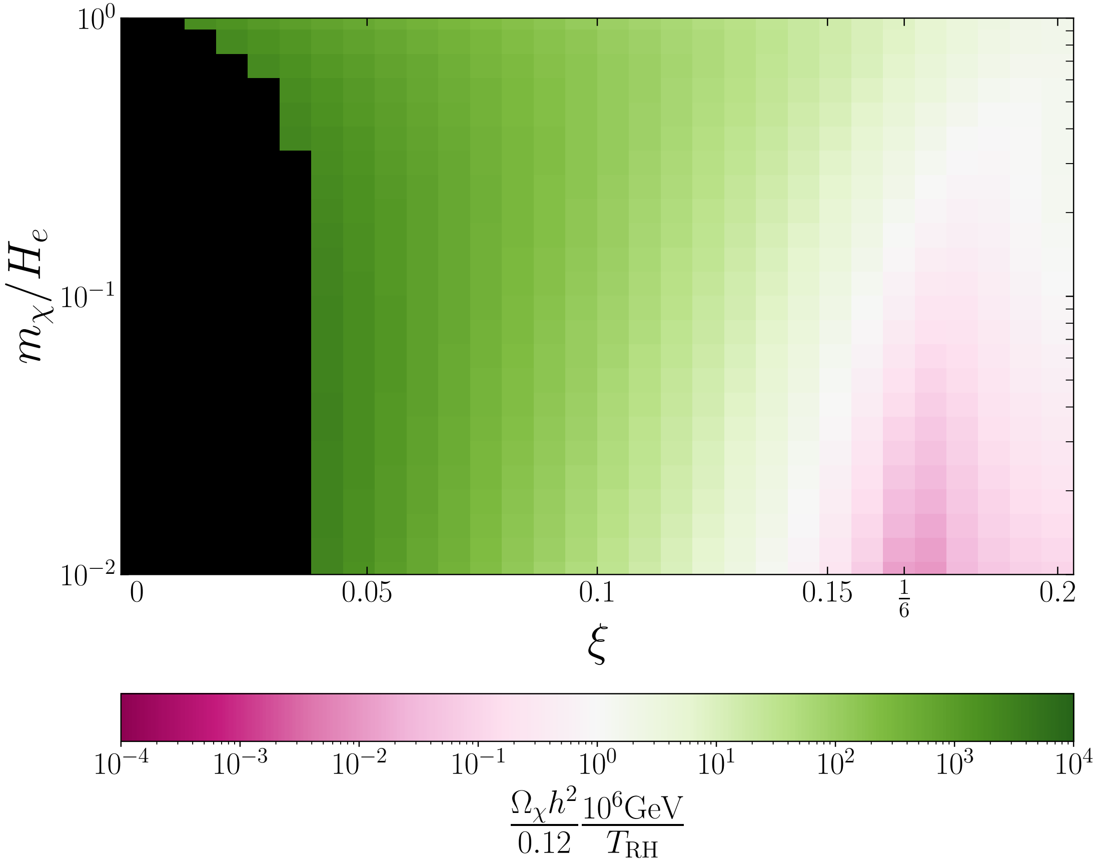
isocurvature avoidance:

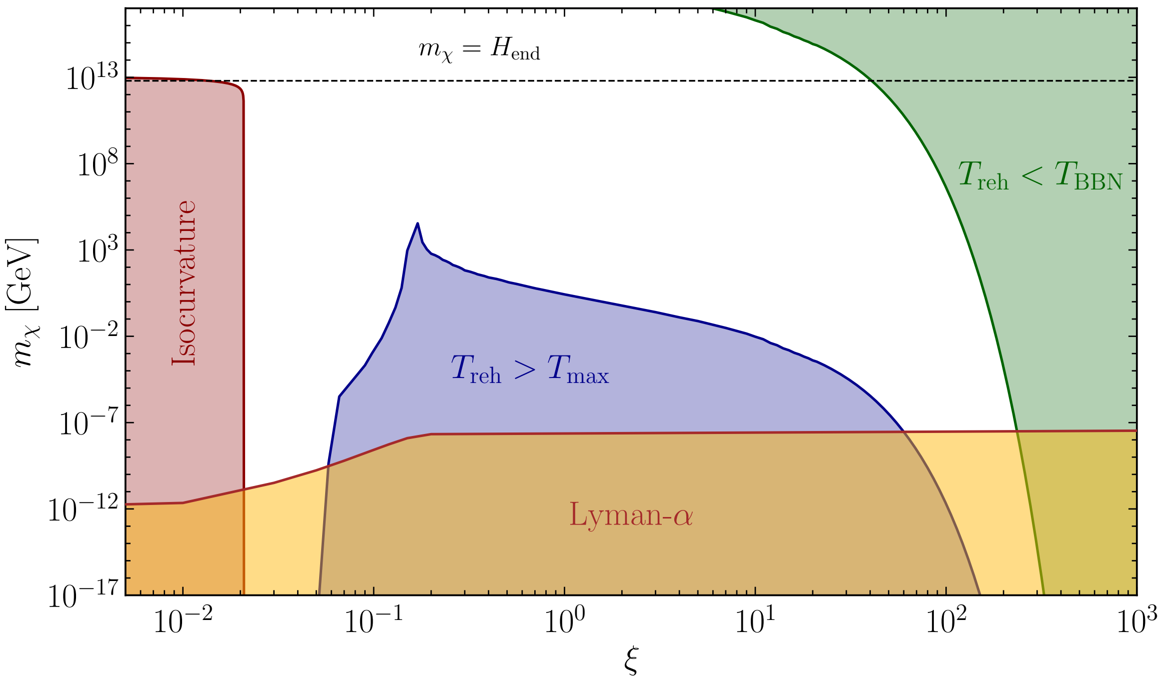
$$m_\chi \gtrsim (0.8 - 1.6)\, m_\phi$$

going non-minimal

[Kolb, AL, McDonough, & Payeur (2022)], [Garcia, Pierre, & Verner (2023)]
see also: [Markkanen, Rajantie, & Tenkanen (2018); Tenkanen (2019)]

$$\mathscr{L} \supset -\tfrac{1}{2} m_\chi^2 \chi^2 + \tfrac{1}{2}\xi R \chi^2$$





isocurvature constraints on ultra-light scalar GPP can be avoided by introducing a "small" non-minimal coupling to gravity

spin-1 particles
& ultra-light vectors

A massive Proca field in FRW

$$S = \int \mathrm{d}^4x \, \sqrt{-g} \left[-\tfrac{1}{4} g^{\mu\rho} g^{\nu\sigma} F_{\mu\nu} F_{\rho\sigma} - \tfrac{1}{2} m^2 g^{\mu\nu} A_\mu A_\nu \right]$$

transverse polarization modes

$$\left(\partial_\eta^2 + \omega_T^2\right)\chi_{T,k} = 0$$

$$\omega_T^2 = k^2 + a^2 m^2$$

identical to a conformally-coupled scalar

longitudinal polarization modes

$$\left(\partial_\eta^2 + \omega_L^2\right)\chi_{L,k} = 0$$

$$\omega_L^2 = k^2 + a^2 m^2 + \frac{1}{6}\frac{k^2 a^2 R}{k^2 + a^2 m^2} + 3\frac{k^2 a^4 H^2 m^2}{(k^2 + a^2 m^2)^2}$$

extra terms from integrating out time-like component

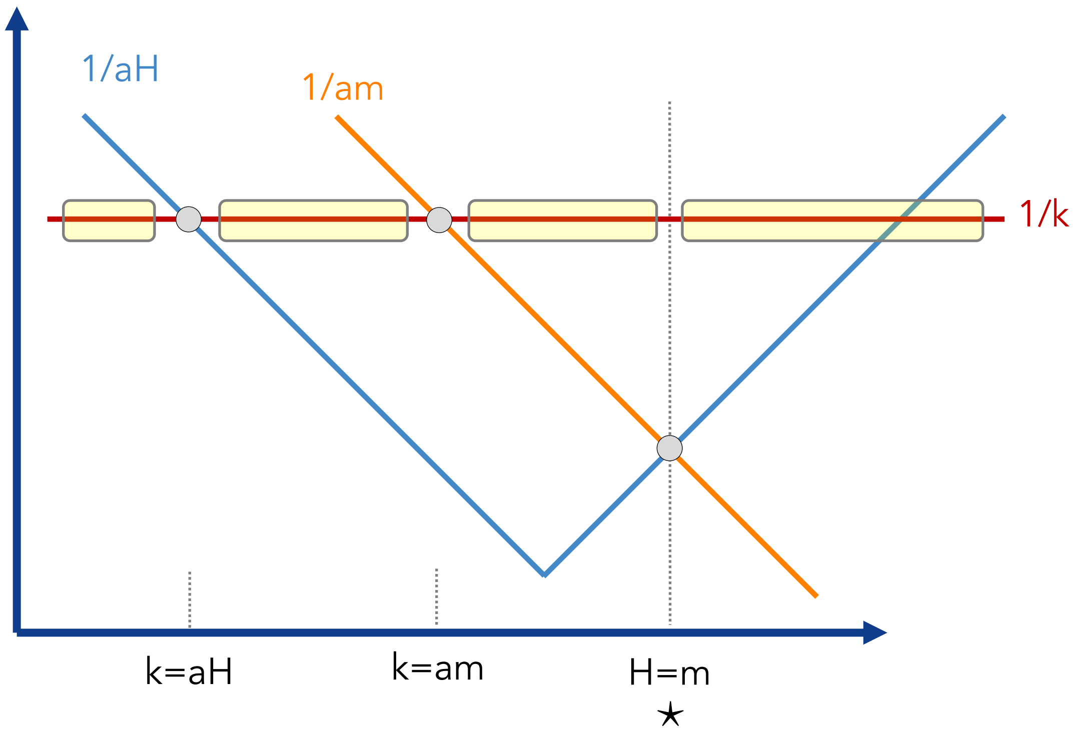
Differentiating vectors & scalars

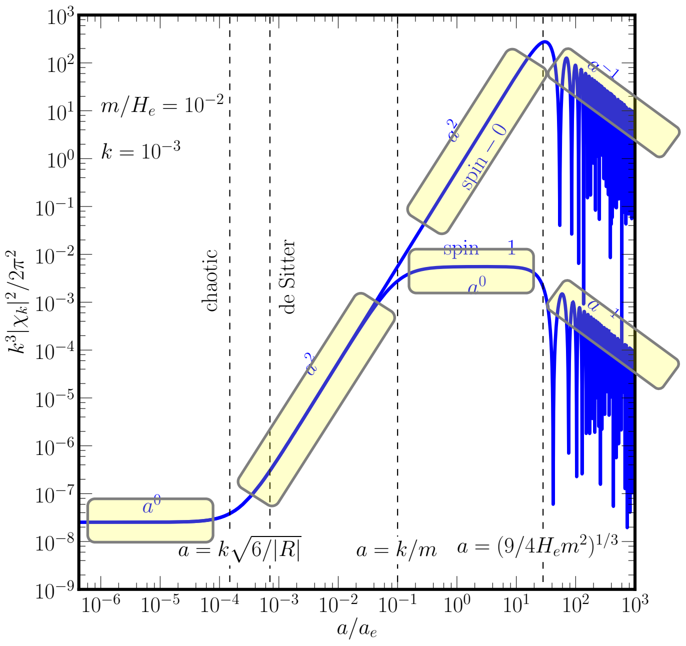
$$\partial_\eta^2 \chi_k + \omega^2 \chi_k = 0$$

$$\chi_k(\eta) \sim \frac{1}{\sqrt{2k}} e^{-ik\eta} \text{ (early)}$$

$$\omega^2_{\text{scalar}} = k^2 + a^2 m^2 + \frac{1}{6} a^2 R$$

$$\omega^2_{L\text{-vector}} = k^2 + a^2 m^2 + \frac{1}{6} \frac{k^2 a^2 R}{k^2 + a^2 m^2} + 3 \frac{k^2 a^4 H^2 m^2}{(k^2 + a^2 m^2)^2}$$





Vectors evolve differently while nonrelativistic & outside the horizon

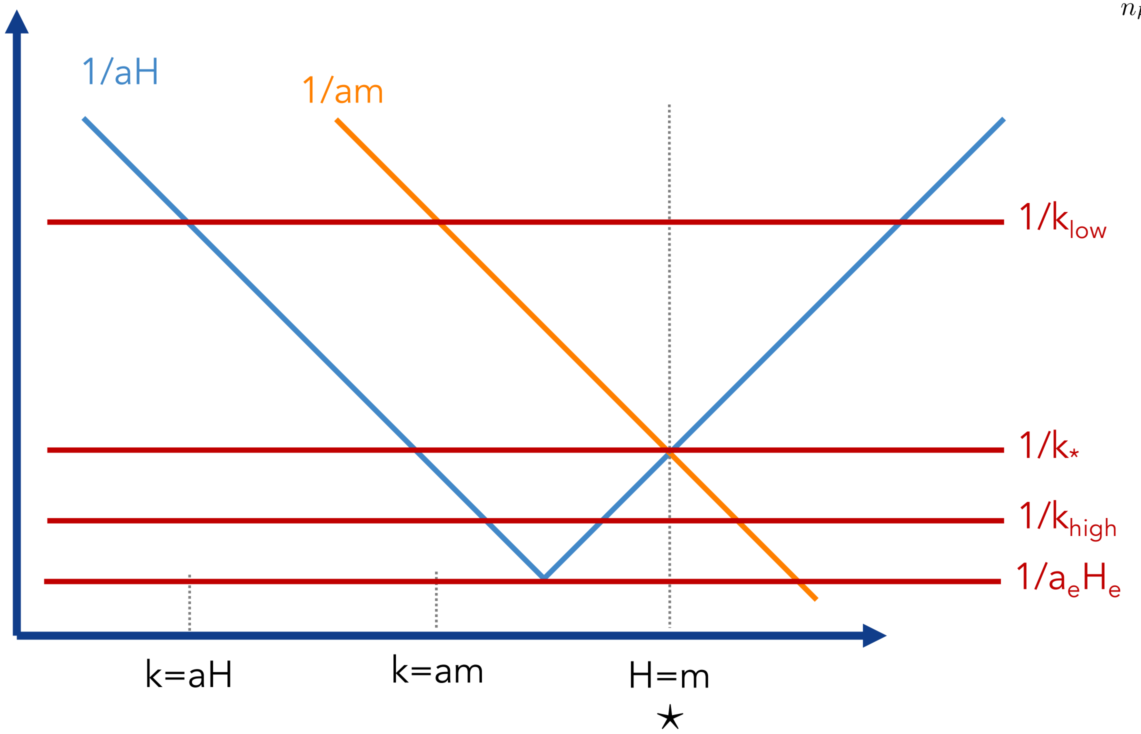
Differentiating vectors & scalars

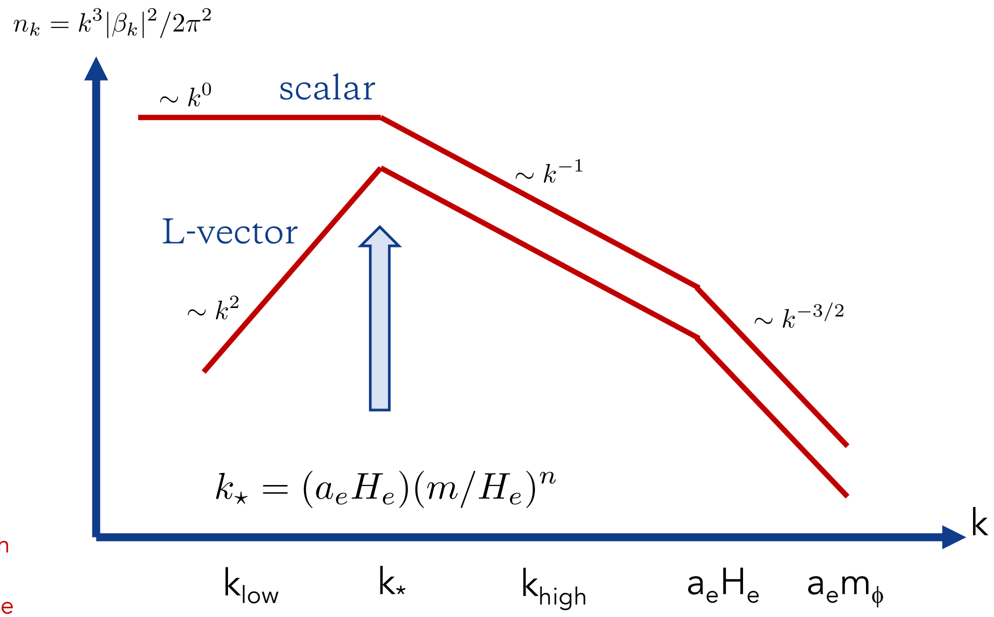
$$\partial_\eta^2 \chi_k + \omega^2 \chi_k = 0$$

$$\chi_k(\eta) \sim \frac{1}{\sqrt{2k}} e^{-ik\eta} \text{ (early)}$$

$$\omega^2_{\text{scalar}} = k^2 + a^2 m^2 + \tfrac{1}{6} a^2 R$$

$$\omega^2_{L\text{-vector}} = k^2 + a^2 m^2 + \frac{1}{6} \frac{k^2 R}{k^2 + a^2 m^2} + 3 \frac{k^2 a^4 H^2 m^2}{(k^2 + a^2 m^2)^2}$$





Vectors are suppressed toward low k.
Most power carried at k_*

Relic abundance

All the spin-1 dark matter is produced gravitationally for $H_{inf} < 10^{14}$ GeV & m > μeV

$$\Omega_\chi h^2 \simeq \begin{cases} (0.130)\left(\frac{H_{\rm inf}}{10^{14}\ \rm GeV}\right)^2\left(\frac{T_{\rm RH}}{10\ \rm GeV}\right) & , \quad t_e \leq t_\star < t_{\rm RH} \\ (0.201)\left(\frac{H_{\rm inf}}{10^{14}\ \rm GeV}\right)^2\left(\frac{m}{\mu\rm eV}\right)^{1/2} & , \quad t_e \leq t_{\rm RH} < t_\star \end{cases}$$