NNLO QCD corrections to $\bar{B} \to X_s\gamma$ without interpolation in m_c

Mikołaj Misiak

University of Warsaw

“Scalars 2025”, Warsaw, September 22-25th, 2025

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The new renormalized corrections to be reported today have been calculated by
M. Czaja, M. Czakon, T. Huber, MM, M. Niggetiedt, A. Rehman, K. Schönwald and M. Steinhauser
[to appear in arXiv:2510.nnnnn]

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5. Summary and outlook

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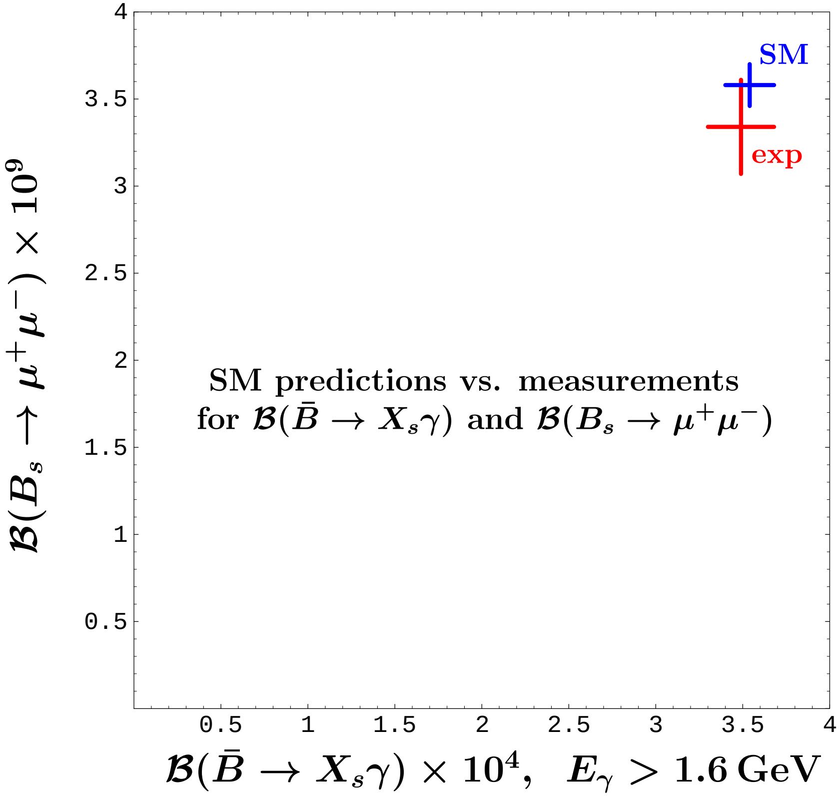

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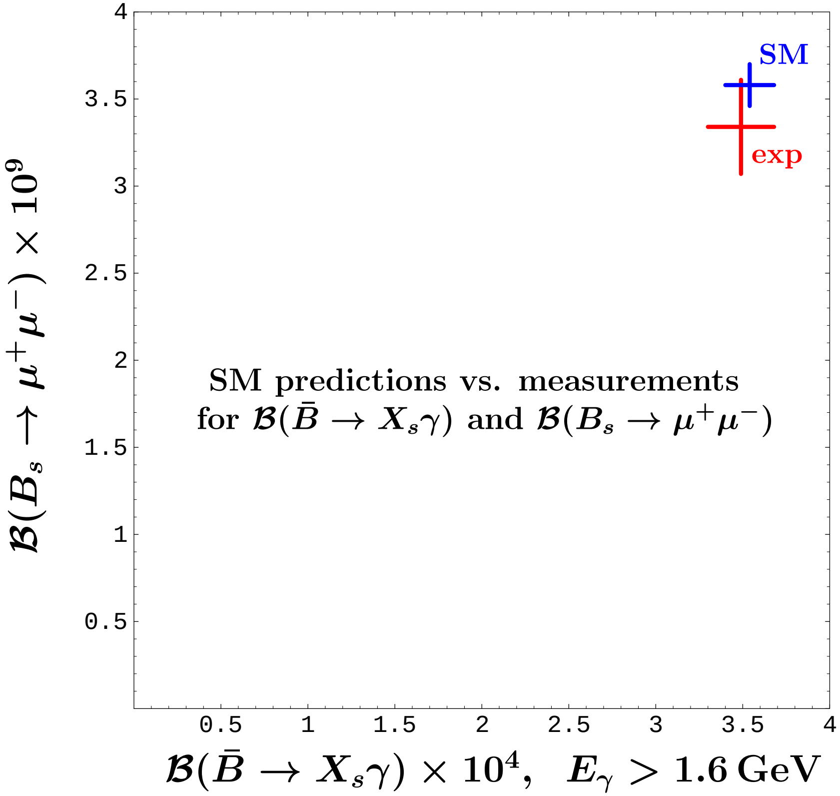

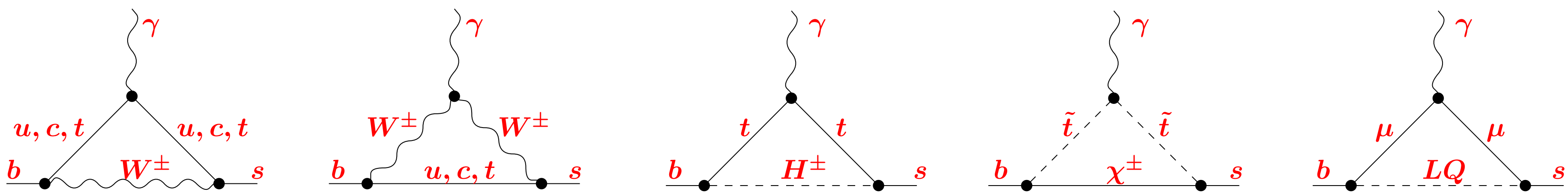

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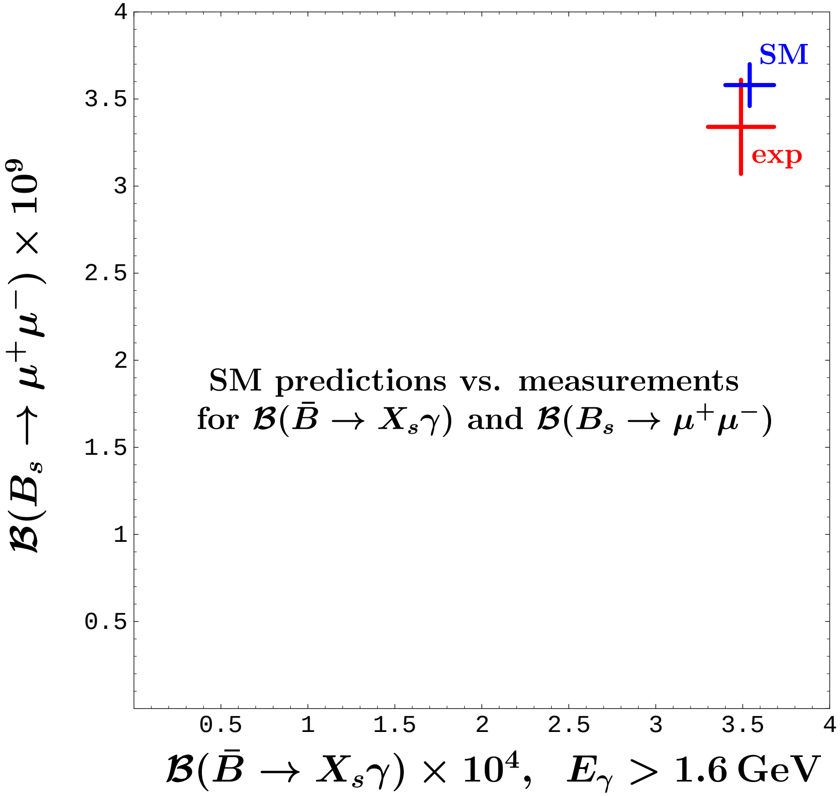

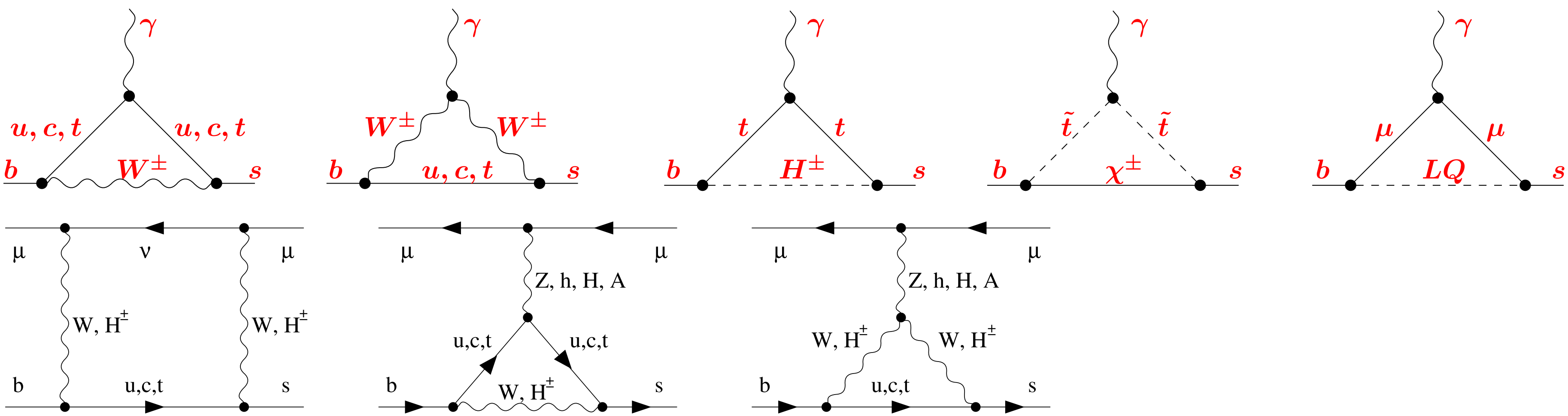

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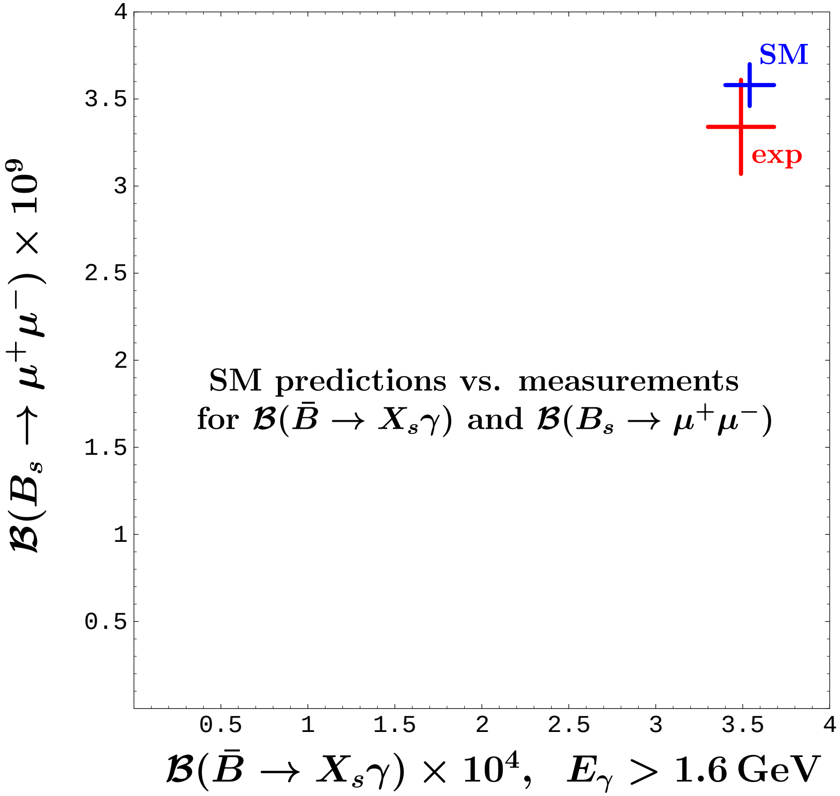
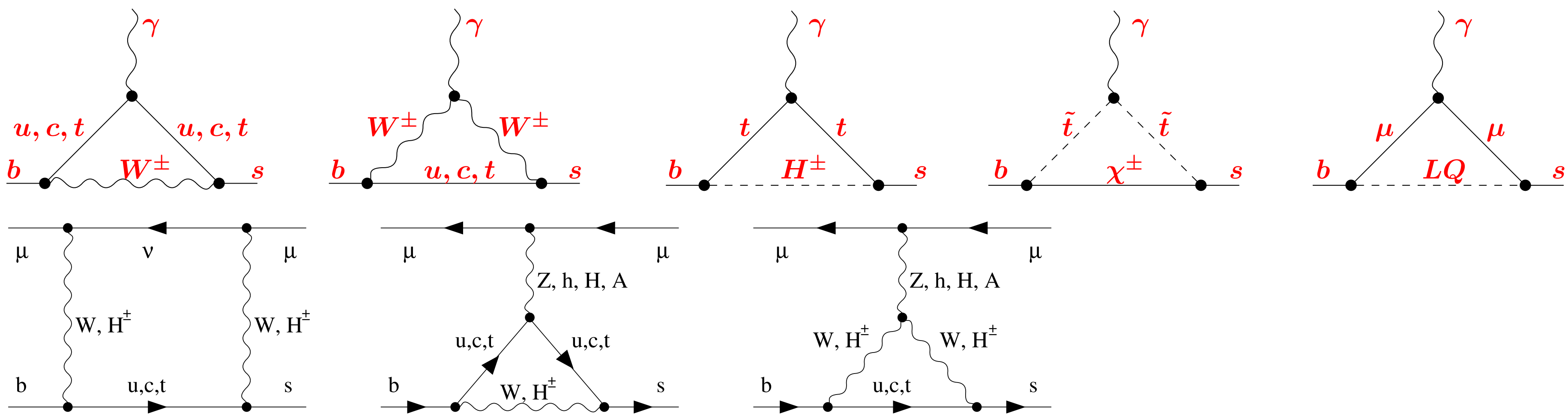


$\mathcal{B}(\bar{B} \to X_s\gamma)^{\text{exp}}_{E_\gamma>1.6} \times 10^4 = 3.49 \pm 0.19 \quad (\pm 5.5\%)$

CLEO, BaBar and Belle measurements combined by PDG [2022,2024] and HFLAV [arXiv:2206.07501].



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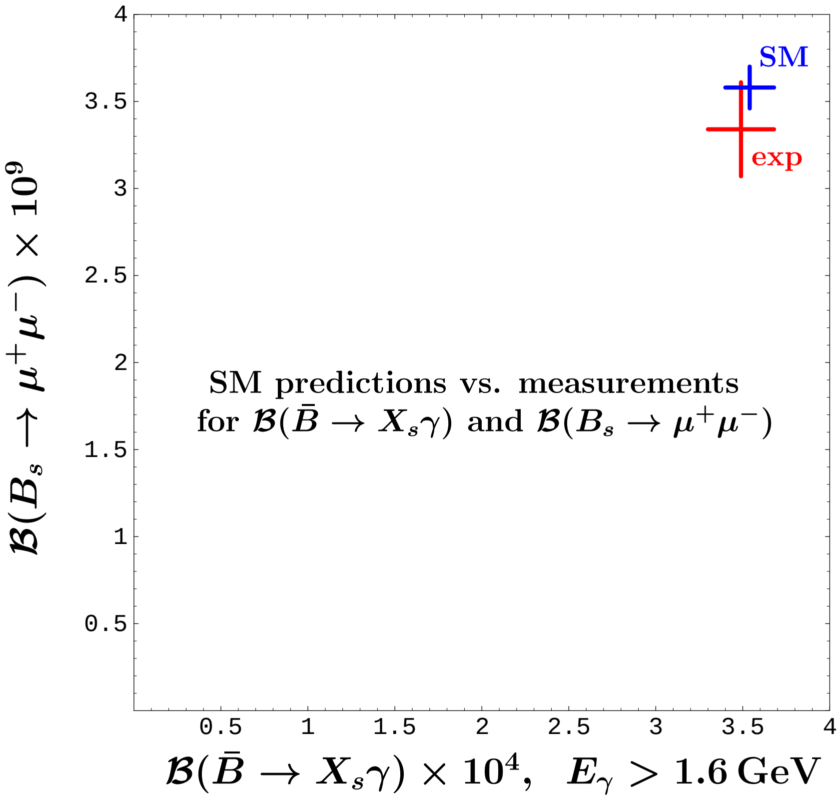

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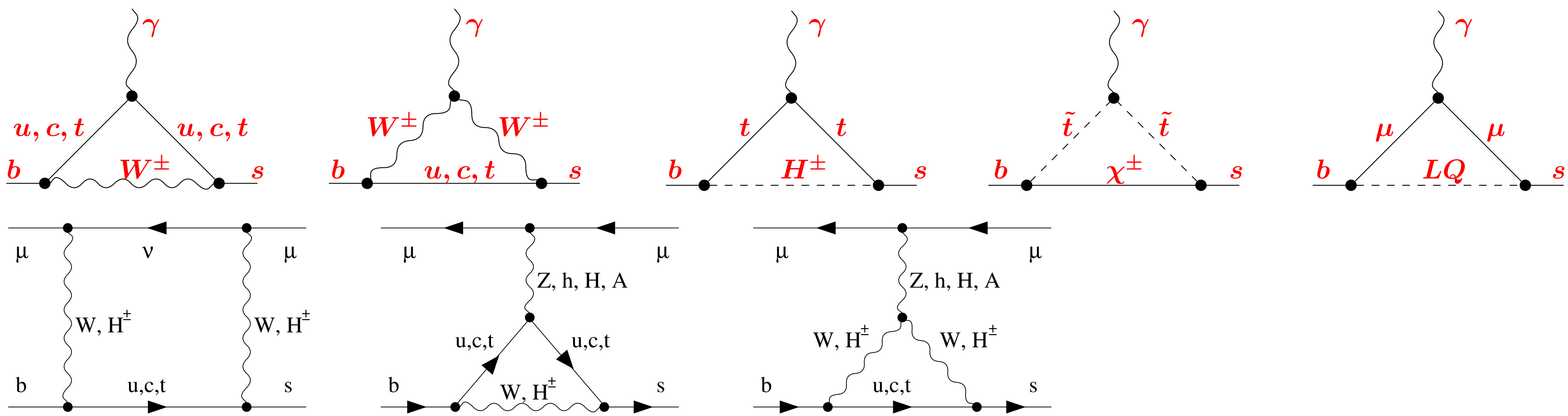
Final for 2510.nnnnn.



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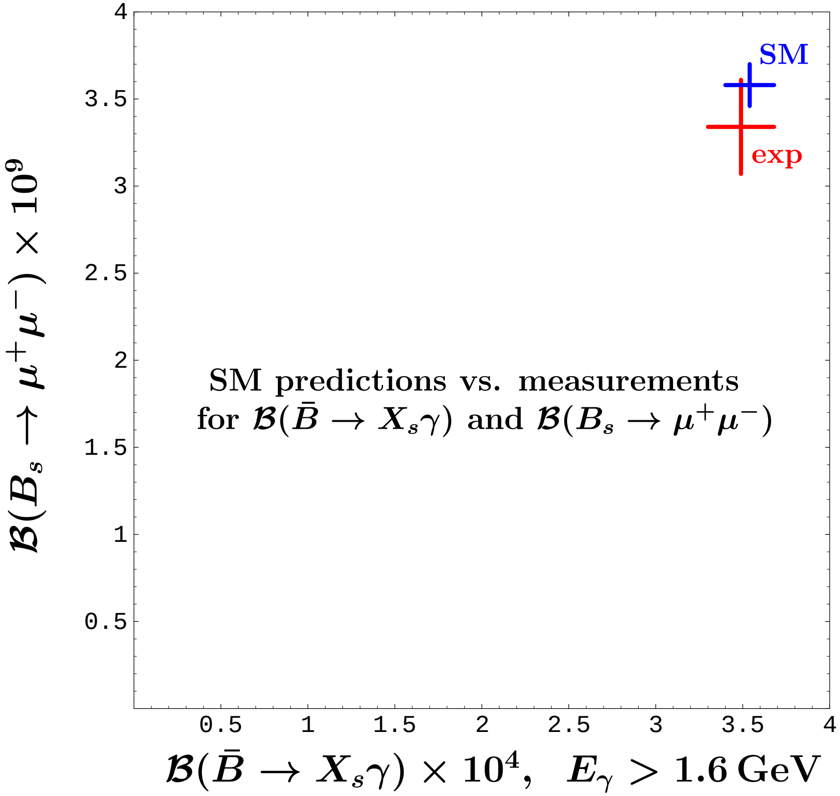

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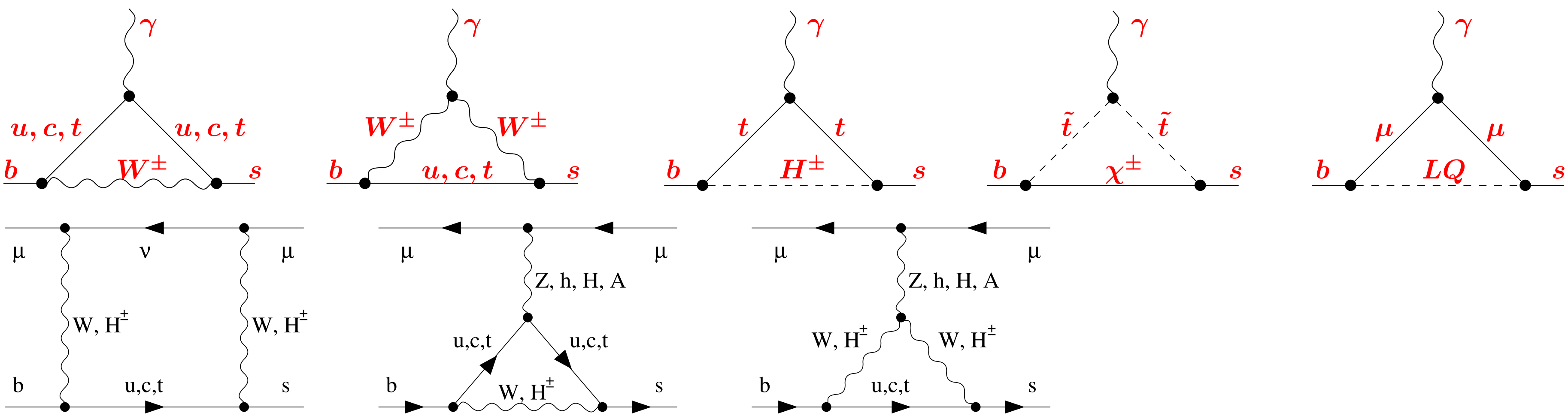

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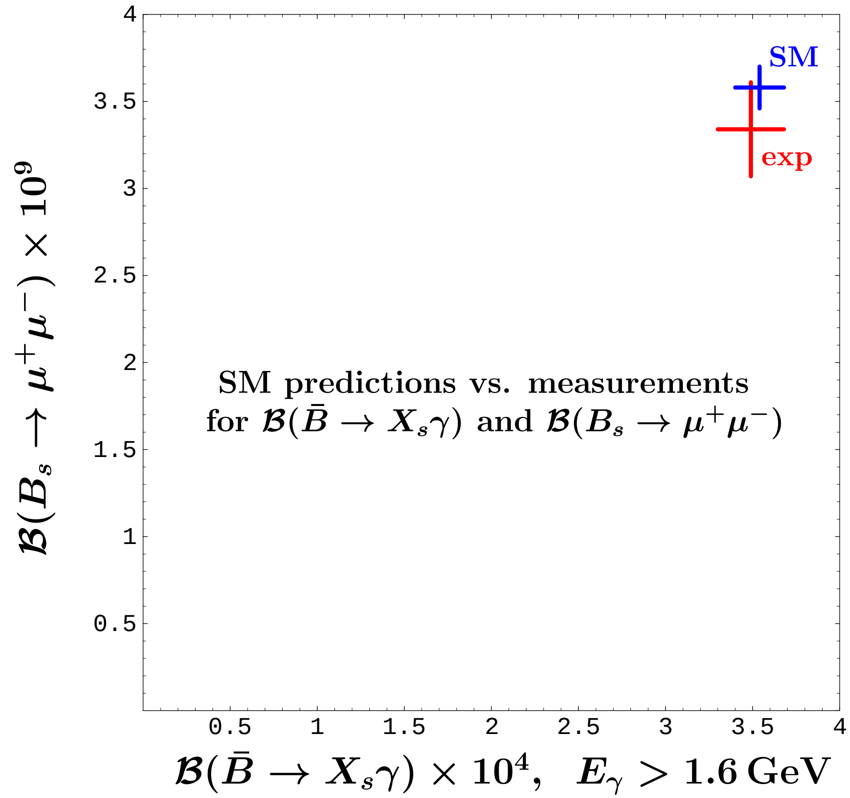

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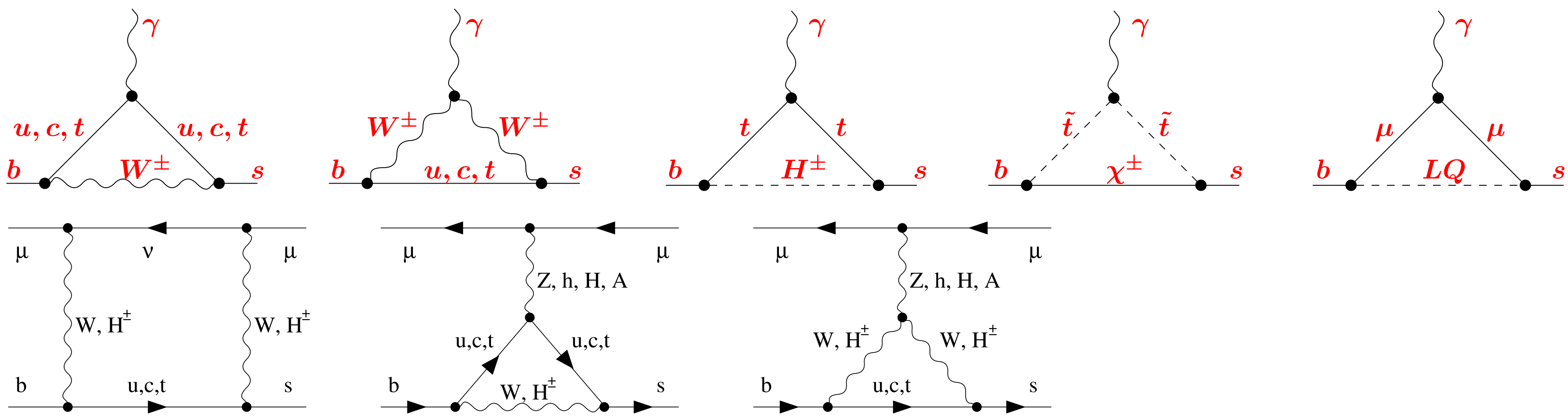
$$\mathcal{B}(B_s \to \mu^+\mu^-)^{\mathrm{SM}} \times 10^9 = 3.58 \pm 0.12 \quad (\pm 3.4\%)$$

arXiv:1311.0903 by C. Bobeth, M. Gorbahn, T. Hermann, MM, E. Stamou, M. Steinhauser with parameter updates and -0.5% QED correction from arXiv:1907.07011 by M. Beneke, C. Bobeth and R. Szafron.



$\mathcal{B}_{s\gamma}$ in the Two-Higgs-Doublet Model II

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$$\mathcal{L}_{\text{Yukawa}} = -\bar{q}_i Y^u_{ij} u_j H_2 - \bar{q}_i Y^d_{ij} d_j H_1 - \bar{l}_i Y^e_{ij} e_j H_1 + \text{h.c.},$$

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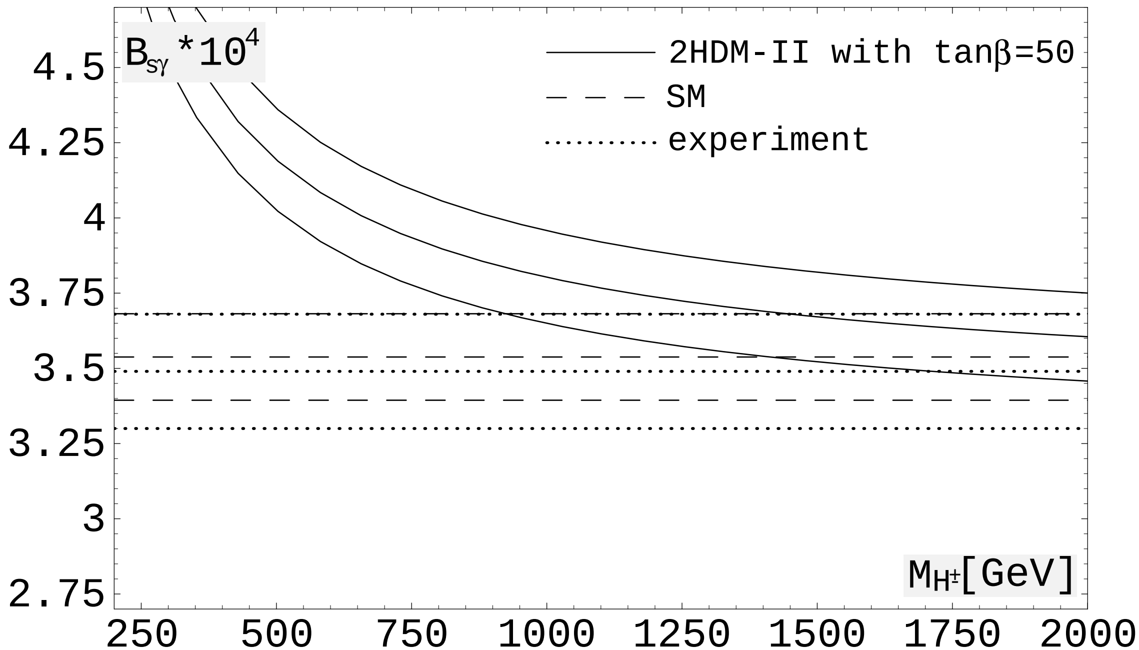

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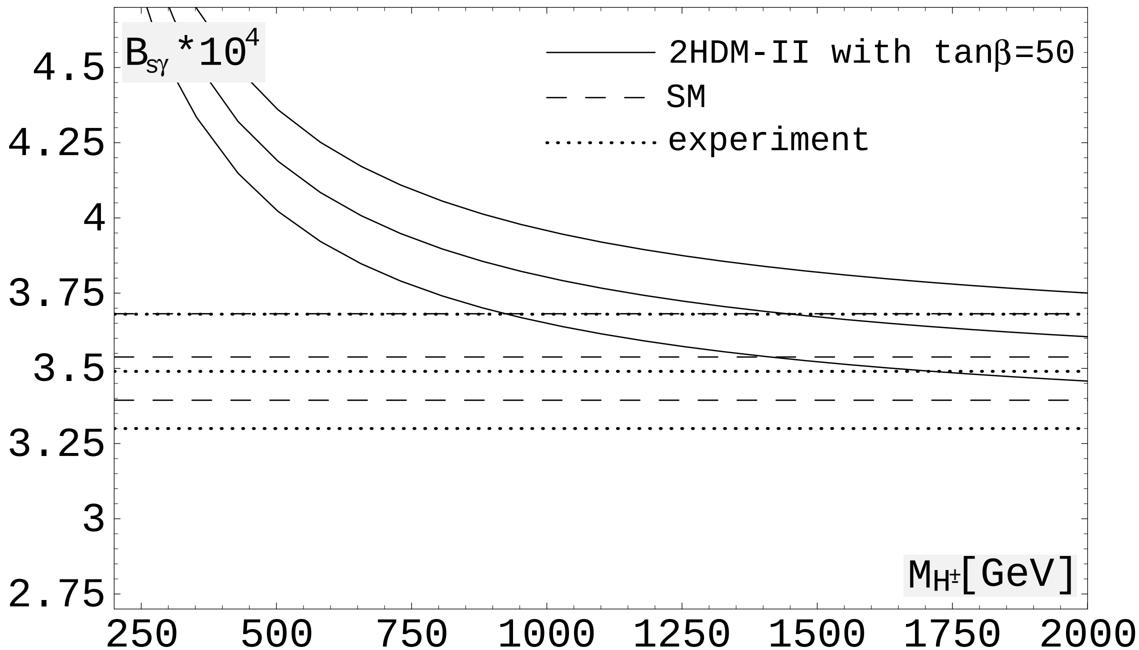
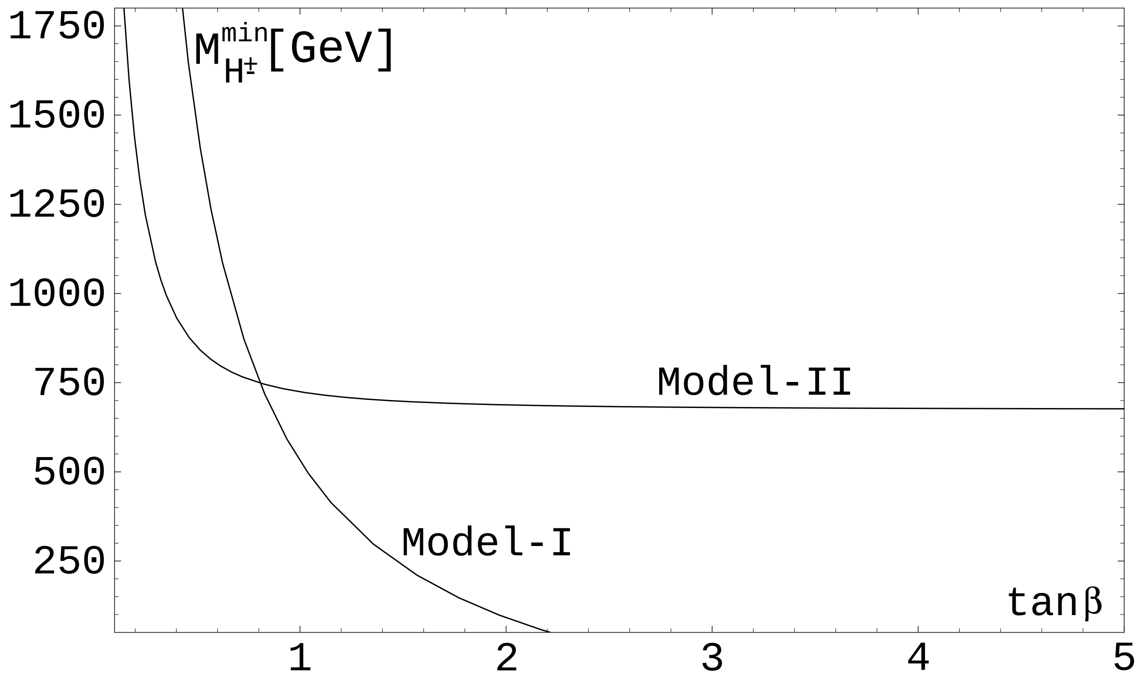


$$M_{H^\pm} > 675 \,\text{GeV} \quad @ \; 95\%\text{C.L.}$$

Exclusion bounds (95%C.L.) in the $\tan\beta$–$M_{H^\pm}$ plane:



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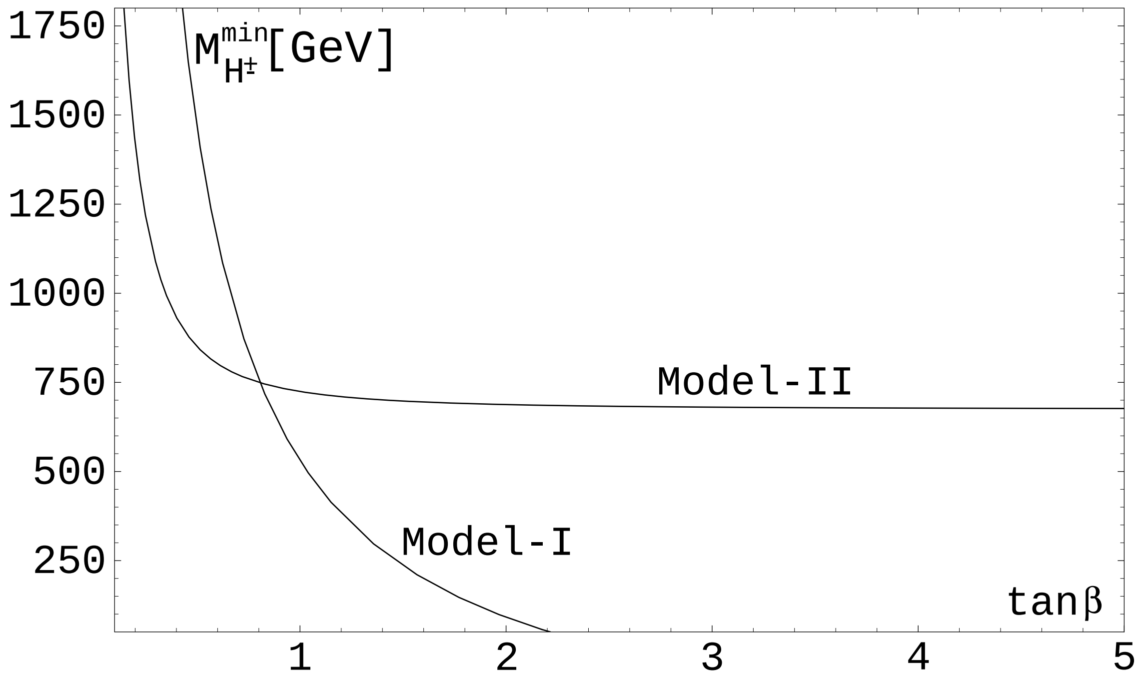

In Model-I:

$$\delta_{H^\pm} A(b \to s\gamma) \quad \sim \quad \left[-F_1\left(\frac{m_t^2}{M_{H^\pm}^2}\right) \;+\; F_2\left(\frac{m_t^2}{M_{H^\pm}^2}\right) \right] \cot^2\beta$$

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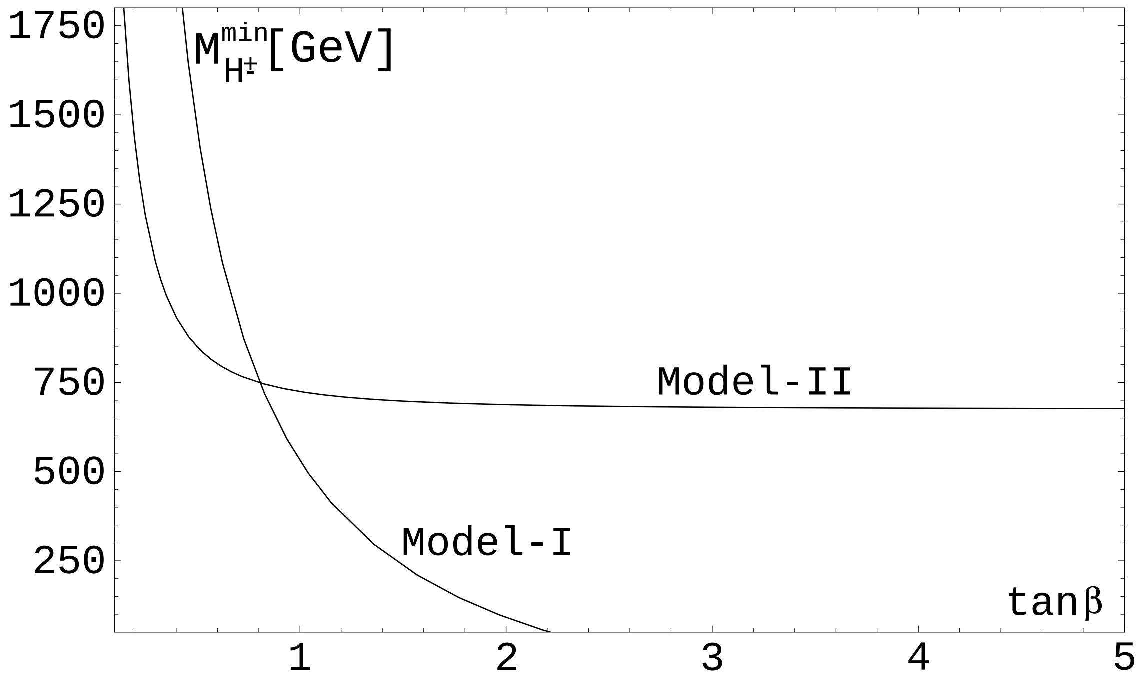

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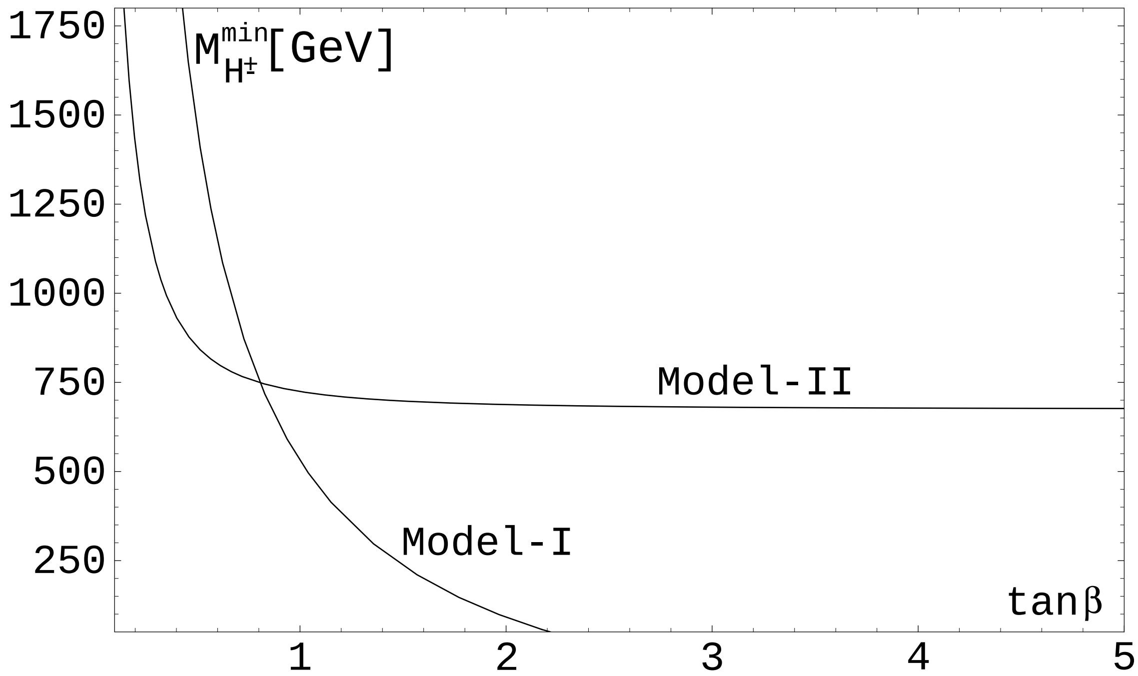

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In a large class of beyond-SM theories: $\mathcal{B}_{s\gamma} \times 10^4 = (3.54 \pm 0.14) - 8.51\, \Delta_{\rm BSM} C_7(\mu_0) - 2.16\, \Delta_{\rm BSM} C_8(\mu_0),$

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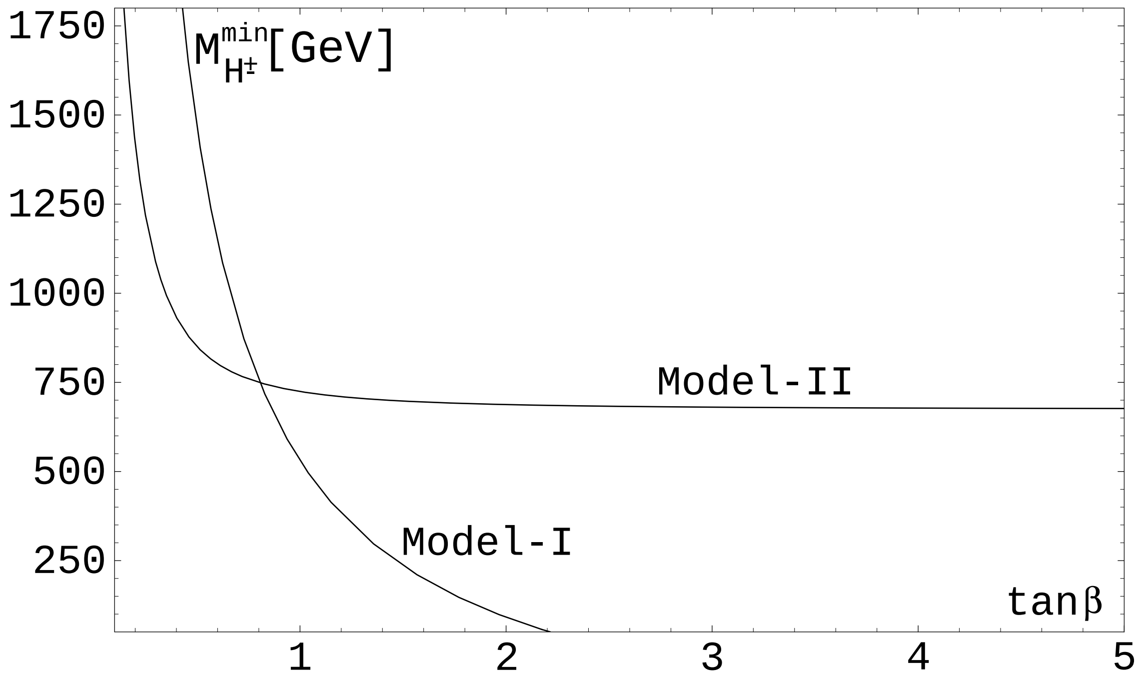

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with $\Delta_{\rm BSM} C_7(\mu_0)$ and $\Delta_{\rm BSM} C_8(\mu_0)$ provided in hep-ph/9904413 up to the NLO in QCD.

Updated SM prediction:

$$\mathcal{B}_{s\gamma}^{\mathrm{SM}} = (3.54 \pm 0.14) \times 10^{-4}$$

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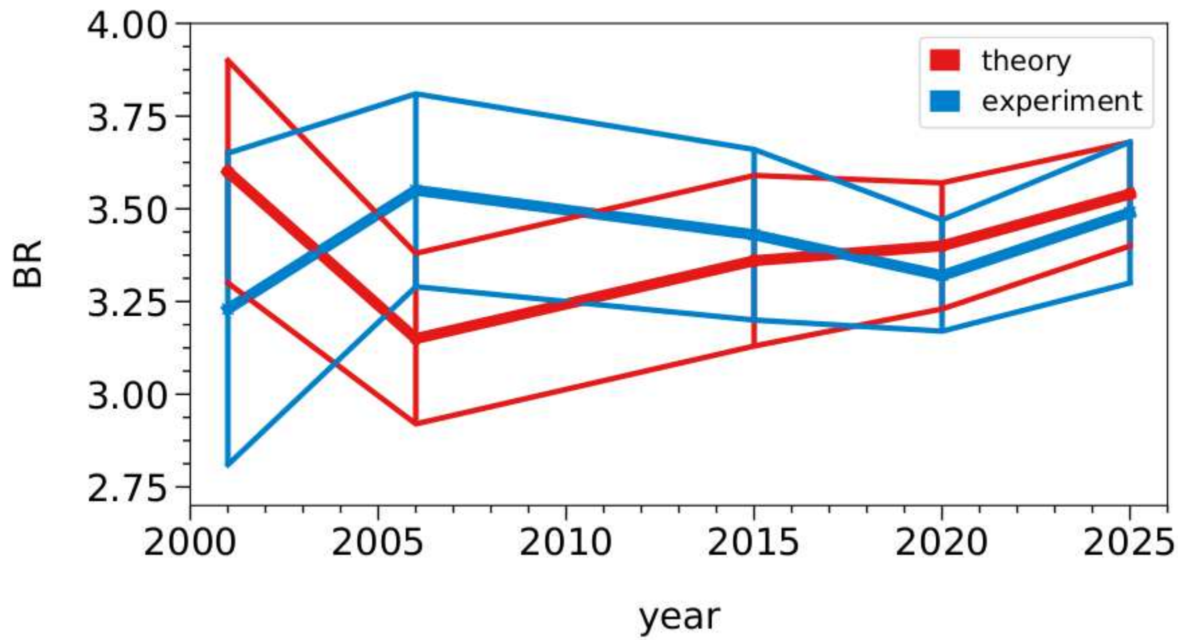

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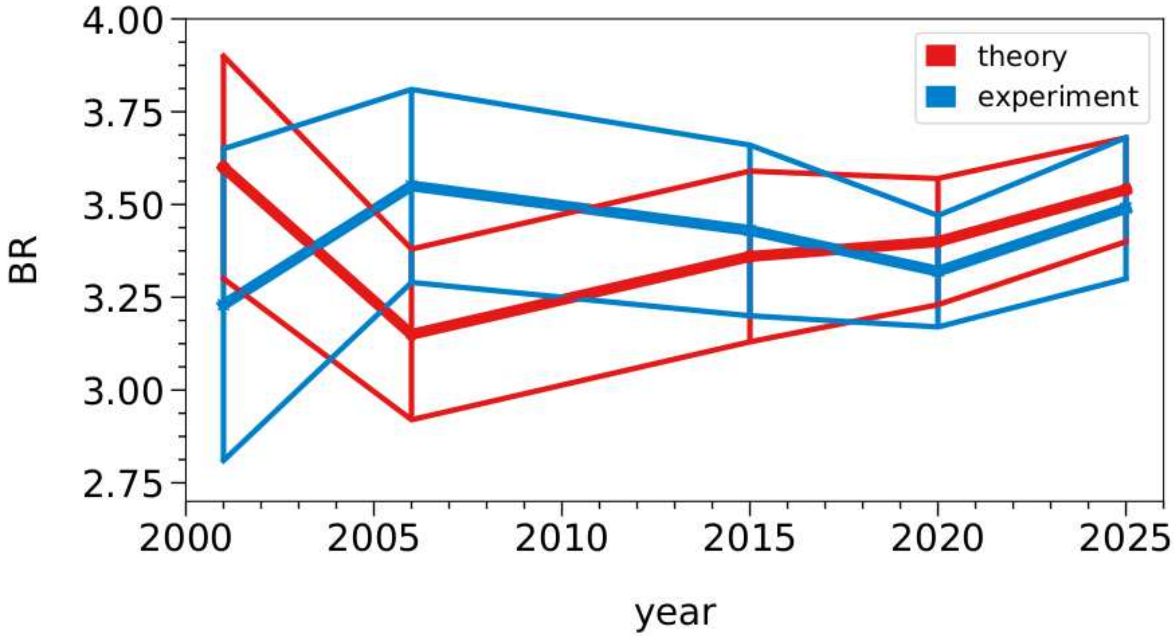

Improvements in the dominant charm-mass dependent NNLO QCD corrections:

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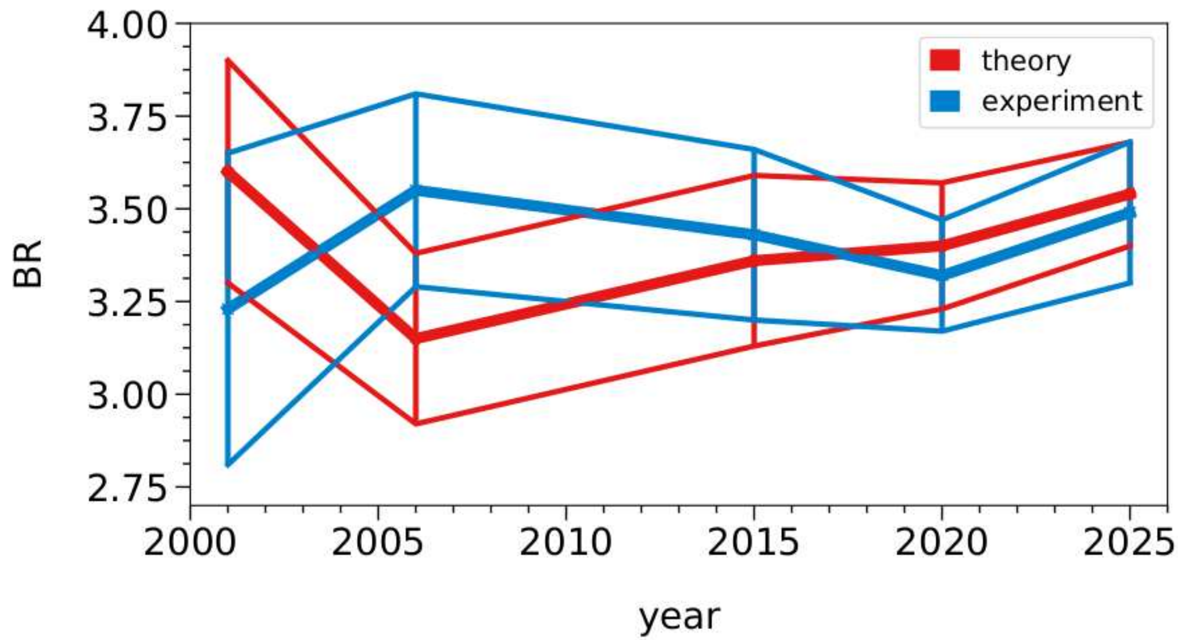

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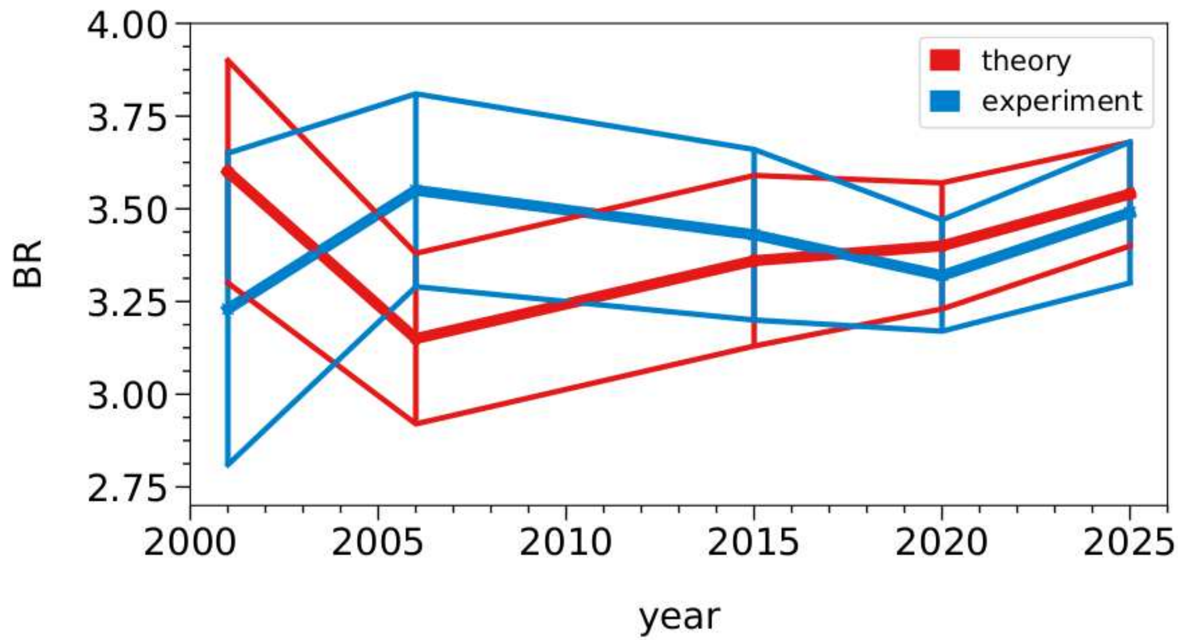

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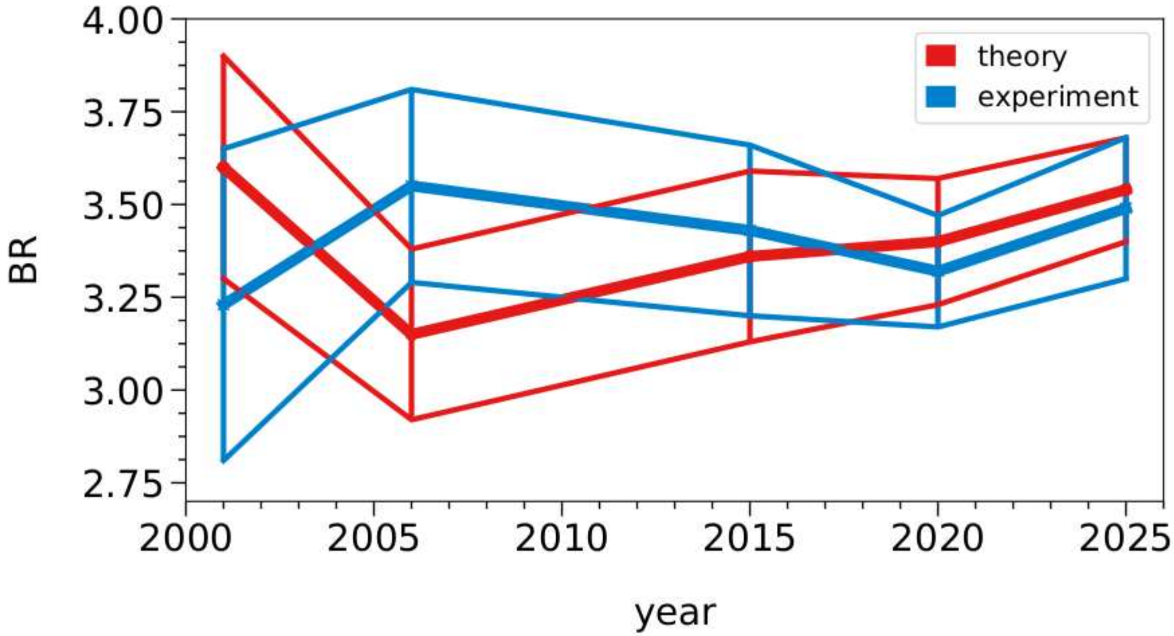

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2000: NLO only,
2006: extrapolation of non-BLM NNLO from the $m_c \gg m_b$ limit,
2015: interpolation of non-BLM NNLO between the $m_c = 0$ and $m_c \gg m_b$ limits,

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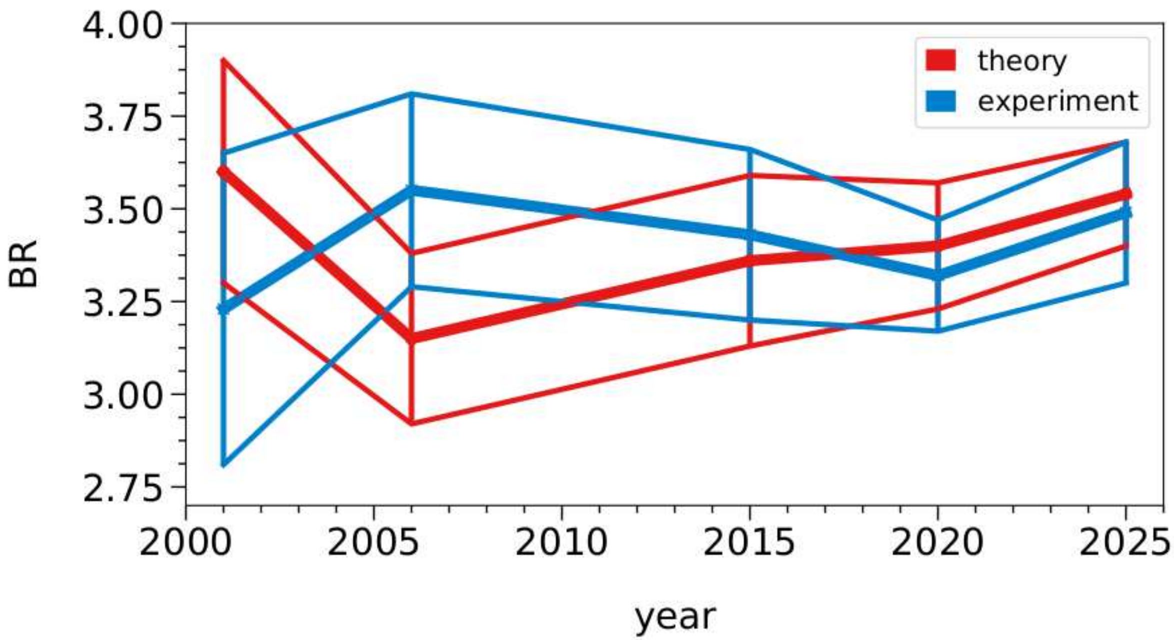

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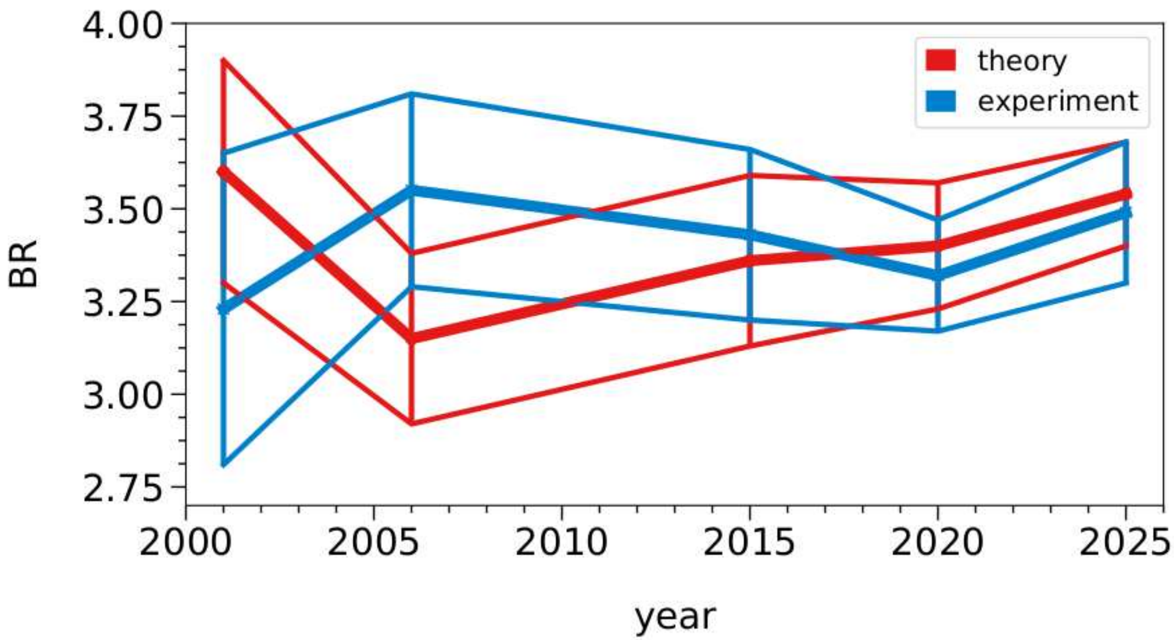

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The perturbative NLO corrections are now formally complete thanks to including formerly missing 4-body and 5-body contributions from arXiv:2510.nnnnn [K. M. Brune, T. Huber, L-T. Moos].

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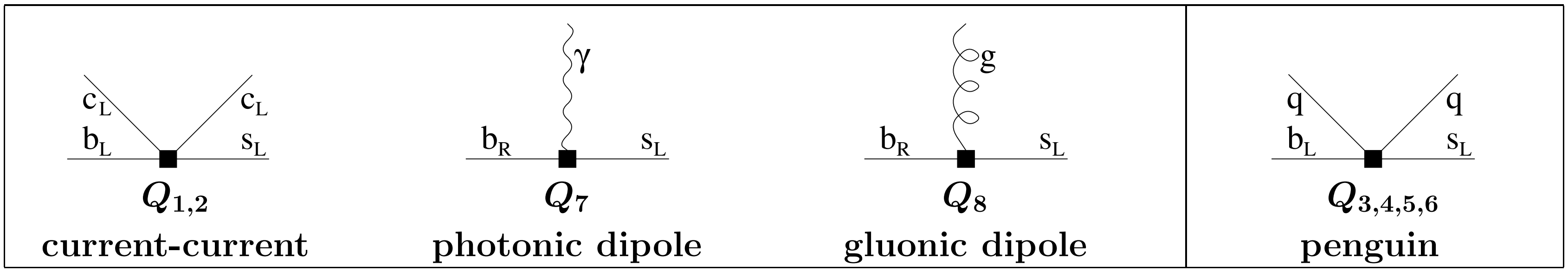

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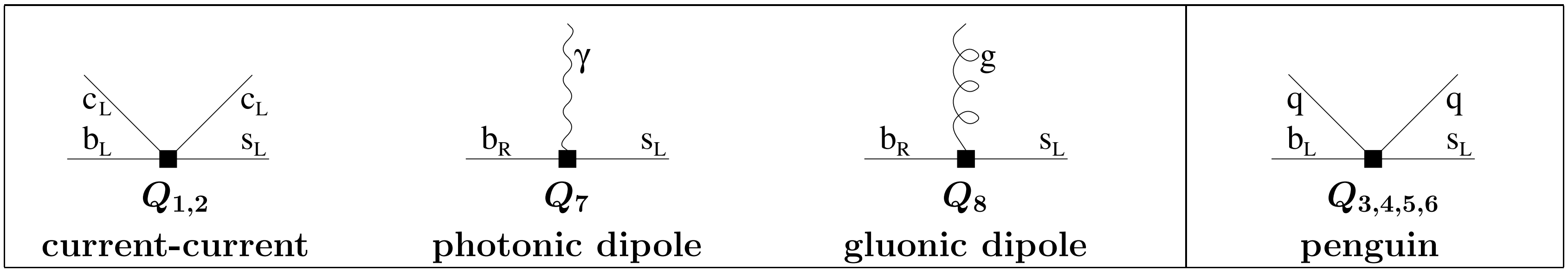

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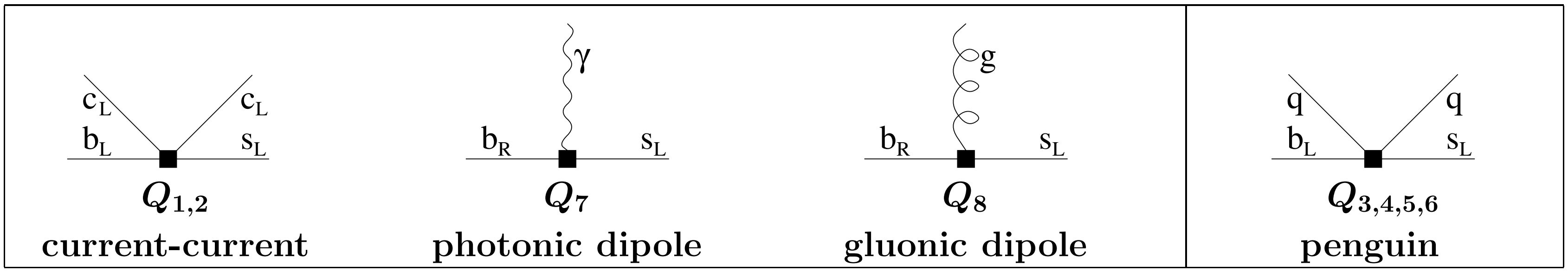

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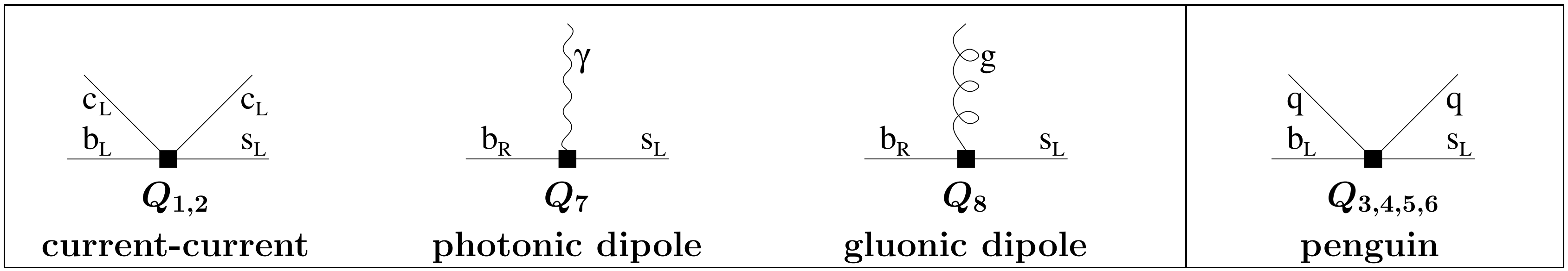

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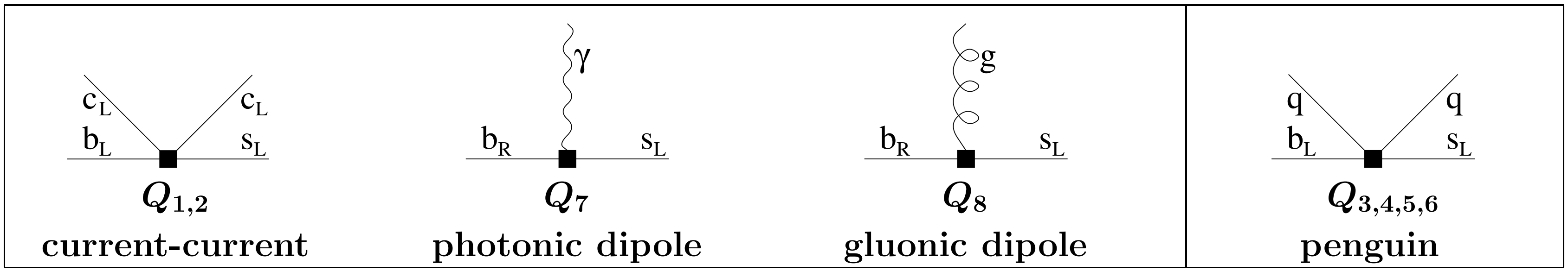

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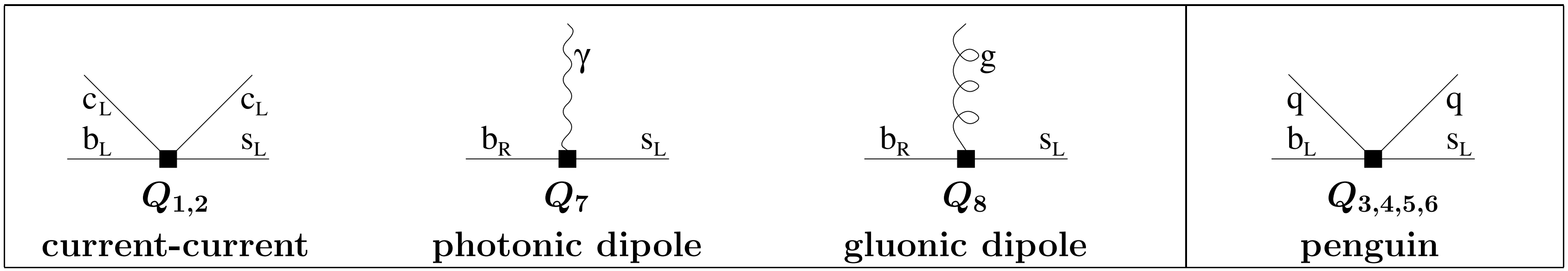

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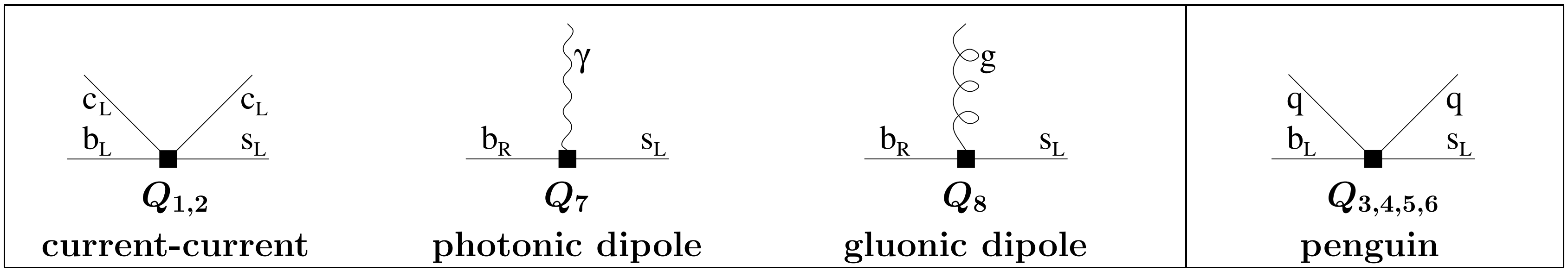

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NLO

A. Ali and C. Greub, ZPC 49 (1991) 431, PLB 259 (1991) 182, PLB 361 (1995) 146 [hep-ph/9506374], C. Greub, T. Hurth and D. Wyler, hep-ph/9602281, hep-ph/9603404.

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NNLO

$G_{77}^{(2)}$: K. Melnikov and A. Mitov, hep-ph/0505097,
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H. M. Asatrian, A. Hovhannisyan, V. Poghosyan, T. Ewerth, C. Greub and T. Hurth, hep-ph/0605009,
H. M. Asatrian, T. Ewerth, A. Ferroglia, P. Gambino and C. Greub, hep-ph/0607316,
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H. M. Asatrian, T. Ewerth, A. Ferroglia, P. Gambino and C. Greub, hep-ph/0607316,
H. M. Asatrian, T. Ewerth, H. Gabrielyan and C. Greub, hep-ph/0611123.

$G_{78}^{(2)}$: H. M. Asatrian, T. Ewerth, A. Ferroglia, C. Greub and G. Ossola, arXiv:1005.5587.

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NLO

A. Ali and C. Greub, ZPC 49 (1991) 431, PLB 259 (1991) 182, PLB 361 (1995) 146 [hep-ph/9506374],
C. Greub, T. Hurth and D. Wyler, hep-ph/9602281, hep-ph/9603404.

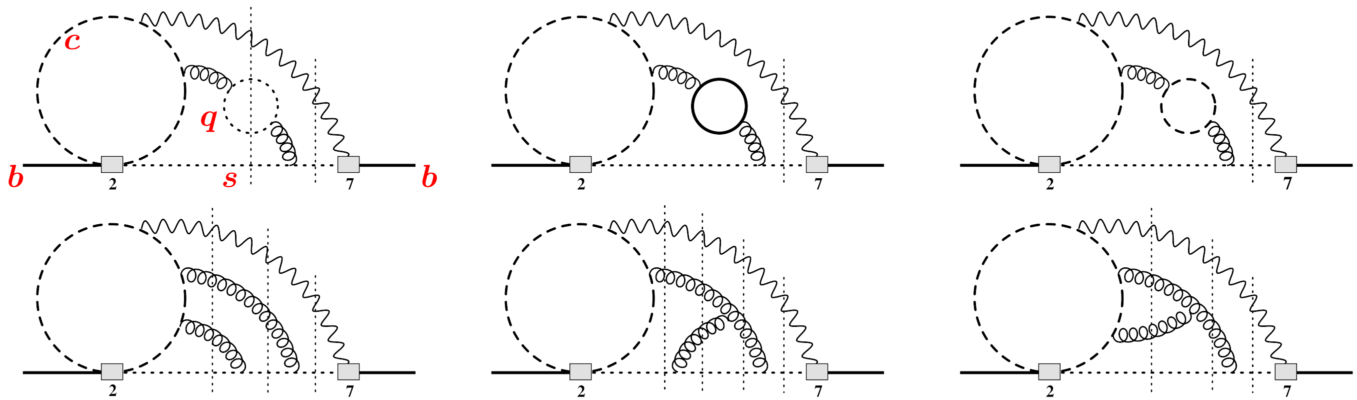
NNLO

$G_{77}^{(2)}$: K. Melnikov and A. Mitov, hep-ph/0505097,
I. Blokland, A. Czarnecki, MM, M. Ślusarczyk and F. Tkachov, hep-ph/0506055,
H. M. Asatrian, A. Hovhannisyan, V. Poghosyan, T. Ewerth, C. Greub and T. Hurth, hep-ph/0605009,
H. M. Asatrian, T. Ewerth, A. Ferroglia, P. Gambino and C. Greub, hep-ph/0607316,
H. M. Asatrian, T. Ewerth, H. Gabrielyan and C. Greub, hep-ph/0611123.

$G_{78}^{(2)}$: H. M. Asatrian, T. Ewerth, A. Ferroglia, C. Greub and G. Ossola, arXiv:1005.5587.

$G_{27}^{(2)}$: Z. Ligeti, M.E. Luke, A.V. Manohar and M.B. Wise, hep-ph/9903305, large-β_0, 4-body,
K. Bieri, C. Greub and M. Steinhauser, hep-ph/0302051, large-β_0, 2-body,
MM and M. Steinhauser, hep-ph/0609241, arXiv:1005.1173, $m_c \gg m_b$,
R. Boughezal, M. Czakon and T. Schutzmeier, arXiv:0707.3090, large-β_0 and massive loops, 2-body,
M. Czakon, P. Fiedler, T. Huber, MM, T. Schutzmeier and M. Steinhauser, arXiv:1503.01791, $m_c = 0$,
MM, A. Rehman and M. Steinhauser, arXiv:1702.07674, arXiv:2002.01548, counterterms, large-β_0 and massive loops,
C. Greub, H. M. Asatrian, F. Saturnino and C. Wiegand, arXiv:2303.01714, physical m_c, partial 2-body,
M. Fael, F. Lange, K. Schönwald and M. Steinhauser, arXiv:2309.14706, physical m_c, full 2-body,
M. Czaja, M. Czakon, T. Huber, MM, M. Niggetiedt, A. Rehman,
K. Schönwald and M. Steinhauser, arXiv:2309.14707, physical m_c, full 2-body,
C. Greub, H. M. Asatrian, H. H. Asatryan, L. Born and J. Eicher, arXiv:2407.17270, physical m_c, full 2-body,
M. Czaja, M. Czakon, T. Huber, MM, M. Niggetiedt, A. Rehman,
K. Schönwald and M. Steinhauser, arXiv:2510.nnnnn, physical m_c, fully inclusive, renormalized result.

Sample propagator diagrams with cuts contributing to G_{27} @ NNLO:



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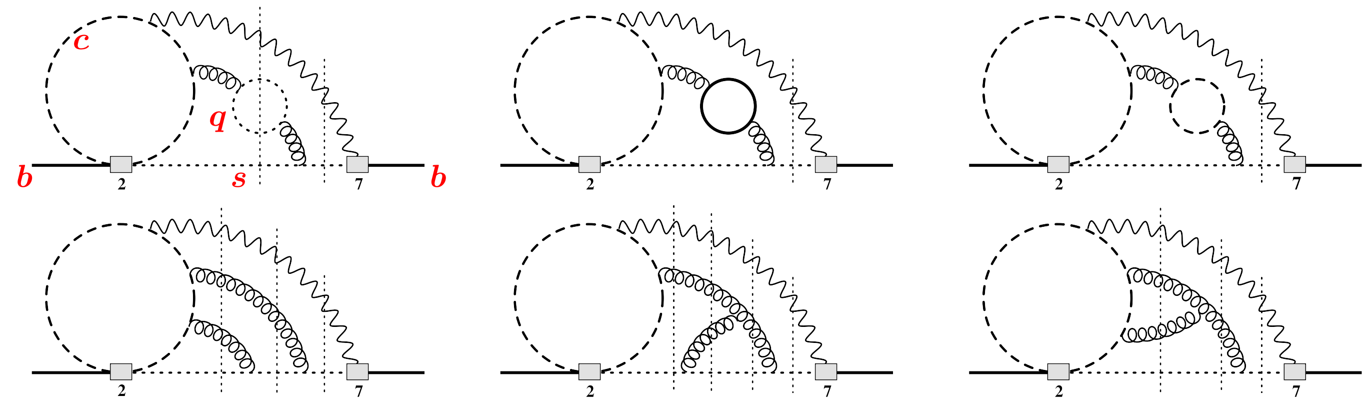

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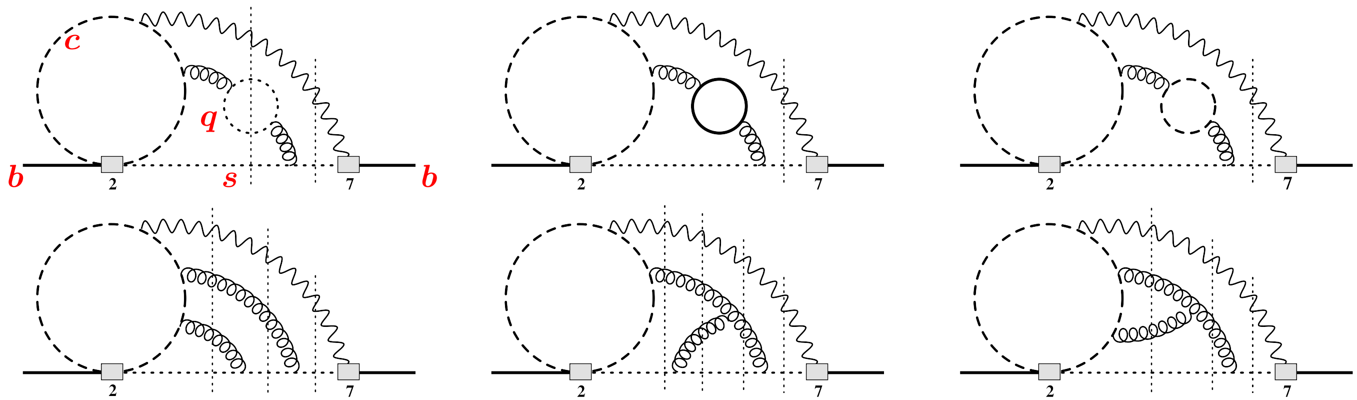

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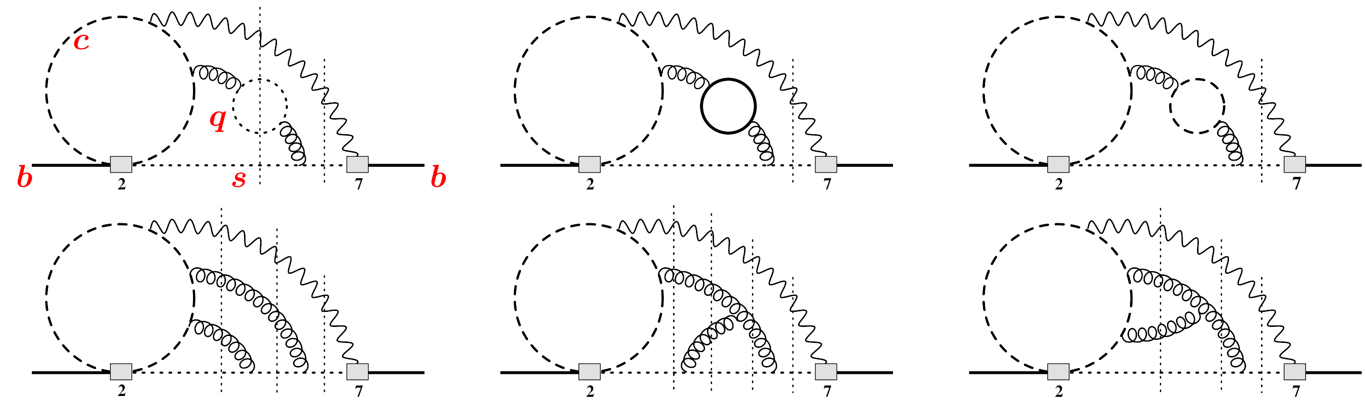

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$$\frac{d}{dz} M_k(z,\epsilon) = \sum_l R_{kl}(z,\epsilon)\, M_l(z,\epsilon), \qquad (*)$$

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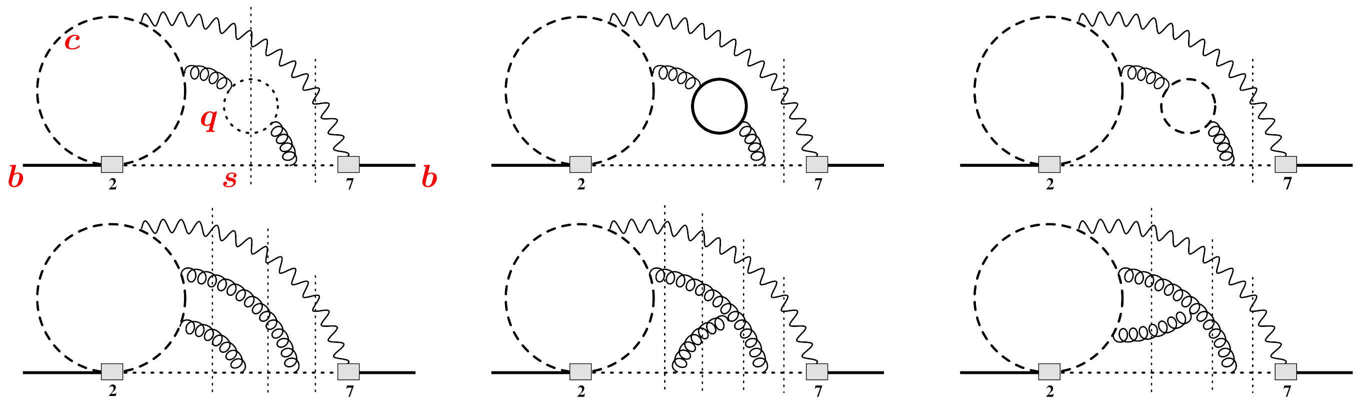

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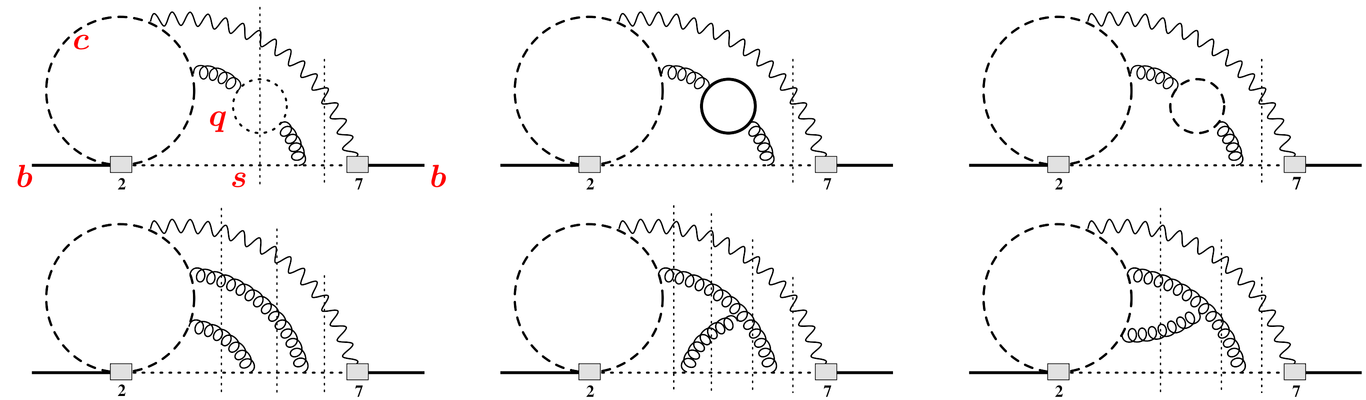

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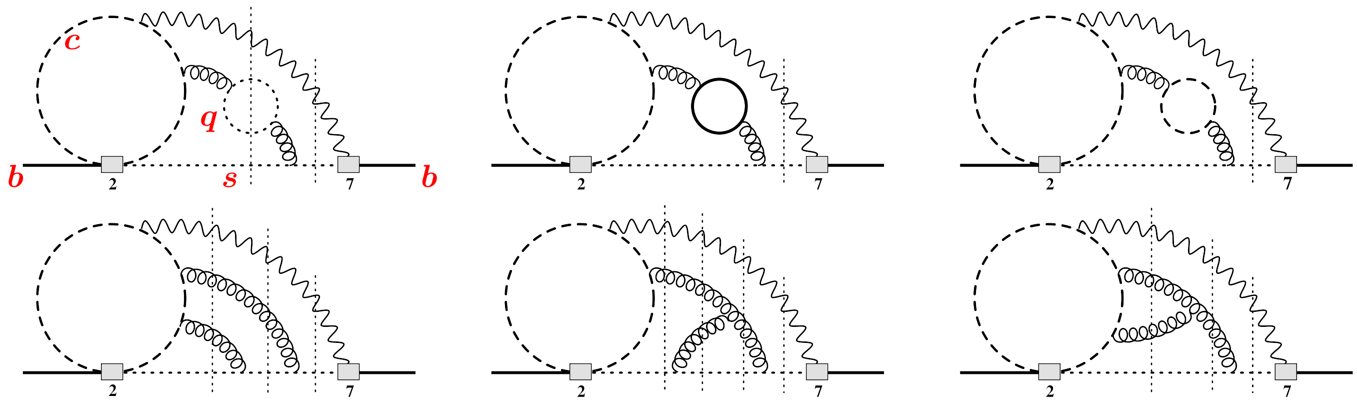

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6. Solving the system $(*)$ numerically [e.g., A.C. Hindmarsch, http://www.netlib.org/odepack] along an ellipse in the complex z plane.

Fully inclusive (2-, 3- and 4-body), renormalized results for $G_{17}^{(2)}$ and $G_{27}^{(2)}$.

M. Czaja, M. Czakon, T. Huber, MM, M. Niggetiedt, A. Rehman, K. Schönwald and M. Steinhauser, arXiv:2510.nnnnn.

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Comparison to the interpolated NNLO correction in arXiv:1503.01791.

$$\frac{\Delta\mathcal{B}_{s\gamma}}{\mathcal{B}_{s\gamma}} \simeq U(z,\delta) \equiv \frac{\alpha_s^2(\mu_b)}{8\pi^2} \frac{C_1^{(0)}(\mu_b)F_1(z,\delta)+\left(C_2^{(0)}(\mu_b)-\frac{1}{6}C_1^{(0)}(\mu_b)\right)F_2(z,\delta)}{C_7^{(0)\text{eff}}(\mu_b)} \qquad \left(z=\frac{m_c^2}{m_b^2}\right)$$

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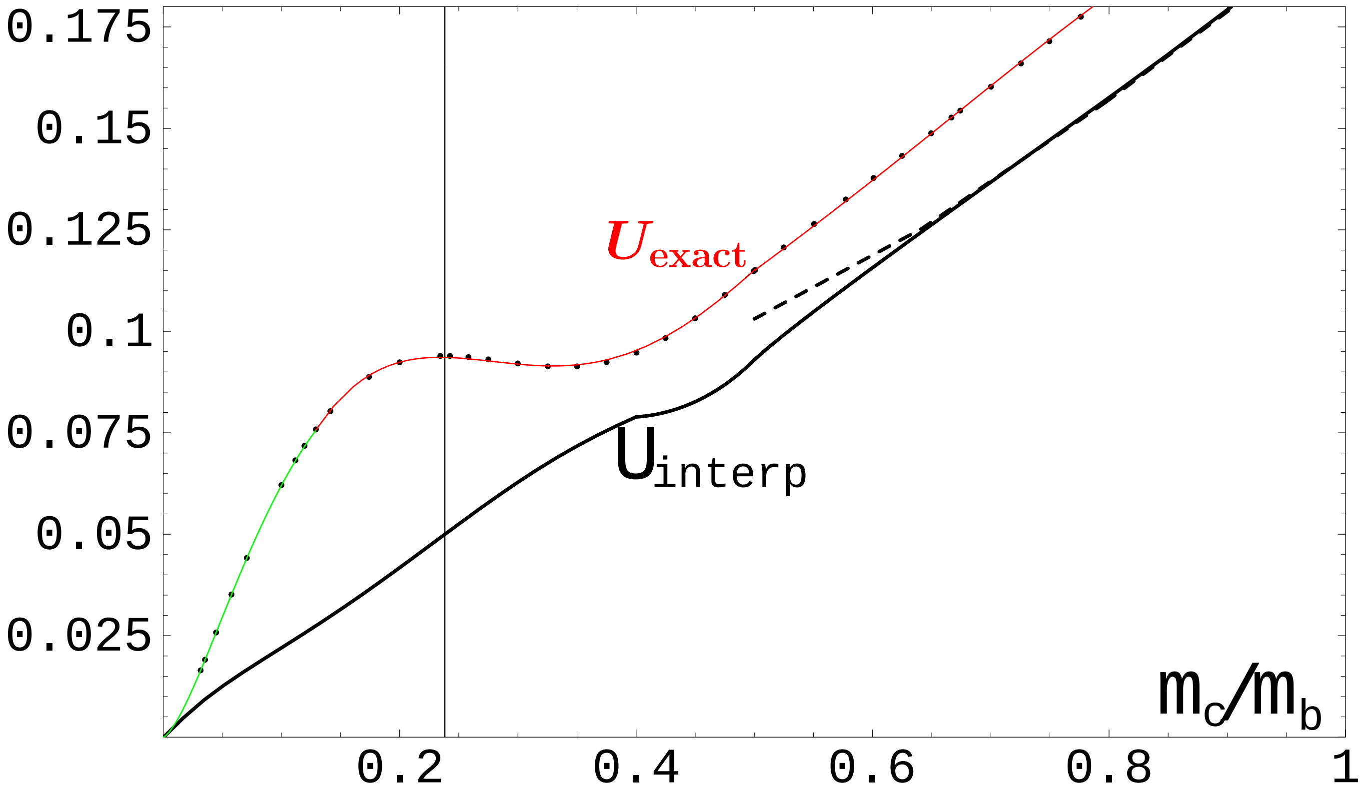
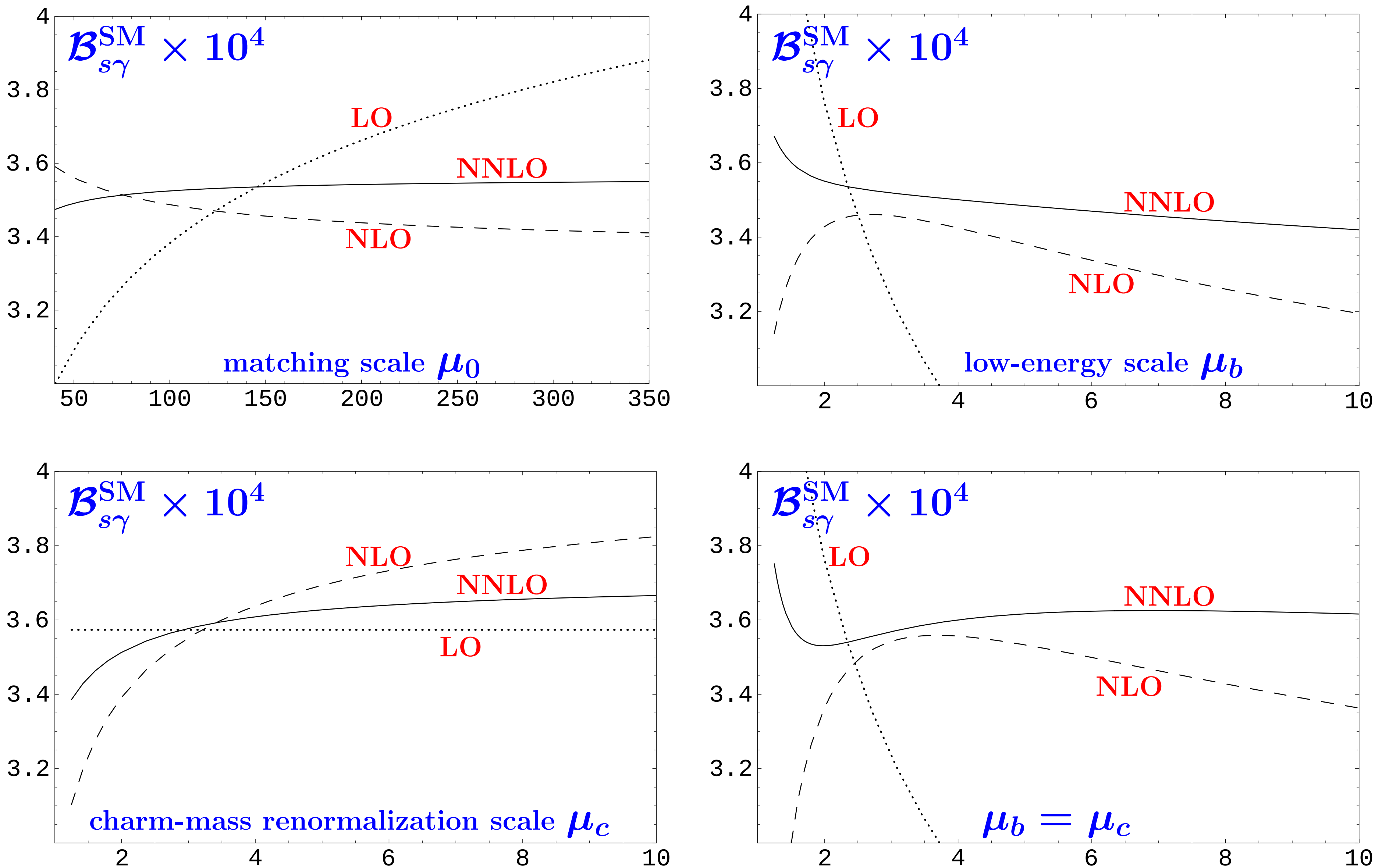
Interpolated and exact results for $\delta = 1$ (no cut on E_γ):



Renormalization scale dependence of $\mathcal{B}_{s\gamma}^{\rm SM}$ for $E_\gamma > 1.6$ GeV



"Central" values: $\mu_0 = 160$ GeV, $\mu_b = \mu_c = \frac{1}{2} m_{b,kin}(1\,\text{GeV}) \simeq 2.29\,\text{GeV}$.

Resolved photon contribution to the Q_7-$Q_{1,2}$ interference.

M.B. Voloshin, hep-ph/9612483; A. Khodjamirian, R. Rückl, G. Stoll and D. Wyler, hep-ph/9702318;
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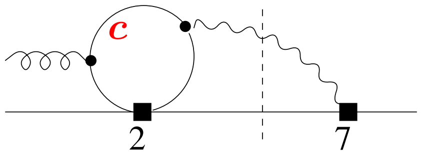
$\langle \bar{B} |$  $| \bar{B} \rangle$

$$\delta \mathrm{N}(\mathrm{E}_0) = (C_2 - \tfrac{1}{6} C_1) C_7 \underbrace{\left[-\frac{\mu_G^2}{27 m_c^2} + \frac{\Lambda_{17}}{m_b} \right]}_{-\frac{\kappa_V \mu_G^2}{27 m_c^2}}$$

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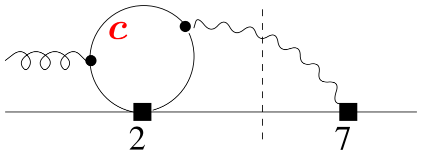
$\langle \bar{B} |$  $| \bar{B} \rangle$

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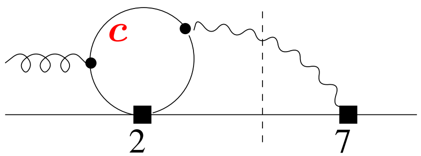
$\langle \bar{B}|$  $|\bar{B}\rangle$

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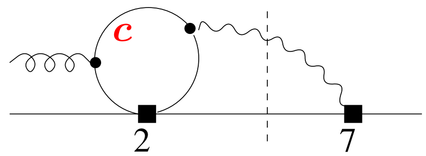
$\langle \bar{B} |$  $| \bar{B} \rangle$

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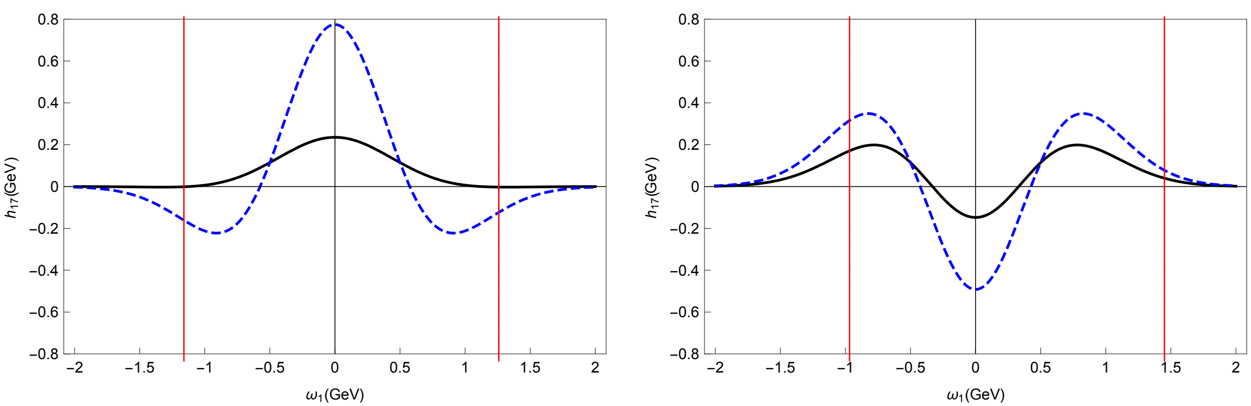

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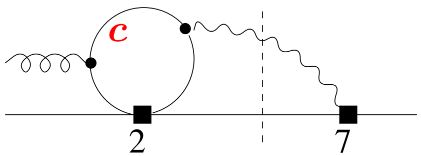
$\langle \bar{B}|$  $|\bar{B}\rangle$

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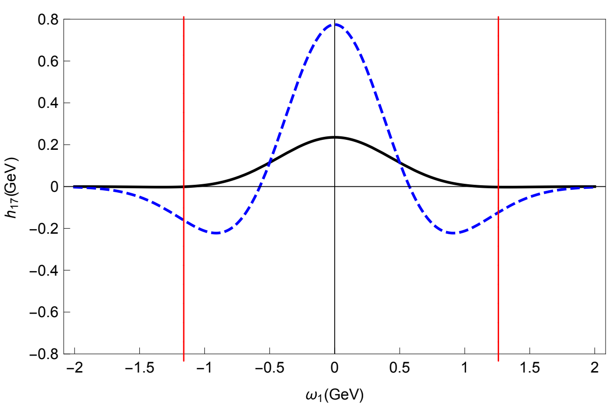
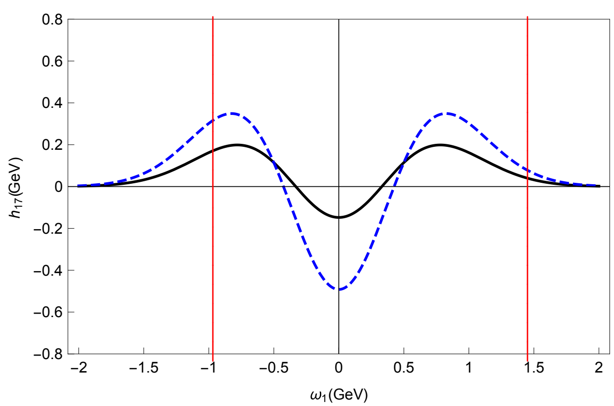




G+P numerically:
$\Lambda_{17} \in [-24, 5]$ MeV for $m_c = 1.17$ GeV.

In our code: $\kappa_V = 1.2 \pm 0.3$.
Warning: scheme for m_c!

Moment constraints vs. models of h_{17}

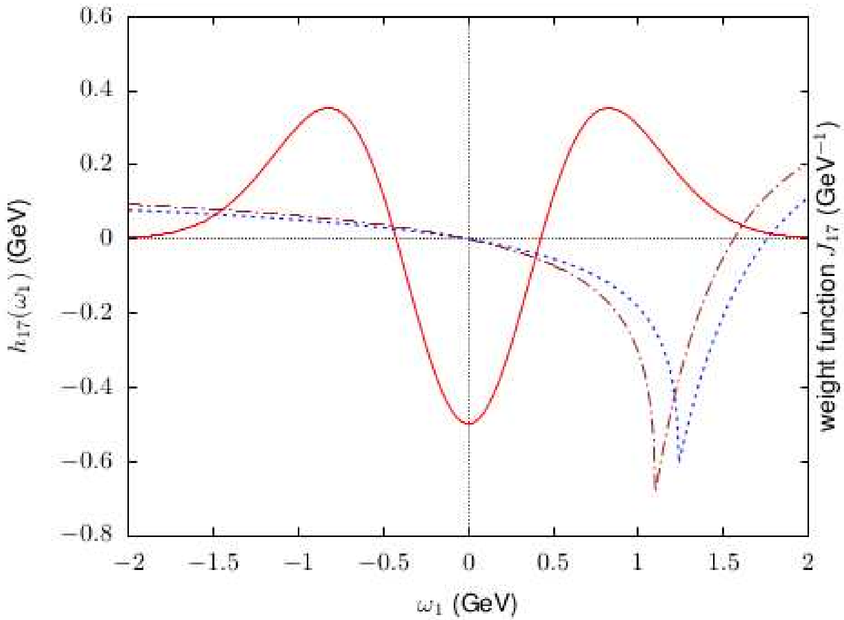
M. Benzke, S.J. Lee, M. Neubert, G. Paz, arXiv:1003.5012 – only the leading moment included.

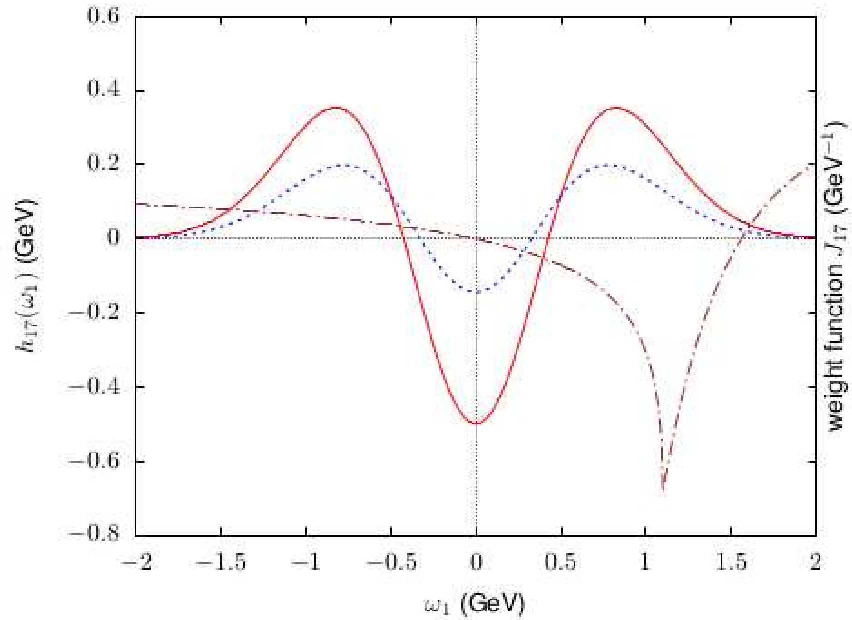
A. Gunawardana, G. Paz, arXiv:1908.02812 – estimates of the subleading moments from LLSA included.

M. Benzke, T. Hurth, arXiv:2006.00624 – as above but with more generous modeling and partial $1/m_b^2$ corrections.

R. Bartocci, P. Böer, T. Hurth, arXiv:2411.16634 – RG evolution of $h_{17}(\omega_1, \mu)$.

Plots from arXiv:2006.00624:





Another recent contribution: T. Hurth and R. Szafron, arXiv:2301.01739 – clarifying the SCET treatment of resolved photons in the Q_8-Q_8 interference.

Summary and outlook

- **Updated SM prediction:**

$$\mathcal{B}_{s\gamma}^{\rm SM} = (3.54 \pm 0.14) \times 10^{-4} \quad (\pm 4.0\%).$$

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- **Non-perturbative outlook:**

Complete the $\mathcal{O}\left(\frac{1}{m_b^2}\right)$ and $\mathcal{O}(\alpha_s)$ resolved-photon corrections in the $Q_{1,2}$-Q_7 interference.

Use arXiv:2301.01739 to update the Q_8-Q_8 interference.

Use arXiv:2211.07663 [B. Dehnadi, I. Novikov, F. J. Tackmann] and the SIMBA analysis in arXiv:2007.04320 to improve HFLAV/PDG averaging and extrapolation to $E_0 = 1.6\,\mathrm{GeV}$.

BACKUP SLIDES

The "hard" contribution to $\bar{B} \to X_s \gamma$

J. Chay, H. Georgi, B. Grinstein PLB 247 (1990) 399.
A.F. Falk, M. Luke, M. Savage, PRD 49 (1994) 3367.

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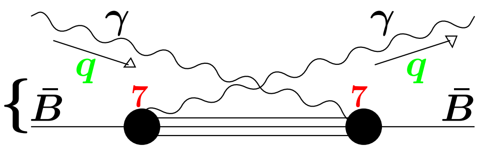
$\mathrm{Im}\{$  $\} \equiv \mathrm{Im}\, A$

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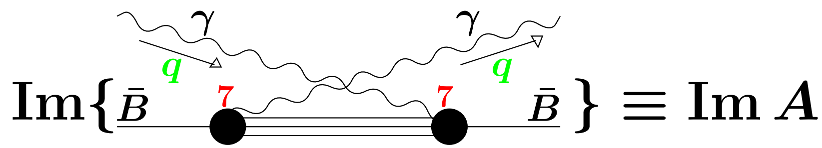

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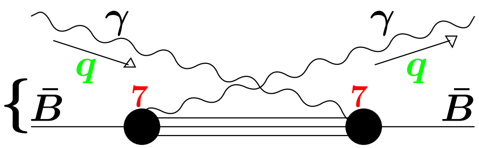
$\mathrm{Im}\{$  $\} \equiv \mathrm{Im}\, A$

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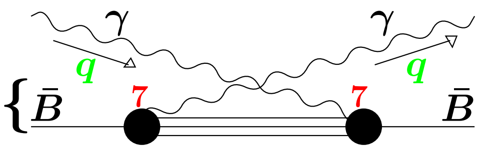

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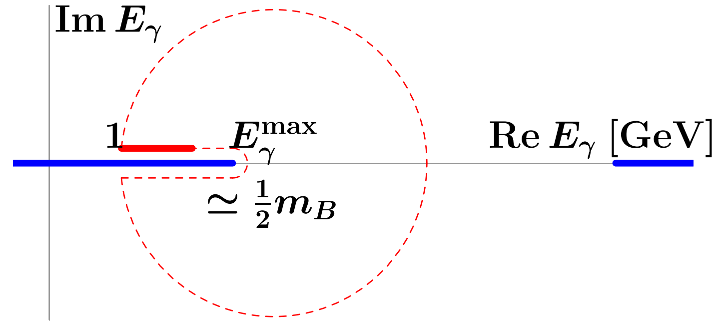

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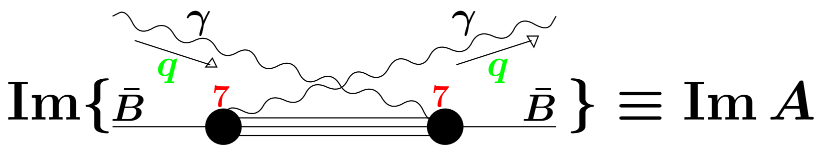

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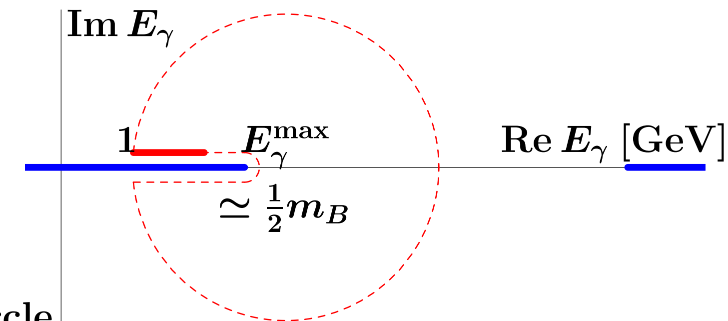

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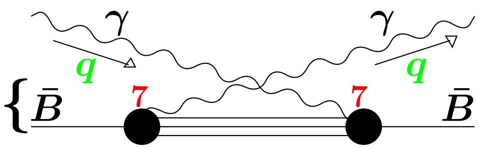
Im{  } $\equiv$ Im A

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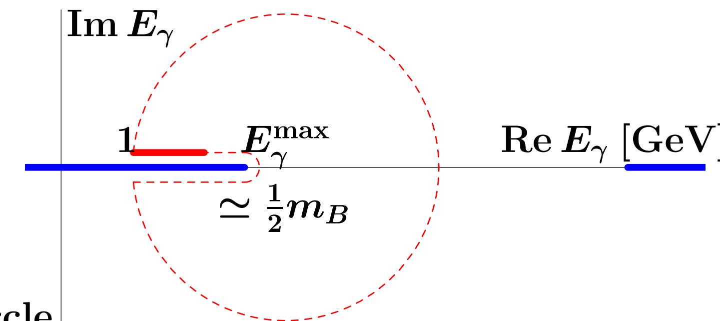

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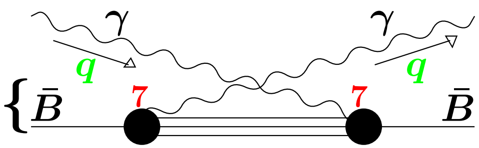
Im{  } ≡ Im A

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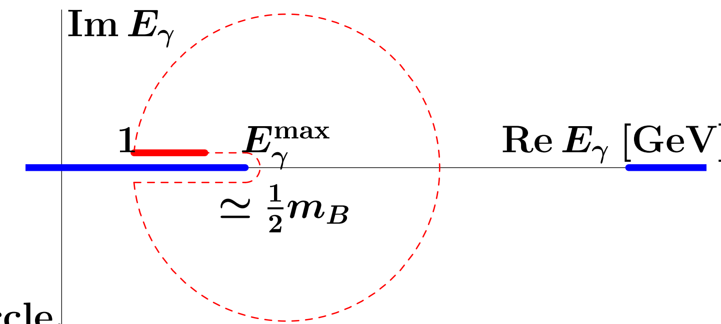

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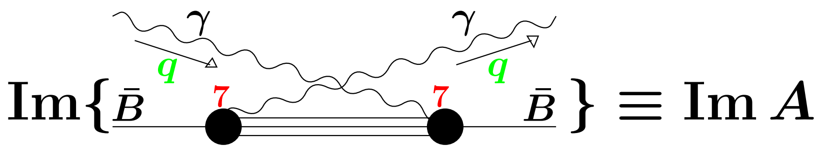

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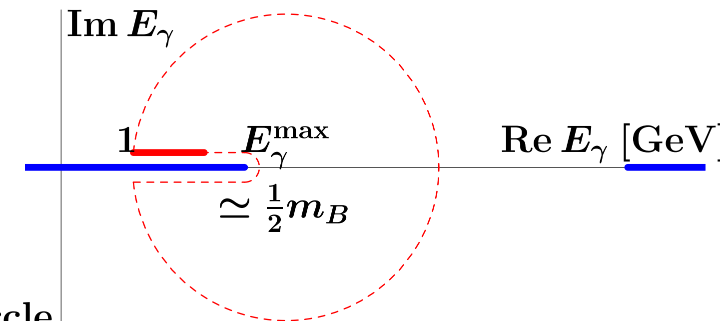

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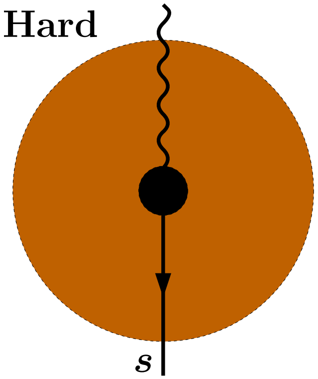
The HQET heavy-quark field: $b_v(x) = \frac{1}{2}(1+\not{v})b(x)\exp(im_b\, v\cdot x)$ with $v = p/m_B$.

Energetic photon production in charmless decays of the $\bar{B}$-meson

$(E_\gamma \gtrsim \frac{m_b}{3} \simeq 1.6\,\text{GeV})$

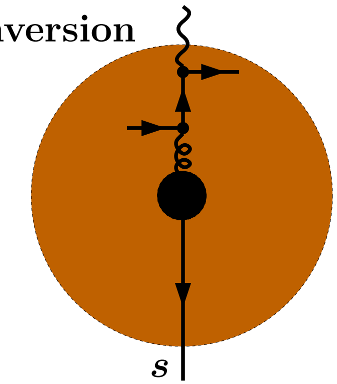
A. Without long-distance charm loops:

1. **Hard**



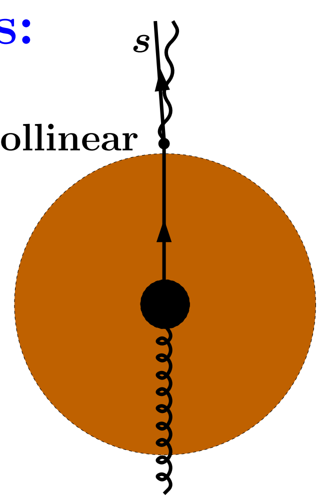
Dominant, well-controlled.

2. **Conversion**



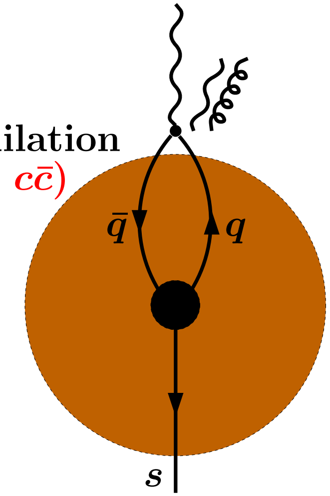
$\mathcal{O}(\alpha_s \Lambda / m_b)$,
$\leftrightarrow$ izospin asymmetry.

3. **Collinear**



Perturbatively $< 1\%$,
$\leftrightarrow$ fragmentation functions.

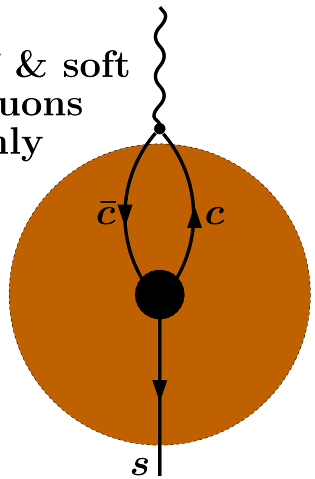
4. **Annihilation** $(q\bar{q} \neq c\bar{c})$



Exp. π^0, η, η', ω subtracted,
perturbatively $\sim 0.1\%$.

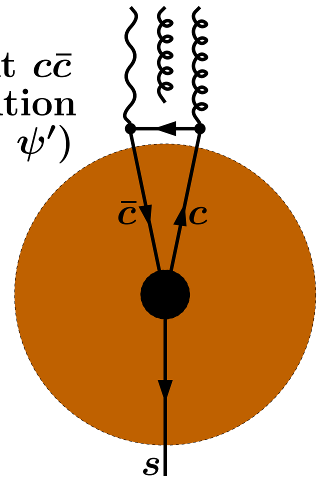
B. With long-distance charm loops:

5. $c\bar{c}$ **& soft gluons only**



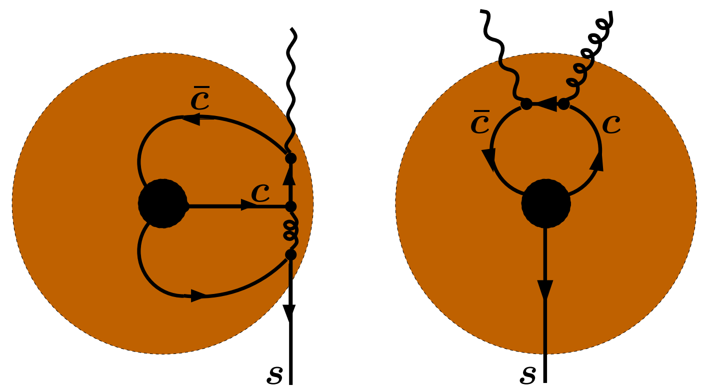
HQET & SCET.

6. **Boosted light $c\bar{c}$ state annihilation (e.g. η_c, J/ψ, ψ')**



Exp. J/ψ subtracted $(< 1\%)$.

7. **Annihilation of $c\bar{c}$ in a heavy $(\bar{c}s)(\bar{q}c)$ state**



Examples of SM diagrams for the matching of $C_7(\mu_0)$ $(\mu_0 \sim M_W, m_t)$

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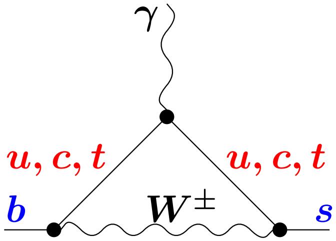

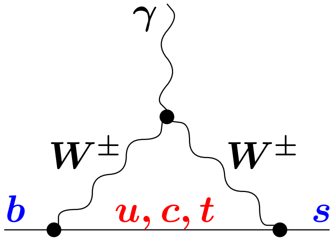

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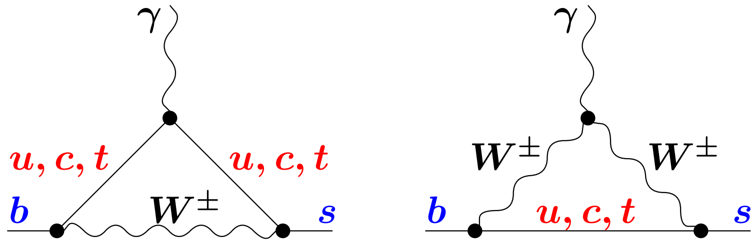

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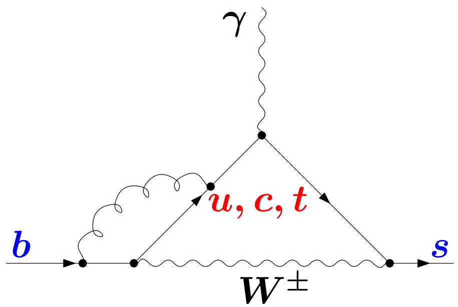

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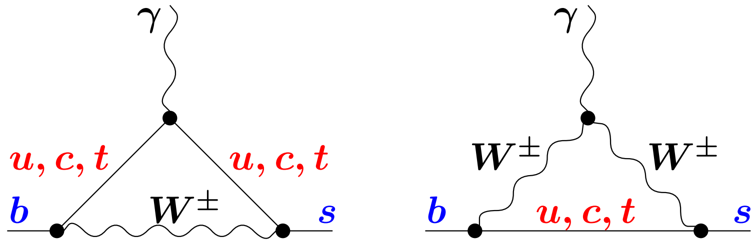

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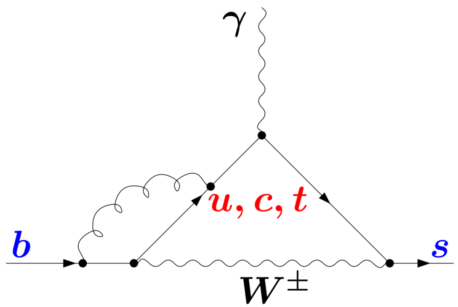

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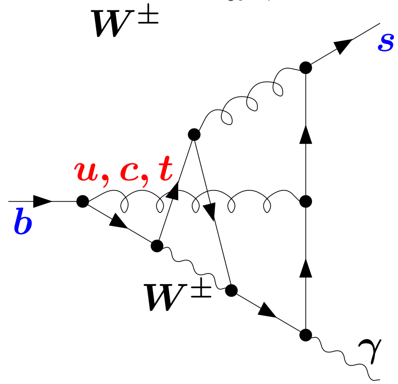

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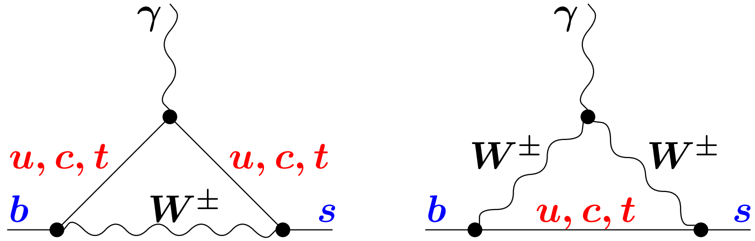

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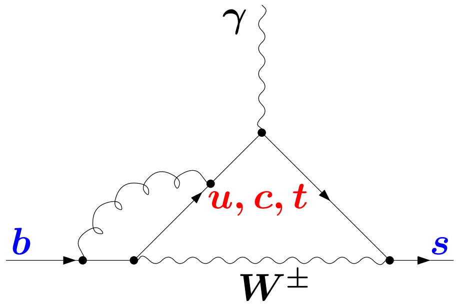

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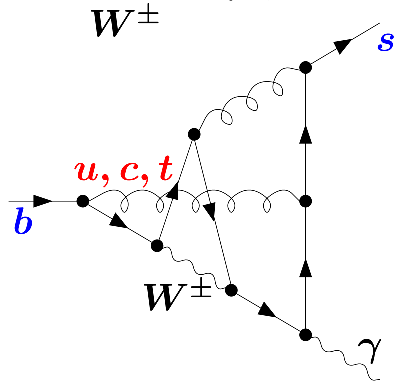

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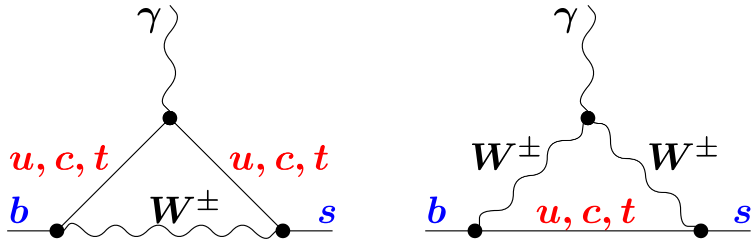

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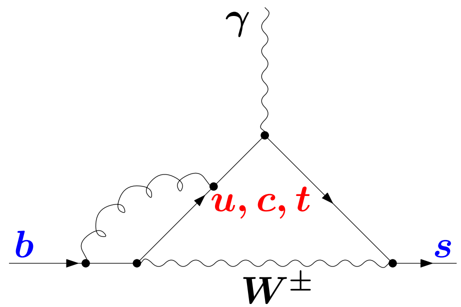

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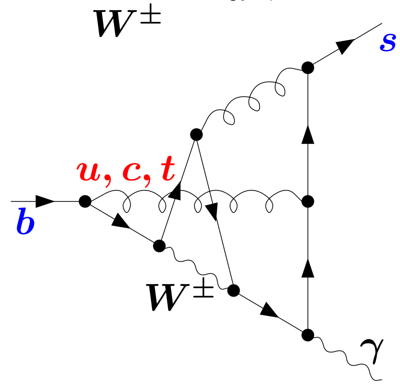

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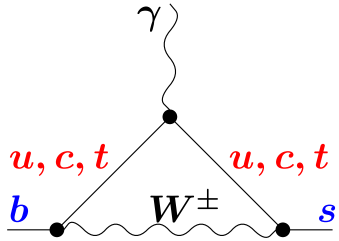

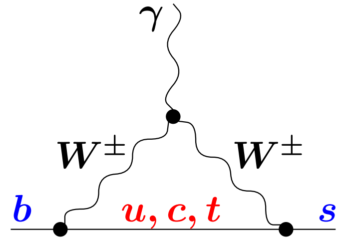

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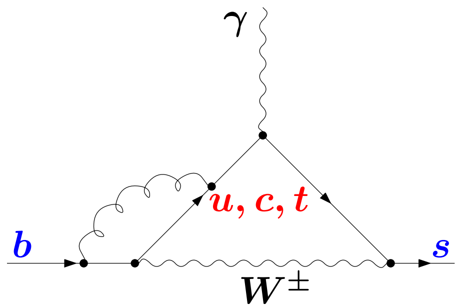

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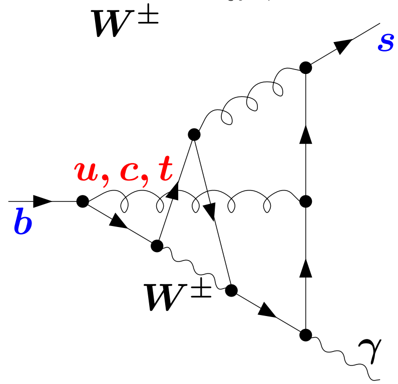

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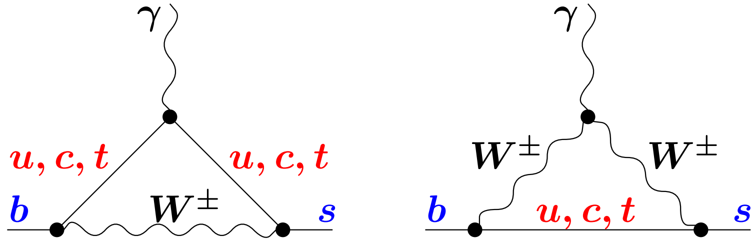

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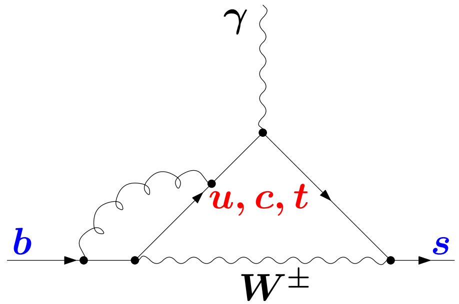

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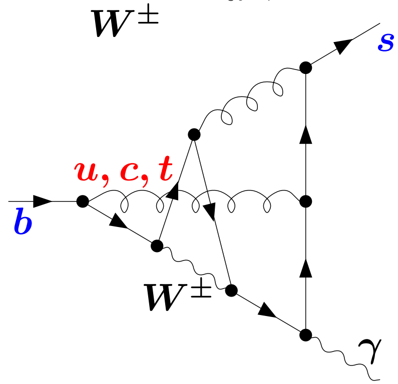

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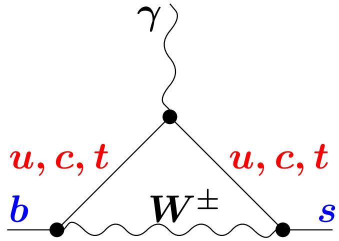

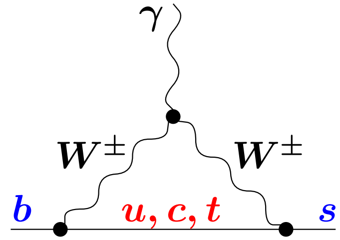

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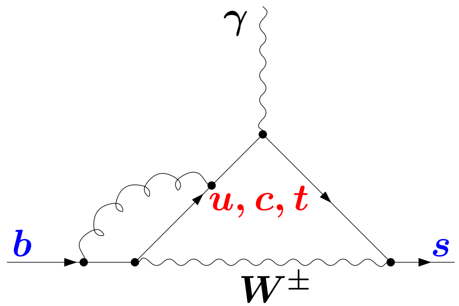

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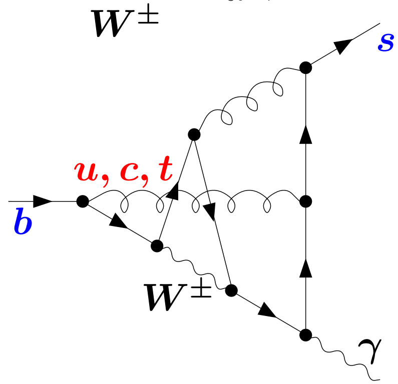

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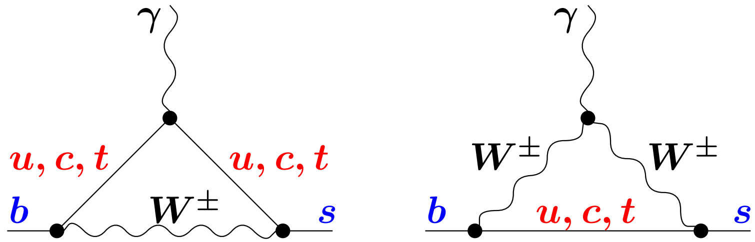

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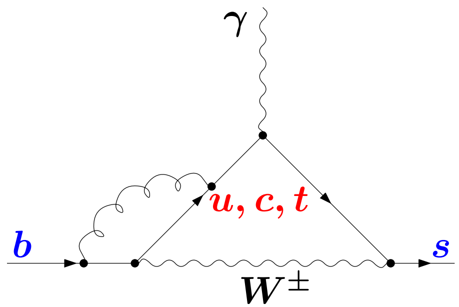

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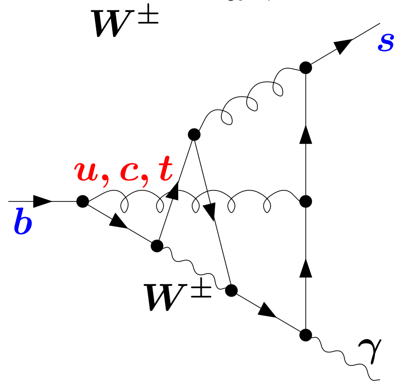

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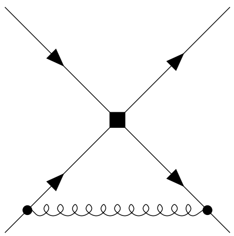

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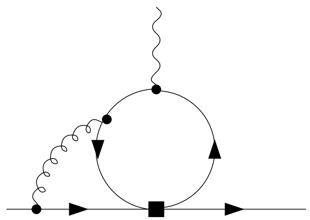
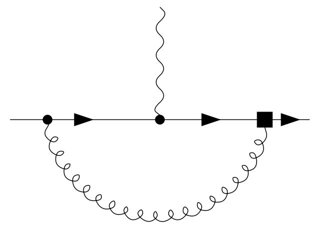


[Grinstein *et al.*, 1990]



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[Grigjanis *et al.*, 1988]

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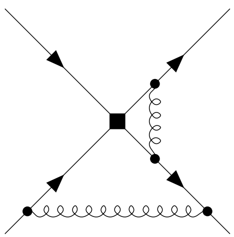

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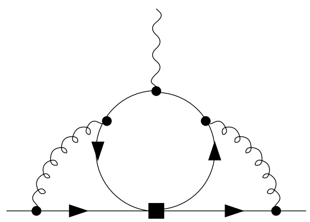

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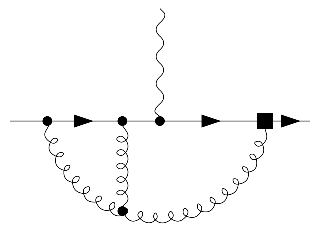

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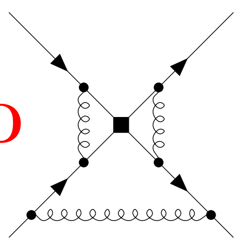

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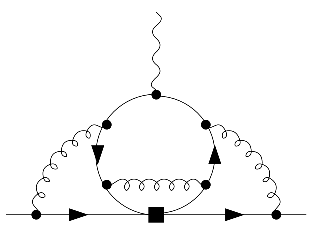

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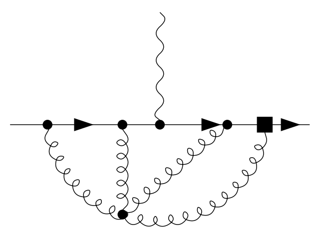

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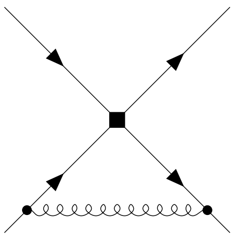
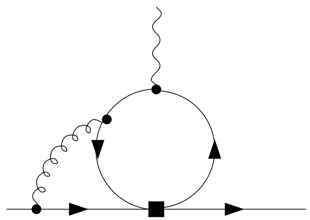
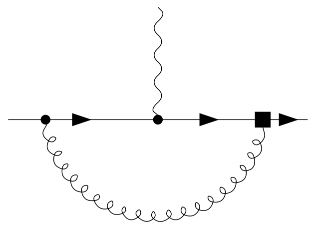
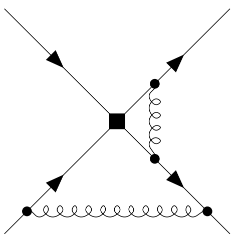
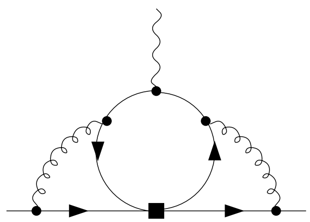
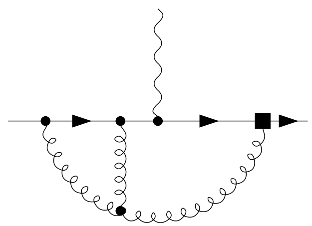
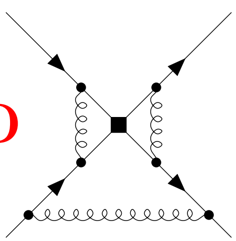
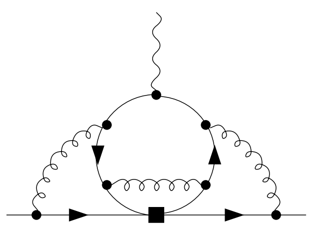
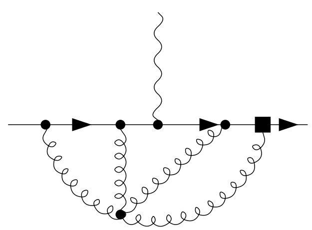
LO	 [Gaillard, Lee, 1974] [Altarelli, Maiani, 1974]	 [Grinstein *et al.*, 1990]	 [Shifman *et al.*, 1978] [Grigjanis *et al.*, 1988]
NLO	 [Altarelli *et al.*, 1981] [Buras, Weisz, 1990]	 [Chetyrkin, MM, Münz, 1997]	 [MM, Münz, 1995]
NNLO	 [Gorbahn, Haisch, 2004]	 [Czakon, Haisch, MM, 2006] $\sim 2 \times 10^4$ diagrams, -4% effect on $\mathcal{B}_{s\gamma}^{\rm SM}$	 [Gorbahn, Haisch, MM, 2005]

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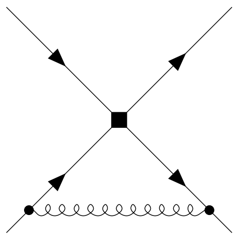
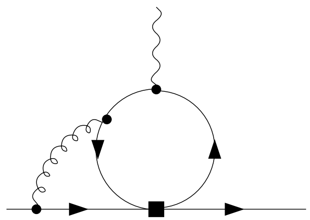
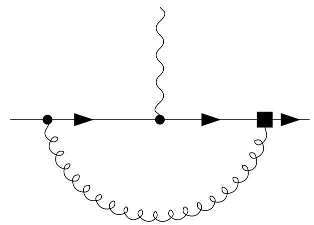
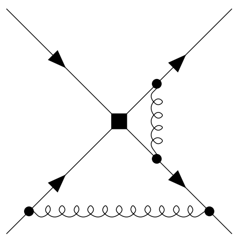
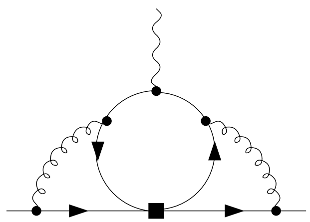
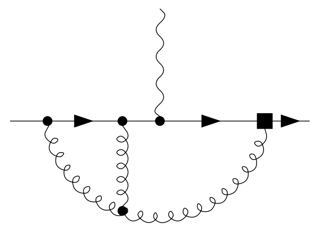
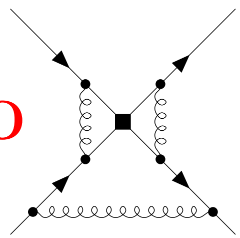
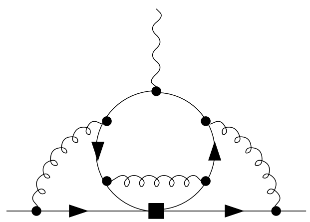
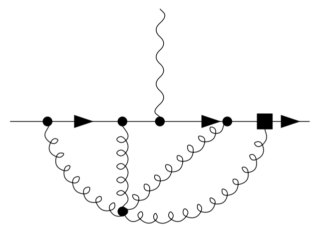
LO	 [Gaillard, Lee, 1974] [Altarelli, Maiani, 1974]	 [Grinstein *et al.*, 1990]	 [Shifman *et al.*, 1978] [Grigjanis *et al.*, 1988]
NLO	 [Altarelli *et al.*, 1981] [Buras, Weisz, 1990]	 [Chetyrkin, MM, Münz, 1997]	 [MM, Münz, 1995]
NNLO	 [Gorbahn, Haisch, 2004]	 [Czakon, Haisch, MM, 2006] $\sim 2 \times 10^4$ diagrams, -4% effect on $\mathcal{B}_{s\gamma}^{\rm SM}$	 [Gorbahn, Haisch, MM, 2005]

Since 2006, all the Wilson coefficients $C_1(\mu_b), \ldots, C_8(\mu_b)$ are known at the NNLO.