Physics Beyond the Standard Model: The case of Vector-Like Quarks

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Physics BSM with vector-like quarks

Quarks with L and R components transforming in the same way under the SM gauge group

Bare mass terms in the Lagrangian are allowed

Mixing of the new quarks with the SM-like quarks gives rise to:

Deviations from unitarity of the VCKM

Z mediated Flavour-Changing-Neutral-Currents

Higgs mediated Flavour-Changing-Neutral-Currents

These new phenomena are suppressed by the ratio of electroweak scale and the masses of the new heavy quarks

Rich variety of new Physics

Possible motivations to introduce isosinglet vector-like quarks

Vector-like fermions arise for instance in grand unified models

Naturally small violation of 3x3 unitarity of the VCKM and non-vanishing but naturally suppressed flavour-changing r
(FCNC)

This opens up many interesting possibilities for rare K and B decays as well as CP asymmetries in neutral B de

Adding isosinglet quarks to the SM leads to new sources of CP violation

In particular one may achieve spontaneous CP violation in this framework with the addition of a complex scalar singlet to the Higgs sector

Possibility of solving the strong CP problem a la Barr and Nelson

Bento, Branco, Parada, 1991

Possibility of having a Common Origin for all CP Violations

Branco, Parada, MNR, 2003

Fundamental properties of the CKM matrix

G. C Branco, L. Lavoura, J. P. Silva "CP Violation" Oxford University Press 1999

$$\mathscr{L}_{CC} = \left(\overline{u}\ \overline{c}\ \overline{t}\right)_L \gamma^\mu \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}_L W_\mu^+ + \text{H.c.}$$

The CKM matrix is complex but not all its phases have physical meaning

$$u_\alpha = e^{i\varphi_\alpha} u'_\alpha, \qquad d_k = e^{i\varphi_k} d'_k$$

There is freedom to rephase the mass eigenstate quark fields. As a result:

$$V'_{\alpha k} = e^{i(\varphi_k - \varphi_\alpha)} V_{\alpha k}$$

Only rephasing invariant quantities have physical meaning.
The simplest rephasing invariants of the CKM matrix are moduli and "quartets"

$$|V_{\alpha k}| \qquad Q_{\alpha i \beta j} \equiv V_{\alpha i} V_{\beta j} V^*_{\alpha j} V^*_{\beta i} \qquad \text{with } \alpha \neq \beta \text{ and } i \neq j.$$

Higher order Invariants can in general be written in terms of these .

Details about Rephasing invariant quantities

Example :

$$Q = V_{us} V_{cb} V_{cs}^{*} V_{ub}^{*}$$

$$\mathrm{Im}\ Q \simeq \lambda^6 \sin(\arg Q)$$ $\arg Q$ is of order 1

λ is essentially the sine of the Cabibbo angle and it is a parameter appearing in the Wolfenstein parametrisation of the CKM matrix

|Im Q| has the same value for all quartets and measures the strength of CP violation in the SM.

In the SM it is not possible to generate the Baryon Asymmetry of the Universe (BAU)

One of the reasons is that in the SM CP violation is too small:

$$\mathcal{I}_{\rm CP} = \mathrm{tr}[y_u y_u^\dagger, y_d y_d^\dagger]^3 \sim \frac{\mathrm{tr}[h_u, h_d]^3}{v^{12}} \sim 10^{-25}$$

In models with VLQs one may have CP odd invariants of much lower mass scale

Example:

$$\mathrm{tr}\left(\left[h_u, h_d^s\right] H_u^{(2)}\right) \qquad H_u^{(r)} \equiv m_u (m_u^\dagger m_u + M_u^\dagger M_u)^{r-1} m_u^\dagger$$

$$h_d \equiv m_d m_d^\dagger$$

Differences between the imaginary parts of the quartets

In the SM, one can show that all imaginary parts of rephasing invariant quartets:

$$V_{us} V_{cb} V_{ub}^* V_{cs}^* = Q_{uscb}$$

$$V_{cd} V_{ts} V_{td}^* V_{cs}^* = Q_{cdts}$$

$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix},$$

have the same modulus

In the presence of VLQs one obtains a different result, for example:

Im Q_2112 - Im Q_1132 = Im Q_1142

A surprising result: In the 3×3 up corner of a V^{CKM} matrix of arbitrary size one has: 5

$9-5=4$ rephasing invariant phases

The following phase convention may be chosen, in general

$$\arg V^{3\times 3} = \begin{bmatrix} 0 & \beta_K & -\gamma \\ \pi & 0 & 0 \\ -\beta & \pi+\beta_s & 0 \end{bmatrix}$$

$$V = \overbrace{\begin{pmatrix} \boxed{\begin{matrix} |V_{ud}| & |V_{us}|\,e^{i\chi'} & |V_{ub}|\,e^{-i\gamma} \\ -|V_{cd}| & |V_{cs}| & |V_{cb}| \\ |V_{td}|\,e^{-i\beta} & -|V_{ts}|\,e^{i\chi} & |V_{tb}| \end{matrix}} & \begin{matrix} \cdots \\ \cdots \\ \cdots \end{matrix} \\ \begin{matrix} \vdots \quad\quad & \vdots \quad\quad & \vdots \end{matrix} & \ddots \end{pmatrix}}^{V_{\rm CKM}} .$$

Confronting experiment

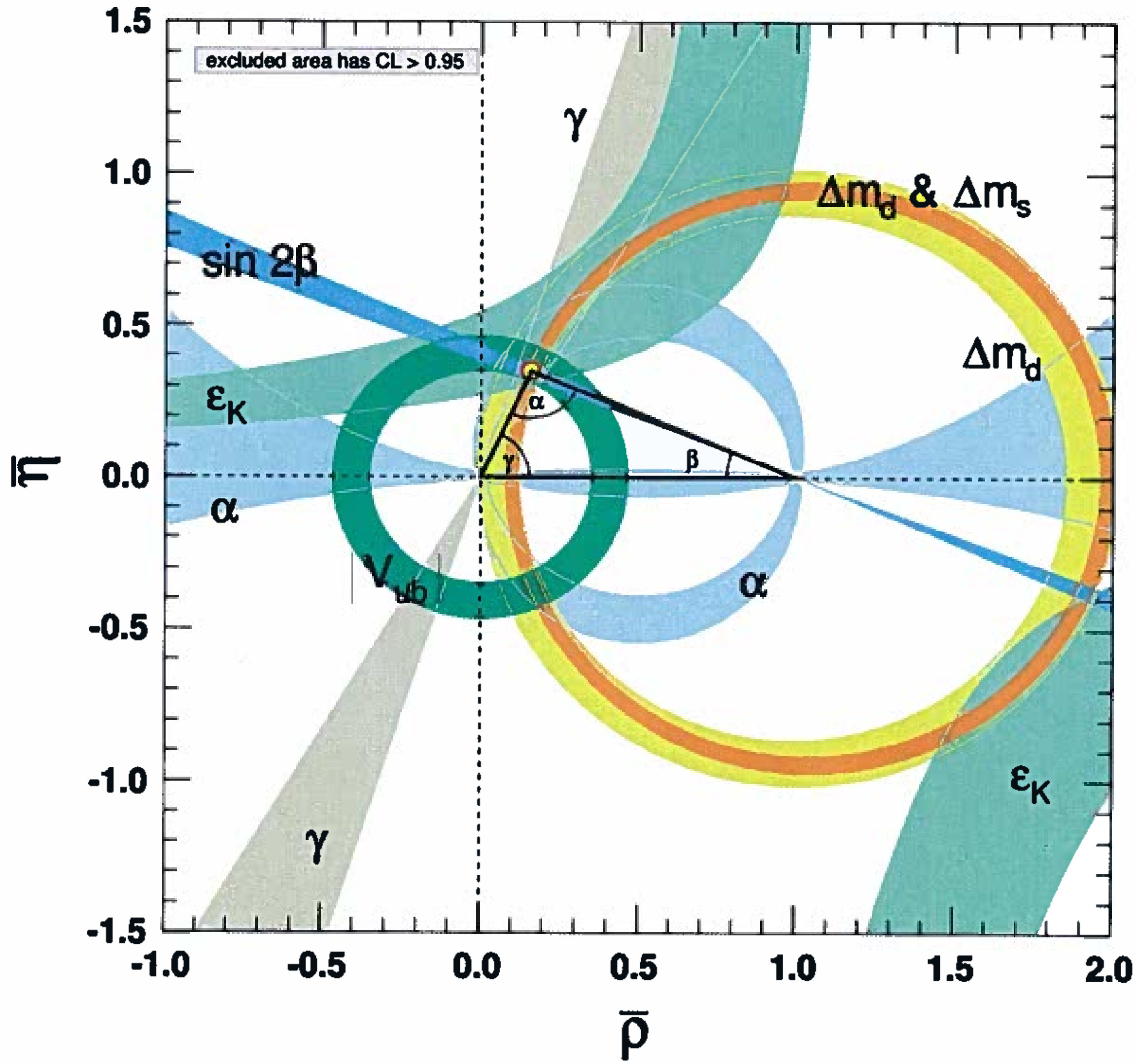

Figure 12.2: Constraints on the $\bar{\rho}, \bar{\eta}$ plane. The shaded areas have 95% CL.

What if there is New Physics?

Changes in the unitarity relations in the presence of VLQs

Moduli differences:

Albergaria, gcb, 2023

In the SM, 3x3 unitarity of the CKM matrix leads to an "asymmetry" defined as:

$$\mathbf{a} \equiv |V_{31}|^2 - |V_{13}|^2 = |V_{23}|^2 - |V_{32}|^2 = |V_{12}|^2 - |V_{21}|^2$$

In an SM extension with one up-type VLQ the quark mixing matrix consists of the first three columns of a 4x4 unitary matrix:

$$V = \begin{pmatrix} V_{11} & V_{12} & V_{13} & V_{14} \\ V_{21} & V_{22} & V_{23} & V_{24} \\ V_{31} & V_{32} & V_{33} & V_{34} \\ V_{41} & V_{42} & V_{43} & V_{44} \end{pmatrix}$$

Changes in the unitarity relations in the presence of VLQs

From unitarity of first row and first column of V, one derives in the SM; with 1 VLQ:

$$a_{12,13} \equiv \left(|V_{12}|^2 - |V_{21}|^2\right) - \left(|V_{31}|^2 - |V_{13}|^2\right) = |V_{41}|^2 - |V_{14}|^2$$

Using unitarity of other rows and columns of V one obtains:

$$a_{12,32} \equiv \left(|V_{12}|^2 - |V_{21}|^2\right) - \left(|V_{23}|^2 - |V_{32}|^2\right) = |V_{24}|^2 - |V_{42}|^2,$$
$$a_{13,23} \equiv \left(|V_{13}|^2 - |V_{31}|^2\right) - \left(|V_{32}|^2 - |V_{23}|^2\right) = |V_{34}|^2 - |V_{43}|^2.$$

From $D_0 - \overline{D_0}$ mixing, we know that, in models with one up-type VLQ, we have

$$|V_{14}|^2 |V_{24}|^2 < (2.1 \pm 1.2) \times 10^{-8}.$$

Recent measurements of $|V_{us}|$ and $|V_{ud}|$ indicate that unitarity of the first row may be violated, $|V_{ud}|^2+|V_{us}|^2+|V_{ub}|^2 < 1$, at the level of two or three standard deviations. This

Experimental data suggests:

B. Belfatto, R. Beradze and Z. Berezhiani, *The CKM unitarity problem: A trace of new physics at the TeV scale?*, *Eur. Phys. J. C* **80** (2020) 149 [1906.02714].

$$\Delta \equiv \Delta_1 = 1 - |V_{ud}|^2 - |V_{us}|^2 - |V_{ub}|^2 = \left|\mathcal{V}^*_{L41}\right|^2 = s^2_{14}$$

$$\sqrt{\Delta} \sim 0.04$$

Several references in our work, e.g, FLAG Review 2021 (Flavour Lattice Averaging group) (2111.09849)

BBB suggested the addition of a down-type (Q=-1/3) vector-like isosinglet quark

In alternative, the introduction of an up-type (Q=2/3) vector-like isosinglet quark is especially interesting since experimental limits on FCNC in the up sector are less stringent than those in the down sector

Recall conversation with Bill Marciano

Yukawa terms and bare mass terms (notation):

$$-\mathcal{L}_u \supset Y^u_{ij}\,\overline{Q}^0_{Li}\,\tilde{\phi}\,u^0_{Rj} + \overline{Y}_i\,\overline{Q}^0_{Li}\,\tilde{\phi}\,T^0_R$$
$$+ \overline{M}_i\,\overline{T}^0_L\,u^0_{Ri} + M\,\overline{T}^0_L\,T^0_R + \text{h.c.}\,,$$

(artificial splitting of R-handed components)

$$-\mathcal{L}_d = Y^d_{ij}\,\overline{Q}^0_{Li}\,\phi\,d^0_{Rj} + \text{h.c.}$$

Following the spontaneous breakdown of electroweak symmetry:

$$-\mathcal{L} \supset \begin{pmatrix}\overline{u}^0_L & \overline{T}^0_L\end{pmatrix} \mathcal{M}_u \begin{pmatrix} u^0_R \\ T^0_R \end{pmatrix} + \overline{d}^0_L\,\mathcal{M}_d\,d^0_R + \text{h.c.}$$

$$m = \frac{v}{\sqrt{2}}Y^u \qquad \overline{m} = \frac{v}{\sqrt{2}}\overline{Y} \qquad v \simeq 246\ \text{GeV}$$

3×3 $\qquad$ 3×1

$$\mathcal{M}_u = \left(\begin{array}{c|c} m & \overline{m} \\ \hline \overline{M} & M \end{array}\right)$$

4×4

Possible to choose WB where the down quark mass matrix is real diagonal

$$\mathcal{V}^\dagger_L\,\mathcal{M}_u\,\mathcal{V}_R = \mathcal{D}_u$$

$$\mathcal{V}_{L,R} \equiv \left(\begin{array}{c} A_{L,R} \\ \hline B_{L,R} \end{array}\right)$$

4×4

Non-Unitary mixing

$$\mathcal{L}_W = -\frac{g}{\sqrt{2}}\overline{u}^0_{Li}\left(\gamma^\mu W^+_\mu\right) d^0_{Li} = -\frac{g}{\sqrt{2}}\overline{u}_{L\alpha}\left(\gamma^\mu W^+_\mu\right)\left(\mathcal{V}^\dagger\right)^{\alpha i} d_{Li}$$

$\mathcal{V}^{CKM}$ corresponds to the 4×3 block of the matrix $\mathcal{V}^\dagger$

$$\mathcal{V}^{CKM} = \left(\mathcal{V}^\dagger\right)^{(4\times 3)} = A_L^\dagger$$

Useful Parametrisation

F. Botella, L-L. Chau. 1986

$$\mathcal{V}^\dagger = O_{34} V_{24} V_{14} \cdot V_4^{PDG} \qquad V_4^{PDG} = \begin{pmatrix} & & & 0 \\ & \left[V^{PDG}\right]^{(3\times 3)} & & 0 \\ & & & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\mathcal{V}_L^\dagger = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & c_{34} & s_{34} \\ 0 & 0 & -s_{34} & c_{34} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & c_{24} & 0 & s_{24}e^{-i\delta_{24}} \\ 0 & 0 & 1 & 0 \\ 0 & -s_{24}e^{i\delta_{24}} & 0 & c_{24} \end{pmatrix} \begin{pmatrix} c_{14} & 0 & 0 & s_{14}e^{-i\delta_{14}} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -s_{14}e^{i\delta_{14}} & 0 & 0 & c_{14} \end{pmatrix} V_4^{PDG}$$

$$\Delta \equiv \Delta_1 = 1 - |V_{ud}|^2 - |V_{us}|^2 - |V_{ub}|^2 = \left|\mathcal{V}^*_{L41}\right|^2 = s_{14}^2 \qquad \sqrt{\Delta} \sim 0.04$$

Couplings of the Z boson

$$\mathcal{L}_Z = -\frac{g}{c_W}\left[\frac{1}{2}\left(\overline{u}^0_{Li}\,\gamma^\mu\,u^0_{Li} - \overline{d}^0_{Li}\,\gamma^\mu\,d^0_{Li}\right) - \frac{2}{3}s^2_W\left(\overline{u}^0_i\,\gamma^\mu\,u^0_i + \overline{T}^0\,\gamma^\mu\,T^0\right) + \frac{1}{3}s^2_W\left(\overline{d}^0_i\,\gamma^\mu\,d^0_i\right)\right] Z_\mu$$

$$\to -\frac{g}{c_W}\left[\frac{1}{2}\left(\overline{u}_L \quad \overline{T}_L\right)\,F^u\,\gamma^\mu\begin{pmatrix}u_L\\T_L\end{pmatrix} - \frac{1}{2}\overline{d}_{Li}\,\gamma^\mu d_{Li} - \frac{2}{3}s^2_W\left(\overline{u}_i\,\gamma^\mu\,u_i + \overline{T}\,\gamma^\mu\,T\right) + \frac{1}{3}s^2_W\left(\overline{d}_i\,\gamma^\mu\,d_i\right)\right] Z_\mu\,,$$

$$F^u = A_L^\dagger A_L = \mathbb{1} - B_L^\dagger B_L$$

Couplings of the Higgs boson

$$\mathcal{L}_h = -\frac{1}{\sqrt{2}}\,\overline{u}^0_{Li}\left(Y^u_{ij}\,u^0_{Rj} + \overline{Y}^u_i\,T^0_R\right)h - \frac{1}{\sqrt{2}}\,Y^d_{ij}\,\overline{d}^0_{Li}\,d^0_{Rj}\,h + \text{h.c.}$$

$$\to -\left(\overline{u}_L \quad \overline{T}_L\right)\,F^u\,\mathcal{D}_u\begin{pmatrix}u_R\\T_R\end{pmatrix}\frac{h}{v} - \overline{d}_L\,\mathcal{D}_d\,d_R\frac{h}{v} + \text{h.c.}\,.$$

Similarly to the case of Z-mediated FCNC, the strength of Higgs-mediated FCNC is controlled by the off-diagonal entries of the matrix F^u and by the ratios m_q/v, ($q =$ = u, c, t, T

Identification of the small numbers in V_{CKM} :

$$|V_{ub}| \approx 3.6 \times 10^{-3}$$

$$|\mathrm{Im}\, Q| \simeq 3 \times 10^{-5}$$

Q → Rephasing invariant quartet of V_{CKM}

In the SM, $|\mathrm{Im}\, Q|$ has the same value for all quartets and gives the strength of CP violation in the SM

Conjecture:

The generation of |V_ub| and ImQ

from New Physics

We propose that the CKM matrix is generated from three different contributions

$$V_{\mathrm{CKM}}^{\mathrm{eff}} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix}_{\mathrm{up}} \cdot \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix}_{\mathrm{NP}} \cdot \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{13} & 0 \\ 0 & 0 & 1 \end{pmatrix}_{\mathrm{down}}$$

In order to implement the structure
we assume that there is a basis where the
down and up quark matrices take the form:

$$M_d = \begin{pmatrix} m^d_{11} & m^d_{12} & 0 \\ m^d_{21} & m^d_{22} & 0 \\ 0 & 0 & m^d_{33} \end{pmatrix} \qquad M_u = \begin{pmatrix} m^u_{11} & 0 & 0 \\ 0 & m^u_{22} & m^u_{23} \\ 0 & m^u_{32} & m^u_{33} \end{pmatrix}$$

It can be shown that one can obtain these patterns through the introduction of a Z_4 symmetry at the Lagrangian level

Without the introduction of New Physics, one simply obtains a simplified and reduced CKM mixing, where

$$V_{\rm CKM} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix}_{\rm up} \cdot \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}_{\rm down} = \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12}c_{23} & c_{23}c_{12} & -s_{23} \\ -s_{23}s_{12} & s_{23}c_{12} & c_{23} \end{pmatrix}$$

At this level one has: $|V_{31}| = |V_{12}|\,|V_{23}|$ and $V_{13} = 0$

Our conjecture offers an explanation why:

|V_31| > |V_13| !!!

$V_{13} = 0$ also leads to vanishing CP violation

Introduce an up-type VLQ

and assume the 4x4 up-type quark matrix to be of the form:

$$\mathcal{M}_u = \begin{pmatrix} 0 & 0 & 0 & m_{14} \\ 0 & m_{22} & m_{23} & m_{24}e^{i\beta} \\ 0 & m_{32}e^{i\alpha} & m_{33} & 0 \\ m_{41} & 0 & -m_{43}e^{i\delta} & M \end{pmatrix}$$

then one can generate:

$$|V^{CKM}_{44}|_{13} \neq 0 \qquad |\mathrm{Im}\, Q_{44}| \neq 0$$

Numerical example:

Mass matrices in GeV at the m_Z scale:

$$M_d = \begin{pmatrix} 0.00292338 & -0.0134741 & 0 \\ 0.000673705 & 0.0584675 & 0 \\ 0 & 0 & 2.9 \end{pmatrix}$$

$$m_d = 0.003, \quad m_s = 0.060, \quad m_b = 2.9,$$

$$\mathcal{M}_u = \begin{pmatrix} 0 & 0 & 0 & 53.7334 \\ 0 & 0.59952 & -6.91815 & 1.250e^{-0.285i} \\ 0 & -0.0239936 & 172.862 & 0 \\ 0.046526 & 0 & 14.886e^{-1.190i} & 1250 \end{pmatrix}$$

$$m_u = 0.002, \quad m_c = 0.60, \quad m_t = 173, \quad m_T = 1251.$$

$$\begin{bmatrix} \bar{u} & \bar{c} & \bar{t} & \bar{T} \end{bmatrix} \begin{bmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \\ V_{Td} & V_{Ts} & V_{Tb} \end{bmatrix} \begin{bmatrix} d \\ s \\ b \end{bmatrix}$$

The CKM matrix is the 4×3 left-sub-matrix of the following full 4×4 mixing matrix

$$|\mathcal{V}| = \begin{pmatrix} 0.97354 & 0.224413 & 0.00370431 & 0.0429468 \\ 0.224536 & 0.973644 & 0.0399975 & 0.000996211 \\ 0.00833917 & 0.0393001 & 0.999192 & 0.00151171 \\ 0.0416344 & 0.0105585 & 0.001674 & 0.999076 \end{pmatrix}$$

These mass matrices lead to:

CP violation rephasing invariant phases are

$$\gamma \equiv \arg\left(-V_{ud}V_{cb}V_{ub}^{*}V_{cd}^{*}\right) \simeq 68.0^{\circ},$$

$$\sin(2\beta) \equiv \sin\left[2\arg\left(-V_{cd}V_{tb}V_{cb}^{*}V_{td}^{*}\right)\right] \simeq 0.746,$$

$$\beta_s \equiv \chi \equiv \arg\left(-V_{ts}V_{cb}V_{cs}^{*}V_{tb}^{*}\right) \simeq 0.020,$$

$$\beta_K \equiv \chi' \equiv \arg\left(-V_{cd}V_{us}V_{cs}^{*}V_{ud}^{*}\right) \simeq 5.71 \times 10^{-4}.$$

CP-odd invariant quantity $I_{\rm CP} = |{\rm Im}Q| \equiv |{\rm Im}\left(V_{ub}V_{cd}V_{ud}^{*}V_{cb}^{*}\right)|$

$$I_{\rm CP} \simeq 3.00 \times 10^{-5}.$$

There is an intriguing similarity between vector like quarks and right-handed neutrinos.

This increases the plausibility of New Physics including

Vector-like quarks

Standard Model (SM) and Neutrino Masses

- In the SM, neutrinos are strictly massless

No Dirac mass ν_R is not introduced

No Majorana mass neither at tree level nor at higher orders due to exact B-L conservation.

Therefore the SM has been ruled out by experiment. So one is led to:

$$\nu SM \equiv SM + \nu_R$$

$\nu SM \equiv SM + \nu_R$

If one follows "the rules" and write the most general neutrino mass terms, one has:

Dirac mass : $g_\nu v \overline{\nu}_L \nu_R + h.c.$

Majorana mass : $M_R \nu_R^T C \nu_R$

Since the Majorana mass term is gauge invariant, one can have $M_R \gg v$. This leads to the Seesaw mechanism with:

$$m_\nu \approx \frac{(m_\nu^D)^2}{M_R}$$

So the Harvard Group dictated that neutrinos have no mass and the "right GUT" was SU(5) where neutrinos are again massless due to accidental B-L conservation

Recall the talk by Murray Gell-Mann at Columbia about SO(10)...

Important feature of ν_R

The Majorana mass term $\nu_R^T C \nu_R$ is $SU(2)_L \times U(1)$ invariant. Therefore M_R can be significantly larger than v.

$$M_R > v$$

Question: Can one have an analogous situation in the quark sector?

Answer: Yes! Vector-like quarks:

Q_L, Q_R transform in the same way under $SU(2)\times U(1)$

$\overline{Q}_L Q_R$ is $SU(2)\times U(1)$ invariant

CONCLUSIONS

Vector-like quarks are very interesting candidates for physics BSM

Very simple extension of the SM, providing striking new experimental effects

Vector-like quarks are "cousins" of right-handed neutrinos which provide through seesaw the most plausible explanation of the smallness of neutrino masses

- Weak point: No firm prediction for the scale of VLQs.

This is a <u>universal weak point</u> in <u>all</u> (so far) proposed New Physics proposals... !!

The SM was an notable exception!

Before gauge interactions the suggestion was IVB with ≈ 2 GeV!

↑ intermediate vector boson...

A Common Origin for all CP Violations

The Lagrangian is CP invariant. CP is spontaneously violated

Field content, Higgs and quark sector:

$$\begin{pmatrix} u^0 \\ d^0 \end{pmatrix}_{iL}, u^0_{iR}, \quad d^0_{\alpha R}, \quad D^0_L, \quad i = 1, 2, 3, \quad \alpha = 1, ..., 4, \quad \phi, \quad S$$

D is a down-type vector-like quark, S is a scalar singlet

A Z_2 symmetry is imposed in order to naturally suppress strong CP a la Barr and Nelson

$$D^0 \to -D^0, \quad S \to -S$$

$SU(2) \times U(1) \times Z_2$ invariant scalar potential

$$V = V_0\,(\phi, S) + (\mu^2 + \lambda_1\, S^* S + \lambda_2\, \phi^\dagger \phi)(S^2 + S^{*2}) + \lambda_3\,(S^4 + S^{*4})$$

V_0 contains all terms that are phase independent and includes the SM Higgs

Real coefficients spontaneous CP violation $\quad \langle \phi^0 \rangle = \dfrac{v}{\sqrt{2}}, \quad \langle S \rangle = \dfrac{V \exp(i\beta)}{\sqrt{2}}$

A Common Origin for all CP Violations

The Yukawa interactions of the quarks are given by: (all coefficients are real)

$$\mathcal{L}_Y = -\sqrt{2}(\overline{u^0}\ \overline{d^0})^i_L (g_{ij}\phi\, d^0_{jR} + h_{ij}\tilde{\phi}\, u^0_{jR}) - M_d \overline{D^0_L}\, D^0_R - \sqrt{2}(f_i\, S + f'_i\, S^*)\, \overline{D^0_L} d^0_{iR} + \text{h.c.}$$

with (i,j = 1, 2, 3) and the down quark mass matrix is now of the form:

$$\mathcal{M}_d = \begin{pmatrix} m_d & 0 \\ \overline{M_d} & M_d \end{pmatrix}$$

Diagonalisation of this mass matrix: Bento, gcb, Parada, 1991

$$\mathcal{M}_d \mathcal{M}_d^\dagger\, U_L = U_L \begin{pmatrix} d_d^2 & \\ & D_d^2 \end{pmatrix} \qquad \text{with} \qquad U_L = \begin{pmatrix} K & R \\ S & T \end{pmatrix}$$

Working in the weak basis where the up quark mass matrix is diagonal)

$$S \simeq -\frac{1}{D_d^2}(\overline{M_d}\, m_d^\dagger)\ K \qquad \text{where here } \overline{M_{dj}} = f_j\, V e^{i\beta} + f'_j V e^{-i\beta}$$

$$D_d^2 \simeq (\overline{M_d}\ \overline{M_d}^\dagger + M_d^2) \qquad \mathcal{H}_{eff} = m_d\, m_d^\dagger - \frac{1}{D_d^2}(m_d\ \overline{M_d}^\dagger)(\overline{M_d}\ m_d^\dagger)$$

- Non decoupling provided the scale of the bare mass term of D does not dominate over the scale of the vev of the scalar singlet, concerning the generation of a complex VCKM

- Suppression of deviations from unitarity irrespective of which scale dominates

A Common Origin for all CP Violations

From the previous page we see that in the Bento, gcb, Parada framework a complex CKM matrix can be generated from spontaneous CP violation at a high energy scale

Concerning strong CP violation:

$$\mathscr{L} \supset \theta_{\rm QCD} \frac{g_s^2}{64\pi^2} \epsilon^{\mu\nu\alpha\beta} G^a_{\mu\nu} G^a_{\alpha\beta}$$

The QCD Lagrangian contains a CP violating term originating from the QCD vacuum

However, CP violation has not been observed in the strong interactions

Furthermore, $\theta_{\rm QCD}$ is a free parameter

Experimentally what is measurable is the combination: $\bar{\theta} = \theta_{\rm QCD} - \theta_{\rm weak}$

$$\theta_{\rm weak} = \arg(\det \mathcal{M}_u \times \det \mathcal{M}_d)$$

Within the SM the electric dipole moment of the neutron which is CP, P and T violating is proportional to $\bar{\theta}$.

It is the fact that $\bar{\theta}$. is tiny that constitutes the strong CP problem: $\bar{\theta} \lesssim 10^{-10}$

No additional symmetry is restored at Lagrangian level is restored when $\bar{\theta} = 0$

A Common Origin for all CP Violations

Strong CP

Nelson-Barr proposal

1. VEVs that break the SM gauge group cannot break CP and they only connect the usual quark fields.
2. VEVs that break CP spontaneously cannot break the SM gauge group and they can only connect SM quark fields with the additional VLQs.

Bento, gcb, Parada model

$$\mathcal{L}_Y = -\sqrt{2}(\overline{u^0}\ \overline{d^0})^i_L(g_{ij}\phi\, d^0_{jR} + h_{ij}\tilde{\phi}\, u^0_{jR}) - M_d \overline{D^0_L}\, D^0_R - \sqrt{2}(f_i\, S + f'_i\, S^*)\, \overline{D^0_L} d^0_{iR} + \text{h.c.}$$

Since CP is a symmetry imposed in the Lagrangian $\quad \theta_{QCD} = 0$

$$\theta_{\text{weak}} = \arg(\det \mathcal{M}_d \times \det m_u).$$

$$\theta_{\text{weak}} = 0 \qquad \mathcal{M}_d = \begin{pmatrix} m_d & 0 \\ M_d & M_d \end{pmatrix}$$

A Common Origin for all CP Violations

Extension to the Leptonic sector: three right handed neutrinos are included

$$\mathcal{L}_l = \overline{\psi_l^0} G_l \phi \; e_R^0 + \overline{\psi_l^0} G_\nu \tilde{\phi} \; \nu_R^0 + \frac{1}{2} \nu_R^{0T} C (f_\nu S + + f_\nu{}' S^*) \nu_R^0 + h.c.$$

all coefficients are real

Imposed symmetry: $\psi_l^0 \to i\psi_l^0, \quad e_R^0 \to i e_R^0, \quad \nu_R^0 \to i \nu_R^0$

The initial Z_2 symmetry is thus promoted to a Z_4 symmetry

$$\mathcal{L}_l = \overline{\psi_l^0} G_l \phi \; e_R^0 + \overline{\psi_l^0} G_\nu \tilde{\phi} \; \nu_R^0 + \frac{1}{2} \nu_R^{0T} C (f_\nu S + + f_\nu{}' S^*) \nu_R^0 + h.c.$$

the symmetry prevents the existence of bare Majorana terms, however these are generated by the couplings to the field S

there are standard model particles transforming non-trivially under the symmetry