Inflationary potentials with the exponential vs polynomial approach to the plateau

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Based on

On the Present Status of Inflationary Cosmology arXiv:2505.13646

RK, Linde

The BAO-CMB Tension and Implications for Inflation arXiv:2507.12459

Ferreira, McDonough, Balkenhol, RK, Knox, Linde

Abstract

A simplest interpretation of the Planck data was that inflationary potentials may have a **plateau**, which is **approached exponentially** fast at large values of the scalar field.

However, **recent data from ACT and SPT, in combination with DESI,** suggest that the inflationary spectral index, $\mathbf{n_s}$, is slightly higher than its value based on Planck data.

We will explain why this small deviation, if confirmed, is very significant: it disfavors some of the most popular inflationary models of the last decade, and suggests that the inflaton potentials may **approach the plateau not exponentially but polynomially**.

Single field inflation models

$$e^{-1}\mathcal{L}_{can} = \frac{1}{2}R - \frac{1}{2}(\partial\phi)^2 - V(\phi)$$

Exponential approach to the plateau

$$V_{\rm exp} = V_0(1 - e^{-\phi/\mu} + \cdots)$$

Exponential α-attractors, T- and E-models

$$\mu^2 = \frac{3\alpha}{2} = \frac{1}{|\mathcal{R}|}\,, \qquad \mathcal{R} = -\frac{2}{3\alpha}$$

Polynomial approach to plateau

$$V_{\rm polyn} = V_0(1 - (\mu/\phi)^k + \cdots)$$

In particular, for polynomial α-attractors

$$\mu^2 = \frac{3\alpha}{2} = \frac{1}{|\mathcal{R}|}\,, \qquad \mathcal{R} = -\frac{2}{3\alpha}$$

n_s - r plane

Primordial power spectra conventionally parameterized as

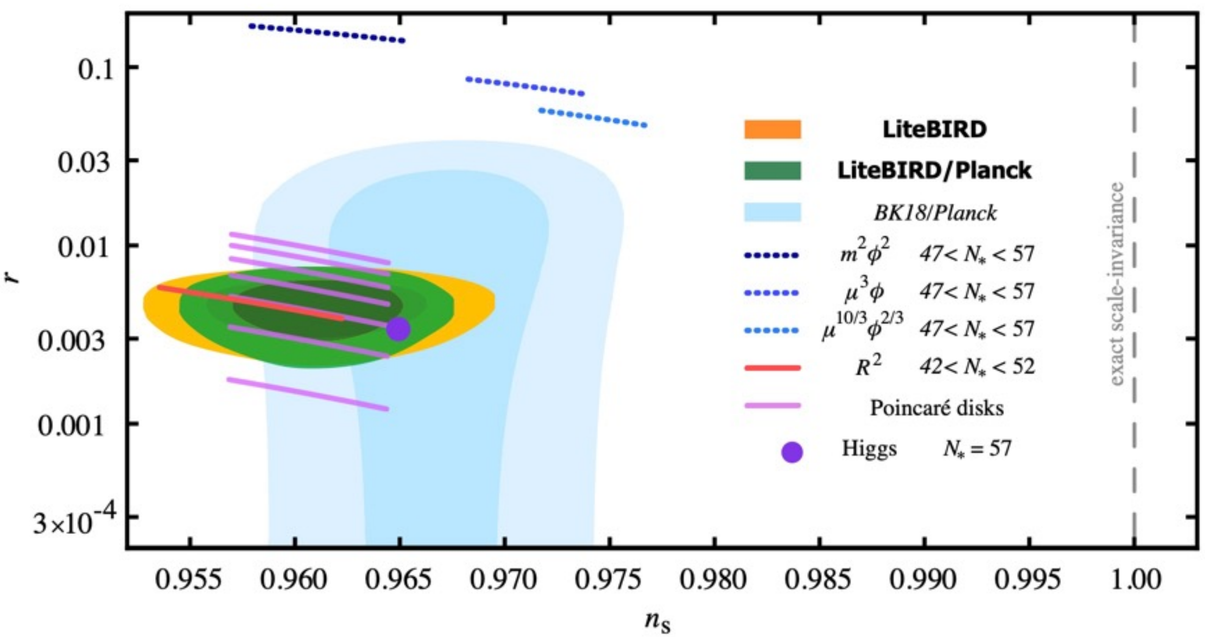
$$\Delta_\zeta^2(k) = \Delta_\zeta^2 \left(\frac{k}{k_*}\right)^{n_s(k)-1}$$

n_s is the spectral index

Ratio of the power in primordial gravitational waves to the power in primordial density perturbations: tensor-to-scalar ratio **r**

$$r = \frac{\Delta_h^2(k)}{\Delta_\zeta^2(k)}$$

Various inflationary models predict values of CMB observables: n_s and **r**

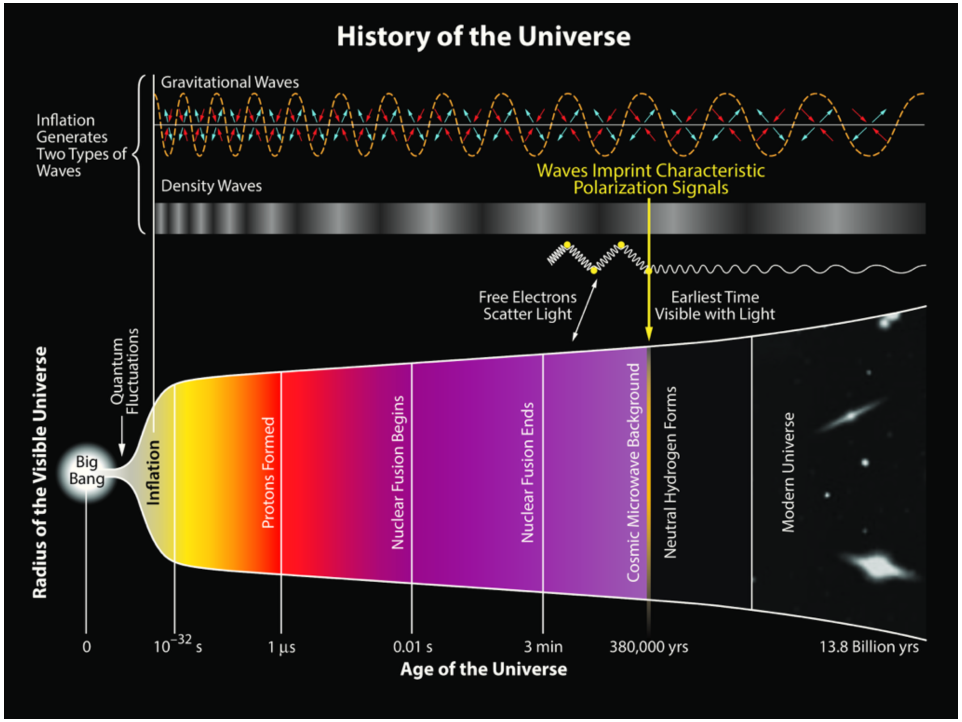


LiteBIRD constraints on the tensor-to-scalar ratio r and the scalar spectral index ns. The red line and the dark purple dot show the predictions of the Starobinsky model and Higgs model of inflaton. The light purple lines show the predictions of the Poincare disk models based on α-attractors

ACT, DESI, and SPT, 2025

$$n_s \approx 0.974$$

“Holy Grail” of observational cosmology is to detect primordial gravitational waves



Find a value of tensor-to-scalar ratio **r** detecting primordial gravitational waves

Bounds on r

BICEP / Keck XIII: Improved Constraints on Primordial Gravitational Waves using Planck, WMAP, and BICEP/Keck Observations through the 2018 Observing Season arXiv:2110.00483

$r_{0.05} < 0.036$ at 95% confidence

The Atacama Cosmology Telescope: DR6 Constraints on Extended Cosmological Models

$r < 0.038$ (95%, P-ACT-LB-BK18) arXiv:2503.14454

One of the conclusions based on the Planck data was that the simplest inflationary potentials that fit the data have a plateau, which is approached exponentially fast at the large values of the scalar field

Planck 2013

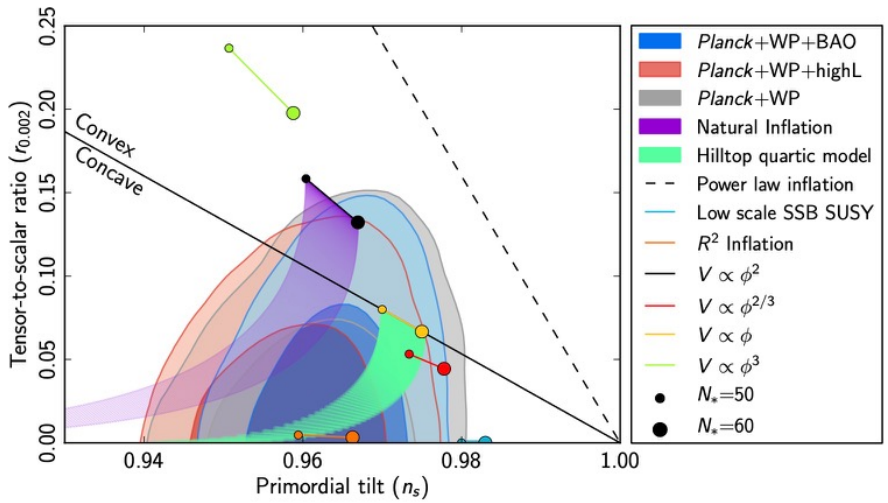


Fig. 1. Marginalized joint 68% and 95% CL regions for n_s and $r_{0.002}$ from *Planck* in combination with other data sets compared to the theoretical predictions of selected inflationary models.

Planck 2015

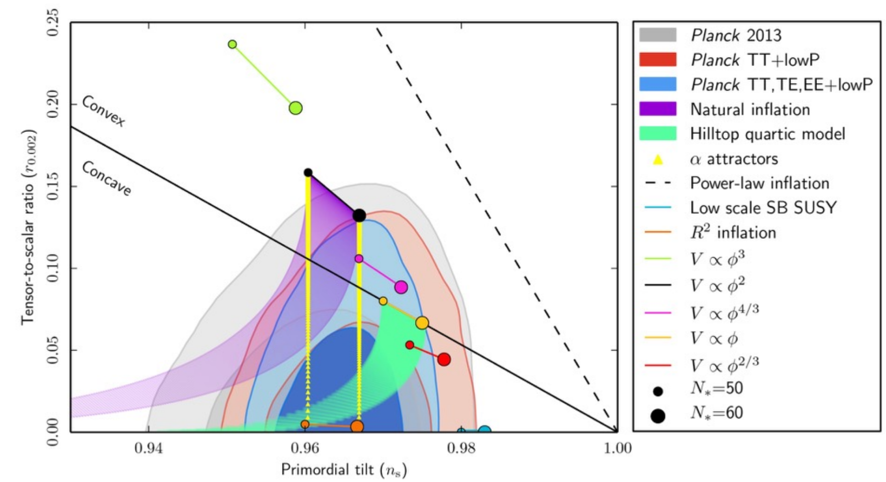


Fig. 12. Marginalized joint 68 % and 95 % CL regions for n_s and r at $k = 0.002\,\mathrm{Mpc}^{-1}$ from *Planck* compared to the theoretical predictions of selected inflationary models. Note that the marginalized joint 68 % and 95 % CL regions have been obtained by assuming $dn_s/d\ln k = 0$.

As shown in Fig. 1, most of the joint 95% allowed region lies below the convex potential limit, and concave models with a red tilt in the range [0.945-0.98] are allowed by *Planck* at 95% CL.

R^2 inflationary model proposed by Starobinsky is the most preferred.
Due to its high tensor- to-scalar ratio, the model ϕ^2 is now strongly disfavored

Now in 2025 r < 0.036

Preference to **plateau** potentials :
no discovery of gravity waves

Still valid

Exponential approach: n_s < 0. 965

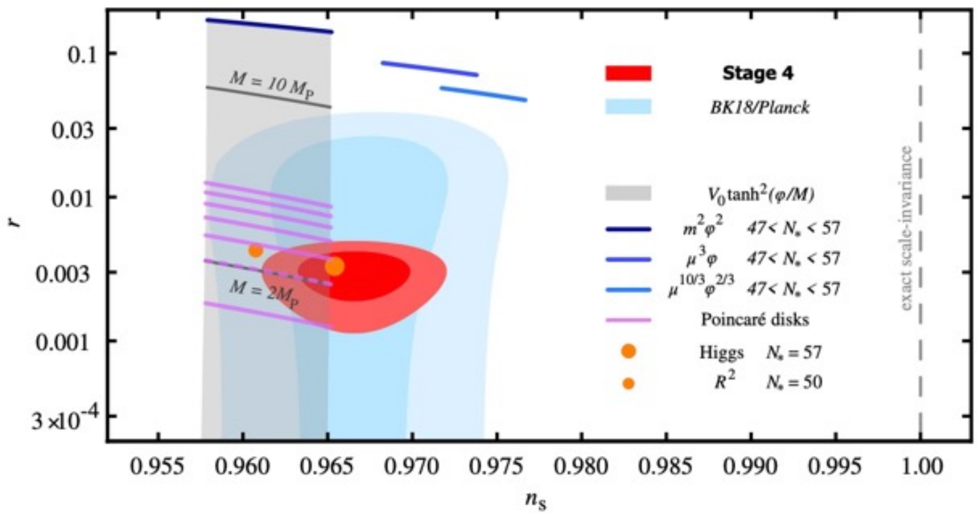
ACT, DESI, SPT: n_s = 0. 974

Polynomial approach to plateau

Snowmass2021 Cosmic Frontier: Cosmic Microwave Background Measurements White Paper

arXiv:2203.07638

Planck+BICEP/Keck Array



In all models here, not yet ruled out by a bound on **r**

$n_s < 0.966$

ACT, DESI, and SPT, 2025

$n_s = 0.974$

Shown are the current best constraints from a combination of the BICEP2/*Keck Array* experiments and Planck for a model with $r = 0.003$.

Models that naturally explain the observed departure from scale invariance separate into two viable classes: **monomial and plateau**.

The **monomial** models φ^p are shown for three values of p as blue lines for $47 < N_* < 57$. The simplest realization of this class is now **disfavored**

The **plateau** models include the tanh^2 form (gray band) as an example, as this form arises in a sub-class of α-attractor models. Some particular realizations of physical models in the plateau class are also shown: Starobinsky model, Higgs inflation and Poincaré disk α-attractors. The differing choices of N_* for Higgs and Starobinsky reflect differing expectations for reheating efficiency.

The meaning of the parameter α in α-attractor models in hyperbolic geometry

$$ds^2 = \frac{dx^2 + dy^2}{(1 - x^2 - y^2)^2}$$

http://mathworld.wolfram.com/PoincareHyperbolicDisk.html

Kahler space curvature

$$R_K = -\frac{2}{3\alpha}$$

$$K = -3\alpha \log(1 - Z\bar{Z})$$

supergravity

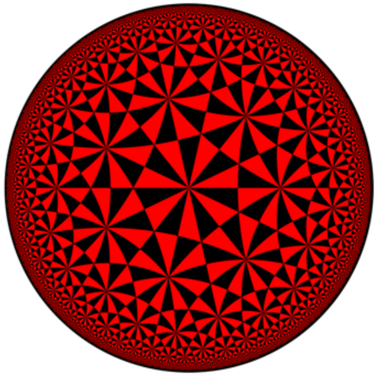
RK, Linde, 2015

Escher in the Sky

3α is R^2 of the Escher disk

3α=1,2,3,4,5,6,7

$$ds^2 = 3\alpha \frac{dZ\, d\bar{Z}}{(1 - Z\bar{Z})^2}$$



For a unit size **Poincare Hyperbolic disk**:

$$r \sim 10^{-3} \qquad \alpha = \frac{1}{3}$$

Next CMB satellite mission targets: 7 Poincare disks

Why attractors?

$$V = V_0 \tanh^{2n}\left(\frac{\phi}{\sqrt{6\alpha}}\right)$$

Simple analytic answers for observables in slow-roll approximation

$$r(\alpha, n, N) = \frac{12\alpha}{N^2 + \frac{N}{2n} g(\alpha, n) + \frac{3}{4}\alpha},$$

$$n_s(\alpha, n, N) = \frac{1 - \frac{2}{N} - \frac{3\alpha}{4N^2} + \frac{1}{2nN}(1 - \frac{1}{N}) g(\alpha, n)}{1 + \frac{1}{2nN} g(\alpha, n) + \frac{3\alpha}{4N^2}},$$

$$g(\alpha, n) \equiv \sqrt{3\alpha(4n^2 + 3\alpha)} .$$

Kahler space curvature

$$R_K = -\frac{2}{3\alpha}$$

Will be measured in the sky

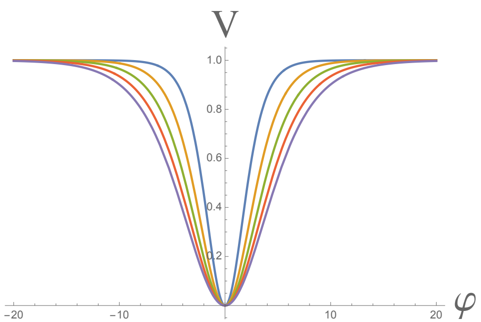
Here N is the number of e-foldings of inflation

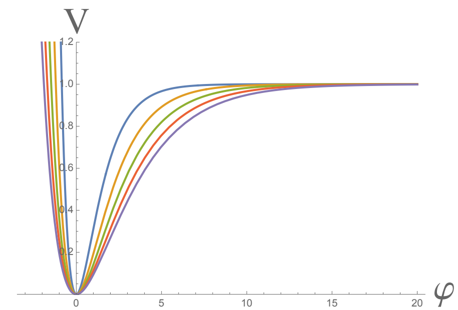
At **large N and small α** we find an attractor value of CMB observables

$N \approx 55$

$$n_s = 1 - \frac{2}{N}, \qquad r = \alpha \frac{12}{N^2}$$

Plateau potentials of α-attractors





$$\frac{1}{2}R - \frac{1}{2}\partial\varphi^2 \;\; - \alpha\mu^2\Big(\tanh\frac{\varphi}{\sqrt{6\alpha}}\Big)^2$$

$$\frac{1}{2}R - \frac{1}{2}\partial\varphi^2 \;\; - \alpha\mu^2\Big(1 - e^{-\sqrt{\frac{2}{3\alpha}}\varphi}\Big)^2$$

Simplest T-model

in canonical variables

Simplest E-model

$$\frac{1}{2}R - 3\alpha\frac{\partial Z\partial\bar{Z}}{(1-Z\bar{Z})^2} - V_0 Z\bar{Z}$$

In geometric variables

$$\frac{1}{2}R - 3\alpha\frac{\partial T\partial\bar{T}}{(T+\bar{T})^2} - V_0(T-1)^2$$

Prediction for BICEP/Keck, CMBS-4 and LiteBIRD experiments

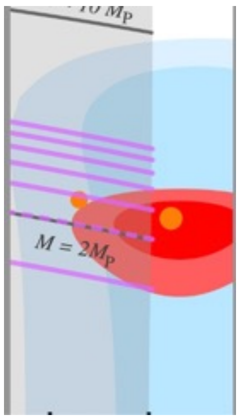
3α =1,2,3,4,5,6,7
KK/string theory origin
of 4D N=1 supergravity

Discrete purple targets

$3\alpha \in \mathbb{R}$

Generic 4D N=1 supergravity

Grey band



3α=3 : Starobinsky and Higgs models

Why **Higgs inflation** (Bezrukov, Shaposhnikov "The Standard Model Higgs boson as the Inflaton" 2007) is related to α-attractors with α=1 and exponential approach to plateau?

The Jordan frame action is

$$S_J = \int d^4x\sqrt{-g}\Big\{ -\frac{M^2+\xi h^2}{2}R + \frac{\partial_\mu h \partial^\mu h}{2} - \frac{\lambda}{4}\left(h^2-v^2\right)^2 \Big\}.$$

After conformal transformation from the Jordan frame to the Einstein frame

$$S_E = \int d^4x\sqrt{-\hat{g}}\Big\{ -\frac{M_P^2}{2}\hat{R} + \frac{\partial_\mu \chi \partial^\mu \chi}{2} - U(\chi) \Big\}$$

the potential for the canonical scalar field χ at large field χ is

$$U(\chi) = \frac{\lambda M_p^4}{4\xi^2}\left(1-\exp\left(-\sqrt{\frac{2}{3M_p}}\chi\right)+\ldots\right)$$

at large field χ the potential has a plateau, and it approaches it exponentially with the factor $\sqrt{\frac{2}{3}}$

α-attractors have the factor $\sqrt{\frac{2}{3\alpha}}$

Hence, in Higgs inflation the values of n_s and r are the same as that of the α-attractors with α=1

Is n_s = 0.974 from **ACT, DESI, and SPT, 2025** a final word?

This is not clear due to BAO-CMB Tension

Ferreira, McDonough, Balkenhol, RK, Knox, Linde arXiv:2507.12459

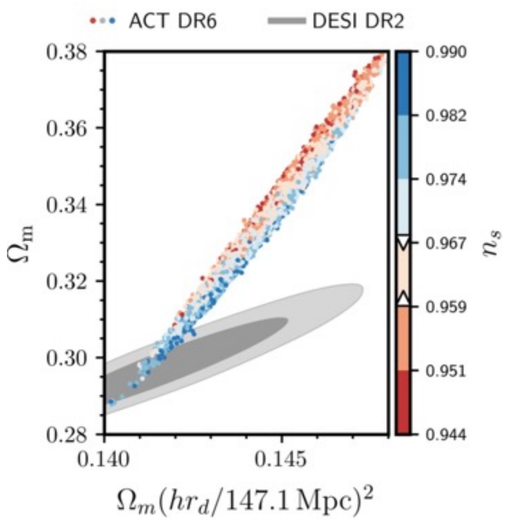
Constraint on n_s derives from the combination of cosmic microwave background (CMB) data with baryon acoustic oscillation (BAO) data. The resulting n_s constraint is shifted significantly upward relative to the constraint from CMB alone.

The consequence is that previously preferred inflationary models are seemingly disfavored by $\gtrsim 2\sigma$.

But there is a $3\sigma - 4\sigma$ tension between CMB datasets and DESI BAO data under the assumption of the standard ΛCDM cosmological model.

Given the crucial role of n_s in discriminating between inflationary models, **we urge caution in interpreting CMB+BAO constraints on n_s until the BAO-CMB tension is resolved**.

The BAO-CMB Tension: tension between the CMB and DESI BAO inferences of the parameters $r_d h$ and Ω_m In the case of ACT, the tension is **3.1σ**, which increases to **3.7σ** if ACT is combined with SPT

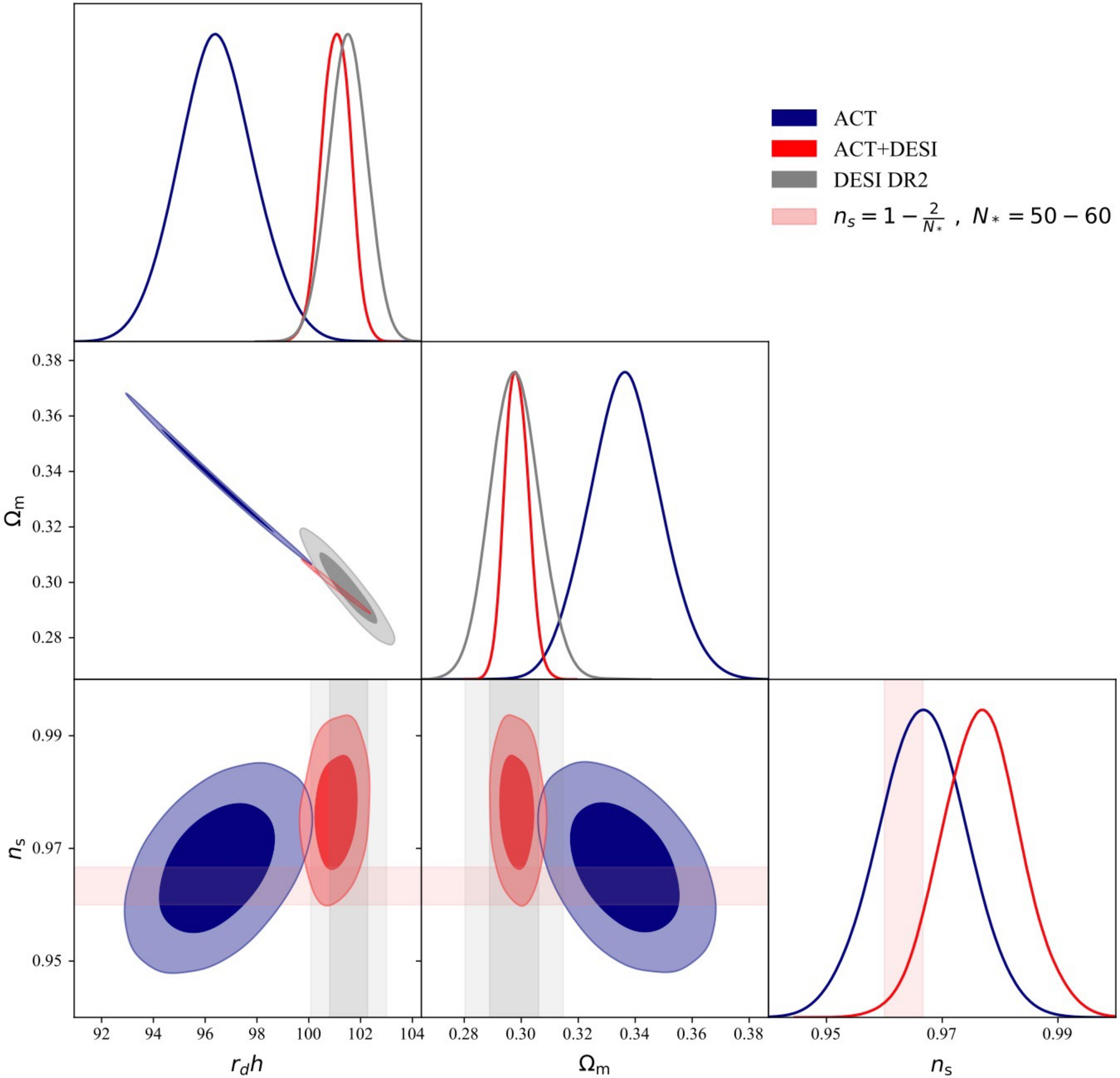


Constraints from ACT DR6 CMB and DESI DR2 BAO Data

Parameter	ACT	ACT + DESI
n_s	0.9666 (0.9664) ± 0.0076	0.9770 (0.9754) ± 0.0070
$r_d h$ [Mpc]	96.5 (96.27) ± 1.5	101.04 (101.05) ± 0.54
Ω_m	0.337 (0.338) ± 0.013	0.2999 (0.2996) ± 0.0040

TABLE I. Parameter constraints (best fit) from the fit of ΛCDM to ACT DR6 primary CMB data alone and in combination with DESI BAO data.

$r_d h$ parameter combines the comoving sound horizon r_d and the dimensionless Hubble constant h



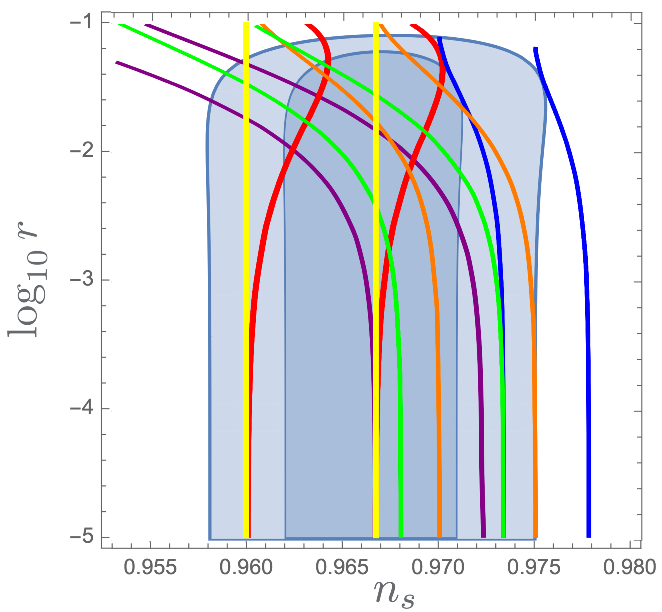
ACT
ACT+DESI
DESI DR2
$n_s = 1 - \frac{2}{N_*}$, $N_* = 50 - 60$
0.38
0.36
0.34
0.32
0.30
0.28
Ω_m
0.99
0.97
0.95
n_s
92 94 96 98 100 102 104
$r_d h$
0.28 0.30 0.32 0.34 0.36 0.38
Ω_m
0.95 0.97 0.99
n_s

We may still look at models compatible with ACT, DESI, SPT data with upward in n_s

CMB-S4 Snowmass2021 Cosmic Frontier: CMB Measurements White Paper", arXiv:2203.07638

All targets at $n_s < 0.966$

Meanwhile many other inflationary models were studied which fill in the right hand side of the blue area above with $n_s > 0.966$. These are known as



$\alpha \to 0 \qquad r \to 0$

D-brane inflation, KKLTI models, pole inflation, polynomial α-attractors

ATTRACTORS:

exponential α-attractor T-models and E-models (yellow and red)	n_s is α-independent at small r, r-depends on α
polynomial attractor models (purple, green, orange, blue)	n_s is α-independent at small r, but depends on k r-depends on α and k

RK, Linde, 1909.04687

Encyclopædia Inflationaris 781pp

Martin, Ringeval, Venin 1303.3787, but v5 2024

May 21, 2001

BI **Brane Inflation** Dvali, Shafi, Solganik Burgess, Majumdar, Nolte, Quevedo, Rajesh, Zhang

Not well defined at small fields

$$V = V_0\left(1 - \left(\frac{\mu}{\phi}\right)^k\right) \qquad \mu \ll 1$$

KKLTI **Inflation**

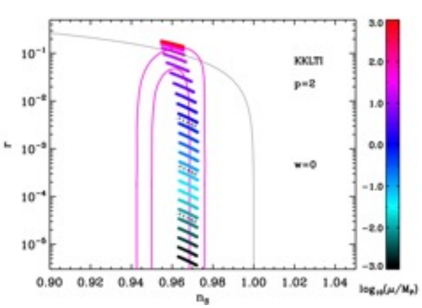
$$V = V_0 \frac{1}{\left(1 + \left(\frac{\mu}{\phi}\right)^k\right)} = V_0 \frac{\phi^k}{\phi^k + \mu^k}$$

Well defined at small fields
Same as BI at large fields

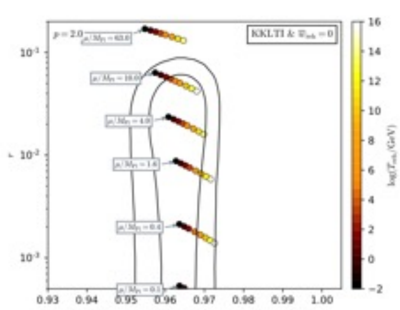
0308055 2003

Inverse harmonic function for $Dp - \bar{D}p$ $p = 7 - k$ inflation $\mu \ll 1$ Kachru, RK, Linde, Maldacena, McAllister, Trivedi

A.49 KKLT Inflation (KKLTI) 2013



A.56 KKLT Inflation (KKLTI) 2024



Encyclopedia treats μ as a phenomenological Parameter in **KKLTI** Inflation

KKLTI model observables at the attractor point and below, at small r

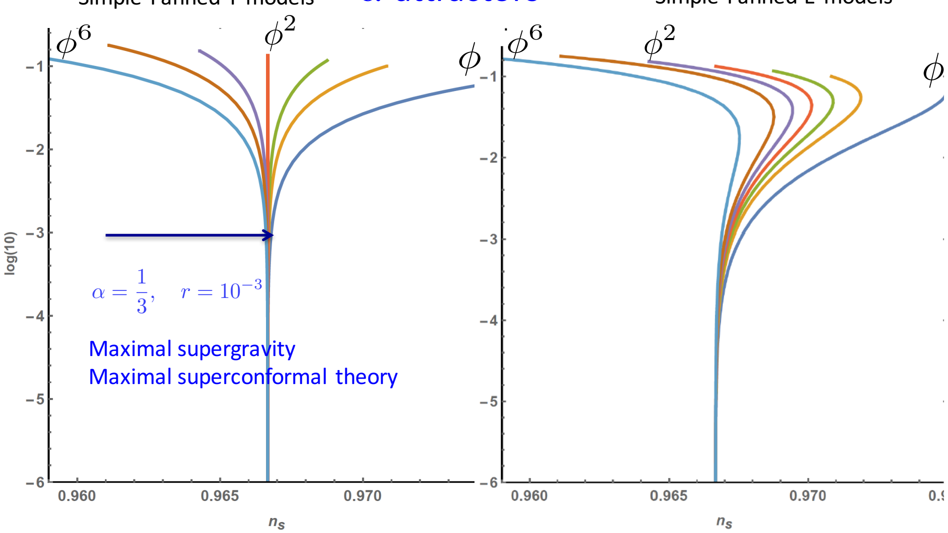
RK, Linde, Yamada, 1811.01023

$$n_s = 1 - \frac{2(k+1)}{(k+2)N}\,, \qquad r = 8\mu^{\frac{2k}{k+2}} k^2 \left(\frac{1}{k(k+2)N}\right)^{\frac{2(k+1)}{k+2}}$$

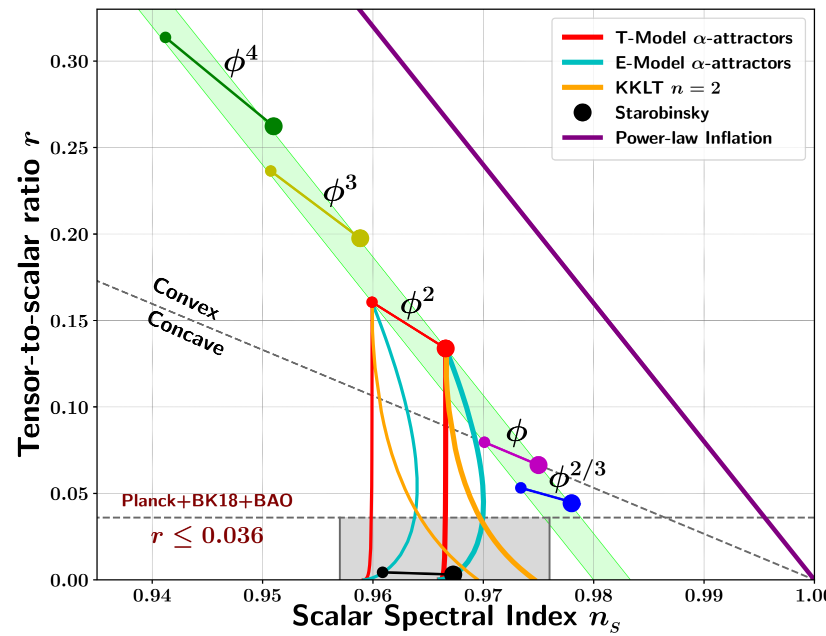
Exponential α-attractors are much simpler

$$n_s = 1 - \frac{2}{N}\,, \qquad r = \alpha\,\frac{12}{N^2}$$

α-attractors



$$\left(\tanh\frac{\varphi}{\sqrt{6\alpha}}\right)^{2n} \qquad \left(1 - e^{\sqrt{\frac{2}{3\alpha}}\varphi}\right)^{2n}$$



S. Mishra, lectures 2403.10606

Pole Inflation

Galante, RK, Linde, Roest , 1412.3797, Terada, 1602.07867

$$\frac{\mathcal{L}}{\sqrt{-g}} = \frac{R}{2} - \frac{a_p}{2}\frac{(\partial\rho)^2}{\rho^p} - V(\rho)$$

Here the pole of order p is at ρ=0 and the residue at the pole is a_p. For p = 2, $a_2 = 3\alpha/2$ this equation describes inflationary regime E-models of α-attractors, but here we consider general values of p.

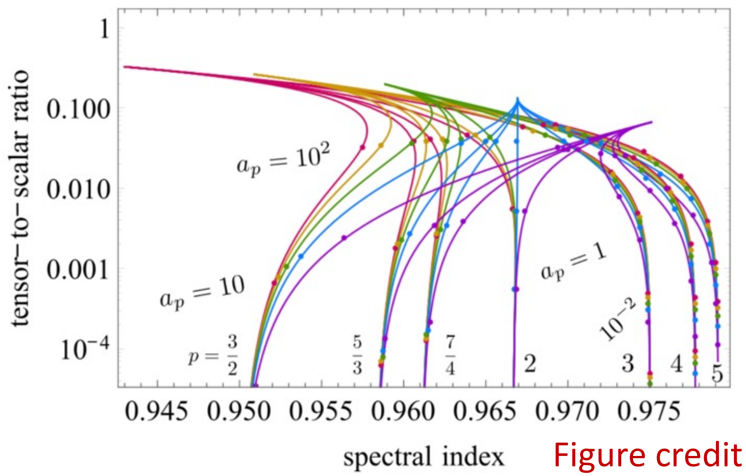


Figure credit to Terada

Pole inflation observables at small r

$$n_s = 1 - \frac{p}{(p-1)N}\,, \qquad r = \frac{8}{a_p}\left(\frac{a_p}{(p-1)N}\right)^{\frac{p}{p-1}}$$

Just a change of variables

Compare with KKLTI

$$k = \frac{2}{p-2}\,, \qquad \mu^2 = a_p\, k^2$$

$$p \neq 2$$

$$n_s = 1 - \frac{2(k+1)}{(k+2)N}\,, \qquad r = 8\mu^{\frac{2k}{k+2}} k^2 \left(\frac{1}{k(k+2)N}\right)^{\frac{2(k+1)}{k+2}}$$

Polynomial α-attractors

RK, Linde 2202.06492

Another version of the model with the same attractor values of observables.
There is a pole of order 2, but the potential is different

$$\frac{\mathcal{L}}{\sqrt{-g}} = \frac{R}{2} - \frac{3\alpha}{4}\frac{(\partial\rho)^2}{\rho^2} - V_0\,\frac{\ln^k \rho}{\ln^k \rho + 1}$$

It has specific values of parameter μ, reflecting the hyperbolic geometry origin of the kinetic term

$$\mu^2 = \frac{3\alpha}{2} = \frac{1}{|\mathcal{R}|}\,, \qquad \mathcal{R} = -\frac{2}{3\alpha}$$

In canonical variables

$$V^{\alpha}_{\rm polyn}(\phi) = V_0\,\frac{|\phi|^k}{(3\alpha/2)^{k/2} + |\phi|^k}$$

Observables as in general KKLTI models with $\mu^2 = \frac{3\alpha}{2}$

Relation between r and (1-n_s) in cosmological attractors

Exponential α-attractors

$$n_s = 1 - \frac{2}{N} \, , \qquad r = \frac{12\alpha}{N^2} \quad \Longrightarrow \quad r = 3\alpha(1-n_s)^2$$

Pole inflation

$$r = 8a_p^{\frac{1}{p-1}} \left(\frac{1-n_s}{p} \right)^{\frac{p}{p-1}}$$

Polynomial attractors, r to (1-n_s) relation for each k

$$r = 2^{\frac{k+4}{k+2}} \left(\frac{\mu^2}{k^2} \right)^{\frac{k}{k+2}} \left(\frac{k}{k+1} (1-n_s) \right)^{\frac{2(k+1)}{k+2}}$$

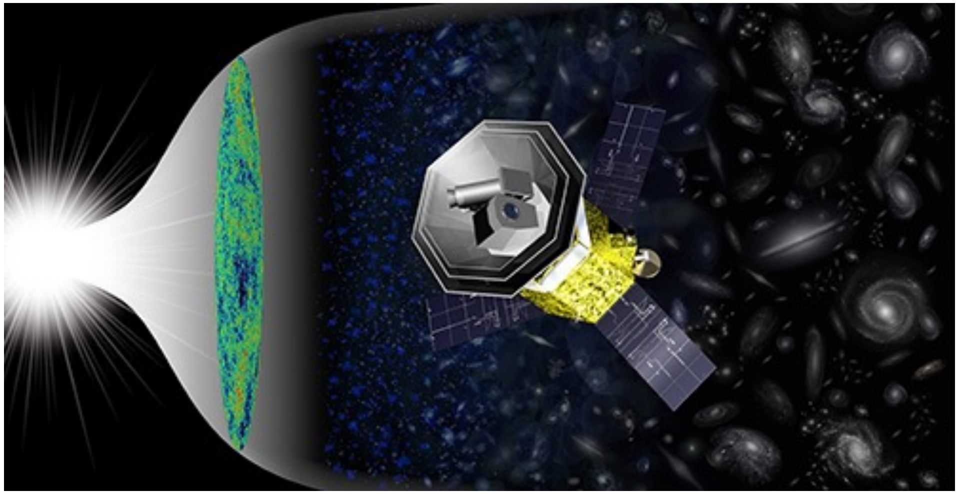
Polynomial α-attractors: same n_s, r with a special choice $\mu^2 = \frac{3\alpha}{2}$

Conclusions

- We have discussed various properties of inflationary attractor models in view of the data from Planck and new data from ACT, DESI, SPT. We explained that the CMB-BAO tension needs to be resolved before we view the increase in n_s as an established fact.

- If the ACT+SPT+DESI result are confirmed, the generic set of attractor models **polynomially approaching the plateau** will be of interest. These models are not as simple as the exponential α-attractors, but nevertheless, they have analytic expressions for n_s and **r** at the attractor point.

- Some of these models (polynomial α-attractors) also have interesting predictions continuous in α, as well as predictions discrete in $3\alpha=1,2,3,4,5,6,7$ for LiteBIRD. These are Poincare disks from the underlying hyperbolic geometry of the moduli space of supergravity, inspired by string theory.



The Lite (Light) spacecraft for the study of B-mode polarization and Inflation from cosmic background Radiation Detection (LiteBIRD)

LiteBIRD is expected to launch in JFY**2032**
(JFY: Japanese fiscal year).