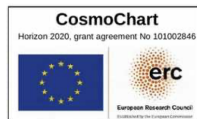


Maximal CP violation from long-range forces

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LPENS & Sorbonne Université, Paris

based on: M.M Flores, K. Petraki, **AS** 25XX.XXXXX



Scalars 2025, Warsaw, 24.09.2025

- 1 Introduction
- 2 Model
- 3 Long-range interactions
- 4 Asymmetries
- 5 Conclusions

Introduction

Contact-type interactions

$$m_{\text{med}} \gtrsim M, \quad \lambda_{\text{B}} \gtrsim m_{\text{med}}^{-1}$$

VS

Long-range interactions

$$m_{\text{med}} \ll M, \quad \lambda_{\text{B}} \ll m_{\text{med}}^{-1}$$

Contact-type interactions

$$m_{\text{med}} \gtrsim M, \quad \lambda_B \gtrsim m_{\text{med}}^{-1}$$

VS

Long-range interactions

$$m_{\text{med}} \ll M, \quad \lambda_B \ll m_{\text{med}}^{-1}$$

Non-perturbative effects



Dark Matter phenomenology

Sommerfeld effect

distortion of scattering-state wavefunctions
alters the cross-sections

J. Hisano, S. Matsumoto, M. M. Nojiri (2003)

Bound States

unstable BS $\Rightarrow$ DM interaction rate,
DM abundance, indirect detection signals

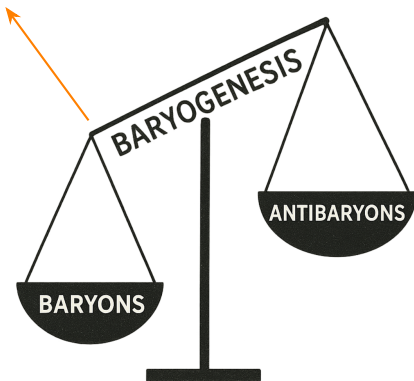
B. von Harling, K. Petraki (2014)

stable BS $\Rightarrow$ elastic scattering, novel indirect
detection signals, inelastic DM-nucleon scattering

K. Petraki, R. R. Volkas (2013)

Baryon-to-entropy
density ratio

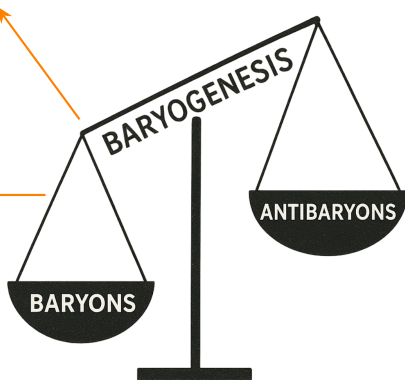
$$Y_B = (8.69 \pm 0.01) \times 10^{-11}$$



Baryon-to-entropy
density ratio

$$Y_B = (8.69 \pm 0.01) \times 10^{-11}$$

Dynamical asymmetry
generation



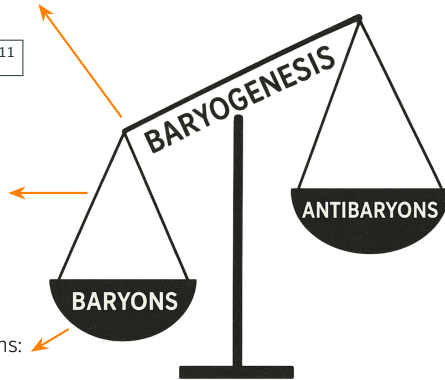
Baryon-to-entropy
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$$Y_B = (8.69 \pm 0.01) \times 10^{-11}$$

Dynamical asymmetry
generation

Three Sakharov conditions:

1. Baryon number violation
2. C and CP symmetry violation
3. Departure from thermal equilibrium



Baryon-to-entropy
density ratio

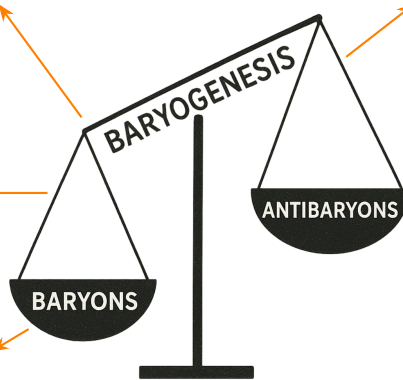
$$Y_B = (8.69 \pm 0.01) \times 10^{-11}$$

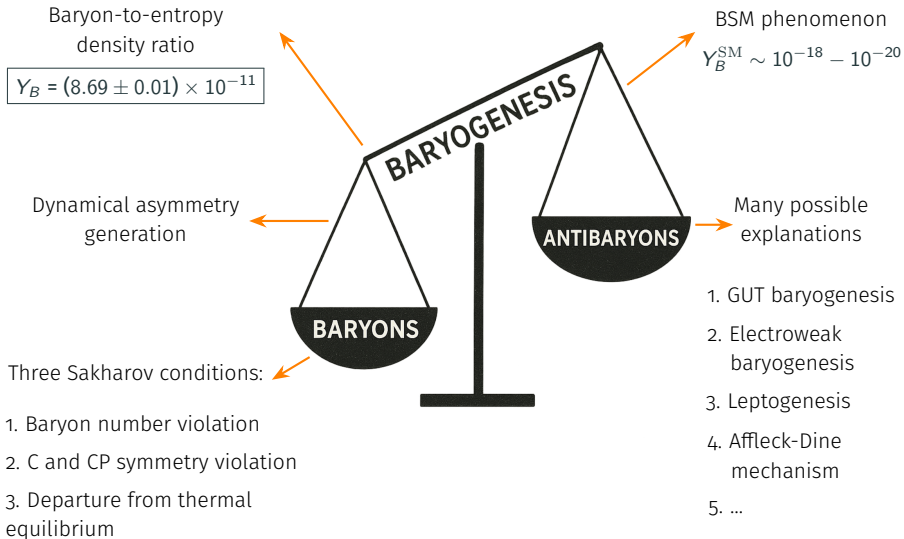
BSM phenomenon
 $Y_B^{\text{SM}} \sim 10^{-18} - 10^{-20}$

Dynamical asymmetry
generation

Three Sakharov conditions:

1. Baryon number violation
2. C and CP symmetry violation
3. Departure from thermal equilibrium





Model

Particle content

We consider the $U_{B-L}(1)$ extension of the Standard Model. The particle content includes:

- two, heavy, non-relativistic and nearly-mass degenerate Dirac fermions $X_{1,2} \sim (1, 1, 0, -1)$,
- doubly-charged scalar $\rho \sim (1, 1, 0, -2)$, which breaks the B-L symmetry
- Gauge boson V_μ of $U_{B-L}(1)$,
- three SM-singlet Weyl fermions χ , which cancel the gauge and gravitational anomalies.

Assumed mass hierarchy

$$\Delta m \equiv m_2 - m_1 \rightarrow 0, m_x \equiv m_2, m_1 \gg v_{B-L}, m_{\text{med}}^{-1} \gg (\mu v_{\text{rel}})^{-1}$$

Lagrangian

The model is described by the Lagrangian density

$$\mathcal{L} = \mathcal{L}_{\text{SM}} + \mathcal{L}_{\text{BSM}}^0 + \mathcal{L}_{\text{BSM}}^{\text{int}} - V_{\text{BSM}}^\rho,$$

where

$$\mathcal{L}_{\text{BSM}}^0 \equiv -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + (\mathcal{D}_\mu \rho)^\dagger (\mathcal{D}^\mu \rho) + \frac{1}{2} \sum_{k=1}^2 \bar{\chi}_k i \not{D} \chi_k + \sum_{i=1}^2 (\bar{\chi}_i i \not{D} \chi_i - m_i \bar{\chi}_i \chi_i),$$

with

$$\mathcal{D}_\mu = \mathcal{D}_\mu^{\text{SM}} + i g_{\text{B-L}} q_{\text{B-L}} V_\mu,$$

Gauge coupling

Yukawa coupling

SM portal

$$\mathcal{L}_{\text{BSM}}^{\text{int}} \equiv - \sum_{i,j=1}^2 \left(\frac{y_{ij}^{\text{R}}}{2\delta_{ij}} \rho^\dagger \bar{\chi}_{\text{R},i}^c \chi_{\text{R},j} + \frac{y_{ij}^{\text{L}}}{2\delta_{ij}} \rho^\dagger \bar{\chi}_{\text{L},i}^c \chi_{\text{L},j} \right) - \sum_{l=1}^3 \sum_{i=1}^2 \lambda_{il}^{\text{X}} \bar{L}_l \tilde{\mathcal{H}} \chi_{\text{R},i} + \text{h.c.},$$

$$V_{\text{BSM}}^\rho \equiv -\mu_\rho^2 |\rho|^2 + \frac{1}{4} \lambda_\rho |\rho|^4 + \lambda_{\rho \mathcal{H}} |\mathcal{H}|^2 |\rho|^2.$$

B-L symmetry
breaking

Asymmetries

The CP asymmetry is generated by the Yukawa interactions

$$\mathcal{L}_{\text{BSM}}^{\text{int}} \equiv - \sum_{i,j=1}^2 \left(\frac{y_{ij}^{\text{R}}}{2\delta_{ij}} \rho^\dagger \bar{X}_{\text{R},i}^c X_{\text{R},j} + \frac{y_{ij}^{\text{L}}}{2\delta_{ij}} \rho^\dagger \bar{X}_{\text{L},i}^c X_{\text{L},j} \right) + h.c.$$

The Yukawa matrices $y^{\text{R,L}}$ are symmetric and generally complex, with $N^2 = 4$ irreducible phases, for $N = 2$ generations.

For convenience, we define

$$y_{ij} \equiv (y_{ij}^{\text{L}} + y_{ij}^{\text{R}})/2, \quad w_{ij} \equiv (y_{ij}^{\text{L}} - y_{ij}^{\text{R}})/2, \quad \alpha_{ij} \equiv |y_{ij}|^2/4\pi, \quad \alpha_{\text{B-L}} \equiv g_{\text{B-L}}^2/4\pi.$$

$N(N-1)/2 = 1$ irreducible phase is associated with the y_{ij} coupling, and $N(N+1)/2 = 3$ with w_{ij} .

We use the parametrization

$$y_{ii} = \sqrt{4\pi\alpha_{ii}} > 0, \quad y_{ij} = \sqrt{4\pi\alpha_{ij}} e^{i\theta_{ij}}, \quad \theta_{ij} \in [0, 2\pi).$$

We also assume that $\boxed{\alpha_{ii}, \alpha_{ij} \gg \alpha_{\text{B-L}}}$.

Asymmetries

The X-number violation occurs via

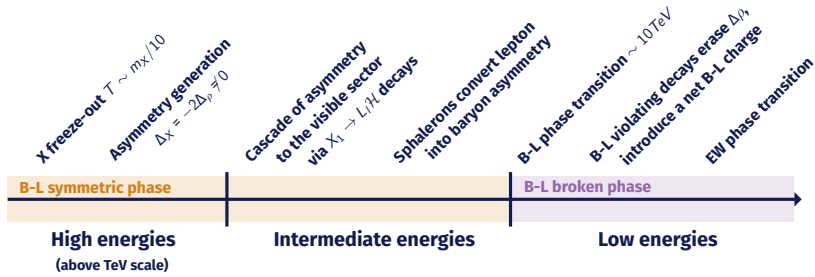
1. $X_2 \rightarrow \bar{X}_1 \rho,$
2. $X_i X_j \rightarrow \rho V_\mu,$
3. $X_i V_\mu \rightarrow \bar{X}_j \rho.$

The CP asymmetry is generated here both

1. perturbatively - through the interference of tree and loop diagrams in various processes $\sim \mathcal{O}(y^6)$
2. non-perturbatively - in long-range $X_i \bar{X}_j \rightarrow X_{i'} \bar{X}_{j'}$ flavour-changing scatterings and $X_1 \bar{X}_2 \rightarrow \rho \rho^*$ annihilations.



$$\Delta X \equiv \Delta X_1 + \Delta X_2 = -2 \Delta \rho \neq 0.$$



Long-range interactions

Long-range interactions in a nutshell

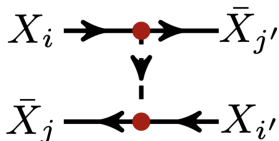
Exchange of light
force mediators



Distortion of the
particle wavefunction

Breakdown of perturbative approach!

Low-momentum transfer in the loops $|\mathbf{q}| \sim \mu v_{\text{rel}} \ll \mu$



Every loop introduces
a factor α/v_{rel}

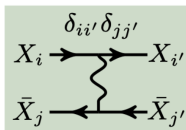
Amplitude enhancement at low velocities
Resummation

Resummation

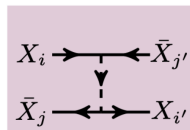
The **2PI** kernel generating the long-range potential

$$\begin{array}{c} X_i \\ \bar{X}_j \end{array} \left[\mathcal{K}_{ij;i'j'}^{X\bar{X}} \right] \begin{array}{c} X_{i'} \\ \bar{X}_{j'} \end{array} =$$

CP-preserving



CP-violating



The resummation of the 2PI kernel leads to the **Dyson-Schwinger** equation for 4-point function

$$\begin{array}{c} X_i \\ \bar{X}_j \end{array} \left[G_{ij;i'j'}^{(4)} \right] \begin{array}{c} X_{i'} \\ \bar{X}_{j'} \end{array} = \sum_{n=-1}^{\infty} \begin{array}{c} X_i \\ \bar{X}_j \end{array} \left[\mathcal{K}_{ij;i_1j_1}^{X\bar{X}} \right] \cdots \left[\mathcal{K}_{i_nj_n;i'j'}^{X\bar{X}} \right] \begin{array}{c} X_{i'} \\ \bar{X}_{j'} \end{array}$$

$$G_{ij,i'j'}^{(4)} = D_{ii'} D_{jj'} \left(1 + \sum_{i'',j''} \mathcal{K}_{ij,i''j''} G_{i''j'',i'j'}^{(4)} \right)$$

K. Petraki, M. Postma,
M. Wiechers (2015)

R. Oncala, K. Petraki
(2011)

Schrödinger equation

In the non-relativistic regime, **Dyson-Schwinger** equation yields a system of coupled **Schrödinger** equations:

$$\sum_{i',j'} \left[-\frac{\nabla^2}{2\mu_{ij}} \delta_{ii'} \delta_{jj'} + [V_{X\bar{X}}(r)]_{i'j'}^{ij} \right] \Psi_{i'j'}^{\mathcal{S}}(\mathbf{r}) = \mathcal{E}_{ij}^{\mathcal{S}} \Psi_{ij}^{\mathcal{S}}(\mathbf{r}),$$

Unrotated wavefunction

$$\mathcal{E}_{ij}^{\mathcal{S}} \equiv E^{\mathcal{S}} - m_i - m_j.$$

Total energy in the CM frame

The projections of a state $\mathcal{S}$ on flavor eigenstates ij

$$\Psi^{\mathcal{S}}(x) = \left(\Psi_{1\bar{1}}^{\mathcal{S}}(x), \Psi_{1\bar{2}}^{\mathcal{S}}(x), \Psi_{2\bar{1}}^{\mathcal{S}}(x), \Psi_{2\bar{2}}^{\mathcal{S}}(x) \right)^T,$$

$$\Psi_{ij}^{\mathcal{S}}(x) \equiv \langle \Omega | T [X_i(x/2) \bar{X}_j(-x/2)] | \mathcal{S} \rangle_{x^0=0}.$$

Long-range potential

XX and $X\bar{X}$ pairs interact via V_μ and ρ boson exchange, which generates long-range potentials

$$V_t(r) = \frac{i}{4M_\mu} \int \frac{d^3q}{(2\pi)^3} i\mathcal{K}_t(q) e^{iq \cdot r}$$

$$V_u(r) = \frac{i(-1)^\ell}{4M_\mu} \int \frac{d^3q}{(2\pi)^3} i\mathcal{K}_u(q) e^{iq \cdot r}$$

The $X\bar{X}$ potential matrix reads

$$\mathbb{V}_{X\bar{X}}(r) = -\frac{\alpha_{B-L}}{r} \times \mathbb{1}$$

$$-(-1)^{\ell+s} \frac{e^{-m_\rho r}}{r} \times \begin{pmatrix} \alpha_{11} & \boxed{A_{11}^*} & A_{11} & \alpha_{12} \\ A_{11} & \alpha_{12} & A & \boxed{A_{22}^*} \\ \boxed{A_{11}^*} & A^* & \alpha_{12} & \boxed{A_{22}} \\ \alpha_{12} & \boxed{A_{22}} & \boxed{A_{22}^*} & \alpha_{22} \end{pmatrix}$$

$$\alpha_{B-L} \equiv \frac{g_{B-L}^2}{4\pi},$$

$$\alpha_{ij} \equiv \frac{|y_{ij}|^2}{4\pi},$$

contains complex phase

$$\boxed{A_{ii} \equiv \frac{y_{ii} y_{12}^*}{4\pi}},$$

$$A \equiv \frac{y_{11} y_{22}^*}{4\pi},$$

Hermitian, but not necessarily real!

Analytic approximation – Schrödinger equations

In the limit $\Delta m \rightarrow 0$, $m_\rho \rightarrow 0$, a system of coupled **Schrödinger equations** can be diagonalized

$$\left[-\frac{\nabla^2}{2\mu} + \mathbb{V}_{\mathbf{x}\tilde{\mathbf{x}}}^{\text{diag}}(r) \right] \hat{\Psi}^S(\mathbf{r}) = \mathcal{E}^S \hat{\Psi}^S(\mathbf{r}),$$

$\rightarrow$ r -independent unitary matrix,
whose columns are $\mathbb{V}_{\mathbf{x}\tilde{\mathbf{x}}}$ eigenvectors

Diagonal
static potential

$$\mathbb{V}_{\mathbf{x}\tilde{\mathbf{x}}}^{\text{diag}}(r) \equiv \mathbb{P}^\dagger \mathbb{V}_{\mathbf{x}\tilde{\mathbf{x}}}(r) \mathbb{P} = -\frac{1}{r} \text{diag}\{a_1, a_2, a_3, a_4\},$$

real eigenvalues

Rotated wavefunction

$$\hat{\Psi}^S(\mathbf{r}) \equiv \mathbb{P}^\dagger \Psi^S(\mathbf{r}),$$

$$\mathcal{E}^S = \mathbf{k}^2 / (2\mu), \quad \mathbf{k} = \mu \mathbf{v}_{\text{rel}}.$$

Analytic approximation — Coulomb wavefunctions

The rotated wavefunction can be expressed as

$$\hat{\Psi}_{\mathbf{k}}^S(\mathbf{r}) = \underbrace{\Phi_{\mathbf{k}}(\mathbf{r})}_{\substack{\text{diagonal } 4 \times 4 \\ \text{matrix}}} \cdot \underbrace{\mathcal{N}^S}_{\substack{\text{vector determining} \\ \text{the asymptotic behavior}}}$$

$$\mathcal{N}^S \equiv (N_{11}^S, N_{12}^S, N_{21}^S, N_{22}^S)^T$$

Asymptotically

$$\Phi_{\mathbf{k}}(\mathbf{r}) \stackrel{r \rightarrow \infty}{\simeq} \overset{\text{Incoming plane-wave}}{e^{i\mathbf{k} \cdot \mathbf{r}} \times \mathbb{1}} + \overset{\text{Outgoing spherical-wave}}{\frac{e^{ikr}}{r}} \times \mathbb{F}_{\mathbf{k}}(\Omega_{\mathbf{r}}),$$

$$\mathbb{F}_{\mathbf{k}} = \text{diag}\{f_{\mathbf{k}}^{a_1}, f_{\mathbf{k}}^{a_2}, f_{\mathbf{k}}^{a_3}, f_{\mathbf{k}}^{a_4}\}.$$

Analytic approximation — Coulomb wavefunctions

We seek solutions that asymptote to a pure flavor state at $r \rightarrow \infty$

$$\Psi_{\mathbf{k}}^{\mathcal{S}}(\mathbf{r}) \stackrel{r \rightarrow \infty}{\simeq} e^{i\mathbf{k} \cdot \mathbf{r}} \times \mathcal{U}^{\mathcal{S}} + \frac{e^{ikr}}{kr} \times \mathcal{T}_{\mathbf{k}}^{\mathcal{S}}(\Omega_{\mathbf{r}}), \quad \mathcal{U}^{\mathcal{S}} \equiv (\delta_{1\bar{1}}^{\mathcal{S}}, \delta_{1\bar{2}}^{\mathcal{S}}, \delta_{2\bar{1}}^{\mathcal{S}}, \delta_{2\bar{2}}^{\mathcal{S}})^{\text{T}}.$$

Hence

$$\mathcal{U}^{\mathcal{S}} = \mathbb{P} \cdot \mathcal{N}^{\mathcal{S}}, \quad \mathcal{T}_{\mathbf{k}}^{\mathcal{S}} = k \mathbb{P} \mathbb{F}_{\mathbf{k}} \mathcal{N}^{\mathcal{S}} = k \mathbb{P} \mathbb{F}_{\mathbf{k}} \mathbb{P}^{\dagger} \mathcal{U}^{\mathcal{S}}.$$

The **transition amplitude** from $\mathcal{S} = \{ij\}$ to $\mathcal{S}' = \{i'j'\}$ is

$$[\mathcal{T}_{\mathbf{k}}]_{\mathcal{S}'}^{\mathcal{S}} = k \mathcal{U}^{\mathcal{S}'\dagger} \mathbb{P} \mathbb{F}_{\mathbf{k}} \mathbb{P}^{\dagger} \mathcal{U}^{\mathcal{S}} = k \sum_a \mathbb{P}^{\mathcal{S}'a} f_{\mathbf{k}}^a (\mathbb{P}^{\mathcal{S}a})^*,$$

and the unrotated wavefunction reads

$$\Psi_{\mathbf{k}}^{\mathcal{S}}(\mathbf{r}) = \mathbb{P} \Phi_{\mathbf{k}}(\mathbf{r}) \mathbb{P}^{\dagger} \mathcal{U}^{\mathcal{S}}.$$

Partial wave analysis

Partial-wave expansion

$$\mathcal{T}_{\mathbf{k}}^S(\Omega_{\mathbf{r}}) = \sum_{\ell=0}^{\infty} (2\ell+1) P_{\ell}(\hat{\mathbf{k}} \cdot \hat{\mathbf{r}}) \mathcal{T}_{|\mathbf{k}|,\ell}^S$$

$$f_{\mathbf{k}}^a(\Omega_{\mathbf{r}}) = \sum_{\ell=0}^{\infty} (2\ell+1) P_{\ell}(\hat{\mathbf{k}} \cdot \hat{\mathbf{r}}) f_{|\mathbf{k}|,\ell}^a$$

For the Coulomb wavefunctions:

$$f_{|\mathbf{k}|,\ell}^a \equiv \frac{e^{i2\Delta_{\ell}(\zeta_a)} - 1}{2i k}, \quad e^{2i\Delta_{\ell}(\zeta_a)} = \frac{\Gamma(1 + \ell + i\zeta_a)}{\Gamma(1 + \ell - i\zeta_a)}$$

Velocity dependence
 $\zeta_a \equiv a/v_{\text{rel}}$

The partial-wave cross-sections

$$\frac{\sigma_{\ell}^{S \rightarrow S'}}{\sigma_{\ell}^U} = \left| [\mathcal{T}_{|\mathbf{k}|,\ell}]_{S'}^S \right|^2,$$

Unitarity bound

$$\sigma_{\ell}^U \equiv 4\pi \frac{2\ell+1}{k^2}$$

Asymmetries

CP asymmetries

We parametrize the CP asymmetries as

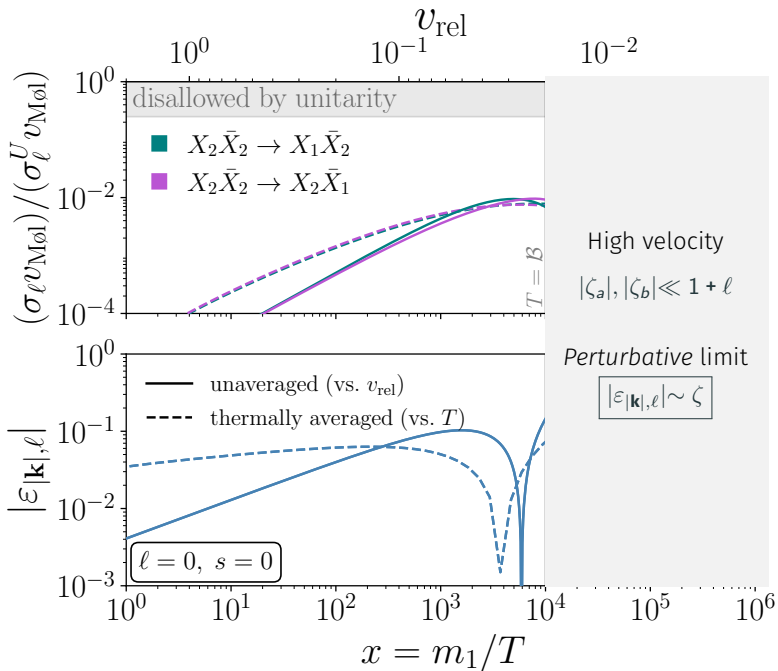
$$[\varepsilon_{|k|,\ell}]_{S \rightarrow S'} = \frac{|[\mathcal{T}_{|k|,\ell}]_{S'}^S|^2 - |[\mathcal{T}_{|k|,\ell}]_{\bar{S}'}^{\bar{S}}|^2}{|[\mathcal{T}_{|k|,\ell}]_{S'}^S|^2 + |[\mathcal{T}_{|k|,\ell}]_{\bar{S}'}^{\bar{S}}|^2}$$

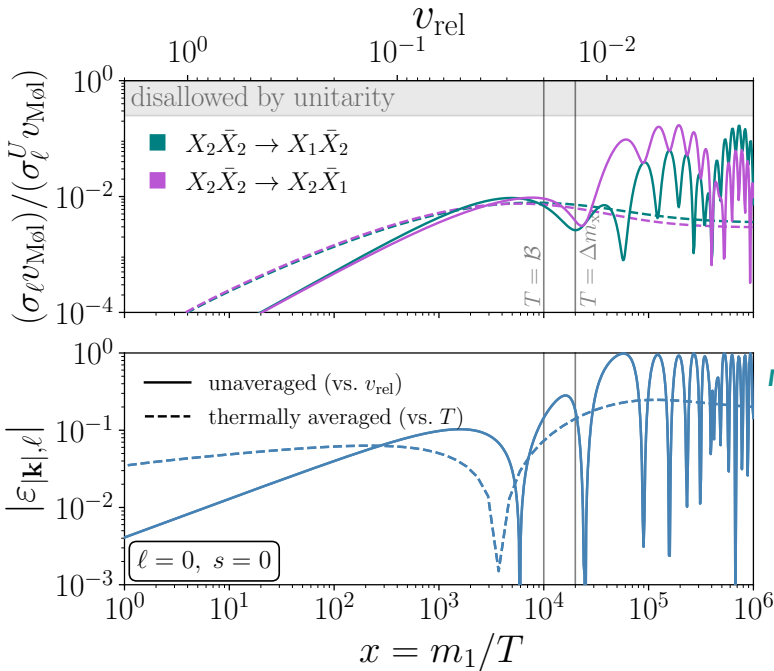
Having complex LG potential is crucial to generate an asymmetry!

$$= \frac{\sum_{a,b} \text{Im} \left[f_{|k|,\ell}^a f_{|k|,\ell}^{b*} \right] \text{Im} \left[\mathbb{P}^{Sa} (\mathbb{P}^{S'a})^* \mathbb{P}^{S'b} (\mathbb{P}^{Sb})^* \right]}{\sum_{a,b} \text{Re} \left[f_{|k|,\ell}^a f_{|k|,\ell}^{b*} \right] \text{Re} \left[\mathbb{P}^{Sa} (\mathbb{P}^{S'a})^* \mathbb{P}^{S'b} (\mathbb{P}^{Sb})^* \right]}.$$

Unitarity & CPT invariance: $[\mathcal{T}_{|k|,\ell}]_{22}^{1\bar{1}} = [\mathcal{T}_{|k|,\ell}]_{1\bar{1}}^{22}, \quad [\mathcal{T}_{|k|,\ell}]_{2\bar{1}}^{12} = [\mathcal{T}_{|k|,\ell}]_{1\bar{2}}^{2\bar{1}}$

CP violation arises in the $j\bar{j} \rightarrow 1\bar{2}, 2\bar{1}$ scatterings.



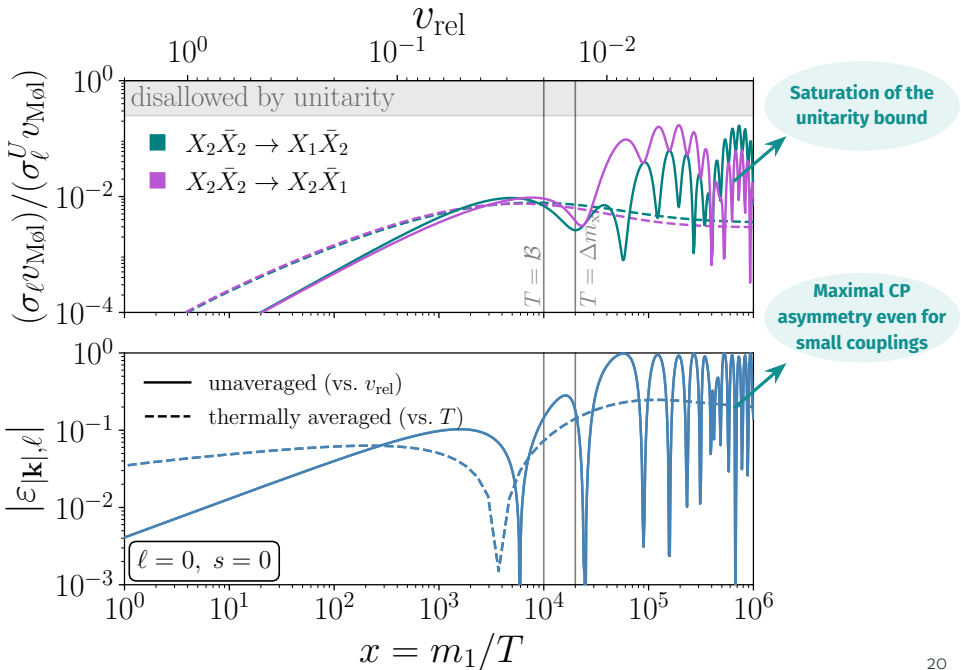


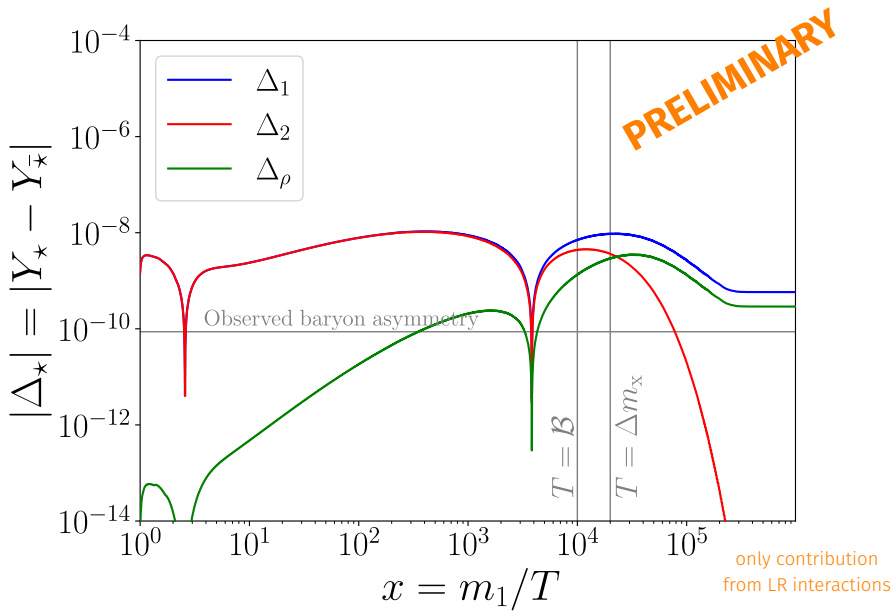
Low velocity

$$|\zeta_\rho| \gtrsim 1 + \ell$$

**Non-perturbative
limit**

$|\varepsilon_{|\mathbf{k}|,\ell}| \sim 1$





Conclusions

Conclusions

- We study a novel baryogenesis scenario involving long-range interactions between non-relativistic particles.
- CP violation necessitates $\mathbb{V}_{X\bar{X}}$ to be complex.
- $|\epsilon_{|\mathbf{k},\ell|}|\sim 1$ can be attained even for weak couplings at low velocities.
- Long-range CP-violating interactions saturate the unitarity bound in a continuum of momenta.
- Non-perturbative dynamics can alter the phenomenological predictions of various well-established theories!