IJCLab Irène Joliot-Curie
Laboratoire de Physique des 2 Infinis

CNRS

Université Paris-Saclay

Adam Falkowski

On-shell approach to gravitational radiation beyond GR

Talk at the Scalars 2025 conference, Warszawa

22 September 2025

Plan:

1. Introduction

2. Quantum Amplitudes and Gravitational Waves

3. Radiation in Scalar-Tensor Theories

AA, Marinellis
2407.16457
2411.12909

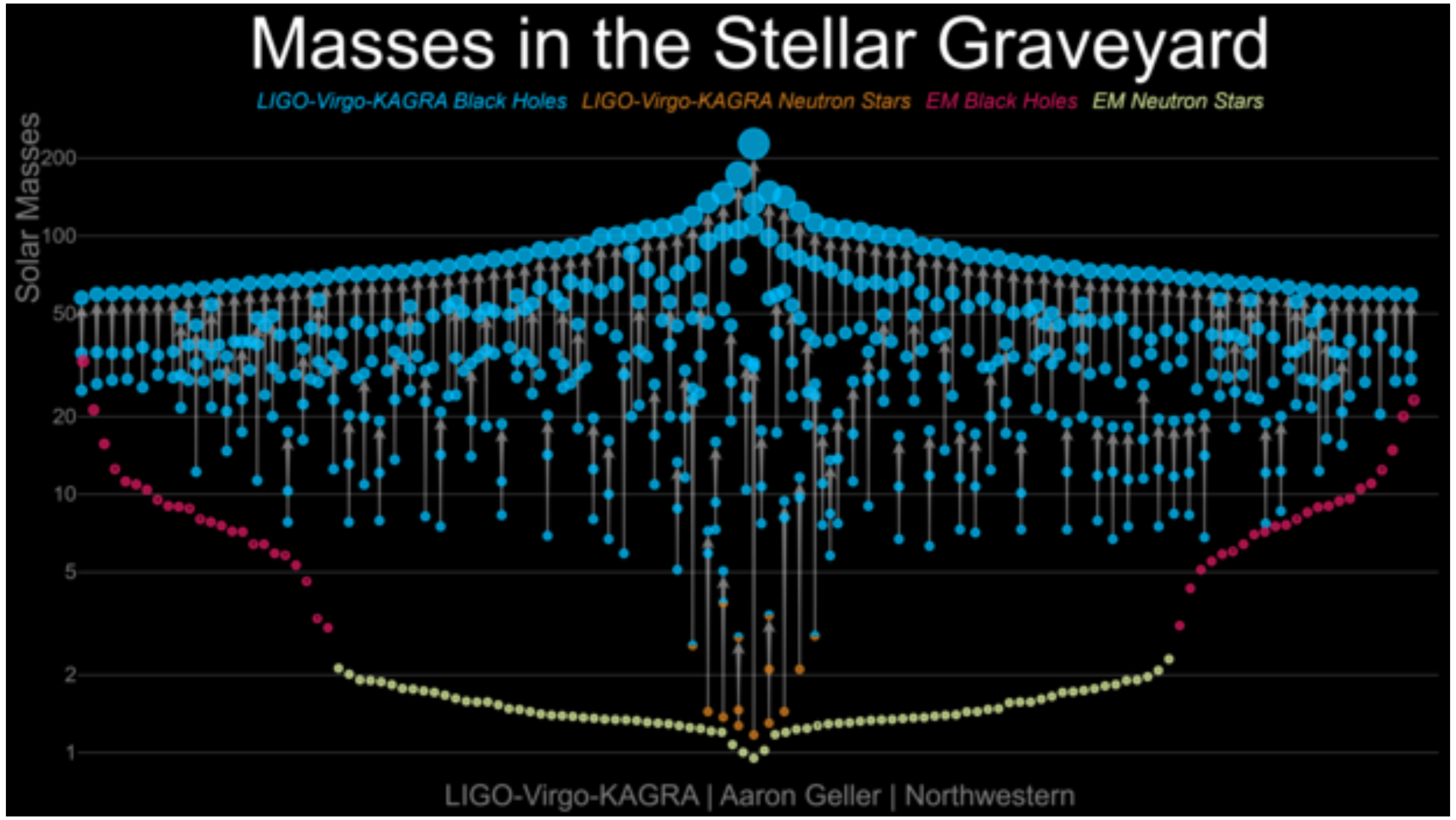
4. Summary

Introduction: gravitational waves detection

Image Credit: EGO*

LIGO: First detection of **Gravitational Waves** (GWs) in **2015**

Many more black hole and neutron star mergers discovered by the LIGO-VIRGO-KAGRA collaboration



Credit:LIGO-Virgo/Aaron Geller/Northwestern University*

New era of precision measurements of GWs

Need for highly accurate GW templates, especially in view of the future upgrades of the LIGO/VIRGO/KAGRA network and future missions such as LISA, Einstein Telescope, Cosmic Explorer, Decigo, Tian-Qin, GEO-HF

New window to test physics beyond general relativity (GR)!

Introduction: gravitational waves detection

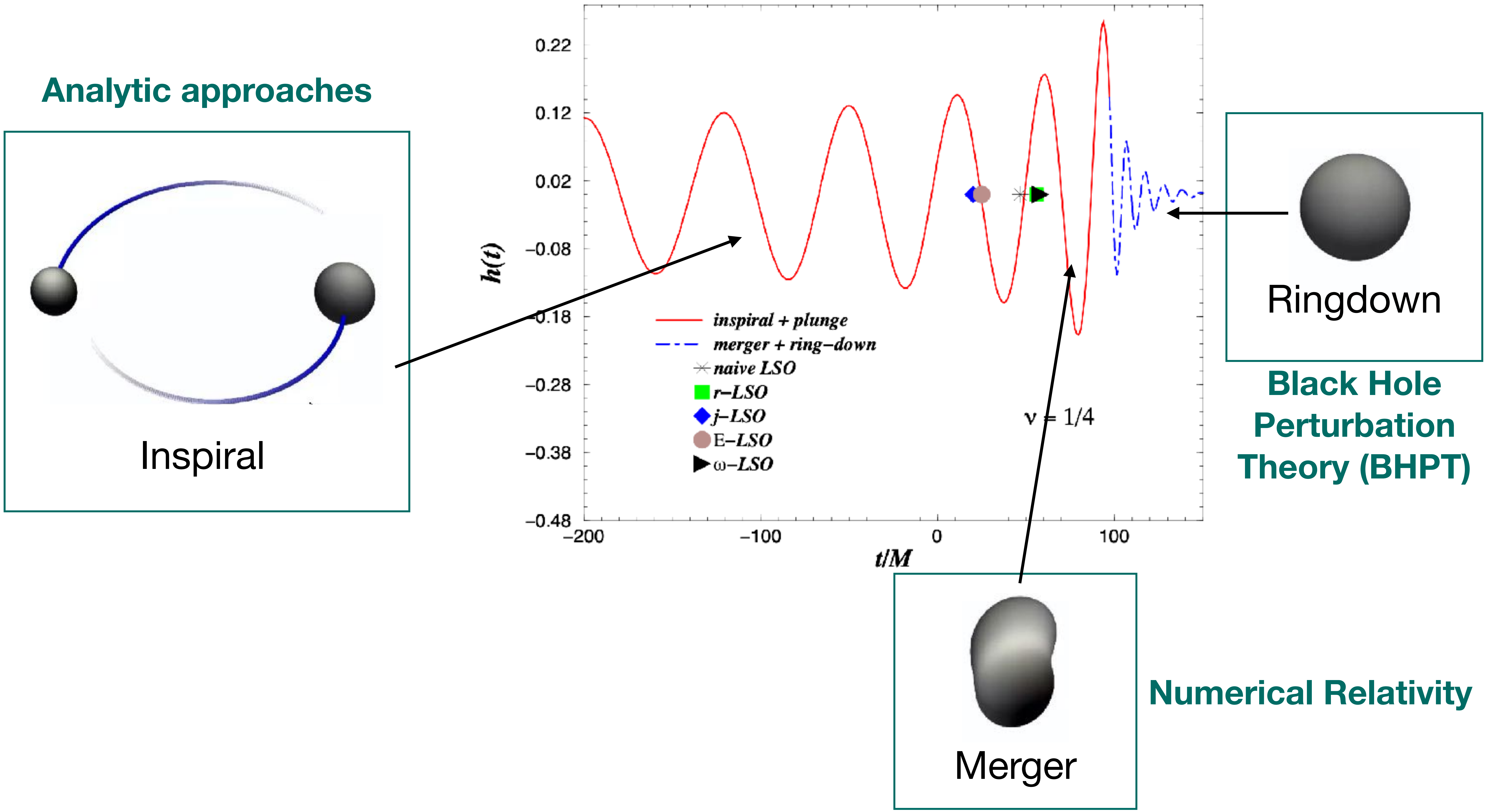
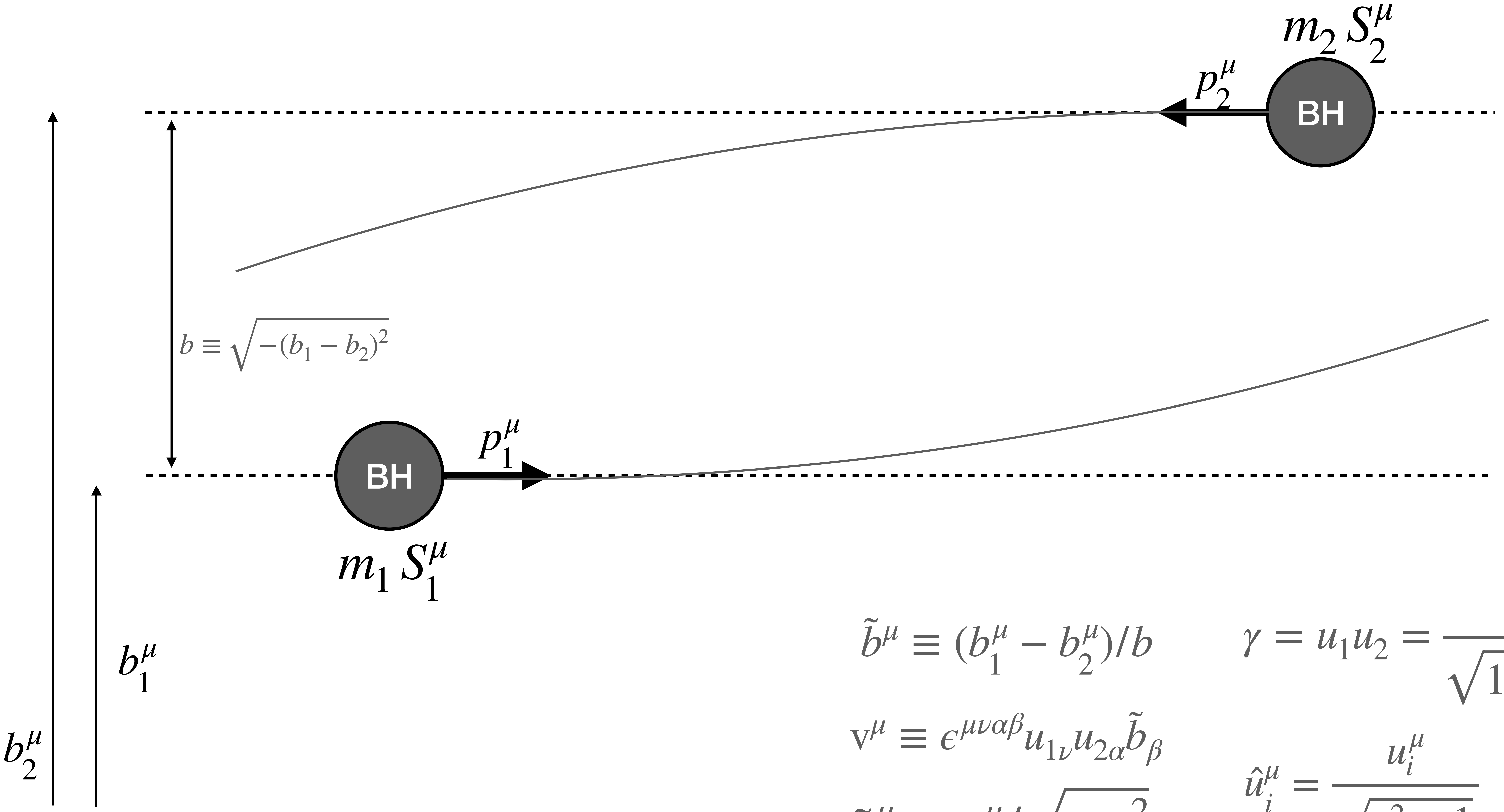


Illustration Credit: Max-Planck-Gesellschaft*

Quantum Amplitudes and Classical Observables



$$u_i^\mu = p_i^\mu / m_i$$
$$a_i^\mu = S_i^\mu / m_i$$

$$\tilde{b}^\mu \equiv (b_1^\mu - b_2^\mu)/b$$
$$\mathrm{v}^\mu \equiv \epsilon^{\mu\nu\alpha\beta} u_{1\nu} u_{2\alpha} \tilde{b}_\beta$$
$$\tilde{\mathrm{v}}^\mu \equiv \mathrm{v}^\mu / \sqrt{-\mathrm{v}^2}$$

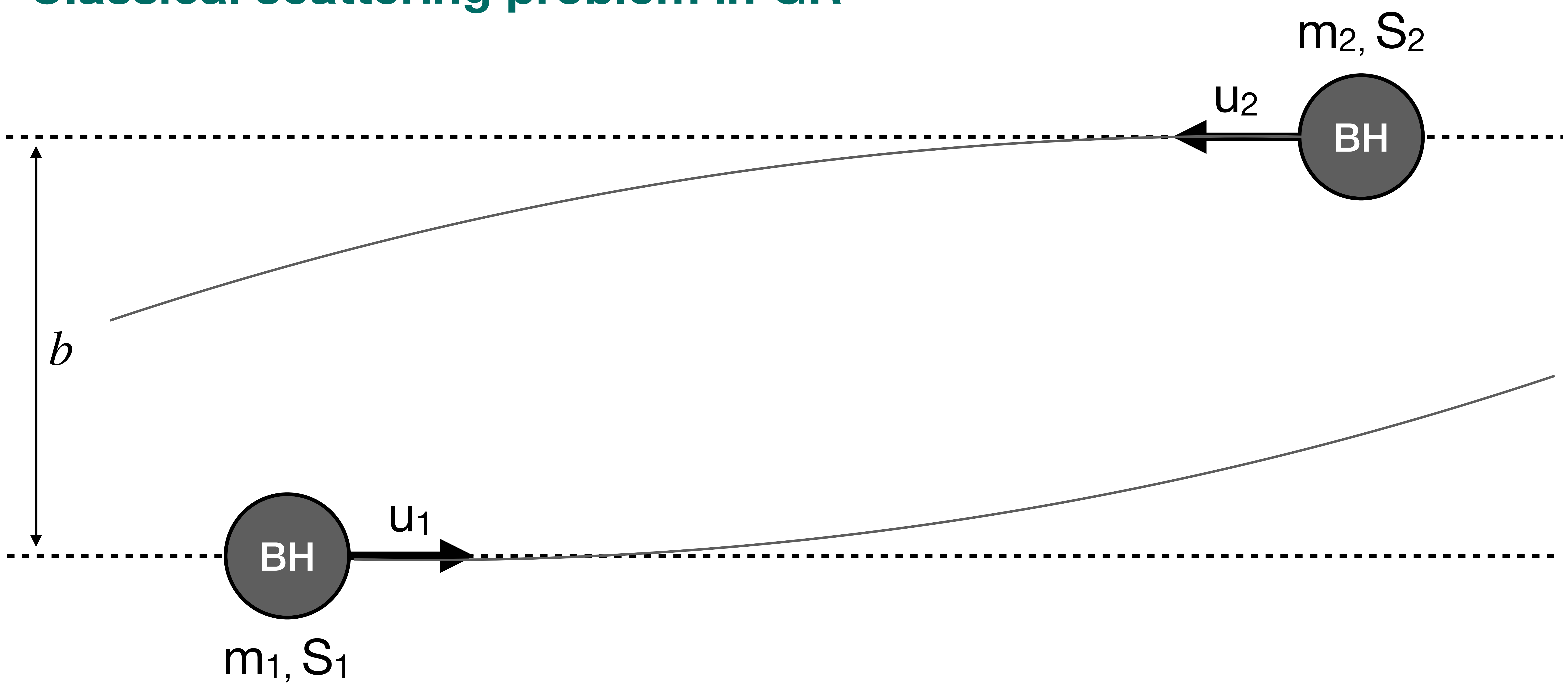
$$\gamma = u_1 u_2 = \frac{1}{\sqrt{1 - \mathrm{v}^2}}$$
$$\hat{u}_i^\mu = \frac{u_i^\mu}{\sqrt{\gamma^2 - 1}}$$

$\hbar = c = 1$

Quantum Amplitudes and Gravitational Radiation

Quantum Amplitudes and Classical Observables

- **Classical scattering problem in GR**



Large separation:

$R_s \ll b$

How can we describe this problem using Scattering Amplitudes computed in QFT?

Black holes represented by point particles interacting as spin-S particles minimally coupled to gravity

Arkani-Hamed, Huang, O'Connell
arXiv:1906.10100

Quantum Amplitudes and Classical Observables

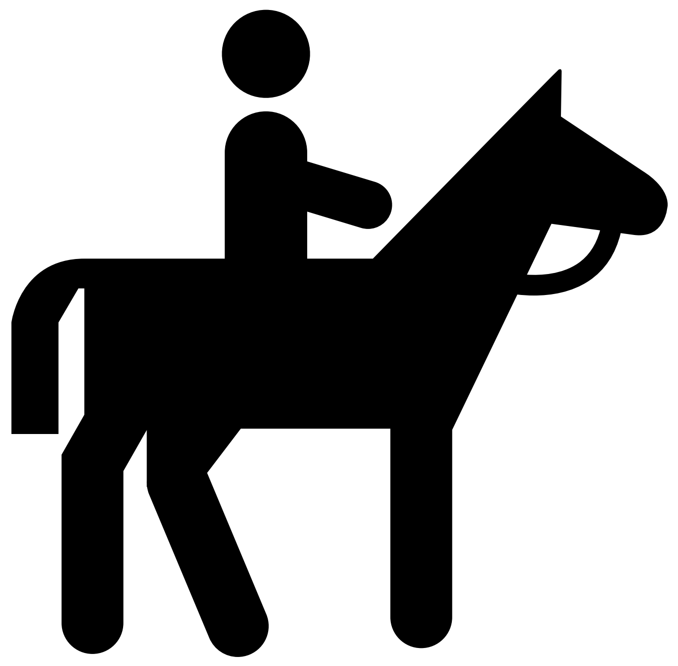
Why employ quantum amplitudes for classical calculations?

- Computations organized in perturbative expansion with Lorentz covariance preserved at each step
- Often, **analytic results** in places where only numerical results previously available
- Can exploit many **modern techniques used in particle physics** to simplify calculation
- Easy to include spin of the colliding objects
- Can be easily applied to relevant theories such as QED or GR
- Can be straightforwardly **extended to beyond GR predictions**

One downside is that amplitudes naturally give scattering observables, while phenomenologically bound systems are more relevant.
The problem of bound-to-boundary continuation is not fully solved yet
Alternative formalism of worldline EFT where this particular problem is absent

Quantum Amplitudes and Radiation Observables

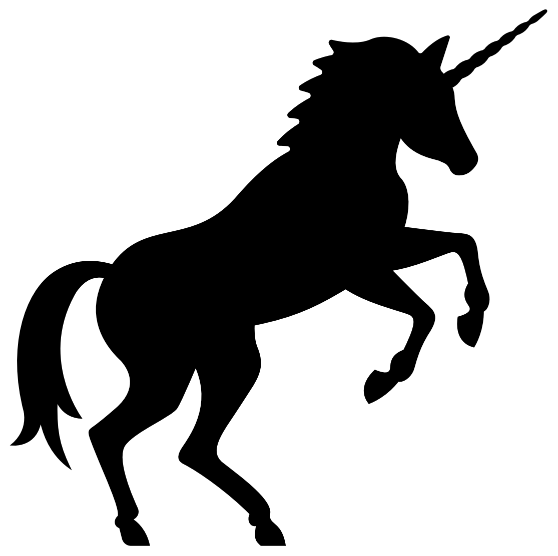
2 workhorses



KMOC formalism

Kosower, Maybee, O'Connell
[arXiv:1811.10950]

Cristofoli, R. Gonzo, D. A. Kosower, D. O'Connell
[arXiv:2107.10193]



On-shell methods

Arkani-Hamed, Huang, Huang
arXiv:1709.04891

KMOC formalism

One can define GR radiation observables as vacuum expectation values of metric field operators and its derivatives

GR

$$\mathcal{R}_{\mu\nu} \equiv {}_{\text{out}}\langle \psi | h_{\mu\nu}(x) | \psi \rangle_{\text{out}}$$

or gauge invariant

$$\mathcal{R}_{\mu\nu\alpha\beta} \equiv {}_{\text{out}}\langle \psi | R_{\mu\nu\alpha\beta}(x) | \psi \rangle_{\text{out}}$$

$$\mathcal{R}_h \equiv {}_{\text{out}}\langle \psi | h^{\mu\nu}(x) | \psi \rangle_{\text{out}} \epsilon^-_{\mu\nu}$$

strain

Given radiation observable $\mathcal{R}_h$ one defines **waveform** W_h as

$$\mathcal{R}_h(x) = \frac{W_h(t)}{|\boldsymbol{x}|} \qquad |\boldsymbol{x}| \to \infty \qquad t \equiv x^0 - |\boldsymbol{x}|$$

retarded time

Furthermore one defines **spectral waveform** or waveshape f_h as Fourier transform:

$$f_h(\omega) = \int dt e^{i\omega t} W_h(t)$$

KMOC formalism

Cristofoli, R. Gonzo, D. A. Kosower, D. O'Connell
[arXiv:2107.10193]

$$\mathcal{R}_h \equiv {}_{\text{out}}\langle\psi|h^{\mu\nu}(x)|\psi\rangle_{\text{out}}\epsilon^-_{\mu\nu}$$

$$|\psi\rangle_{\text{out}} = S|\psi\rangle_{\text{in}} \quad \Rightarrow \quad \mathcal{R}_h = {}_{\text{in}}\langle\psi|S^\dagger h^{\mu\nu}(x)S|\psi\rangle_{\text{in}}\epsilon^-_{\mu\nu}$$

"in-in observable"

$$|\psi\rangle_{\text{in}} = \Pi_{i=1,2}[\int d\Phi(p_i)f_i(p_i)e^{ip_ib_i}]|p_1p_2\rangle_{\text{in}}$$

The radiation observables depends on an amplitude-like object

$$h_{\mu\nu} \sim \int d\Phi_k\, a_{\text{in}}(k)\, e^{-ikx} + \text{h.c} \quad \Rightarrow \quad \mathcal{R}_h \supset {}_{\text{in}}\langle\psi|S^\dagger a_{\text{in}}(k)S|\psi\rangle_{\text{in}}$$

Caron-Huot, Giroux, Hannesdottir, Mizera
arXiv:2310.12199

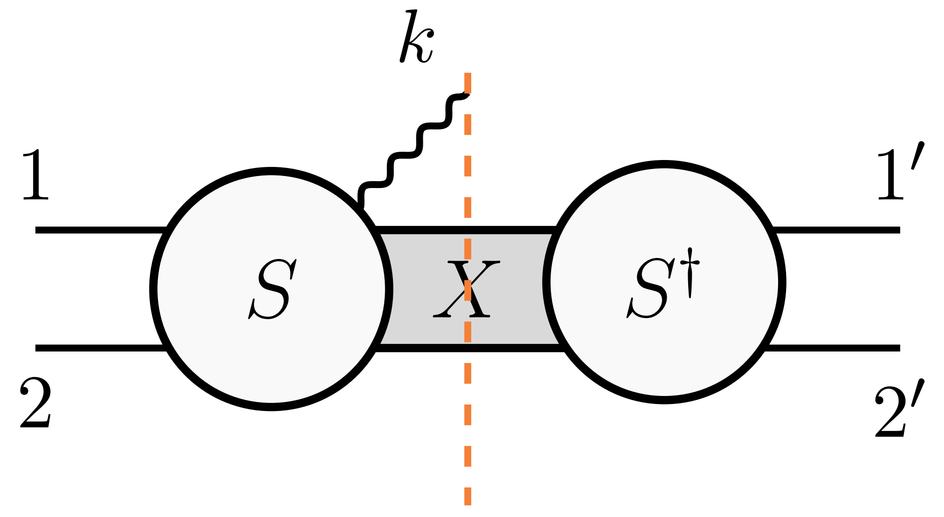
"generalized amplitude",
distinct from ordinary S matrix elements

$${}_{\text{out}}\langle\psi'|\psi\rangle_{\text{in}} = {}_{\text{in}}\langle\psi'|S^\dagger|\psi\rangle_{\text{in}}$$

It can be related to usual S matrix elements via

$${}_{\text{in}}\langle\psi|S^\dagger a_{\text{in}}(k)S|\psi\rangle_{\text{in}} = \int d\Phi_X \;\; {}_{\text{in}}\langle\psi|S^\dagger|X\rangle_{\text{in}}\; {}_{\text{in}}\langle X|a_{\text{in}}(k)S|\psi\rangle_{\text{in}} = \int d\Phi_X \;\; {}_{\text{in}}\langle\psi|S^\dagger|X\rangle_{\text{in}}\; {}_{\text{in}}\langle Xk|S|\psi\rangle_{\text{in}}$$

$\text{Exp}_k =$



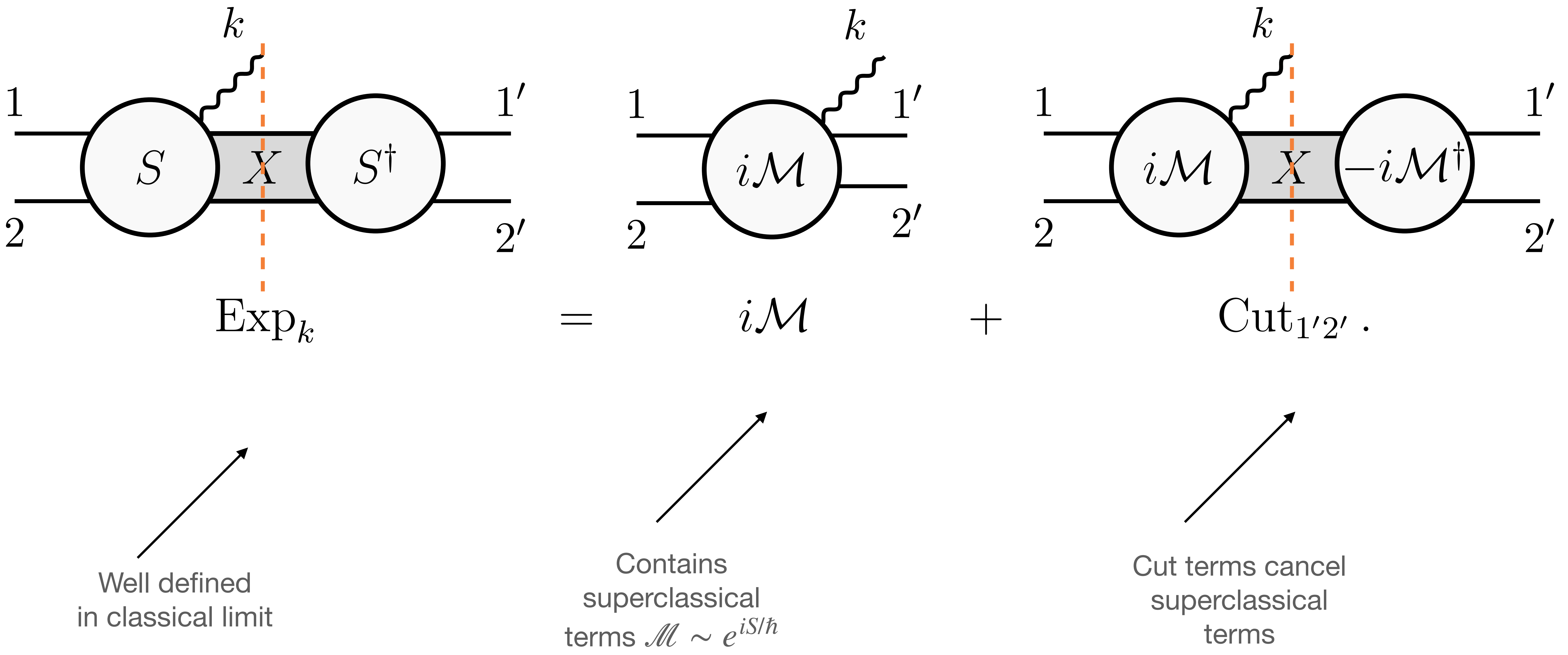
In the following the "in" label dropped to reduce clutter

Quantum Amplitudes and Radiation Observables

$R_h \sim \int d\Phi_X \;\; {}_{\text{in}}\langle \psi | S^\dagger | X \rangle_{\text{in}} \; {}_{\text{in}}\langle Xk | S | \psi \rangle_{\text{in}}$

In terms of usual amplitudes $S = 1 + i\delta^4(p)\mathscr{M}$

$$R_h \sim \langle \psi k | \mathscr{M} | \psi \rangle + \int d\Phi_X \;\; \langle \psi | \mathscr{M}^\dagger | X \rangle \langle Xk | \mathscr{M} | \psi \rangle$$



Quantum Amplitudes and Radiation Observables

Classical limit of R_h is the leading term under the classical (soft) scaling

momenta of matter particles representing classical object	$p_i \to \hbar^0 p_i$
masses of matter particles	$m_i \to \hbar^0 m_i$
momentum transfer for matter particles $q_i = p_i' - p_i$	$q_i \to \hbar^1 q_i$
momenta of radiation quanta	$k_n \to \hbar^1 k_n$
spins of matter particles	$S_i \to \hbar^{-1} S_i$

Waveforms from amplitudes

The rest is an exercise in wave function integration and extracting the leading 1/|x| behaviour

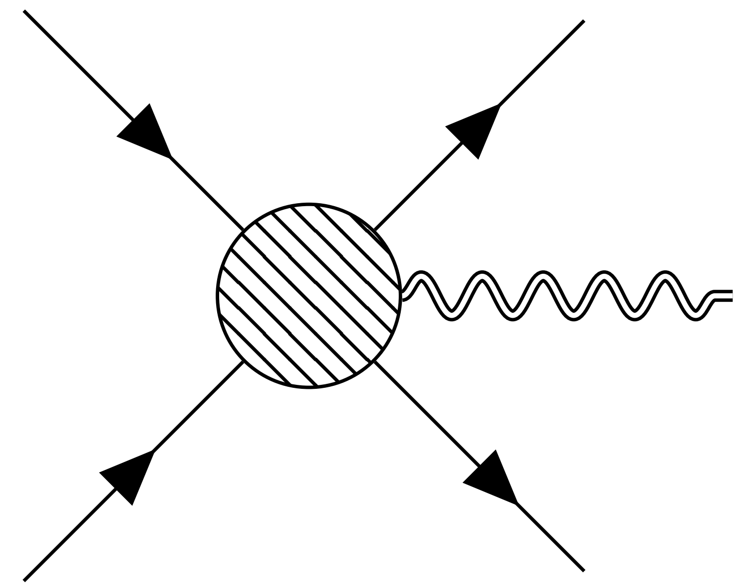
$$|\psi\rangle_{\text{in}} = \int \Pi_{i=1,2}[d\Phi(p_i) f_i(p_i) e^{ip_i b_i}] \, |p_1 p_2\rangle_{\text{in}}$$

At leading PM order KMOC formalism relates spectral waveform to integral of 5-point amplitude.

$$f_h(\omega) = \frac{1}{64\pi^3 m_1 m_2} \int d\mu \mathcal{M}^{\text{cl}}_{\text{tree}}[p_1 + w_1, p_2 + w_2 \to p_1, p_2, k]_{|k=\omega n}$$

$$d\mu \equiv \delta^4(w_1 + w_2 - k)\Pi_{i=1,2}[e^{ib_i w_i} d^4 w_i \delta(u_i w_i)]$$

Integration measure



In fact, not full 5-point amplitude but just its certain residues are needed to calculate the integral!

$$\text{R} \equiv \text{Res}_{w_2^2 \to 0} \mathcal{M}^{\text{cl}}_{\text{tree}}[p_1 + w_1, p_2 + w_2 \to p_1, p_2, k_h]$$

$$f_h(\omega) = -\frac{1}{64\pi^2 m_1 m_2 \sqrt{\gamma^2 - 1}} \int_{-\infty}^{\infty} \frac{dz e^{ib_1 k + iz(\hat{u}_1 k)b}}{\sqrt{z^2+1}} \frac{1}{2} \Big\{$$

$$R\Big(w_2 \to (\hat{u}_1 k)\big[\gamma \hat{u}_2 - \hat{u}_1 + z\tilde{b} + i\sqrt{z^2+1}\tilde{\text{v}}\big]\Big) + R\Big(w_2 \to (\hat{u}_1 k)\big[\gamma \hat{u}_2 - \hat{u}_1 + z\tilde{b} - i\sqrt{z^2+1}\tilde{\text{v}}\big]\Big)\Big\}$$

$$+(1 \leftrightarrow 2)$$

-

De Angelis, Novichkov, Gonzo [arXiv:2309.17429]

Waveforms from amplitudes

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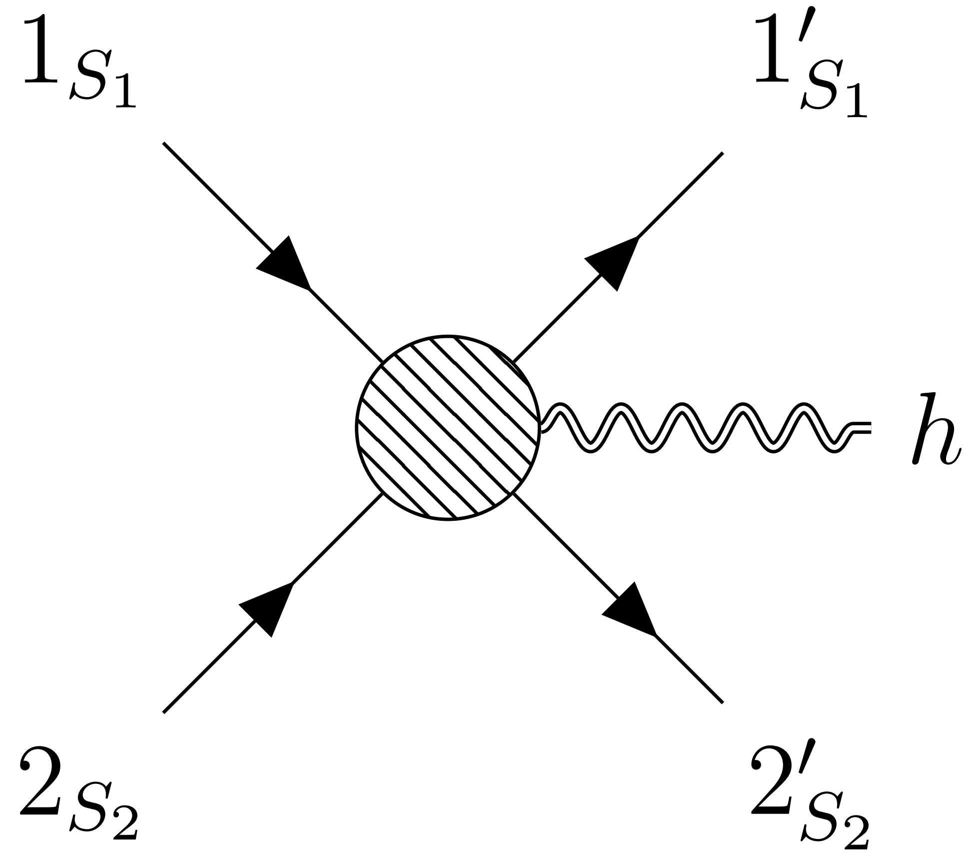
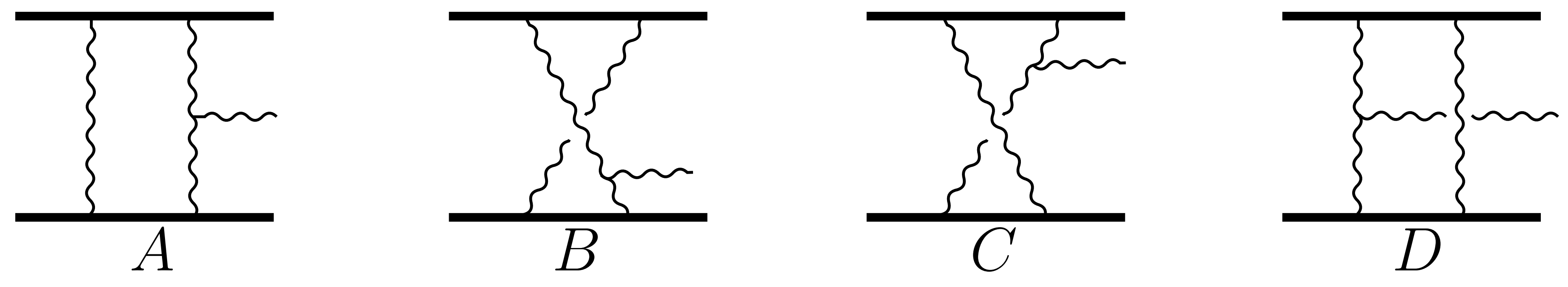
$$d\mu \equiv \delta^4(w_1 + w_2 - k)\Pi_{i=1,2}[e^{ib_i w_i} d^4 w_i \delta(u_i w_i)]$$

Integration measure



Calculation can be extended beyond tree level. Current state of the art is one loop



2303.06211
2303.06111
2303.06112
2303.07006
2310.12199

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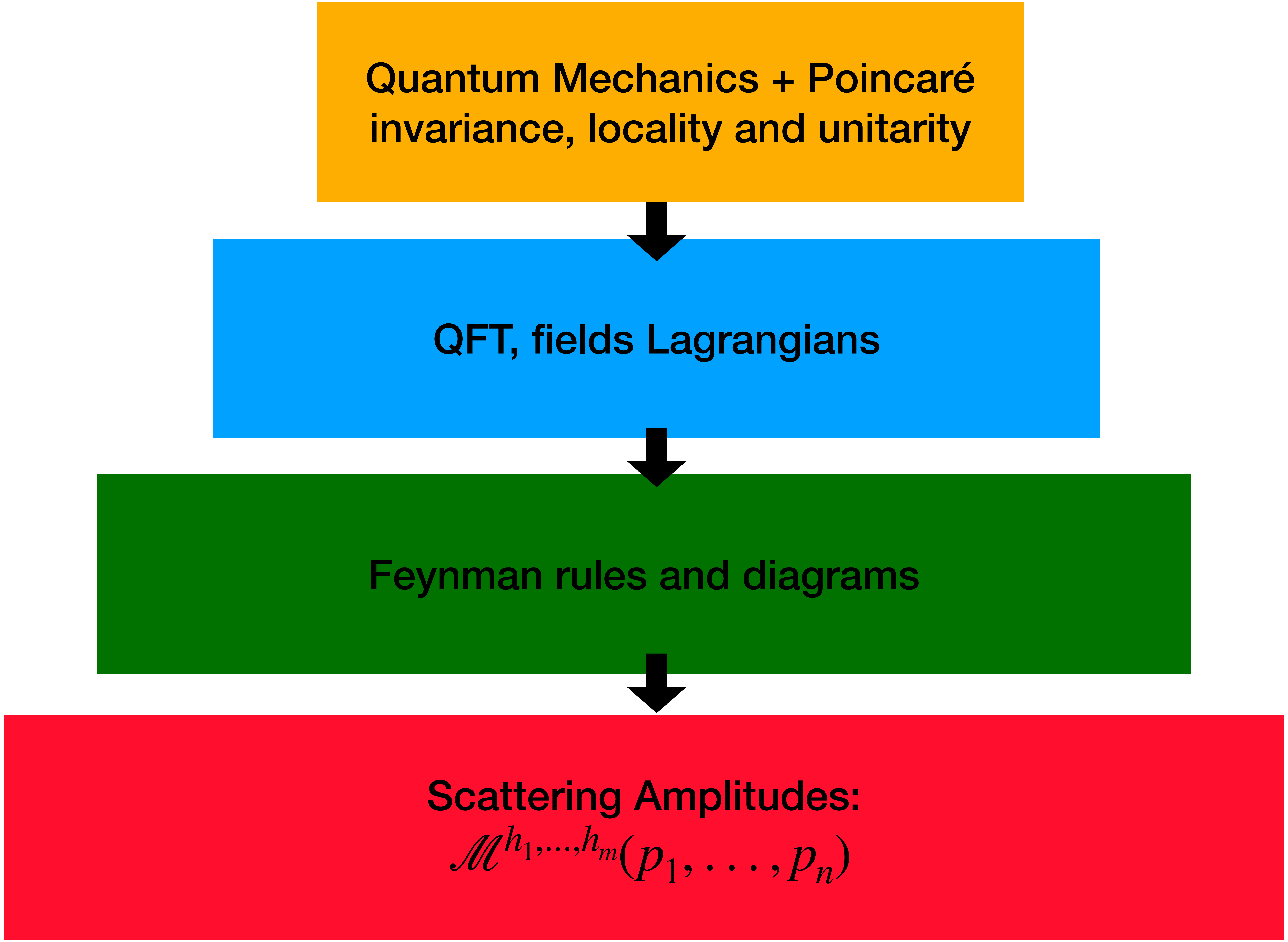
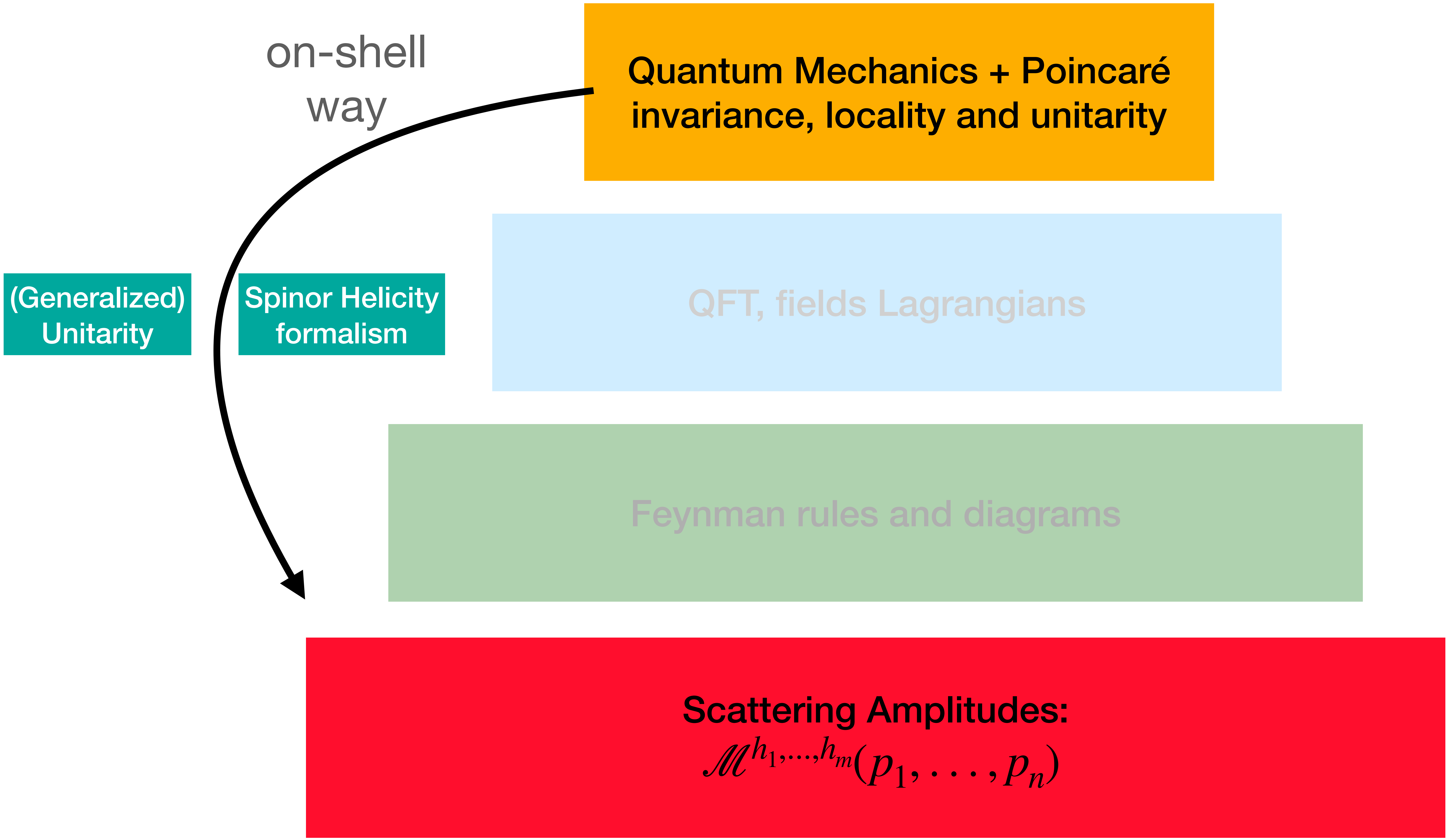
On-shell calculation of gravitational amplitudes

Quantum Amplitudes and Classical Observables: On-shell methods



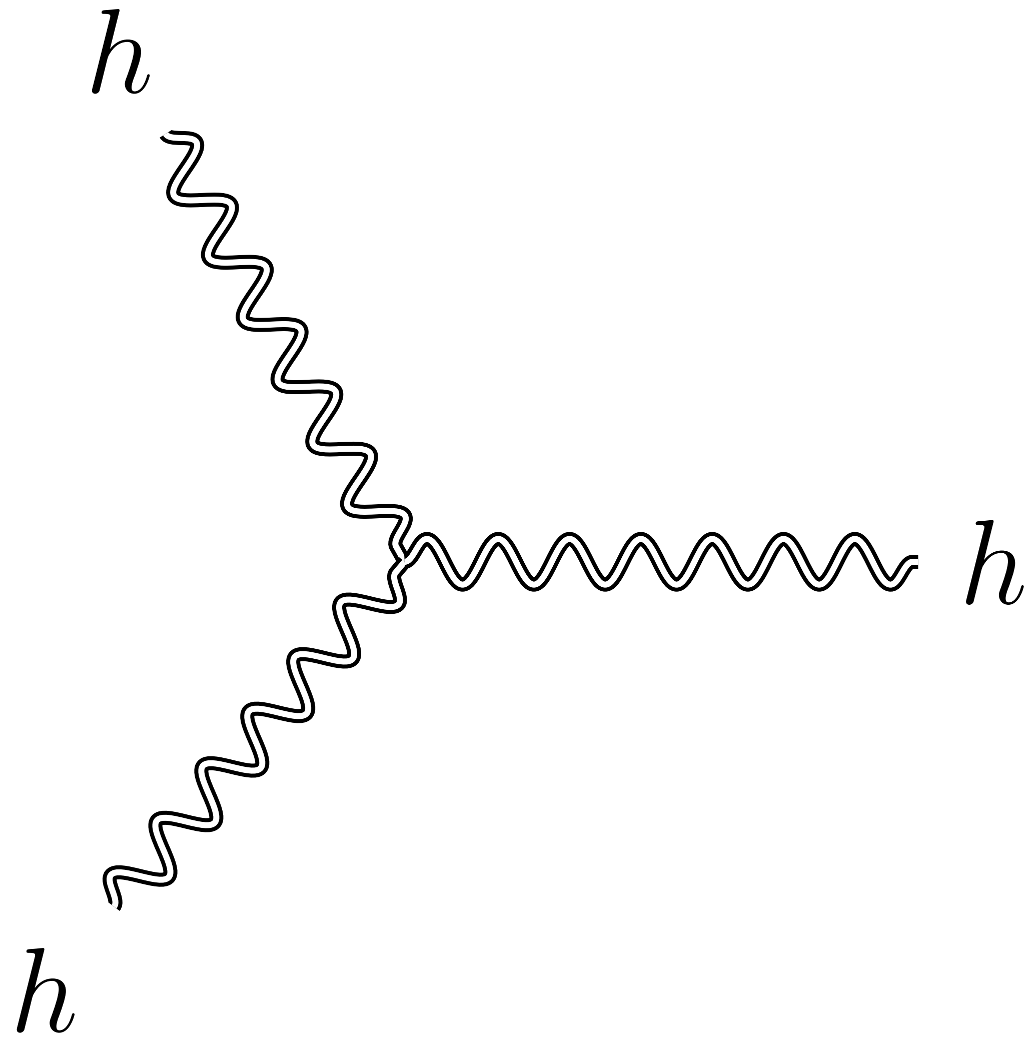
Quantum Amplitudes and Classical Observables: On-shell methods



Quantum Amplitudes and Classical Observables: On-shell methods

Basis building block are **on-shell 3-point amplitudes**

Pure general relativity



,

$$\mathcal{M}(1_h^-,2_h^-,3_h^+) = -\frac{1}{M_{Pl}}\frac{\langle 12\rangle^6}{\langle 13\rangle^2\langle 23\rangle^2}$$

$$\mathcal{M}(1_h^+,2_h^+,3_h^-) = -\frac{1}{M_{Pl}}\frac{[12]^6}{[13]^2[23]^2}\,.$$

$$|n\rangle \equiv \lambda_n$$

$$|n] \equiv \tilde{\lambda}_n$$

$$p_n\sigma = \lambda_n\tilde{\lambda}_n$$

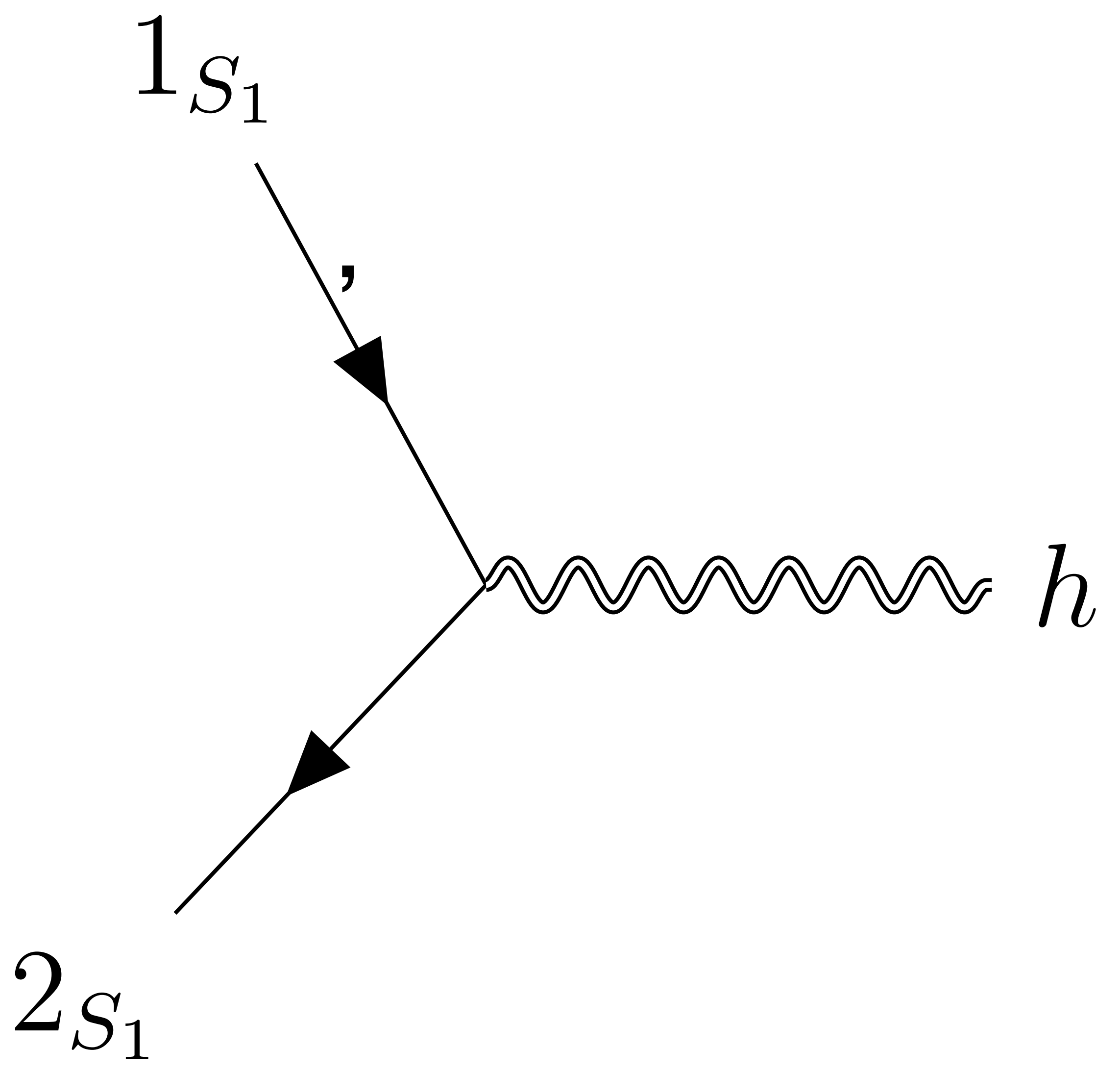
Quantum Amplitudes and Classical Observables: On-shell methods

Basis building block are **on-shell 3-point amplitudes**

Spinning matter representing black holes

$$\mathscr{M}\left[1_{\Phi}2_{\bar{\Phi}}3_h^-\right] = -\frac{\langle 3|p_1|\tilde{\zeta}]^2}{M_{Pl}[3\tilde{\zeta}]^2}\frac{[\mathbf{21}]^{2S}}{m^{2S}},$$

$$\mathscr{M}\left[1_{\Phi}2_{\bar{\Phi}}3_h^+\right] = -\frac{\langle \zeta|p_1|3]^2}{M_{Pl}\langle 3\zeta\rangle^2}\frac{\langle\mathbf{21}\rangle^{2S}}{m^{2S}},$$



$$|n\rangle \equiv \lambda_n$$

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Quantum Amplitudes and Classical Observables: On-shell methods

Basis building block are **on-shell 3-point amplitudes**

Spinning matter representing black holes

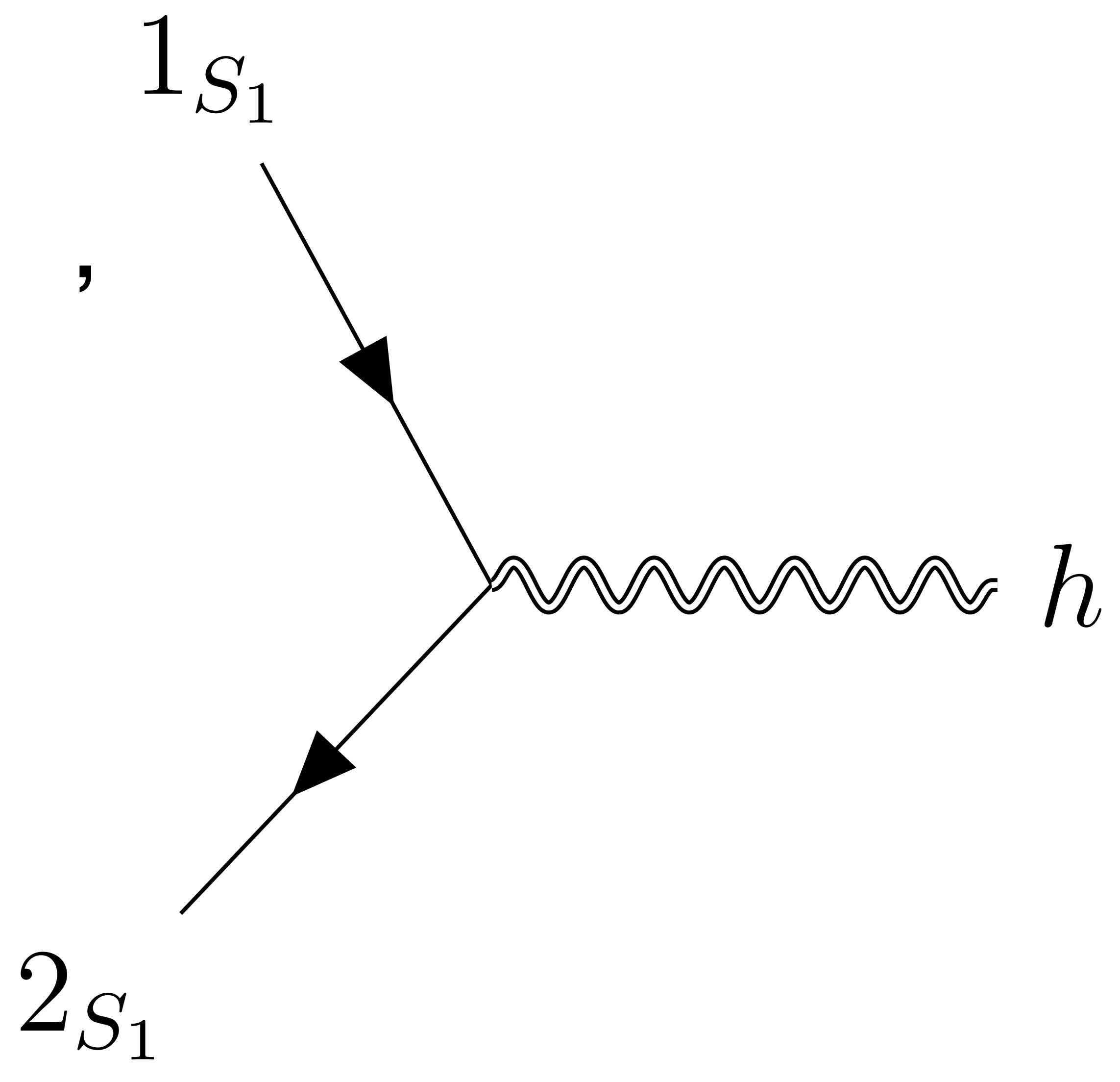
Classical limit

$$\mathcal{M}^{\rm cl}\left[1_\Phi 2_{\bar\Phi} 3_h^-\right] = -\frac{\langle 3|p_1|\tilde\zeta]^2}{M_{Pl}[3\tilde\zeta]^2}\exp(+p_3 a_1)$$

$$\mathcal{M}^{\rm cl}\left[1_\Phi 2_{\bar\Phi} 3_h^+\right] = -\frac{\langle \zeta|p_1|3]^2}{M_{Pl}\langle 3\zeta\rangle^2}\exp(-p_3 a_1)$$

,

Spin vector



$|n\rangle \equiv \lambda_n$

$|n] \equiv \tilde\lambda_n$

$p_n\sigma = \lambda_n\tilde\lambda_n$

Quantum Amplitudes and Classical Observables: Compton amplitudes

$$\mathscr{M}^{\rm cl}[1_\Phi 2_{\bar\Phi} 3_h^-] = -\frac{\langle 3|p_1|\tilde\zeta]^2}{M_{Pl}[3\tilde\zeta]^2}\exp(+p_3 a_1)$$

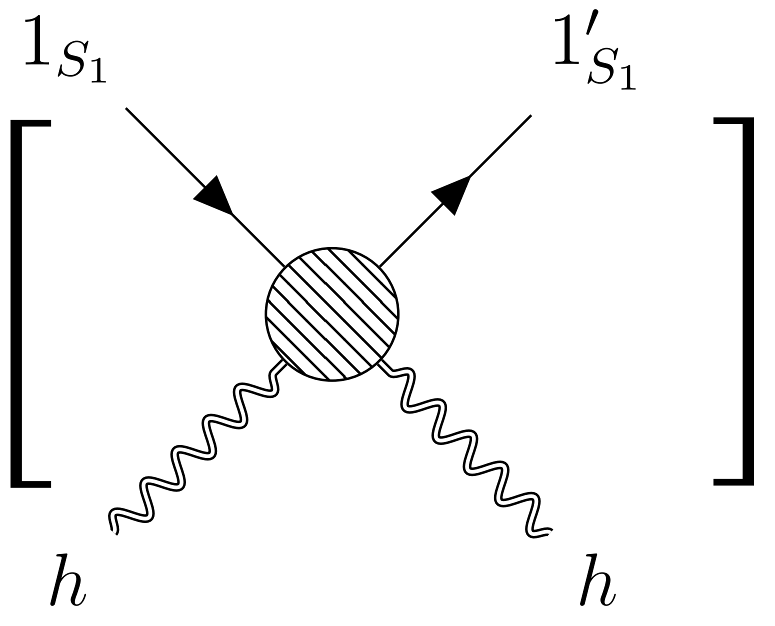
$$\mathscr{M}^{\rm cl}[1_\Phi 2_{\bar\Phi} 3_h^+] = -\frac{\langle \zeta|p_1|3]^2}{M_{Pl}\langle 3\zeta\rangle^2}\exp(-p_3 a_1)$$

$$\mathscr{M}(1_h^-, 2_h^-, 3_h^+) = -\frac{1}{M_{Pl}}\frac{\langle 12\rangle^6}{\langle 13\rangle^2\langle 23\rangle^2}$$

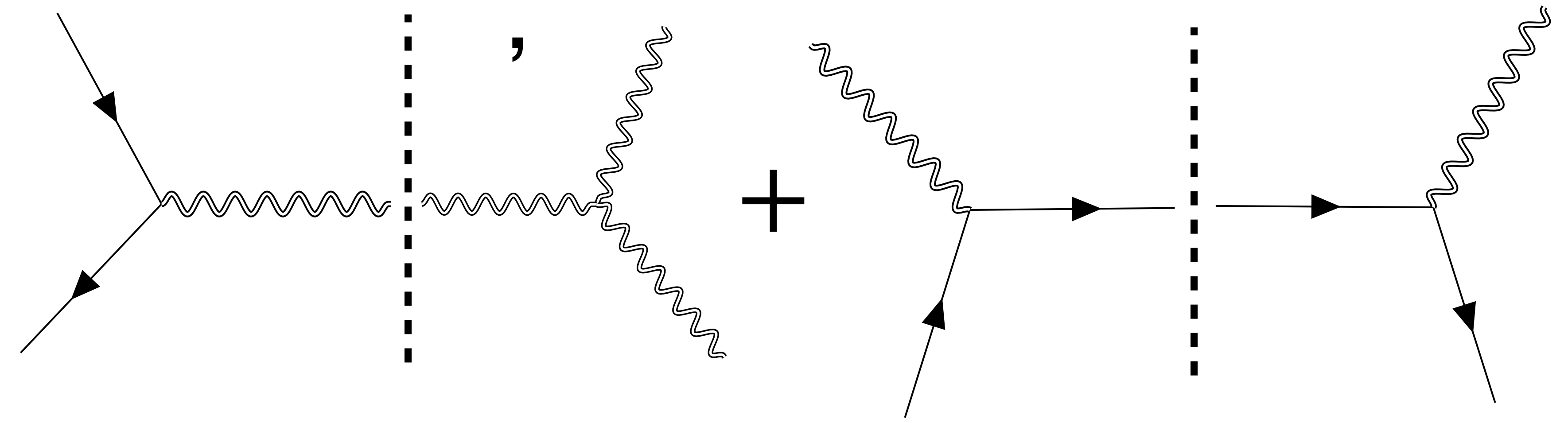
$$\mathscr{M}(1_h^+, 2_h^+, 3_h^-) = -\frac{1}{M_{Pl}}\frac{[12]^6}{[13]^2[23]^2}.$$

→ Build higher-point amplitudes (up to contact terms)
from their **residues** at kinematic poles in the **complex** plane

$$\sum_{i=2,3,4} \text{Res}_{(p_1+p_i)^2\to 0}\Big[$$



$$\Big]\Big|_{tree} = -\Big[$$



$$+(t\leftrightarrow u)\Big]$$

$$\mathscr{M}_U[1_h^- 2_h^+ 3_\Phi 4_{\bar\Phi}] = \frac{\langle 1|p_3|2]^4}{M_{\rm pl}^2 s(t-m^2)(u-m^2)}e^{a_3\tilde x} \qquad \mathscr{M}_U[1_h^- 2_h^- 3_\Phi 4_{\bar\Phi}] = \frac{m^4\langle 12\rangle^4}{M_{\rm Pl}^2 s(t-m^2)(u-m^2)}e^{a_3 q}$$

Contact terms start beyond quartic order in spin

$$q = p_1 + p_2$$

$$\tilde x^\mu \equiv \frac{i\epsilon^{\mu\nu\alpha\beta}(\lambda_1\sigma^\nu\tilde\lambda_2)p_3^\alpha p_4^\beta}{(\lambda_1 p_3\sigma\tilde\lambda_2)}$$

Quantum Amplitudes and Classical Observables: Compton amplitudes

$$\mathscr{M}^{\mathrm{cl}}[1_\Phi 2_{\bar\Phi} 3_h^-] = -\frac{\langle 3|p_1|\tilde\zeta]^2}{M_{Pl}[3\tilde\zeta]^2}\exp(+p_3 a_1)$$

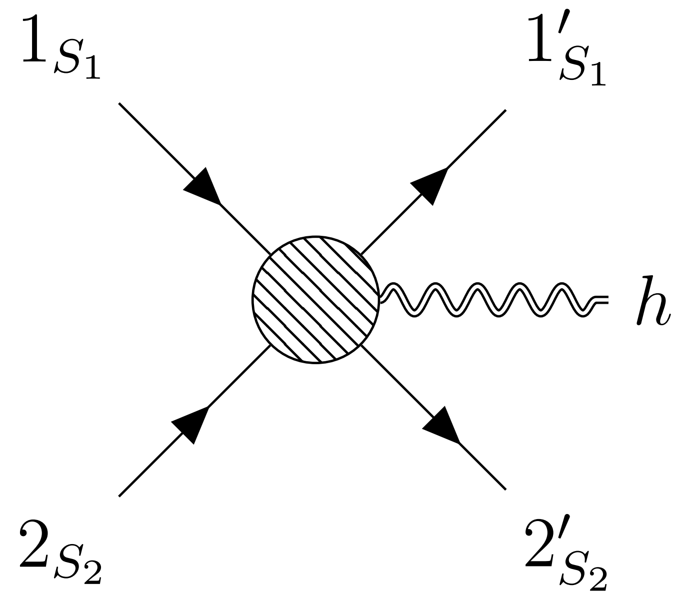
$$\mathscr{M}^{\mathrm{cl}}[1_\Phi 2_{\bar\Phi} 3_h^+] = -\frac{\langle \zeta|p_1|3]^2}{M_{Pl}\langle 3\zeta\rangle^2}\exp(-p_3 a_1)$$

$$\mathscr{M}(1_h^-,2_h^-,3_h^+) = -\frac{1}{M_{Pl}}\frac{\langle 12\rangle^6}{\langle 13\rangle^2\langle 23\rangle^2}$$

$$\mathscr{M}(1_h^+,2_h^+,3_h^-) = -\frac{1}{M_{Pl}}\frac{[12]^6}{[13]^2[23]^2}.$$

$$\mathscr{M}_U[1_h^- 2_h^+ 3_\Phi 4_{\bar\Phi}] = \frac{(\lambda_1 p_3 \sigma \tilde\lambda_2)^4}{M_{\mathrm{pl}}^2 s(t-m^2)(u-m^2)} e^{a_3 \tilde x} \qquad \mathscr{M}_U[1_h^- 2_h^- 3_\Phi 4_{\bar\Phi}] = \frac{m^4(\lambda_1\lambda_2)^4}{M_{\mathrm{Pl}}^2 s(t-m^2)(u-m^2)} e^{a_3 q}$$

→ Build higher-point amplitudes (up to contact terms)
from their **residues** at kinematic poles in the **complex** plane



$$R^s \equiv \mathrm{Res}_{w_2^2\to 0}\mathscr{M}[(p_1+w_1)_{\Phi_1}(p_2+w_2)_{\Phi_2}(-p_1)_{\bar\Phi_1}(-p_2)_{\bar\Phi_2}(-k)_h^s]$$
$$= -\sum_{s'}\mathscr{M}\left[(-k)_h^s (w_2)_h^{s'}(p_1+w_1)_{\Phi_1}(-p_1)_{\bar\Phi_1}\right]\mathscr{M}\left[(-w_2)_h^{-s'}(p_2+w_2)_{\Phi_2}(-p_2)_{\bar\Phi_2}\right]$$

$$R_{(0)}^- = \frac{\left[2(p_1p_2)\Lambda[w_2,p_1] - m_1^2\Lambda[w_2,p_2] + 2(p_1k)\Lambda[p_1,p_2]\right]^2 + m_1^4\Lambda[w_2,p_2]^2}{8M_{\mathrm{Pl}}^3(w_2k)(p_1k)^2}$$

$$\Lambda[a,b,\ldots] \equiv (\lambda_k a\sigma b\bar\sigma\ldots\lambda_k)$$

$$R_{(1)}^- = -\frac{1}{8M_{\mathrm{Pl}}^3(w_2k)(p_1k)^2}\Big\{\left[a_1k + a_1w_2 + a_2w_2\right]\left[2(p_1p_2)\Lambda[w_2,p_1] - m_1^2\Lambda[w_2,p_2] + 2(p_1k)\Lambda[p_1,p_2]\right]^2$$
$$+m_1^4\left[a_1k - a_1w_2 - a_2w_2\right]\Lambda[w_2,p_2]^2 - 2(p_1k)\left[2(p_1p_2)\Lambda[w_2,p_1] - m_1^2\Lambda[w_2,p_2] + 2(p_1k)\Lambda[p_1,p_2]\right]\Lambda[p_1,p_2,w_2,a_1]\Big\}.$$

$$q = p_1 + p_2$$

$$\tilde x^\mu \equiv \frac{i\epsilon^{\mu\nu\alpha\beta}(\lambda_1\sigma^\nu\tilde\lambda_2)p_3^\alpha p_4^\beta}{(\lambda_1 p_3\sigma\tilde\lambda_2)}$$

$$R_{(2)}^- = \ldots$$

Quantum Amplitudes and Classical Observables: gravitational waves

Waveform in GR calculated as expansion in spin $W_h = W_h^{(0)} + W_h^{(1)} + \ldots$

$$T_i \equiv \frac{t - b_i n}{(\hat{u}_i n) b}$$

Kovacs Thorne
Astrophys. J. 217 (1977) 252
Astrophys. J. 224 (1978) 62.

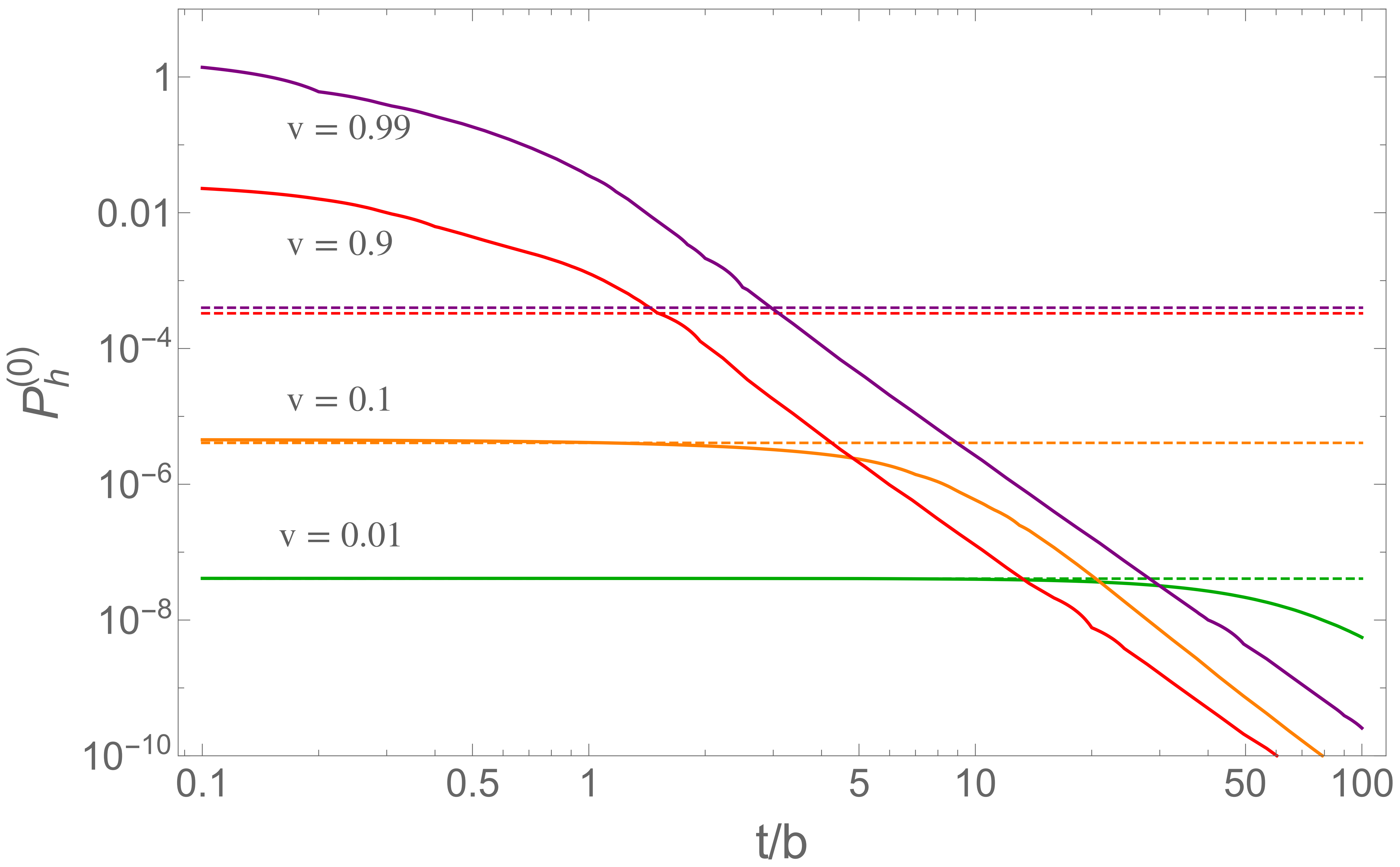
At leading order

$$W_h^{(0)} = -\frac{m_1 m_2}{512\pi^2 M_{\rm Pl}^3 b (\hat{u}_1 n)^2 \sqrt{\gamma^2 - 1}} \frac{1}{\sqrt{z^2+1}} \mathscr{R}\left\{ \frac{\left(\mathscr{F}_1^-(z)\right)^2 + \left(\mathscr{F}_2^-(z)\right)^2}{\gamma(\hat{u}_2 n) - (\hat{u}_1 n) + z(\tilde{b} n) + i\sqrt{z^2+1}(\tilde{v} n)} \right\}_{|z=T_1} + (1 \leftrightarrow 2)$$

$$\mathscr{F}_1^-(z) = \langle n | \left(\hat{u}_1 \hat{u}_2 + 2\gamma z \hat{u}_1 \tilde{b} + z \tilde{b} \hat{u}_2 + 2i\gamma\sqrt{z^2+1}\, \hat{u}_1 \tilde{v} + i\sqrt{z^2+1}\, \tilde{v} \hat{u}_2 \right) | n \rangle$$

$$\mathscr{F}_2^-(z) = \langle n | \left(\hat{u}_1 - z\tilde{b} - i\sqrt{z^2+1}\, \tilde{\rm v} \right) \hat{u}_2 | n \rangle$$

Emitted power in gravitational waves $\dfrac{dP_h}{d\Omega} = 2\,|\partial_t W_h|^2$



For $|t| \lesssim b/{\rm v}$ in the rest frame of particle 2

$$\partial_t W_h^{(0)} = -\, e^{-2i\phi} \frac{m_1 m_2}{16\pi^2 M_{\rm Pl}^3 b^2} \left\{ (\hat{\mathbf{v}}\hat{\boldsymbol{n}})(\hat{\boldsymbol{b}}\hat{\boldsymbol{n}}) + i(\hat{\mathbf{v}} \times \hat{\boldsymbol{b}}) \cdot \hat{\boldsymbol{n}} \right\} {\rm v} + O({\rm v}^2)$$

velocity suppression

Emitted power

$$\frac{dP_h^{(0)}}{d\Omega} = \frac{m_1^2 m_2^2}{128\pi^4 M_{\rm Pl}^6 b^4} \left[(\hat{\mathbf{v}}\hat{\boldsymbol{n}})^2 (\hat{\boldsymbol{b}}\hat{\boldsymbol{n}})^2 + (\hat{\mathbf{v}} \times \hat{\boldsymbol{b}} \cdot \hat{\boldsymbol{n}})^2 \right] {\rm v}^2 + \mathscr{O}({\rm v}^3)$$

Total emitted power

$$P_h^{(0)} = \frac{m_1^2 m_2^2}{80\pi^3 M_{\rm Pl}^6 b^4} {\rm v}^2$$

Radiation in scalar-tensor theories

Scalar-Tensor Theories

→ **Scalar-tensor theories** have long been a popular direction to study **extensions of GR**

→ They consist of gravity theories with the introduction of an additional **massless scalar** degree of freedom

$$S_{GR}[g_{\mu\nu}] \to S_{ST}[g_{\mu\nu}, \phi]$$

Example: Scalar Gauss-Bonnet and Dynamical Chern Simons gravity

$$S = \int d^4x \frac{M_{Pl}^2}{2}\sqrt{-g}R + S_{SGB,DCS}[\phi, g_{\mu\nu}] + S_m[\Psi_m, \mathscr{A}(\phi)g_{\mu\nu}]$$

Gauss Bonnet invariant

$$R^{\mu\nu\rho\sigma}R_{\mu\nu\rho\sigma} - 4R^{\mu\nu}R_{\mu\nu} + R^2$$

Chern Simons invariant

$$R^{\mu\nu\rho\sigma}\tilde{R}_{\mu\nu\rho\sigma} \quad , \quad \tilde{R}^{\mu}{}_{\nu\rho\sigma} = \frac{1}{2}\varepsilon_{\rho\sigma}{}^{\alpha\beta}R^{\mu}{}_{\nu\alpha\beta}$$

- $$S_{SGB,DCS} = \int d^4x\sqrt{-g}\left[\frac{M_{\rm Pl}^2}{\Lambda^2}\left(f(\phi)\mathscr{G} + \tilde{f}(\phi)R\tilde{R}\right) + \frac{1}{2}\left(\partial^\mu\phi\partial_\mu\phi\right)\right]$$

$$f(\phi) = \text{const} + \alpha\frac{\phi}{M_{\rm Pl}} + O(\phi^2) \qquad \tilde{f}(\phi) = \text{const} + \tilde{\alpha}\frac{\phi}{M_{\rm Pl}} + O(\phi^2)$$

GW observations constrain them!

Phys.Rev.Lett. 126 (2021) 18, 181101 [Silva, Holgado, Cárdenas-Avendaño, Yunes]
Phys.Rev.D 107 (2023) 4, 044030 [Silva, Ghosh, Buonan
arXiv: 2406.13654 [Julié, Pompili, Buonanno]

<u>Experimental window:</u>

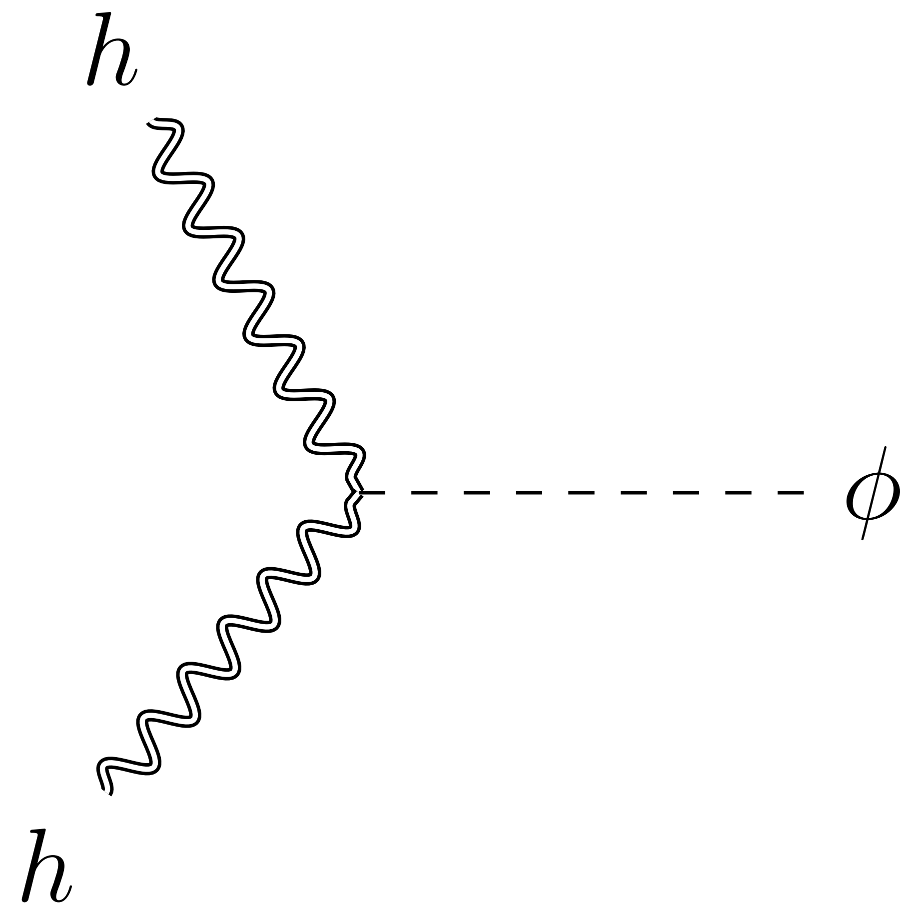
$$\frac{\sqrt{\alpha}}{\Lambda} \lesssim 0.22km \quad , \quad \frac{\sqrt{\tilde{\alpha}}}{\Lambda} \lesssim 9.5km$$

Scalar-Tensor Theories: shift-symmetric limit

- $S_{SGB,DCS} = \int d^4x\sqrt{-g}\left[\frac{M_{Pl}^2}{\Lambda^2}\left(f(\phi)\mathcal{G} + \tilde{f}(\phi)R\tilde{R}\right) + \frac{1}{2}\left(\partial^\mu\phi\partial_\mu\phi\right)\right]$

$$f(\phi) = \text{const} + \alpha\frac{\phi}{M_{\text{Pl}}} + \mathcal{O}(\phi^2) \qquad \tilde{f}(\phi) = \text{const} + \tilde{\alpha}\frac{\phi}{M_{\text{Pl}}} + \mathcal{O}(\phi^2)$$

The linear term in ϕ induces a cubic interaction between scalar and gravity corresponding to 3-point amplitudes



$$\mathcal{M}\left[1_h^- 2_h^- 3_\phi\right] = \frac{2\hat{\alpha}}{\Lambda^2 M_{\text{pl}}}\langle 12\rangle^4$$

$$\mathcal{M}\left[1_h^+ 2_h^+ 3_\phi\right] = \frac{2\hat{\alpha}^*}{\Lambda^2 M_{\text{pl}}}[12]^4$$

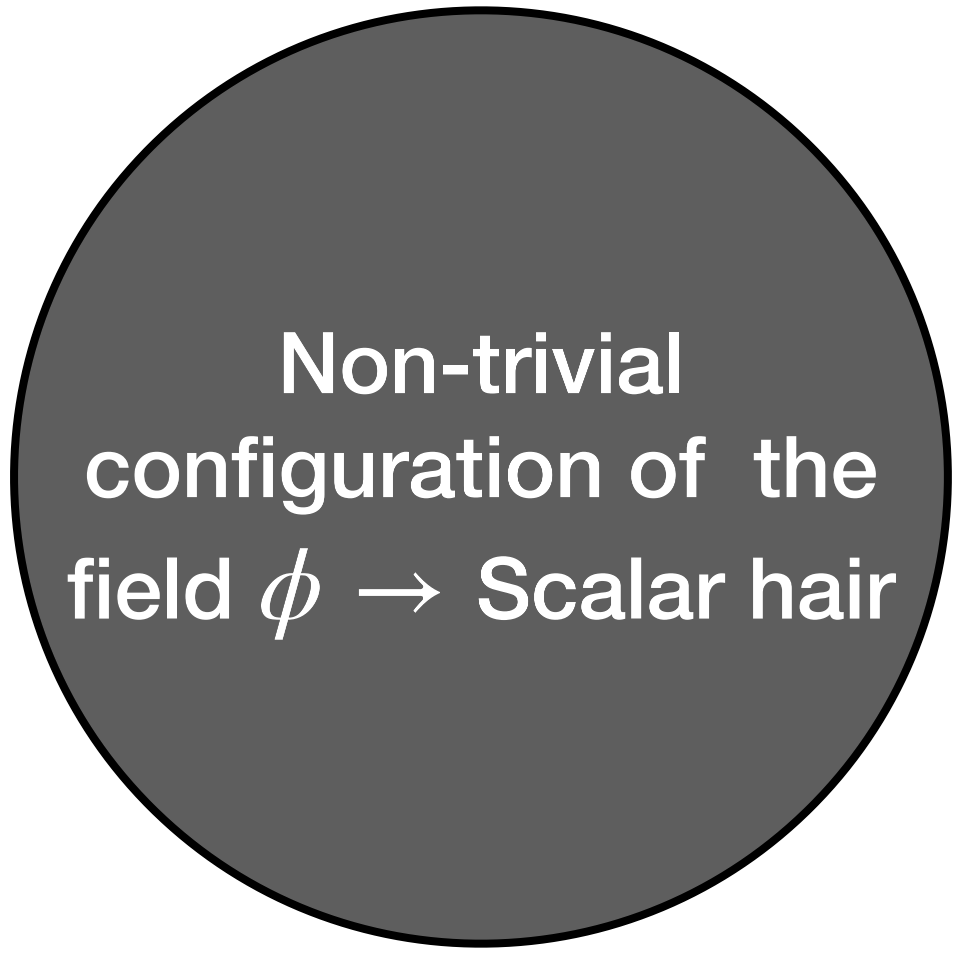
$$\mathcal{M}\left[1_h^- 2_h^+ 3_\phi\right] = 0$$

$$\hat{\alpha} \equiv \alpha + i\tilde{\alpha}$$

However in large classes of scalar-tensor theories the leading corrections to gravitational radiation comes from a different interaction...

Scalar-Tensor Theories: scalar hair

Compact objects can acquire scalar hair in scalar-tensor theories
This is the case for black holes in SGB and DCS



BH solution in ST theory

Far zone

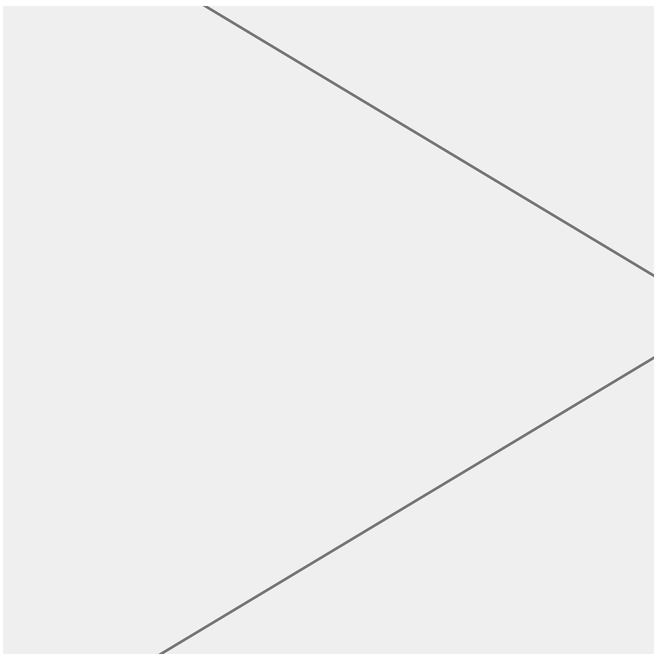
$x \rightarrow \infty$

"Monopole hair" "Dipole hair"

$$\phi = \frac{c_i}{r} + \frac{d_i}{r^2} + \dots$$

$$c_n \sim \alpha \frac{M_{\mathrm{Pl}}^4}{\Lambda^2 m_n^2}$$

How can we model this behaviour with amplitudes?



Scalar-Tensor Theories: scalar hair on shell

AA, Marinellis
2411.12909

We model the black hole as a point-particle interacting with the scalar field via an effective metric (scalar conformal coupling)

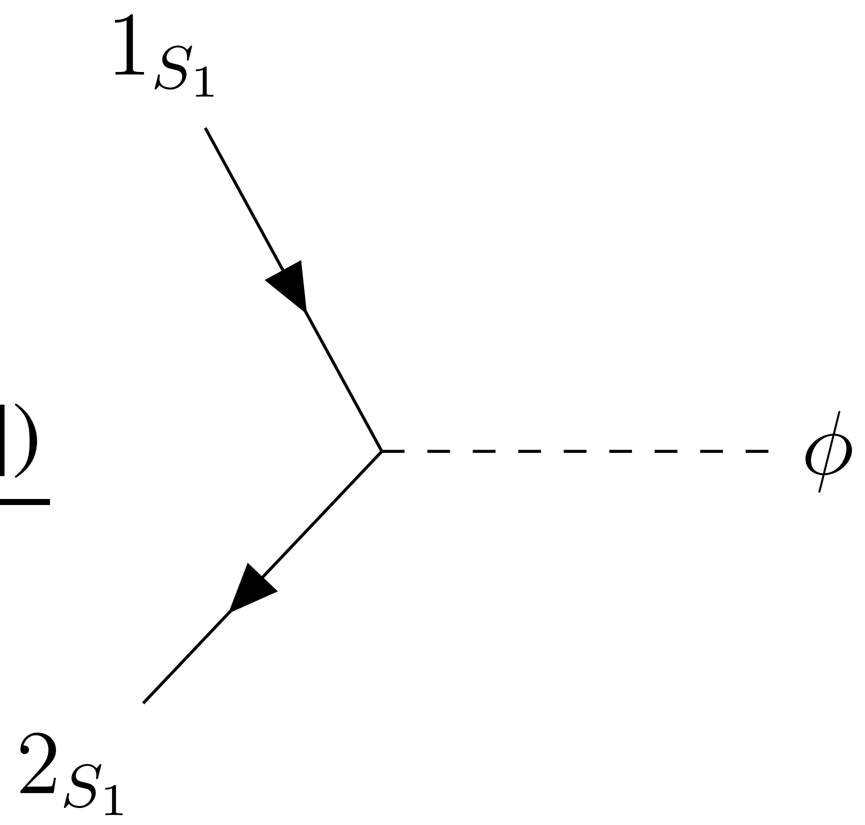
$$\tilde{g}_{\mu\nu} = \exp\left[C\left(\frac{\phi}{M_{Pl}}\right)\right] g_{\mu\nu}$$

$$\boxed{\exp\left[C\left(\frac{\phi}{M_{Pl}}\right)\right] \approx 1 + c\frac{\phi}{M_{Pl}}}$$

3-point amplitudes for arbitrary spinning black holes:

$$\mathcal{M}_{3,bos.}[1_{\Phi_n}, 2_{\bar{\Phi}_n} 3_\phi] = -\frac{c_n}{M_{Pl}} \frac{\langle \mathbf{21} \rangle^{S_n} [\mathbf{21}]^{S_n}}{m_n^{2S_n - 2}}$$

$$\mathcal{M}_{3,ferm.}[1_{\Psi_n}, 2_{\bar{\Psi}_n} 3_\phi] = -\frac{c_n}{M_{Pl}} \frac{\langle \mathbf{21} \rangle^{S_n - 1/2} [\mathbf{21}]^{S_n - 1/2} (\langle \mathbf{21} \rangle + [\mathbf{21}])}{m_n^{2S-2}}$$



In the classical limit
conformal coupling is spin-independent!

$$\mathcal{M}^{\text{cl}}[1_{\Phi_n}, 2_{\bar{\Phi}_n} 3_\phi] = -\frac{c_n m_n^2}{M_{Pl}}$$

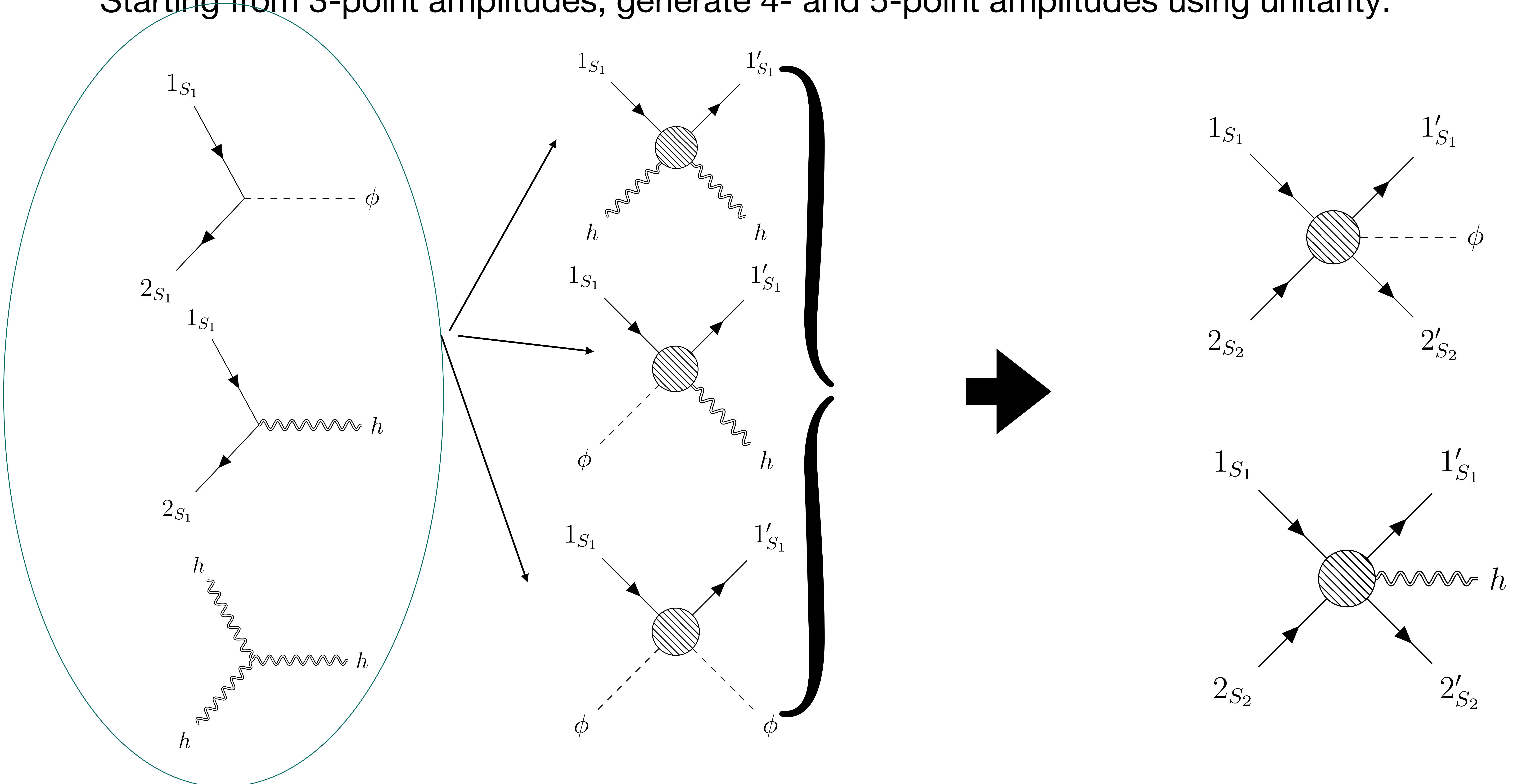
At the lagrangian level for any spin coupling can be obtained by mass redefinition:

$$m \to e^{C/2} m$$

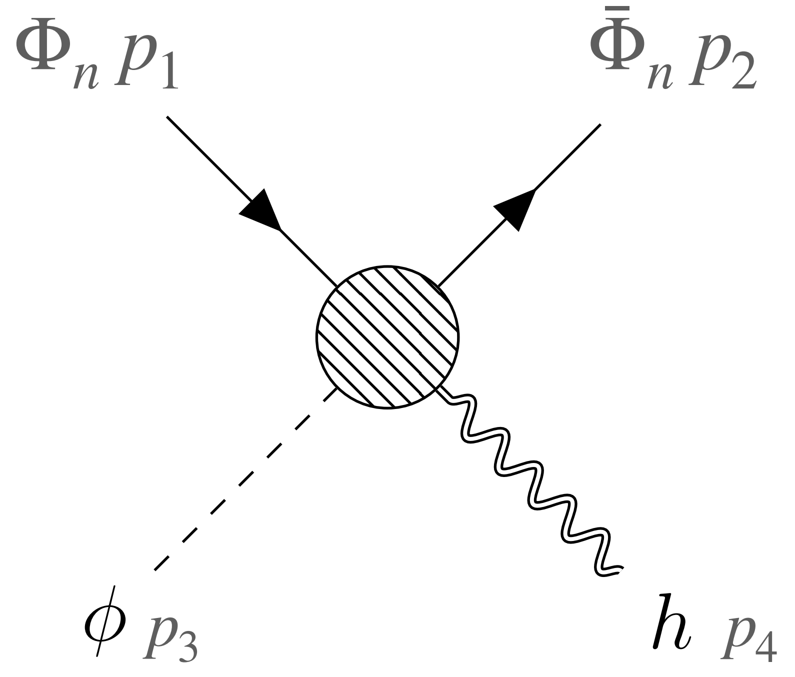
Classical conformal coupling maps to monopole charge in scalar tensor theory

Scalar-Tensor Theories: amplitudes

Starting from 3-point amplitudes, generate 4- and 5-point amplitudes using unitarity:

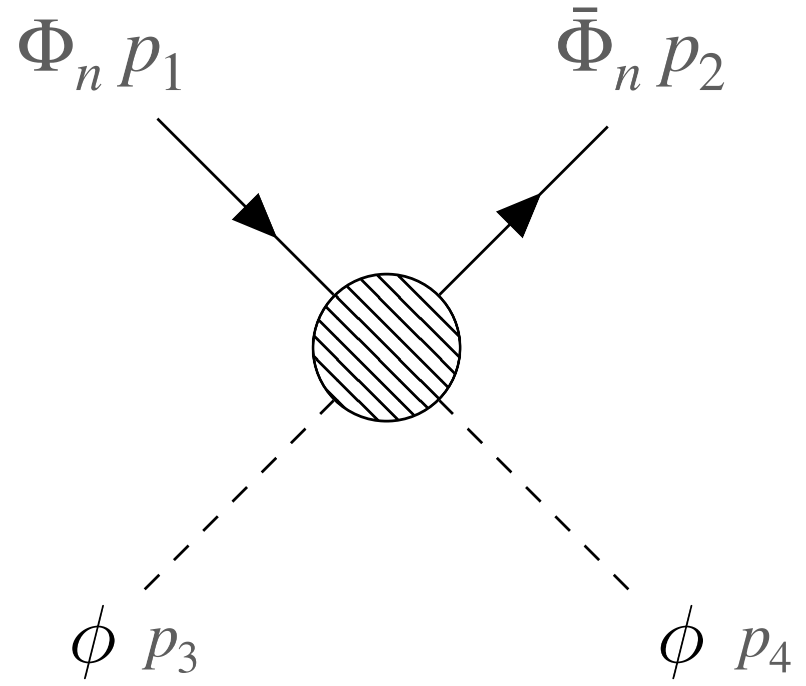


Scalar-Tensor Theories: amplitudes



$$\mathcal{M}_U^{\rm cl}\left[1_{\Phi_n} 2_{\bar{\Phi}_n} 3_\phi 4_h^-\right] = -\frac{c_n m_n^2}{M_{\rm Pl}^2} \frac{\langle 4 | p_3 p_1 | 4 \rangle^2}{s(t-m_n^2)^2}$$

Contact terms start at quadratic order in spin



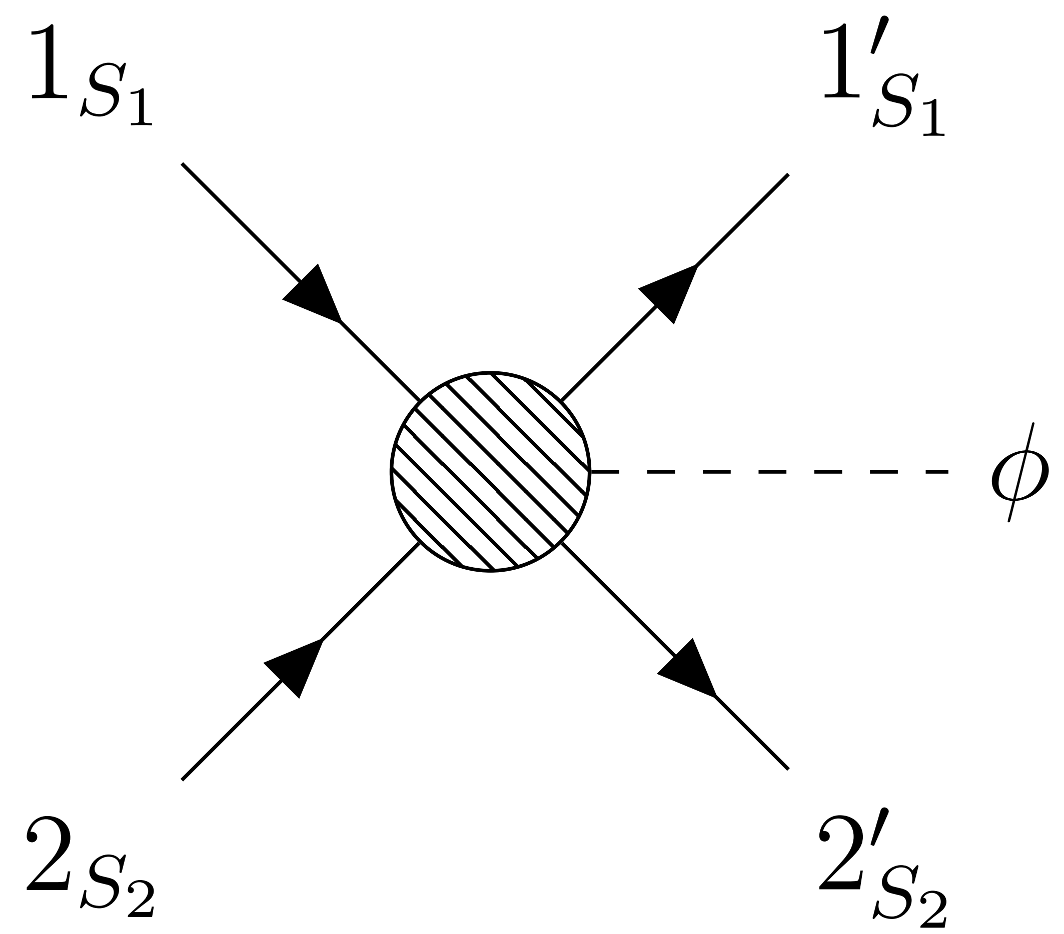
$$\mathcal{M}_U^{\rm cl}\left[1_{\Phi_n} 2_{\bar{\Phi}_n} 3_\phi 4_\phi\right] = \frac{t-m_n^2}{M_{\rm Pl}^2 s}\left\{-(t-m_n^2)\cosh(q\cdot a_n) + 2i\varepsilon_{\mu\nu\rho\sigma} p_1^\mu p_3^\nu p_4^\rho a_n^\sigma \frac{\sinh(q\cdot a_n)}{q\cdot a_n}\right\} + \mathcal{O}(c_n^2)$$

Contact terms start at zeroth-order in spin

$$\mathcal{M}_C^{\rm cl,(0)}\left[1_{\Phi_n} 2_{\bar{\Phi}_n} 3_\phi 4_\phi\right] = \frac{C_n^{(0)} m_n^2}{M_{\rm Pl}^2}$$

$$\mathcal{M}_C^{\rm cl,(1)}\left[1_{\Phi_n} 2_{\bar{\Phi}_n} 3_\phi 4_\phi\right] = -i\frac{C_n^{(1)} m_n^2}{M_{\rm Pl}^2}(q a_n)$$

Scalar-Tensor Theories: amplitudes



$$R_X = -\sum_{i=h,\phi} \mathscr{M}^{\rm cl}_X[(p_1+w_1)_{\Phi_1}(-p_1)_{\bar\Phi_1}(-k)_\phi(w_2)_i]\mathscr{M}^{\rm cl}[(p_2+w_2)_{\Phi_2}(-p_2)_{\bar\Phi_2}(-w_2)_i]$$

One finds the pole part of the residue to all orders in spin, plus contact terms in systematic expansion in spin vector

Part originating from pole terms in 4-point amplitude

$$R_U = \frac{2}{M_{\rm Pl}^3}\left\{\frac{(kw_2)[(p_1p_2)^2 - \frac{1}{2}m_1^2m_2^2] + m_2^2(p_1k)^2 - 2(p_1p_2)(p_1k)(p_2k)}{(p_1k)^2}c_1m_1^2\cosh(w_2a_2)\right.$$
$$+ i\frac{c_1m_1^2(p_1p_2)}{(p_1k)^2}p_1^\mu p_2^\nu k^\rho w_2^\sigma \varepsilon_{\mu\nu\rho\sigma}\sinh(w_2a_2) - 2\frac{c_2m_2^2(p_1k)^2\cosh(w_1a_1) + c_1m_1^2(p_2k)^2\cosh(w_2a_2)}{w_1^2}$$
$$\left.+\frac{2i\varepsilon_{\mu\nu\rho\sigma}p_1^\mu k^\nu w_2^\rho}{w_1^2}\left[a_1^\sigma(p_1k)\frac{c_2m_2^2\sinh(w_1a_1)}{w_1a_1} + p_2^\sigma\frac{c_1m_1^2(p_2k)}{p_1k}\sinh(w_2a_2)\right]\right\}$$

Part originating from contact terms in 4-point amplitude

$$R_C^{(0)} = \frac{C_1^{(0)}c_2m_1^2m_2^2}{M_{\rm Pl}^3} \qquad R_C^{(1)} = -i\frac{C_1^{(1)}c_2m_1^2m_2^2}{M_{\rm Pl}^3}(w_1a_1) \qquad R_C^{(2)} = \ldots$$

Scalar-Tensor Theories: scalar waveforms

Scalar radiation observable $\mathscr{R}_\phi \equiv {}_{\text{out}}\langle\psi|\phi(x)|\psi\rangle_{\text{out}}$

Scalar waveform $\mathscr{R}_X(x) = \dfrac{W_X(t)}{|\boldsymbol{x}|}$ $|\boldsymbol{x}| \to \infty$ $t \equiv x^0 - |\boldsymbol{x}|$

Using KMOC one can relate the scalar waveshape to the residues of the 5-point scalar emission amplitude

$$\mathrm{R} \equiv \mathrm{Res}_{w_2^2\to 0}\mathscr{M}^{\text{cl}}_{\text{tree}}[p_1 + w_1, p_2 + w_2 \to p_1, p_2, k_\phi]$$

$$f_\phi(\omega) = -\frac{1}{64\pi^2 m_1 m_2\sqrt{\gamma^2-1}}\int_{-\infty}^{\infty}\frac{dz e^{ib_1k+iz(\hat{u}_1k)b}}{\sqrt{z^2+1}}\frac{1}{2}\Big\{$$

$$R\big(w_2 \to (\hat{u}_1k)\big[\gamma\hat{u}_2 - \hat{u}_1 + z\tilde{b} + i\sqrt{z^2+1}\,\tilde{\mathrm{v}}\big]\big) + R\big(w_2 \to (\hat{u}_1k)\big[\gamma\hat{u}_2 - \hat{u}_1 + z\tilde{b} - i\sqrt{z^2+1}\,\tilde{\mathrm{v}}\big]\big)\Big\} \qquad k = \omega n$$

$$+(1 \leftrightarrow 2)$$

From this, waveform is calculated via inverse Fourier transform

$$W_\phi(t) = \int_{-\infty}^{\infty}\frac{d\omega}{2\pi}e^{-i\omega t}f_\phi(\omega)$$

Scalar-Tensor Theories: scalar waveforms

$$W_\phi = W_\phi^{(0)} + W_\phi^{(1)} + \dots$$

$$T_i \equiv \frac{t - b_i n}{(\hat{u}_i n) b} \qquad \hat{u}_i = \frac{u_i}{\sqrt{\gamma^2 - 1}} \qquad \gamma = \frac{1}{\sqrt{1 - \mathrm{v}^2}}$$

LO Scalar Waveforms-Spinless part:

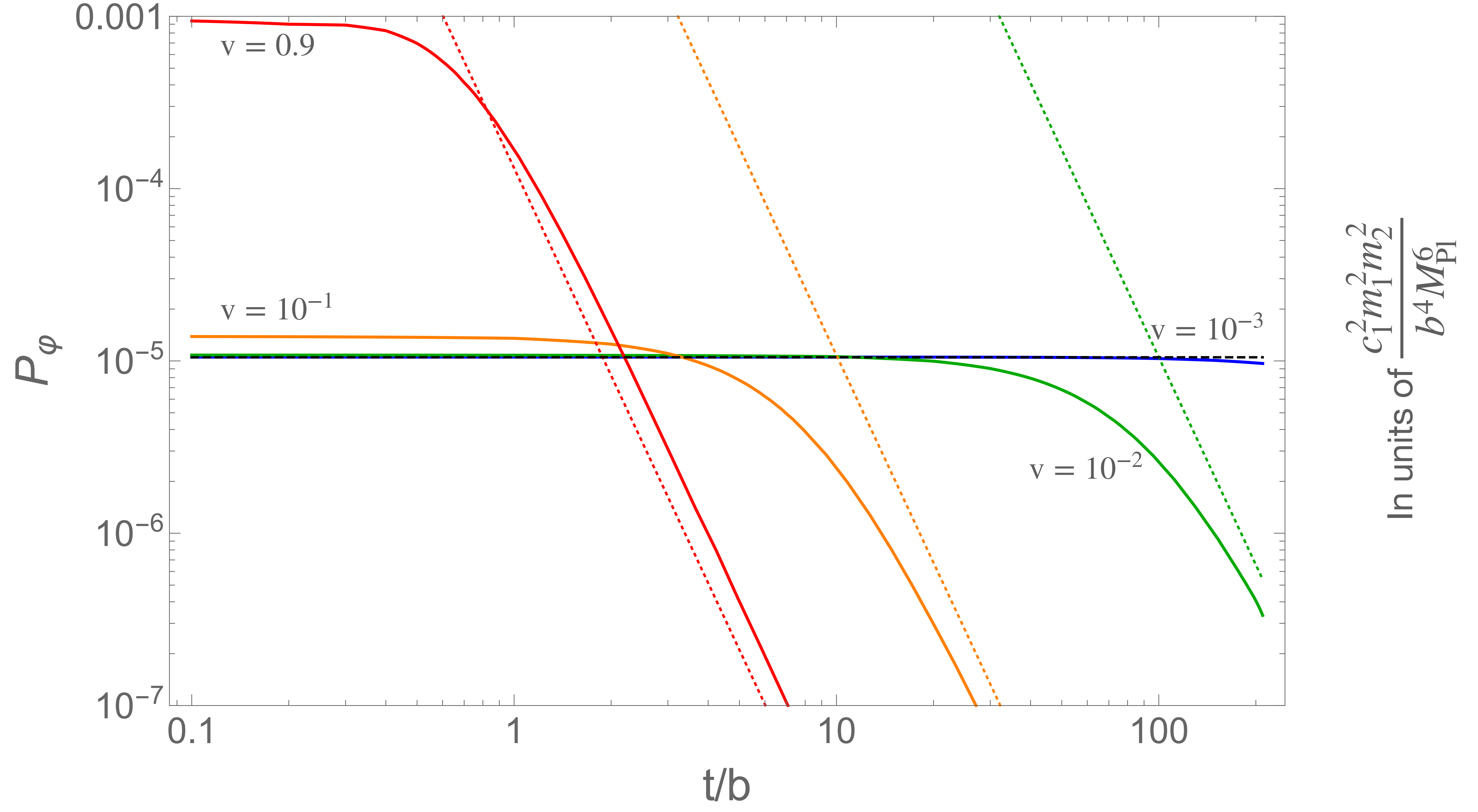
$$W_\phi^{(0)} = -\frac{m_1 m_2}{32\pi^2 M_{Pl}^3 \sqrt{\gamma^2 - 1}(\hat{u}_1 n)^2 b} \frac{1}{\sqrt{1 + T_1^2}} \left\{ \frac{(\gamma^2 - 1)[c_1(\hat{u}_2 n)^2 + c_2(\hat{u}_1 n)^2][\gamma(\hat{u}_2 n) - (\hat{u}_1 n) + (\tilde{b}n)T_1]}{[-(\hat{u}_1 n) + \gamma(\hat{u}_2 n) + (\tilde{b}n)T_1]^2 + (\tilde{v}n)^2(1 + T_1^2)} \right.$$

$$\left. - \frac{c_1}{2} \frac{(\hat{u}_1 n) + (2\gamma^2 - 3)\gamma(\hat{u}_2 n) - (2\gamma^2 - 1)(\tilde{b}n)T_1}{\gamma^2 - 1} + \frac{C_1^{(0)}}{2} c_2(\hat{u}_1 n) \right\} + (1 \leftrightarrow 2)$$

Contact term in 4-point matter-scalar scattering amplitude

Emitted power

$$\frac{dP_\phi}{d\Omega} = (\partial_t W_\phi)^2$$



Scalar-Tensor Theories: scalar waveforms

$$W_\phi = W_\phi^{(0)} + W_\phi^{(1)} + \dots$$

$$T_i \equiv \frac{t - b_i n}{(\hat{u}_i n) b} \qquad \hat{u}_i = \frac{u_i}{\sqrt{\gamma^2 - 1}} \qquad \gamma = u_1 u_2 = \frac{1}{\sqrt{1 - \mathrm{v}^2}}$$

LO Scalar Waveforms-Spinless part

$$W_\phi^{(0)} = -\frac{m_1 m_2}{32\pi^2 M_{Pl}^3 \sqrt{\gamma^2 - 1} (\hat{u}_1 n)^2 b} \frac{1}{\sqrt{1 + T_1^2}} \left\{ \frac{(\gamma^2 - 1)[c_1 (\hat{u}_2 n)^2 + c_2 (\hat{u}_1 n)^2][\gamma(\hat{u}_2 n) - (\hat{u}_1 n) + (\tilde{b} n) T_1]}{[-(\hat{u}_1 n) + \gamma(\hat{u}_2 n) + (\tilde{b} n) T_1]^2 + (\tilde{v} n)^2 (1 + T_1^2)} \right.$$

$$\left. - \frac{c_1}{2} \frac{(\hat{u}_1 n) + (2\gamma^2 - 3)\gamma(\hat{u}_2 n) - (2\gamma^2 - 1)(\tilde{b} n) T_1}{\gamma^2 - 1} + \frac{C_1^{(0)}}{2} c_2 (\hat{u}_1 n) \right\} + (1 \leftrightarrow 2)$$

Center-of-mass frame, non-relativistic limit of small relative velocity v.
PN expansion of the waveform independent of contact terms at this order

$$W_\phi^{(0)} = \frac{m_1 m_2 (c_1 - c_2)}{64\pi^2 M_{\rm Pl}^3 b} \left\{ -\frac{(\hat{\mathbf{v}}\hat{\mathbf{n}})}{\mathrm{v}} + (\hat{\mathbf{b}}\hat{\mathbf{n}}) \frac{t}{b} \right\} + O(\mathrm{v})$$

Phys.Rev.D 85 (2012) 064022, Phys.Rev.D 93 (2016) 2, 029902 [Yagi, Stein, Yunes, Tanaka]
Class.Quant.Grav. 39 (2022) 3, 035002 [Shiralilou, Hinderer, Nissanke, Ortiz, Witek]

Emitted power in the limit of of small velocity

$$P_\phi^{(0)} = \frac{m_1^2 m_2^2}{3072\pi^3 M_{\rm Pl}^6 b^4} (c_1 - c_2)^2 + O(\mathrm{v})$$

→ **Kepler's law**

$$b^{-1} \to \frac{8\pi M_{\rm Pl}^2 \mathrm{v}^2}{m_1 + m_2}$$

$$P_\phi^{(0)} = \frac{4\pi m_1^2 m_2^2 M_{\rm Pl}^2 (c_1 - c_2)^2}{3(m_1 + m_2)^4} \mathrm{v}^8 + O(\mathrm{v}^9)$$

Agreement with existing classical results for SGB!

In particular, $\frac{P_\phi^{(0)}}{P_h(0)} \sim \frac{1}{\mathrm{v}^2}$

Scalar-Tensor Theories: scalar waveforms

Comparison beyond leading PN order for quasi-circular orders

Amplitudes

$$P_\phi^{(0)} = \frac{m_1^2 m_2^2}{3072\pi^3 M_{\rm Pl}^6 b^4}(c_1 - c_2)^2 + \frac{m_1^2 m_2^2}{\pi^3 M_{\rm Pl}^6 b^4}\left[\frac{(c_1+c_2)^2}{3840} + \frac{(c_1+c_2)(c_1-c_2)}{3840}\frac{m_1-m_2}{m_1+m_2} + (6-\eta)\frac{(c_1-c_2)^2}{7680} + \frac{(c_1-c_2)\big(m_2 c_2 C_1^{(0)} - m_1 c_1 C_2^{(0)}\big)}{1536(m_1+m_2)} - \frac{(c_1-c_2)^2}{1536}\frac{t^2}{b^2}\right]\mathrm{v}^2,$$

Classical

Class.Quant.Grav. 39 (2022) 3, 035002 [Shiralilou, Hinderer, Nissanke, Ortiz, Witek]

$$\dot{E}_S = \frac{\eta^2}{G\bar{\alpha}c^3}\left(\frac{G\bar{\alpha}m}{r}\right)^4\Bigg\{\frac{4}{3}\mathcal{S}_-^2 + \frac{8}{15\,c^2}\left(\frac{G\bar{\alpha}m}{r}\right)\Bigg[\left(-23+\eta-10\bar{\gamma}-10\beta_+ + 10\frac{\Delta m}{m}\beta_-\right)\mathcal{S}_-^2 - 2\frac{\Delta m}{m}\mathcal{S}_+\mathcal{S}_-\Bigg] + v^2\Bigg[2\mathcal{S}_+^2 + 2\frac{\Delta m}{m}\mathcal{S}_+\mathcal{S}_- + (6-\eta+5\bar{\gamma})\mathcal{S}_-^2 - \frac{10}{\bar{\gamma}}\frac{\Delta m}{m}\mathcal{S}_-\left(\mathcal{S}_+\beta_+ + \mathcal{S}_-\beta_-\right) + \frac{10}{\bar{\gamma}}\mathcal{S}_-\left(\mathcal{S}_-\beta_+ + \mathcal{S}_+\beta_-\right)\Bigg] + \dot{r}^2\Bigg[\frac{23}{2}\mathcal{S}_+^2 - 8\frac{\Delta m}{m}\mathcal{S}_+\mathcal{S}_- + \left(9\eta - \frac{37}{2} - 10\bar{\gamma}\right)\mathcal{S}_-^2 - \frac{80}{\bar{\gamma}}\mathcal{S}_+\left(\mathcal{S}_+\beta_+ + \mathcal{S}_-\beta_-\right) + \frac{30}{\bar{\gamma}}\frac{\Delta m}{m}\mathcal{S}_-\left(\mathcal{S}_+\beta_+ + \mathcal{S}_-\beta_-\right) - \frac{10}{\bar{\gamma}}\mathcal{S}_-\left(\mathcal{S}_-\beta_+ + \mathcal{S}_+\beta_-\right) + \frac{120}{\bar{\gamma}^2}\left(\mathcal{S}_+\beta_+ + \mathcal{S}_-\beta_-\right)^2\Bigg]\Bigg) - \frac{\Delta m}{m}\frac{\eta}{6\,c^2}\left(\frac{\alpha f'(\phi_0)\mathcal{S}_-\mathcal{S}_+}{\sqrt{\bar{\alpha}}r^2}\right)\left(\mathcal{S}_+ + \frac{\Delta m}{m}\mathcal{S}_-\right)\left[-9\dot{r}^2 + 3v^2 - \frac{2G\bar{\alpha}m}{r}\right] + \mathcal{O}(c^{-3})\Bigg\}.$$

higher PM

irrelevant for quasi-circular orbits

suppressed by more powers of distance

Scalar-Tensor Theories: scalar waveforms

Using amplitudes it is straightforward to continue beyond linear order in spin

$$W_\phi = W_\phi^{(0)} + W_\phi^{(1)} + \dots$$

$$a_i \equiv \frac{S_i}{m_i}$$

one more power of impact parameter

$$W_\phi^{(1)} = \frac{m_1 m_2}{32\pi^2 M_{\rm Pl}^3 b^2 (\hat{u}_1 n)^2} \frac{\partial}{\partial z}\left(\frac{1}{\sqrt{z^2+1}} \mathrm{Re}\left\{ c_1 \left[z(\tilde{v}n) - i\sqrt{z^2+1}(\tilde{b}n)\right]\left[-(\hat{u}_1 a_2) + z(\tilde{b}a_2) + i\sqrt{z^2+1}(\tilde{v}a_2)\right] \right.\right.$$
$$\times \left[\frac{\gamma}{\gamma^2-1} - \frac{(\hat{u}_2 n)}{\gamma(\hat{u}_2 n) - (\hat{u}_1 n) + z(\tilde{b}n) + i\sqrt{z^2+1}(\tilde{v}n)}\right] - \frac{c_2(\hat{u}_1 n)}{\gamma(\hat{u}_2 n) - (\hat{u}_1 n) + z(\tilde{b}n) + i\sqrt{z^2+1}(\tilde{v}n)}$$
$$\times \left[\left[i\sqrt{z^2+1}(\tilde{b}n) - z(\tilde{v}n)\right](\hat{u}_2 a_1) + \left[\gamma(\tilde{v}n) + i\sqrt{z^2+1}\left(\gamma(\hat{u}_1 n) - \hat{u}_2 n\right)\right](\tilde{b}a_1) + \left[z\left(\hat{u}_2 n - \gamma(\hat{u}_1 n)\right) - \gamma(\tilde{b}n)\right](\tilde{v}a_1)\right]$$
$$\left.\left. + \frac{C_1^{(1)} c_2}{2\sqrt{\gamma^2-1}} \left[(\hat{u}_1 n)\left[\gamma(\hat{u}_2 a_1) + z(\tilde{b}a_1)\right] - (a_1 n)\right] \right\} \right) \Big|_{z=T_1} + (1 \leftrightarrow 2)$$

New contact term

$$W_\phi^{(1)} = \frac{m_1 m_2}{32\pi^2 M_{\rm Pl}^3 b^2} \left\{ \left[c_2 \boldsymbol{a_1} - c_1 \boldsymbol{a_2}\right] \cdot \hat{\mathbf{v}}\left(\hat{\mathbf{v}} \times \hat{\boldsymbol{b}} \cdot \hat{\boldsymbol{n}}\right) + \frac{1}{2}\left[c_1 C_2^{(1)} \boldsymbol{a}_2 - c_2 C_1^{(1)} \boldsymbol{a}_1\right] \cdot \hat{\boldsymbol{b}} \right.$$
$$\left. + \mathrm{v}\left[\left[c_1 \boldsymbol{a_2} - c_2 \boldsymbol{a_1}\right] \cdot \left[2\hat{\boldsymbol{b}}\left(\hat{\mathbf{v}} \times \hat{\boldsymbol{b}} \cdot \hat{\boldsymbol{n}}\right) + \hat{\mathbf{v}} \times \hat{\boldsymbol{b}}\left(\hat{\boldsymbol{b}}\hat{\boldsymbol{n}}\right)\right] + \frac{1}{2}\left[c_1 C_2^{(1)} \boldsymbol{a}_2 - c_2 C_1^{(1)} \boldsymbol{a}_1\right] \cdot \hat{\mathbf{v}}\right] \frac{t}{b} \right\}$$
$$+ O(\mathrm{v}^2) .$$

Linear-in-spin correction to emitted power

$$P_\phi^{(1)} = \frac{m_1^2 m_2^2 (c_1 - c_2)}{768\pi^3 M_{\rm Pl}^6 b^5} (\hat{\mathbf{v}} \times \hat{\boldsymbol{b}}) \cdot \left[c_1 \boldsymbol{a_2} - c_2 \boldsymbol{a_1}\right] \mathrm{v}$$

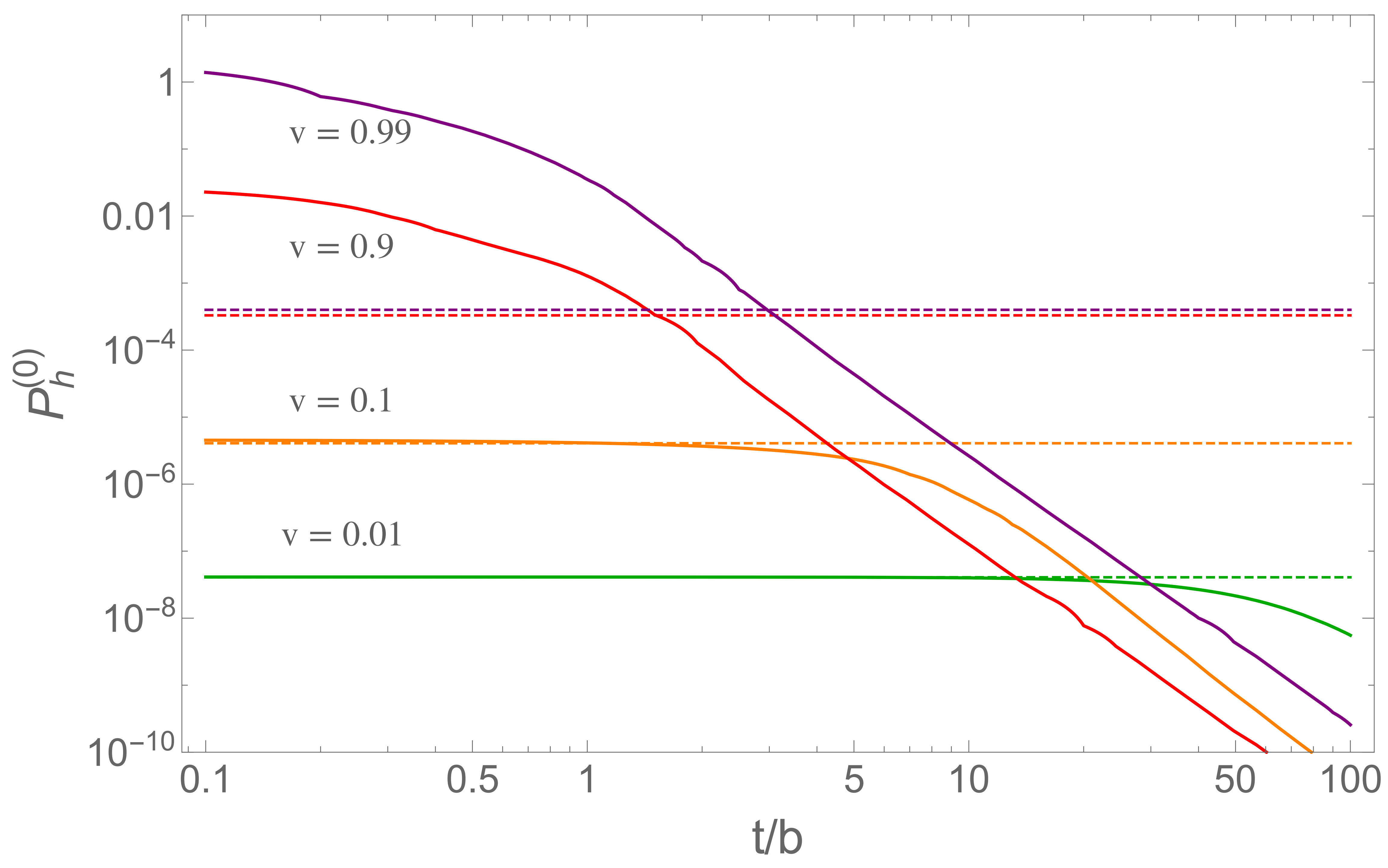
Scalar-Tensor Theories: gravitational waveforms

$$W_h = W_h^{(0)} + W_h^{(1)} + \dots$$

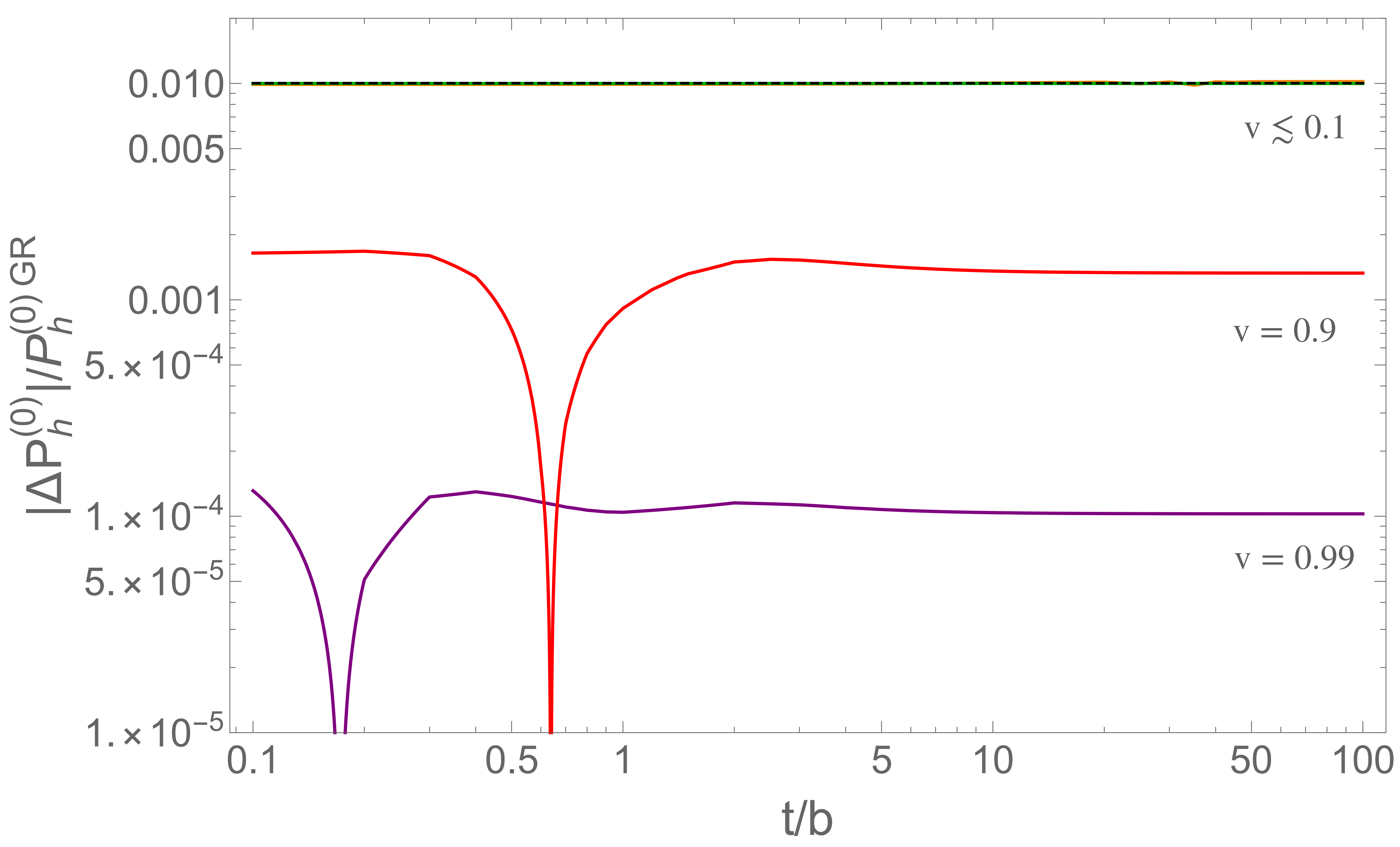
LO Gravitational Waveforms-Spinless part:

$$\Delta W_h^{(0)} = -\frac{c_1 c_2 m_1 m_2}{512\pi^2 M_{Pl}^3 \sqrt{\gamma^2 - 1}(\hat{u}_1 n)^2 b} \frac{1}{\sqrt{1+T_1^2}} \mathrm{Re}\left\{ \frac{(\lambda_n[\gamma \hat{u}_2 \sigma + T_1 \tilde{b}\sigma + i\sqrt{T_1^2+1}\tilde{v}\sigma]\hat{u}_1 \bar{\sigma}\lambda_n)^2}{\gamma(\hat{u}_2 n) - (\hat{u}_1 n) + T_1(\tilde{b}n) + i\sqrt{T_1^2+1}(\tilde{v}n)} \right\} + (1 \leftrightarrow 2)\,.$$

Emitted power in gravitational waves $\quad \frac{dP_h}{d\Omega} = 2|\partial_t W_h|^2$



Correction of emitted power due to scalar exchange



Scalar-Tensor Theories: gravitational waveforms

$$W_h = W_h^{(0)} + W_h^{(1)} + \dots$$

LO Gravitational Waveforms-Spinless part:

$$\Delta W_h^{(0)} = -\frac{c_1 c_2 m_1 m_2}{512\pi^2 M_{Pl}^3 \sqrt{\gamma^2-1}(\hat{u}_1 n)^2 b} \frac{1}{\sqrt{1+T_1^2}} \mathrm{Re}\left\{ \frac{(\lambda_n[\gamma \hat{u}_2 \sigma + T_1 \tilde{b}\sigma + i\sqrt{T_1^2+1}\tilde{v}\sigma]\hat{u}_1 \bar{\sigma}\lambda_n)^2}{\gamma(\hat{u}_2 n) - (\hat{u}_1 n) + T_1(\tilde{b}n) + i\sqrt{T_1^2+1}(\tilde{v}n)} \right\} + (1 \leftrightarrow 2)\,.$$

$$\left.\frac{dP_h}{d\Omega}\right|_{\mathcal{O}(a^0)} \sim \frac{\mathrm{v}^2}{b^4}$$

For closed orbits →

$$\left.\frac{dP_h}{d\Omega}\right|_{\mathcal{O}(a^0)} \sim \mathrm{v}^{10}$$

→ **Suppression compared to scalar radiation in agreement with classical literature**

Phys.Rev.D 85 (2012) 064022, Phys.Rev.D 93 (2016) 2, 029902 (erratum) [Yagi, Stein, Yunes, Tanaka]

Class.Quant.Grav. 39 (2022) 3, 035002 [Shiralilou, Hinderer, Nissanke, Ortiz, Witek]

In fact, at leading PN order emitted power rescaled compared to GR by factor $1 + c_1 c_2$

Summary

- Classical observables often can be efficiently calculated using quantum amplitudes in the framework of the KMOC formalism
- One very fruitful application of this formalism is to calculate corrections to the gravitational potential and waveforms from systems of compact objects in general relativity
- The formalism can be readily extended to scalar-tensor theories of gravity, where both gravitational and scalar radiation is present
- Many results for emitted power in gravitational and scalar waves found in the classical literature can reproduced in the KMOC approach
- We also derive novel results at higher PN and spin orders

Backup

Quantum Amplitudes and Radiation Observables

Expansion in QFT vs in classical GR for 2-to-2 scattering

	0PN	1PN	2PN	3PN	4PN
1PM	$\frac{1}{M_{\rm Pl}^2}$	$\frac{{\rm v}^2}{M_{\rm Pl}^2}$	$\frac{{\rm v}^4}{M_{\rm Pl}^2}$	$\frac{{\rm v}^6}{M_{\rm Pl}^2}$	$\frac{{\rm v}^8}{M_{\rm Pl}^2}$
2PM		$\frac{1}{M_{\rm Pl}^4}$	$\frac{{\rm v}^2}{M_{\rm Pl}^4}$	$\frac{{\rm v}^4}{M_{\rm Pl}^4}$	$\frac{{\rm v}^6}{M_{\rm Pl}^4}$
3PM			$\frac{1}{M_{\rm Pl}^6}$	$\frac{{\rm v}^2}{M_{\rm Pl}^6}$	$\frac{{\rm v}^4}{M_{\rm Pl}^6}$
4PM				$\frac{1}{M_{\rm Pl}^8}$	$\frac{{\rm v}^2}{M_{\rm Pl}^8}$
5PM					$\frac{1}{M_{\rm Pl}^{10}}$

tree level $\sim \frac{1}{bM_{\rm Pl}^2}$

1-loop $\sim \frac{1}{b^2M_{\rm Pl}^4} \sim \frac{R_s}{b}\frac{1}{bM_{\rm Pl}^2}$

2-loop $\sim \frac{1}{b^3M_{\rm Pl}^6} \sim \left(\frac{R_s}{b}\right)^2\frac{1}{bM_{\rm Pl}^2}$

3-loop

4-loop

Loop expansion

Velocity expansion

Kepler's law

$${\rm v}^2 \sim \frac{1}{bM_{\rm Pl}^2}$$

Schwarzschild radius

$$R_s \sim \frac{m}{M_{\rm Pl}^2}$$

Scalar-Tensor Theories: contact term domination

For previous results, take the limit of large contact terms $C_i \to \infty, \quad c_i \to 0, \quad C_i c_i$ finite

$$\partial_t W_\phi^{(0)} = \mathcal{O}(\mathrm{v}^2)$$

$$\partial_t W_\phi^{(1)} = \frac{m_1 m_2}{64\pi^2 M_{\mathrm{Pl}}^3 b^3}\left(\left[c_1 C_2^{(1)} \boldsymbol{a}_2 - c_2 C_1^{(1)} \boldsymbol{a}_1\right] \cdot \hat{\mathbf{v}}\right)\mathrm{v} + \mathcal{O}(\mathrm{v}^2)\,.$$

Emitted power in scalar radiation

$$P_\phi = \frac{m_1^2 m_2^2}{1024\pi^3 M_{\mathrm{Pl}}^6 b^6}\left(\left[c_1 C_2^{(1)} \boldsymbol{a}_2 - c_2 C_1^{(1)} \boldsymbol{a}_1\right] \cdot \hat{\mathbf{v}}\right)^2 \mathrm{v}^2 + \mathcal{O}(\mathrm{v}^3)$$

This resembles the formula valid for DCS gravity $(\alpha = 0, \quad \tilde{\alpha} \neq 0)$: $P_\phi \sim (\boldsymbol{a}\mathrm{v})^2 + \#\boldsymbol{a}^2\mathrm{v}^2$

Tanaka, Yagi, Yunes
arXiv:1208.5102

But we don't know how to produce the second term above

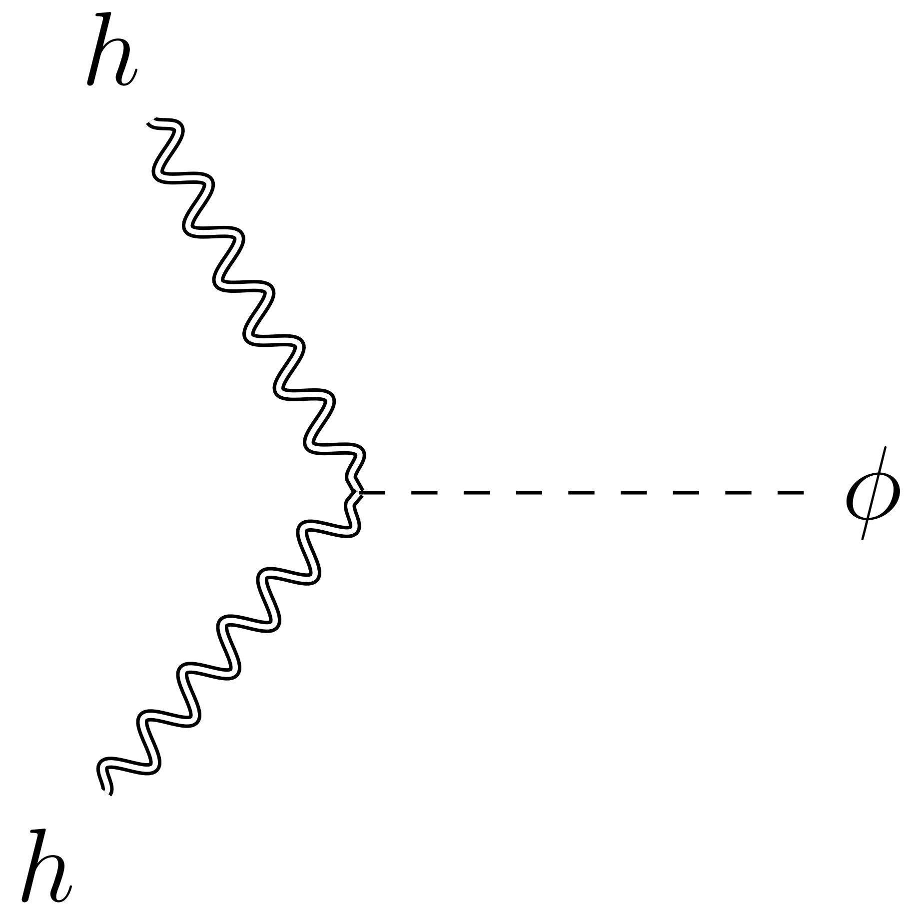
Waveforms for scattering of compact objects without scalar hair

Scalar-Tensor Theories

- $S_{SGB,DCS} = \int d^4x\sqrt{-g}\left[\frac{M_{Pl}^2}{\Lambda^2}\left(f(\phi)\mathcal{G} + \tilde{f}(\phi)R\tilde{R}\right) + \frac{1}{2}\left(\partial^\mu\phi\partial_\mu\phi\right)\right]$

$$f(\phi) = \text{const} + \alpha\frac{\phi}{M_{\text{Pl}}} + O(\phi^2) \qquad \tilde{f}(\phi) = \text{const} + \tilde{\alpha}\frac{\phi}{M_{\text{Pl}}} + O(\phi^2)$$

The linear term induces a shift-symmetric cubic interaction between scalar and gravity corresponding to 3-point amplitudes



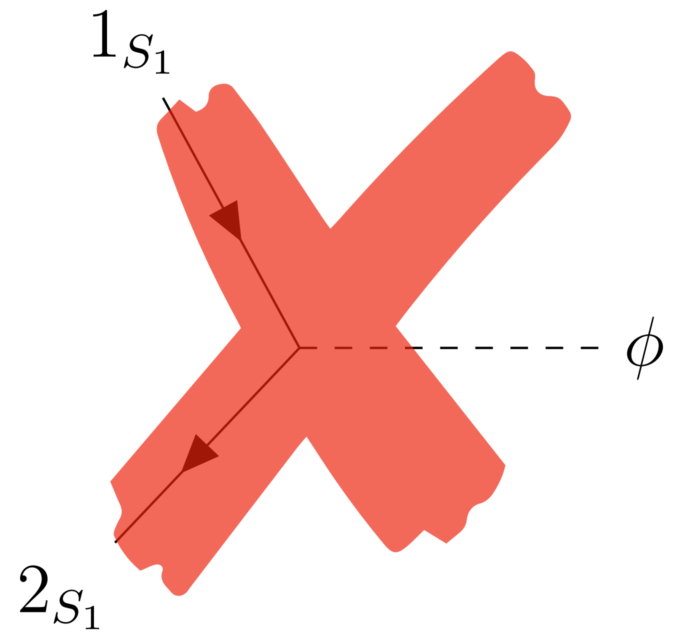
$$\mathcal{M}\left[1_h^- 2_h^- 3_\phi\right] = \frac{2\hat{\alpha}}{\Lambda^2 M_{\text{pl}}}\langle 12\rangle^4$$

$$\mathcal{M}\left[1_h^+ 2_h^+ 3_\phi\right] = \frac{2\hat{\alpha}^*}{\Lambda^2 M_{\text{pl}}}[12]^4$$

$$\mathcal{M}\left[1_h^- 2_h^+ 3_\phi\right] = 0$$

$\hat{\alpha} \equiv \alpha + i\tilde{\alpha}$

Here we assume no scalar hair



Scalar-Tensor Theories

- $S_{SGB,DCS} = \int d^4x\sqrt{-g}\left[\frac{M_{Pl}}{\Lambda^2}\left(f(\phi)\mathcal{G} + \tilde{f}(\phi)R\tilde{R}\right) + \frac{1}{2}\left(\partial^\mu\phi\partial_\mu\phi\right)\right]$

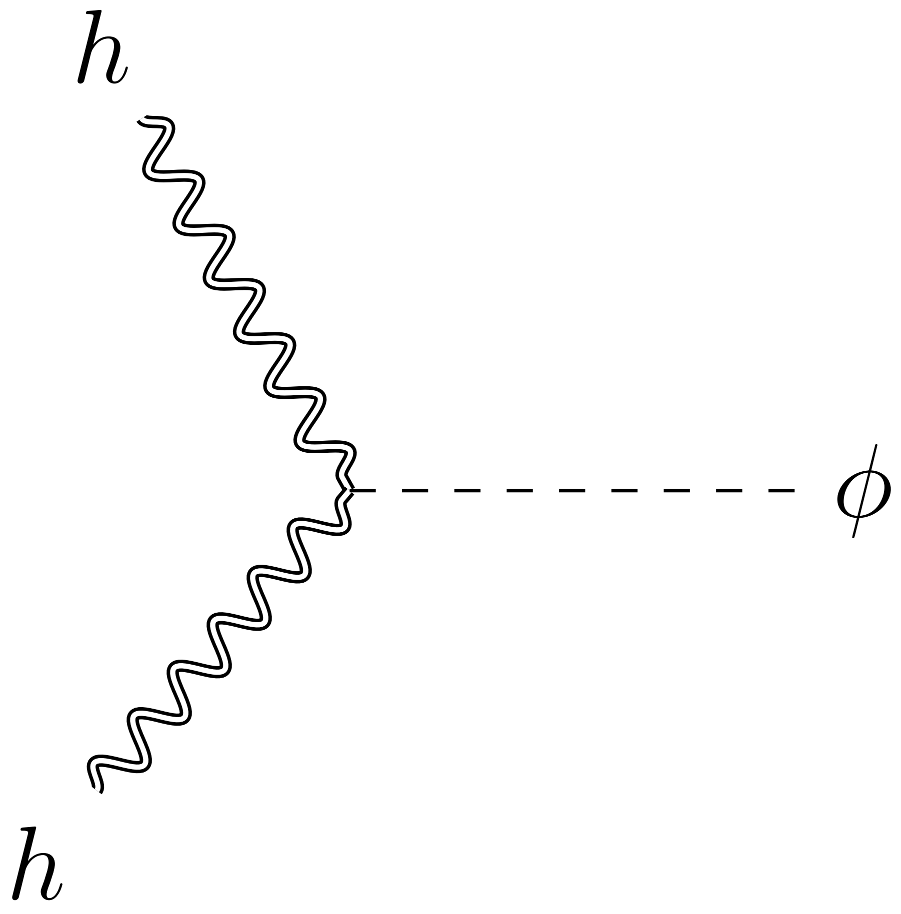
We assume scalar couplings to gravity are shift-symmetric:

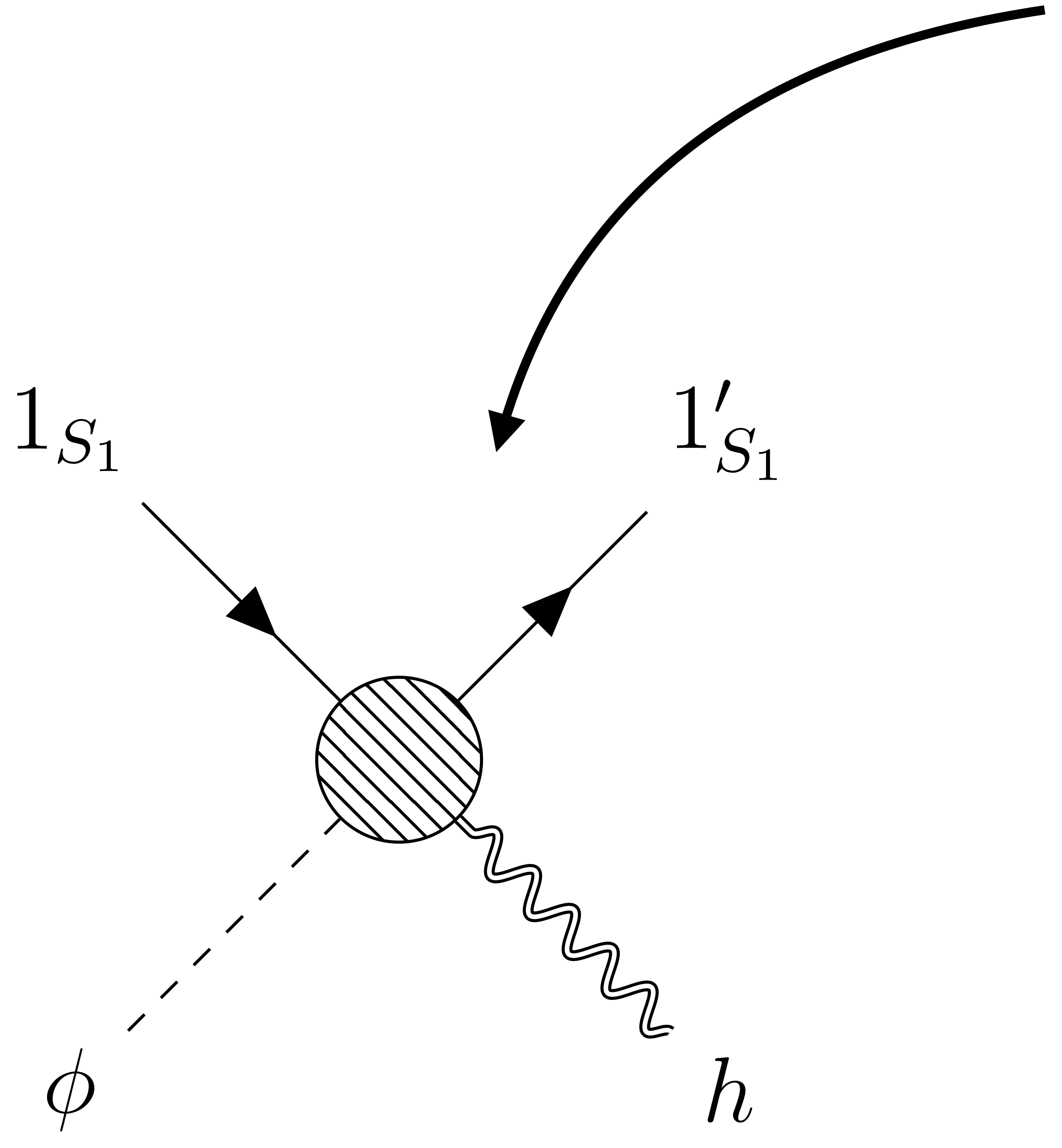
$f(\phi) = \text{const} + \alpha\phi \qquad \tilde{f}(\phi) = \text{const} + \alpha\tilde{\phi}$

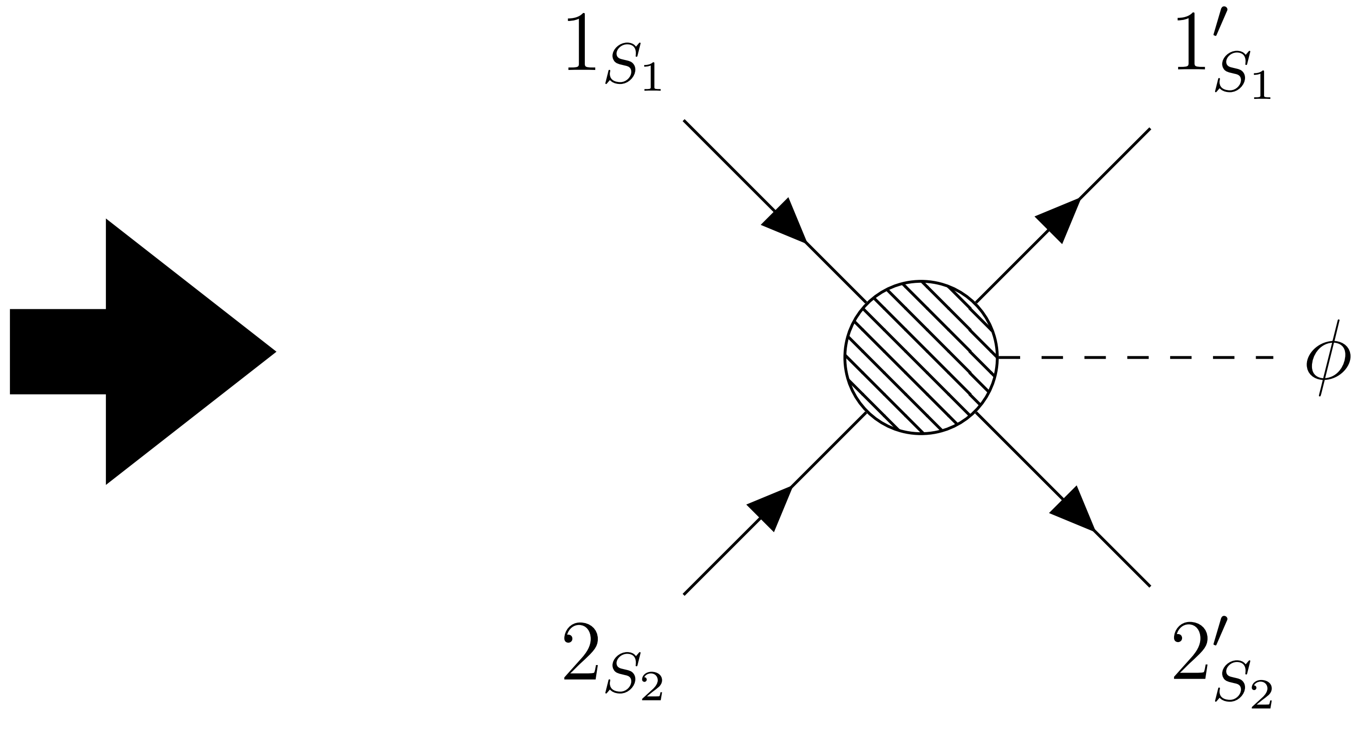
$$\mathscr{M}\left[1_h^- 2_h^- 3_\phi\right] = \frac{2\hat{\alpha}}{\Lambda^2 M_{\rm pl}}\langle 12\rangle^4 \qquad \hat{\alpha} \equiv \alpha + i\tilde{\alpha}$$

$$\mathscr{M}\left[1_h^+ 2_h^+ 3_\phi\right] = \frac{2\hat{\alpha}^*}{\Lambda^2 M_{\rm pl}}[12]^4$$

$$\mathscr{M}\left[1_h^- 2_h^+ 3_\phi\right] = 0$$







Scalar-Tensor Theories

In this case, the relevant 4-point amplitude

$q = p_1 + p_2 = -p_3 - p_4$

$$\mathcal{M}_U^{\rm cl}[1_{\Phi_i} 2_{\bar{\Phi}_i} 3_h^- 4_\phi] = \frac{2\hat{\alpha}\langle 3|p_1 q|3\rangle^2}{M_{\rm Pl}^2 \Lambda^2 q^2} e^{q a_1}$$

scales as $\mathcal{M}_U^{\rm cl} \to \mathcal{M}_U^{\rm cl}\hbar^2$ under classical scaling,
as opposed to $\mathcal{M}_U^{\rm cl} \to \mathcal{M}_U^{\rm cl}\hbar^0$ for Compton amplitudes in GR or QED
of for scalar analogous amplitude in the presence of scalar hair

$$\mathcal{M}_U^{\rm cl}[1_{\Phi_n} 2_{\bar{\Phi}_n} 3_\phi 4_h^-] = -\frac{c_n m_n^2}{M_{\rm Pl}^2} \frac{\langle 4|p_3 p_1|4\rangle^2}{s(t-m_n^2)^2}$$

Additional suppression does not mean however that the effect is necessarily quantum,
but places more stringent constraints on resolvability of radiation

Bellazzini, Isabella, Riva
arXiv:2211.00085

We find the effect is resolvable when

$$R_s \ll b \ll b_{\rm max}, \qquad b_{\rm max} = R_s \frac{M_{\rm Pl}}{m^{1/3}\Lambda^{2/3}}$$

Cf. resolvability on nPM corrections in GR

$$\frac{m^2}{M_{\rm Pl}^2}\left(\frac{R_s}{b}\right)^{n-1} \gg 1$$

Scalar-Tensor Theories

LO scalar waveform in CP-even limit :

$$
\begin{aligned}
W_\phi = \frac{m_1 m_2}{8\pi^2 b^3 M_{Pl}^3 \Lambda^2 (\hat{u}_1 n)^2 \sqrt{\gamma^2-1}} \Bigg(\alpha \Bigg\{ -\frac{d^2}{dz^2}\Bigg[\frac{1}{\sqrt{z^2+1}} \frac{2(\gamma^2-1)^2 (\hat{u}_2 n)^2 \big[\gamma(\hat{u}_2 n) - (\hat{u}_1 n) + (\tilde{b} n) z\big]}{\big[\gamma(\hat{u}_2 n) - (\hat{u}_1 n) + (\tilde{b} n) z\big]^2 + (\tilde{v} n)^2 (z^2+1)} \Bigg] + \big[(\hat{u}_1 n) + \gamma\big(2\gamma^2 - 3\big)(\hat{u}_2 n)\big] \frac{2z^2-1}{(z^2+1)^{5/2}} + \big(2\gamma^2-1\big)(\tilde{b} n) \frac{3z}{(z^2+1)^{5/2}} \Bigg\} \\
+ \frac{2\alpha}{b} \frac{d^3}{dz^3} \mathrm{Re} \Bigg\{ \frac{1}{\sqrt{z^2+1}} \frac{(\hat{u}_2 n) - \gamma(\hat{u}_1 n) + \gamma(\tilde{b} n) z + i\gamma(\tilde{v} n)\sqrt{z^2+1}}{\gamma(\hat{u}_2 n) - (\hat{u}_1 n) + (\tilde{b} n) z + i(\tilde{v} n)\sqrt{z^2+1}} \Big[z(\tilde{v} n) - i\sqrt{z^2+1}(\tilde{b} n) \Big] \times \Bigg[\frac{(a_1 n)}{(\hat{u}_1 n)} - \gamma(a_1 \hat{u}_2) - (a_2 \hat{u}_1) - (a_1^A - a_2^A)(z\tilde{b}^A + i\sqrt{z^2+1}\,\tilde{v}^A) \Bigg] \Bigg\} \\
- \sqrt{\gamma^2-1} \frac{C_1}{b} \frac{d^3}{dz^3} \Bigg\{ \frac{1}{\sqrt{z^2+1}} \mathrm{Re} \Bigg[\Bigg(z\big[(\tilde{v} n) a_1^A + (a_1 \tilde{v}) n^A\big] - i\sqrt{z^2+1}\big[(\tilde{b} n) a_1^A + (a_1 \tilde{b}) n^A\big] \Bigg) \times \Bigg(\hat{u}_2^A - \gamma \hat{u}_1^A + \gamma\big[z\tilde{b}^A + i\sqrt{z^2+1}\,\tilde{v}^A\big] \Bigg) \Bigg] \Bigg\} \Bigg) \Bigg|_{z=T_1} + (1 \leftrightarrow 2) + \mathcal{O}(a^2)\,.
\end{aligned}
$$

Scalar-Tensor Theories

LO scalar waveform for scalar Gauss-Bonnet:

$$W_\phi = \frac{m_1 m_2}{8\pi^2 b^3 M_{Pl}^3 \Lambda^2 (\hat{u}_1 n)^2 \sqrt{\gamma^2-1}} \Bigg(\alpha \Bigg\{ -\frac{d^2}{dz^2}\Bigg[\frac{1}{\sqrt{z^2+1}} \frac{2(\gamma^2-1)^2(\hat{u}_2 n)^2\big[\gamma(\hat{u}_2 n)-(\hat{u}_1 n)+(\tilde{b} n)z\big]}{\big[\gamma(\hat{u}_2 n)-(\hat{u}_1 n)+(\tilde{b} n) z\big]^2+(\tilde{v} n)^2(z^2+1)}\Bigg] + \big[(\hat{u}_1 n)+\gamma\big(2\gamma^2-3\big)(\hat{u}_2 n)\big]\frac{2z^2-1}{(z^2+1)^{5/2}} + \big(2\gamma^2-1\big)(\tilde{b} n)\frac{3z}{(z^2+1)^{5/2}}\Bigg\}$$

$$+\frac{2\alpha}{b}\frac{d^3}{dz^3}\mathrm{Re}\Bigg\{\frac{1}{\sqrt{z^2+1}}\frac{(\hat{u}_2 n)-\gamma(\hat{u}_1 n)+\gamma(\tilde{b} n)z+i\gamma(\tilde{v} n)\sqrt{z^2+1}}{\gamma(\hat{u}_2 n)-(\hat{u}_1 n)+(\tilde{b} n)z+i(\tilde{v} n)\sqrt{z^2+1}}\Big[z(\tilde{v} n)-i\sqrt{z^2+1}(\tilde{b} n)\Big]\times\Big[\frac{(a_1 n)}{(\hat{u}_1 n)}-\gamma(a_1\hat{u}_2)-(a_2\hat{u}_1)-(a_1^A-a_2^A)(z\tilde{b}^A+i\sqrt{z^2+1}\tilde{v}^A)\Big]\Bigg\}$$

$$-\sqrt{\gamma^2-1}\frac{C_1}{b}\frac{d^3}{dz^3}\Bigg\{\frac{1}{\sqrt{z^2+1}}\mathrm{Re}\Bigg[\Bigg(z\big[(\tilde{v} n)a_1^A+(a_1\tilde{v})n^A\big]-i\sqrt{z^2+1}\big[(\tilde{b} n)a_1^A+(a_1\tilde{b})n^A\big]\Bigg)\times\Bigg(\hat{u}_2^A-\gamma\hat{u}_1^A+\gamma\big[z\tilde{b}^A+i\sqrt{z^2+1}\tilde{v}^A\big]\Bigg)\Bigg]\Bigg\}\Bigg)\Bigg|_{z=T_1} + (1\leftrightarrow 2)+\mathcal{O}(a^2)\,.$$

Connect to observables: Power emitted in scalar radiation

$$\frac{dP_\phi}{d\Omega} = (\partial_t W_\phi)^2$$

$$\frac{dP_\phi}{d\Omega}\Bigg|_{\mathcal{O}(a^0)} \sim \frac{\mathrm{v}^6}{b^8} \quad \xrightarrow{\text{For closed orbits}} \quad \frac{dP_\phi}{d\Omega}\Bigg|_{\mathcal{O}(a^0)} \sim \mathrm{v}^{22}$$

$$\frac{dP_\phi}{d\Omega}\Bigg|_{\mathcal{O}(a^1)} \sim \frac{\mathrm{v}^4}{b^{10}} \quad \xrightarrow{\text{For closed orbits}} \quad \frac{dP_\phi}{d\Omega}\Bigg|_{\mathcal{O}(a^1)} \sim \mathrm{v}^{24}$$

Much **bigger suppression** compared to $P_\phi \sim \mathrm{v}^8$ in the presence of scalar hair