SCALARS 2025

SUPERCOOLED PHASE TRANSITIONS FROM RADIATIVE SYMMETRY BREAKING

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UNIVERSITY OF WARSAW

Based on work in collaboration with:
A. Karam, M. Kierkla, T.V.I. Tenkanen, J. van de Vis, P. Schicho, A. Gonstal, M. Lewicki, B. Sójka,
JHEP 03 (2023) 007, JHEP 02 (2024) 234, PLB 860 (2025) 139155, *JHEP* 07 (2025) 153, *JHEP* 08 (2025) 039

LOOKING BACK IN TIME

[Image source: BBC.CO.UK]

UNOBSERVABLE UNIVERSE (PAST)

POTENTIALLY OBSERVABLE UNIVERSE (PAST)

THE BEGINNING
The Universe begins 13.7 billion years ago with an event known as the Big Bang.

Both time and space are created in this event.

FRACTION OF A SECOND
Rapid expansion occurs during a billionth of a billionth of a billionth of a billionth of a second – the visible Universe is the size of a grapefruit.

1 SECOND
The Large Hadron Collider at CERN is recreating the conditions that prevailed a fraction of a second after the Big Bang.

100 – 1000 SECONDS
Nuclei of hydrogen, helium, lithium and other light elements form.

300,000 YEARS
We can detect radiation from the early formation of the Universe back as far as this point. Before this, the Universe is opaque: it's as if a veil has been pulled over it.

A FEW HUNDRED MILLION YEARS
Matter clumps together under its own gravity forming the first protogalaxies and within them, the first stars.

Stars are nuclear furnaces in which heavier elements such as carbon, oxygen, silicon and iron are formed. Massive stars exploding as supernovae create even heavier elements. Such explosions send material into space ready to be incorporated into future generations of stars and planets.

A FEW BILLION YEARS
Initially, the expansion of the U
few billion years after the Big B
accelerate. The acceleration is
force known as 'dark energy',
completely unknown.

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A FEW BILLION YEARS

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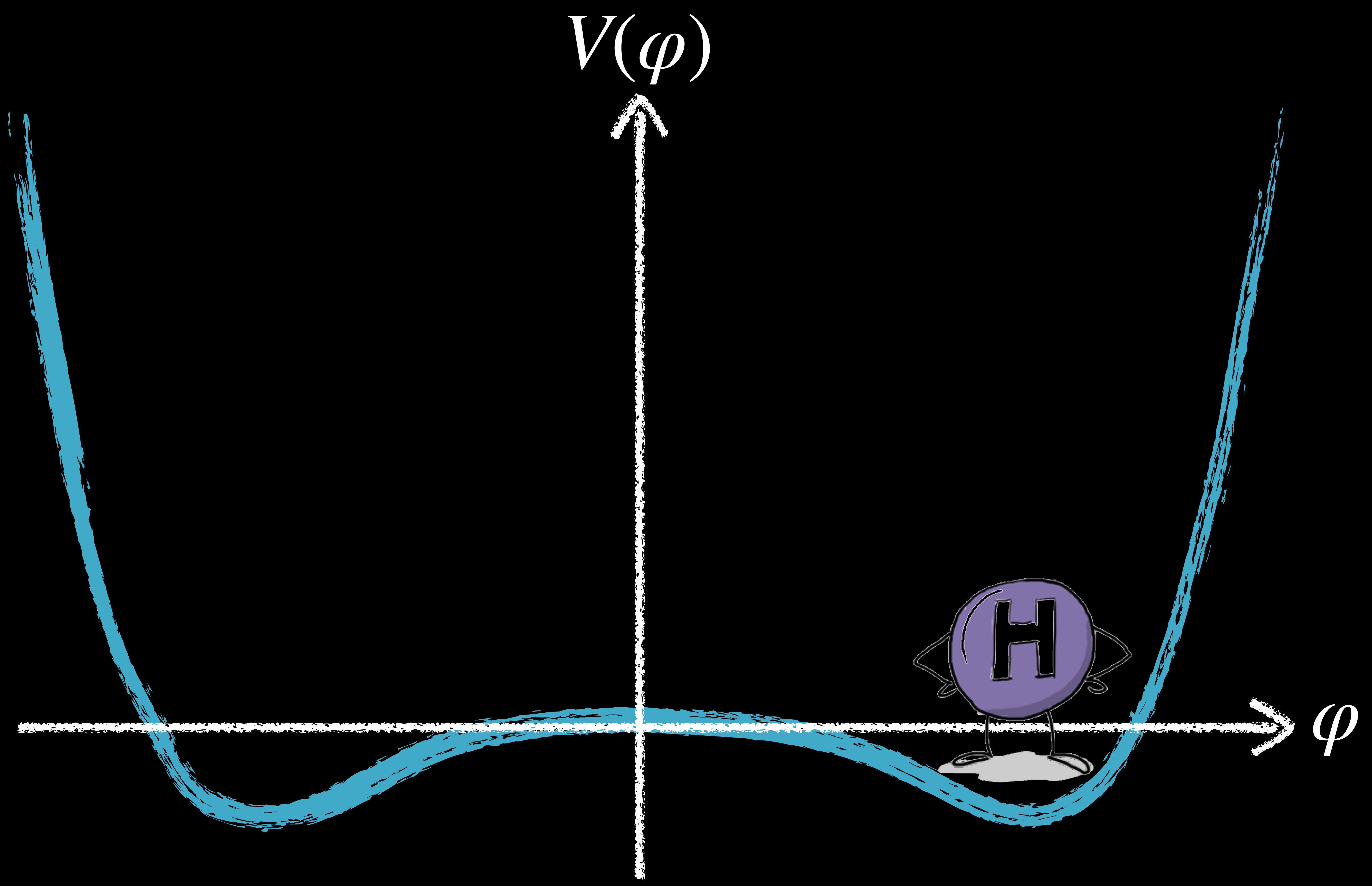
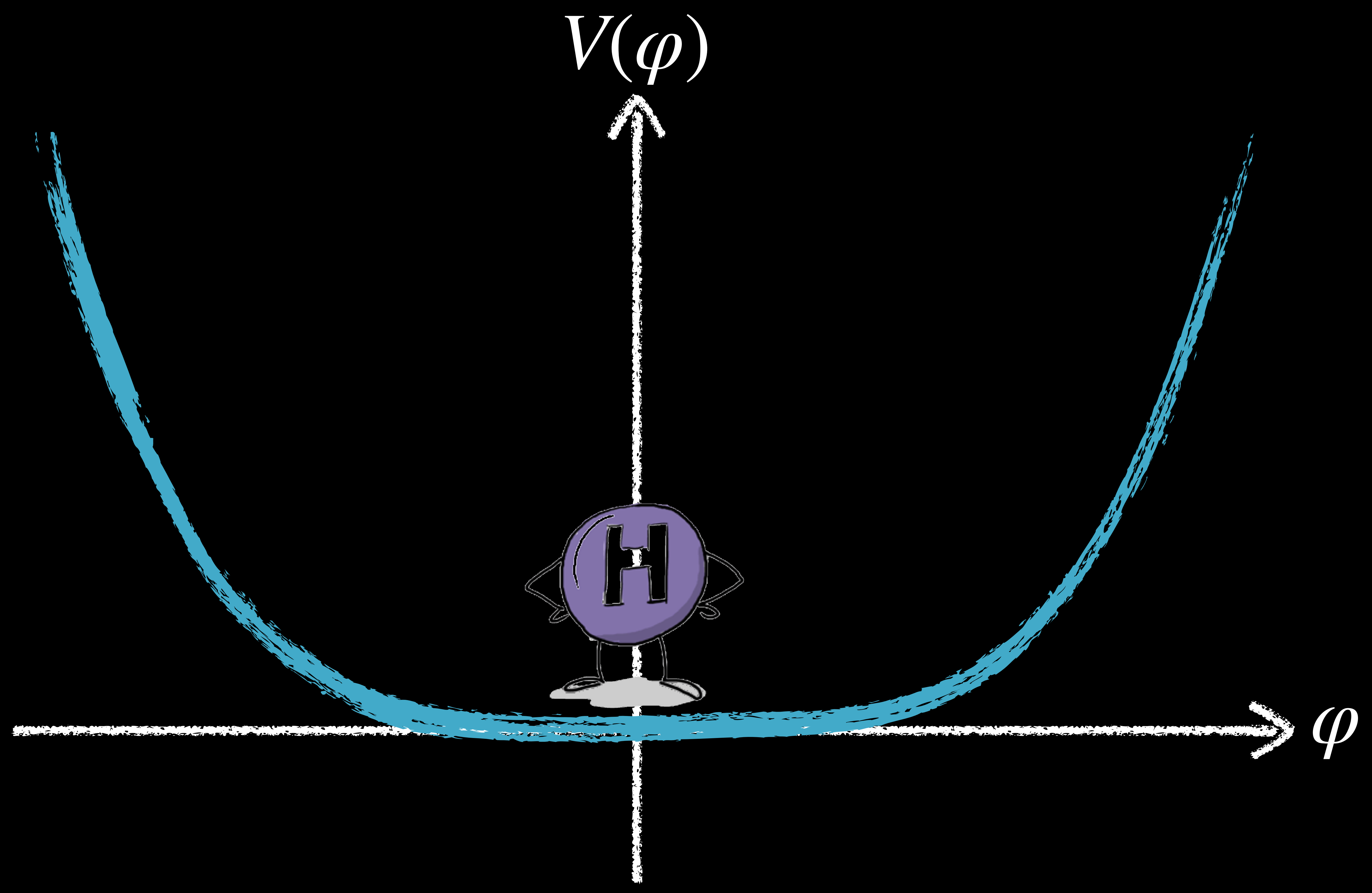
EXPERIMENT: HIGGS EXISTS

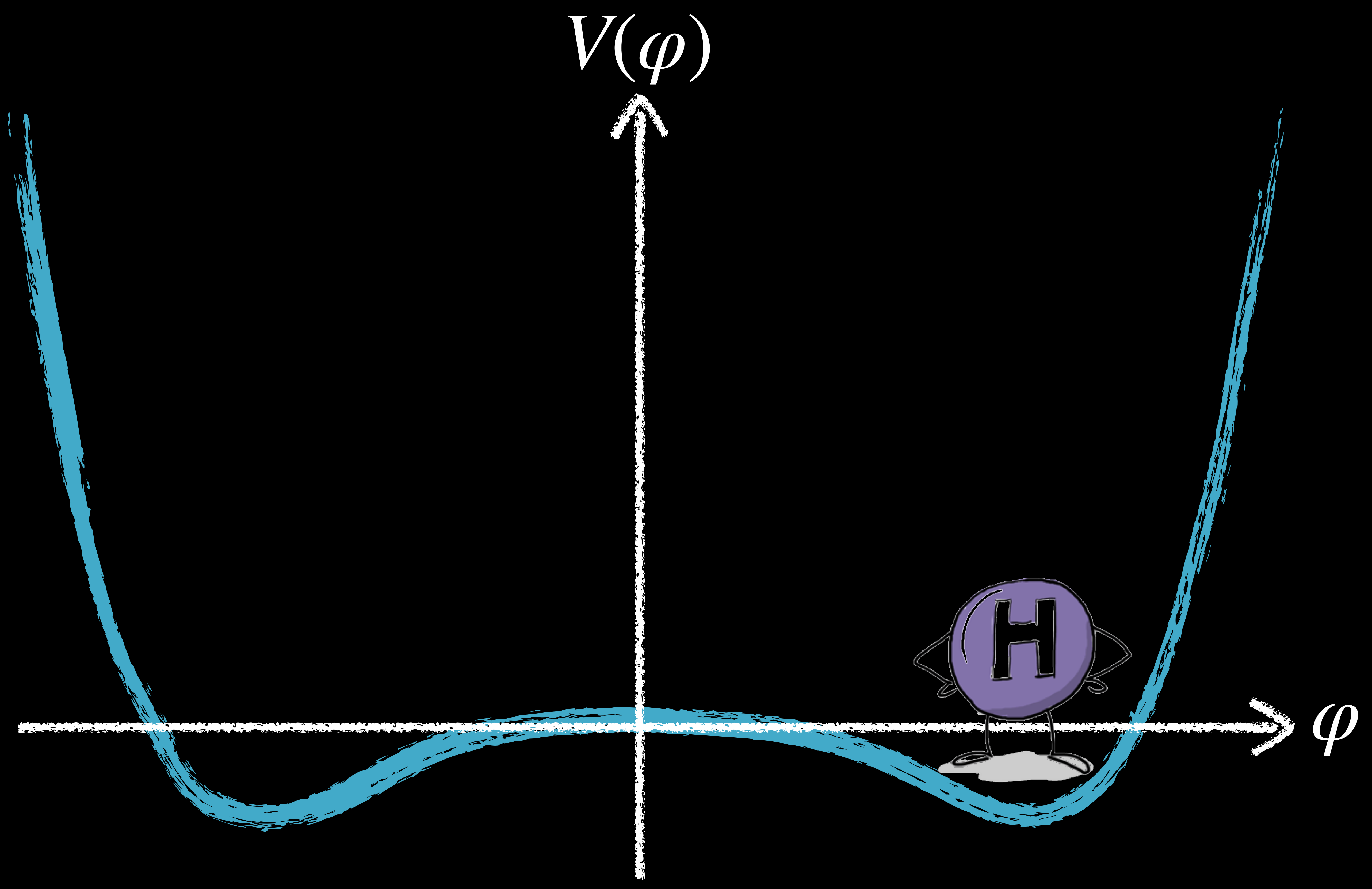


[Image from PhD Comics]

THEORY: SYMMETRY RESTORED AT HIGH T



PHASE TRANSITION HAPPENED!



[Image from PhD Comics]

In the SM the PT is a crossover.

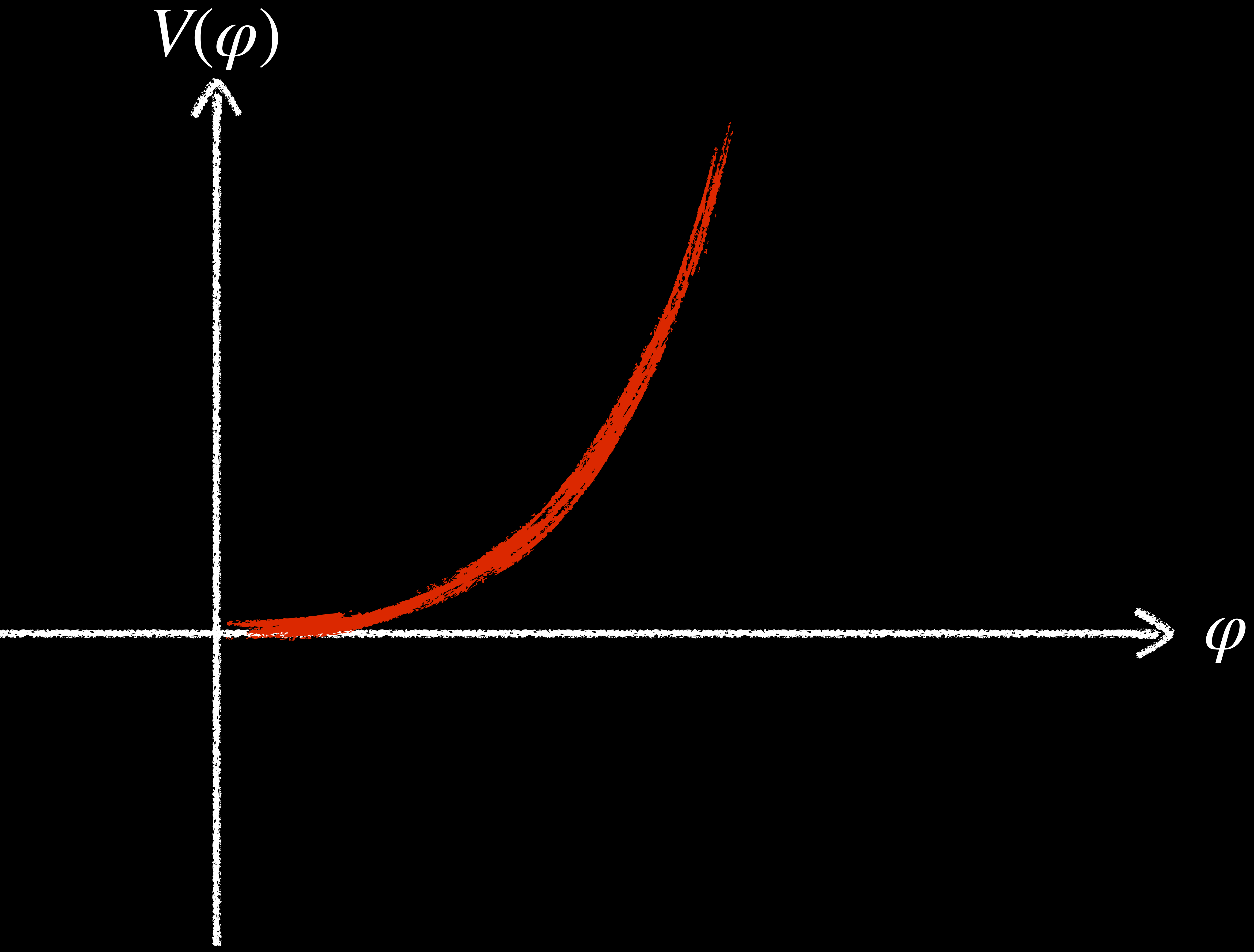
The search for a first-order PT is a search for New Physics!

scalar

[K. Kajantie, M. Laine, K. Rummukainen and M. E. Shaposhnikov, Phys. Rev. Lett. 77, 2887 (1996); K. Rummukainen, M. Tsypin, K. Kajantie, M. Laine and M. E. Shaposhnikov, Nucl. Phys. B 532, 283 (1998); F. Csikor, Z. Fodor and J. Heitger, Phys. Rev. Lett. 82, 21 (1999)]

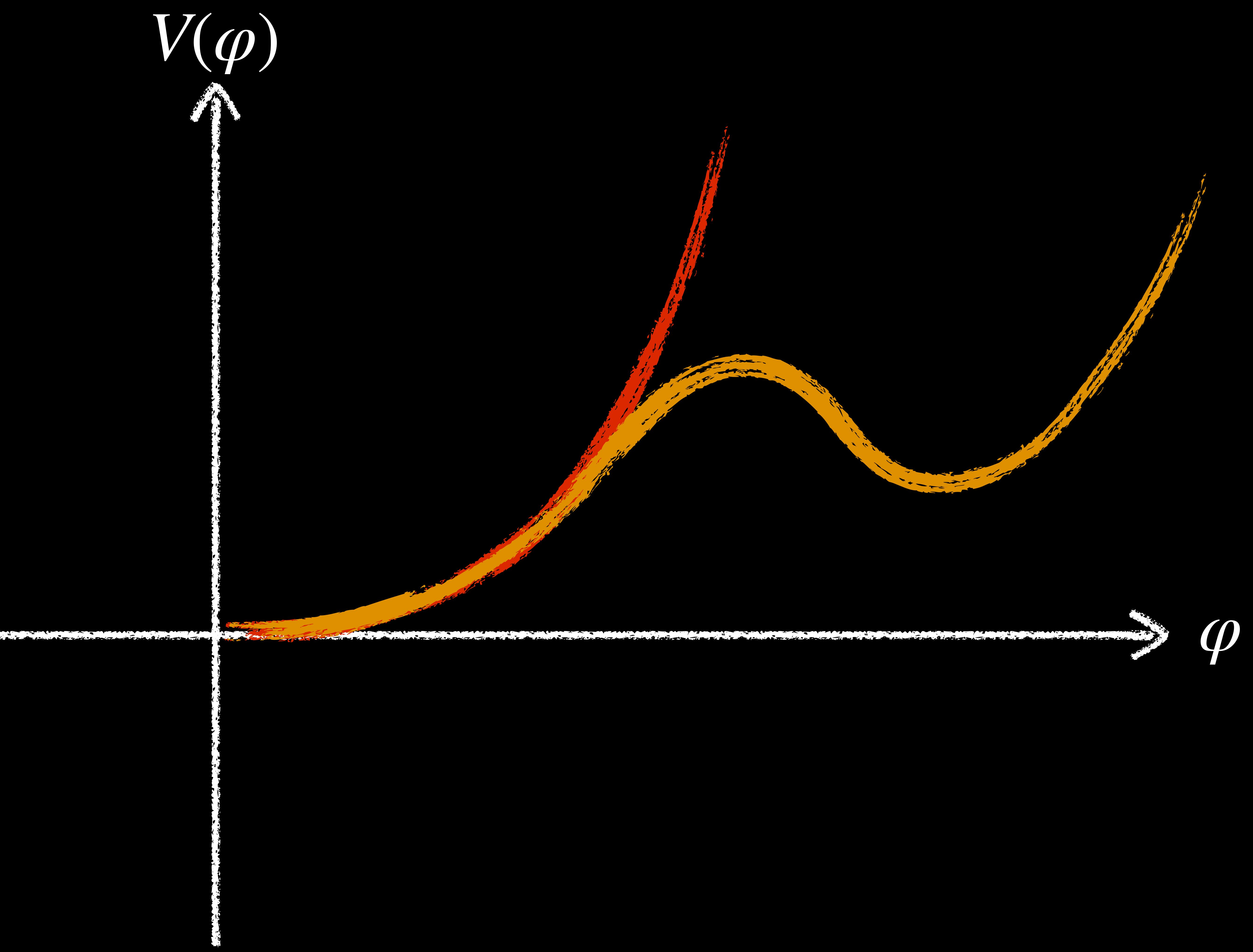
PHASE TRANSITION IN THE EARLY UNIVERSE

FIRST-ORDER PHASE TRANSITION



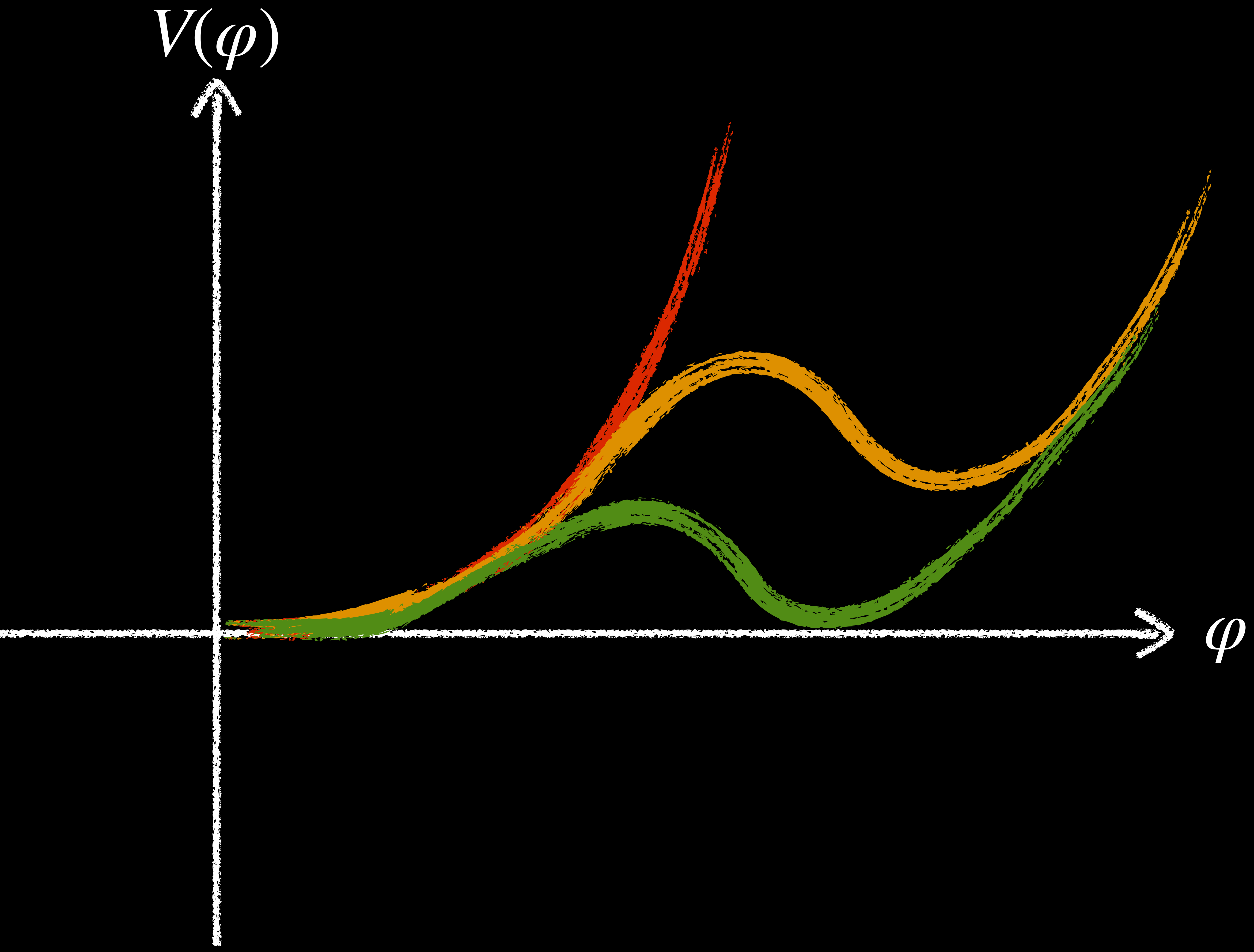
high temperature:
EW and conformal
symmetry restored

FIRST-ORDER PHASE TRANSITION



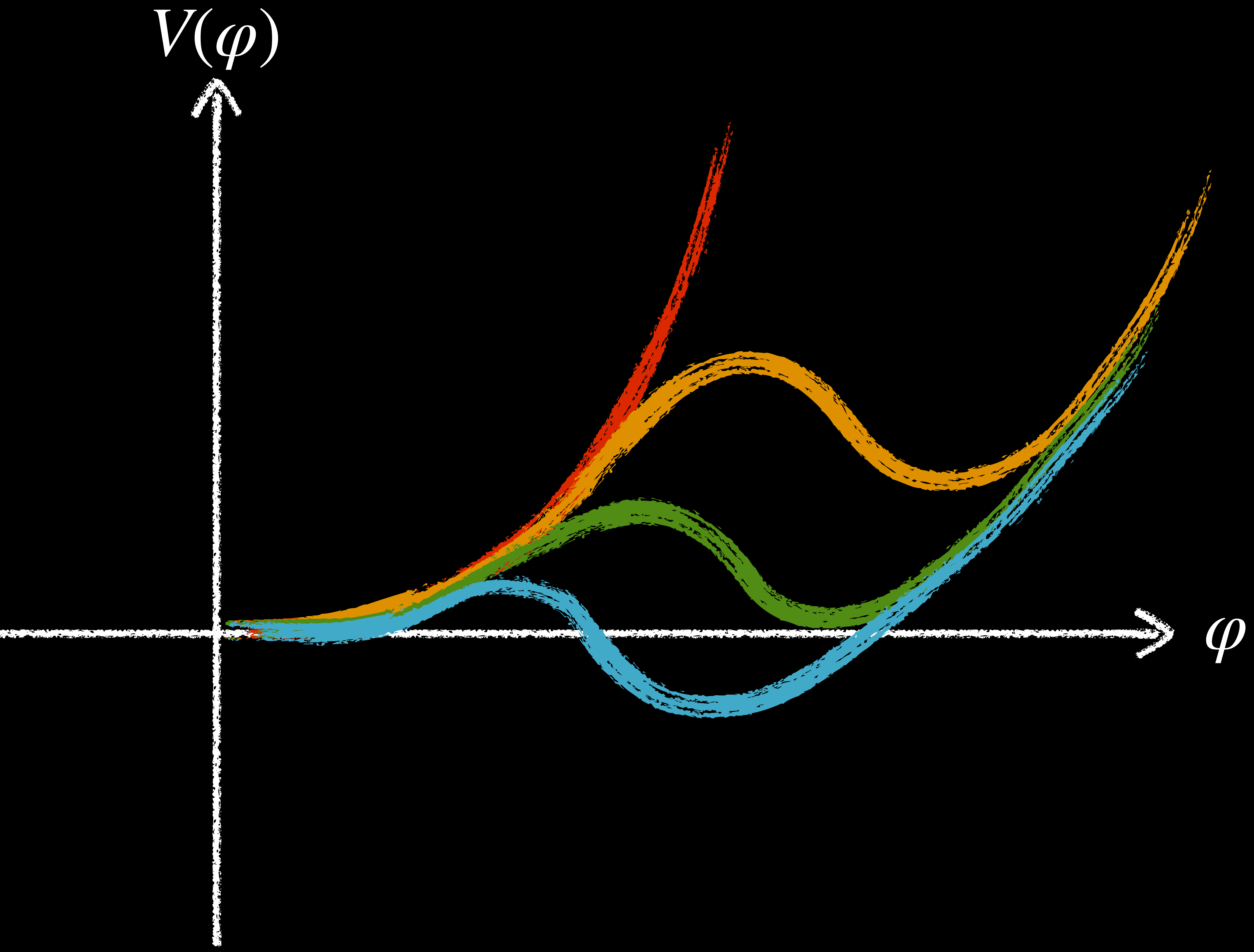
Secondary minimum forms

FIRST-ORDER PHASE TRANSITION



critical temperature:
two degenerate
minima

FIRST-ORDER PHASE TRANSITION



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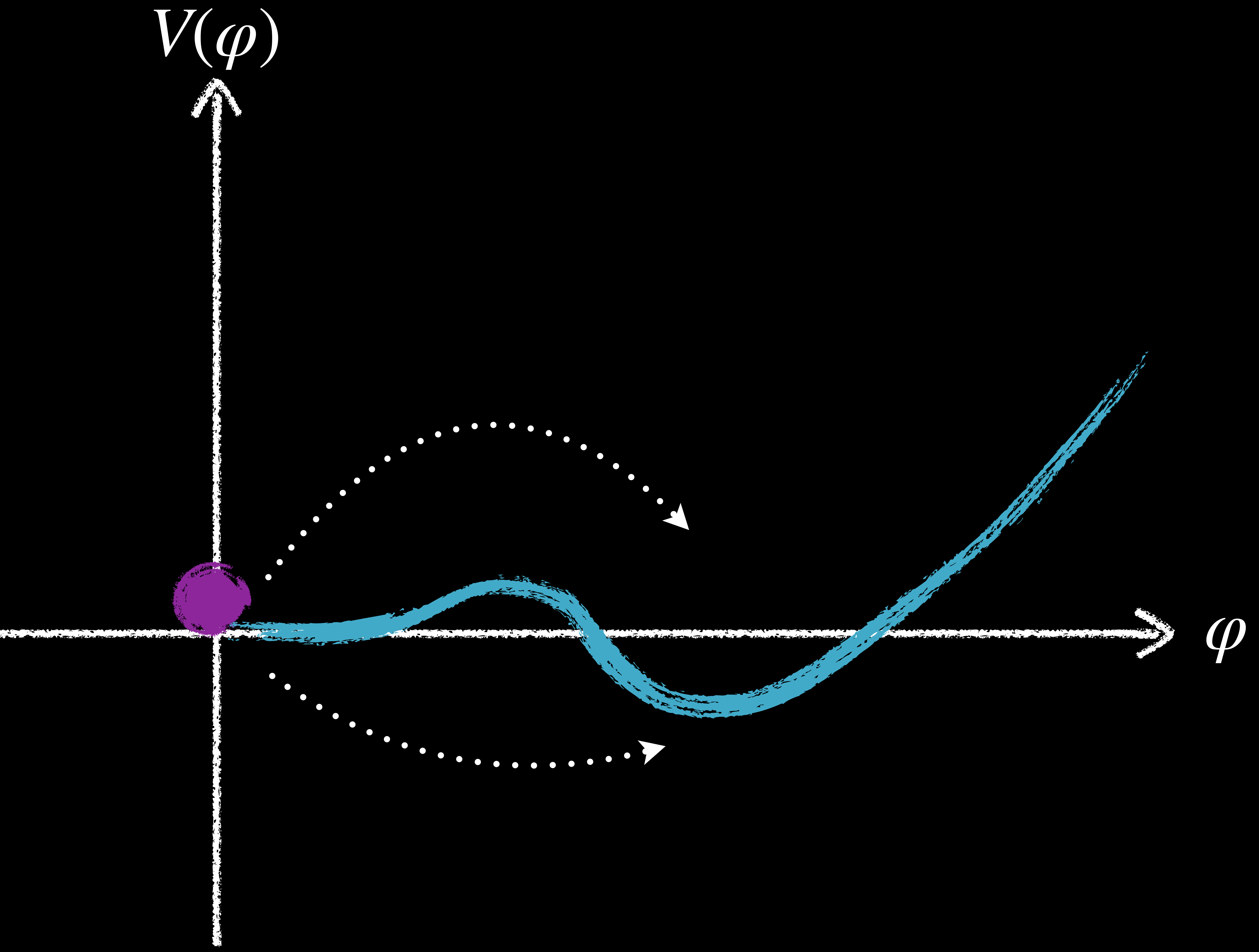

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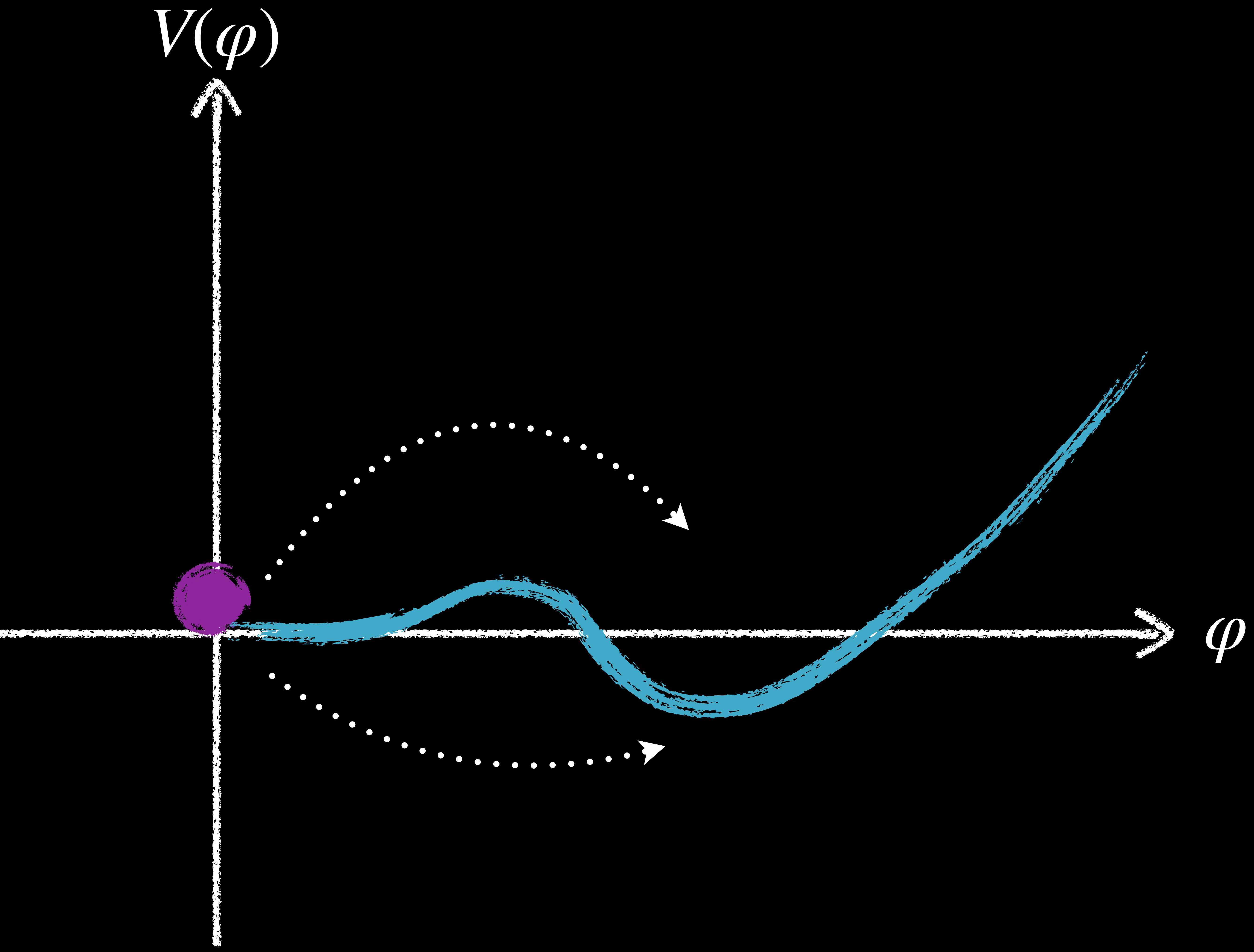
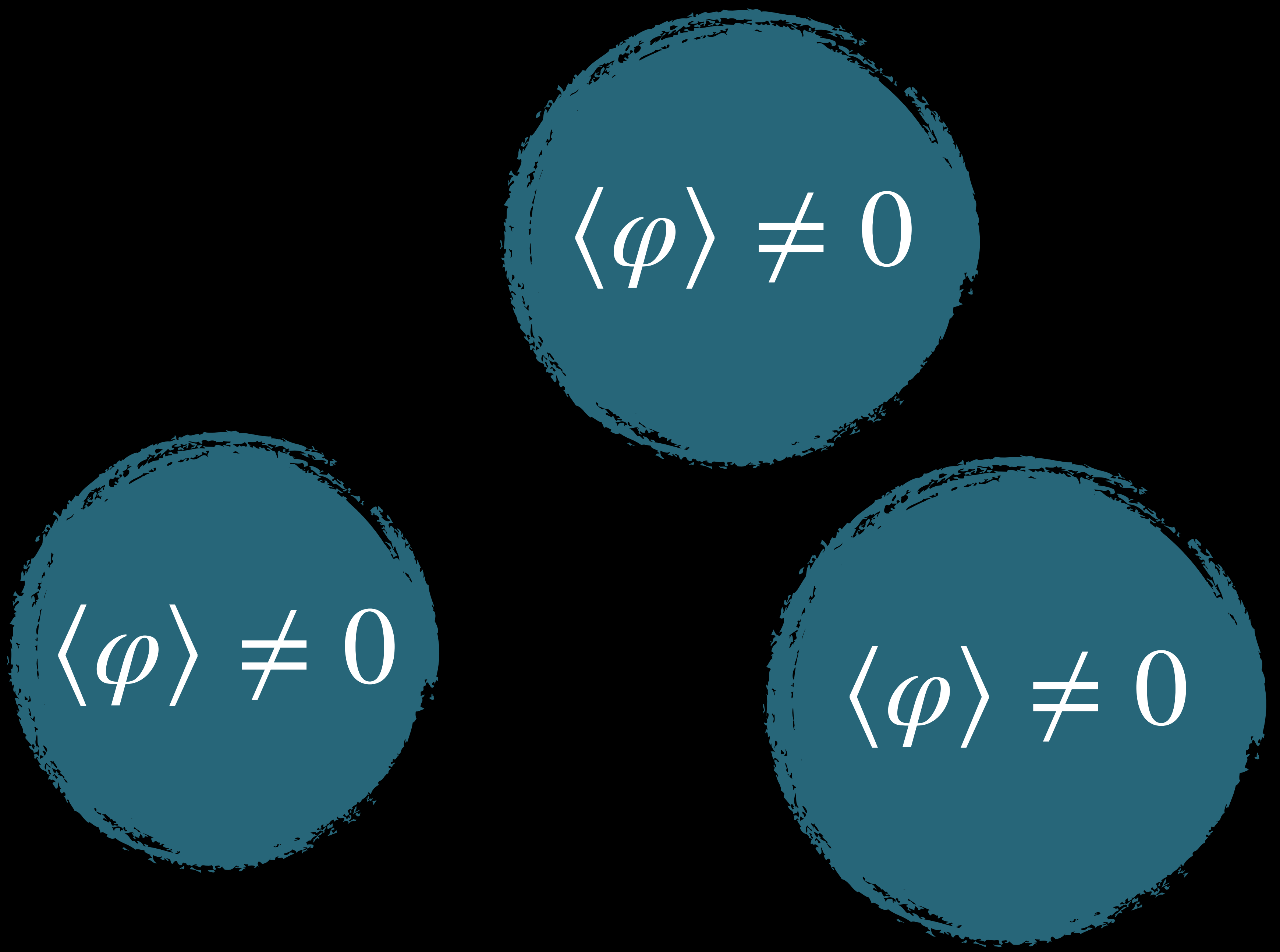
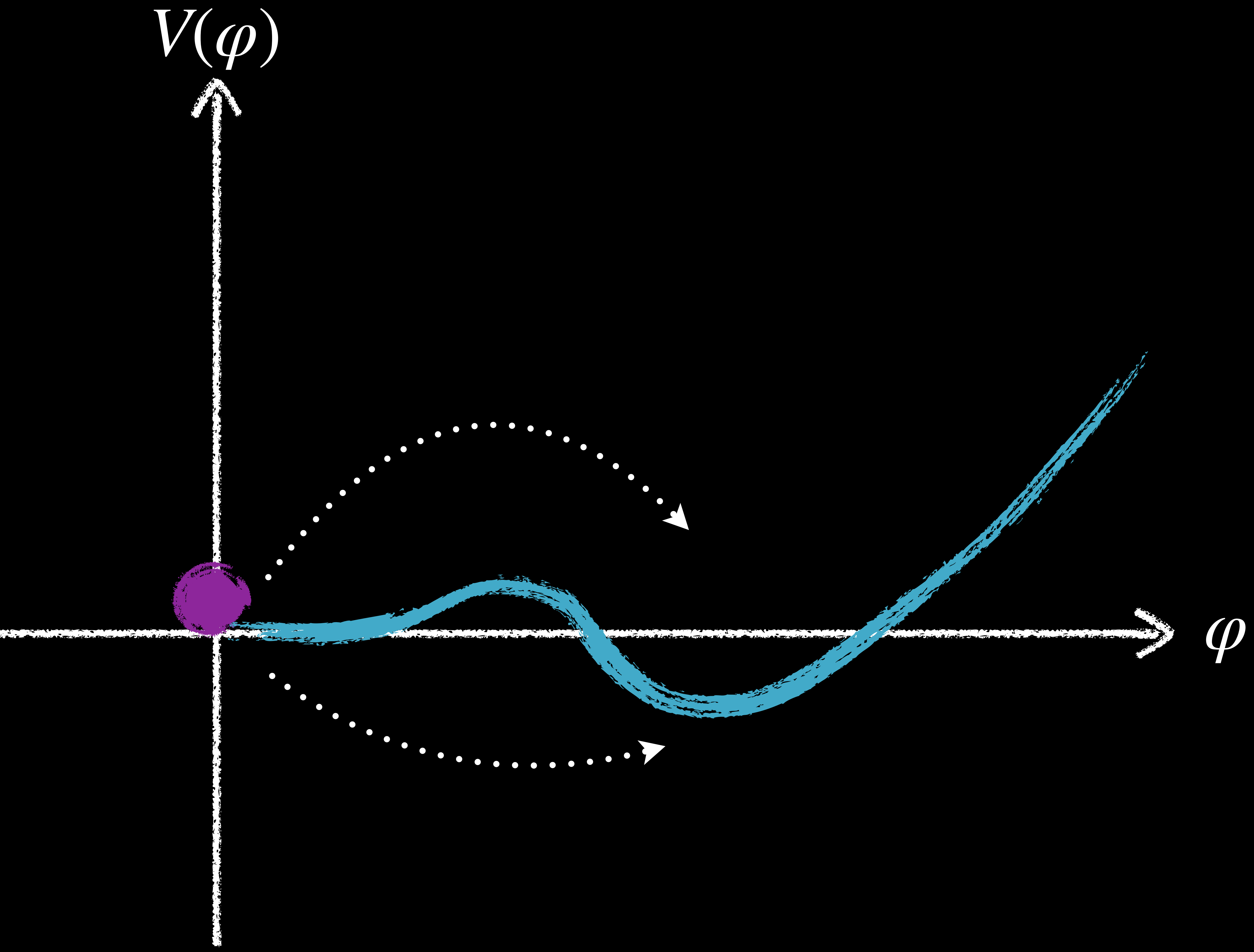


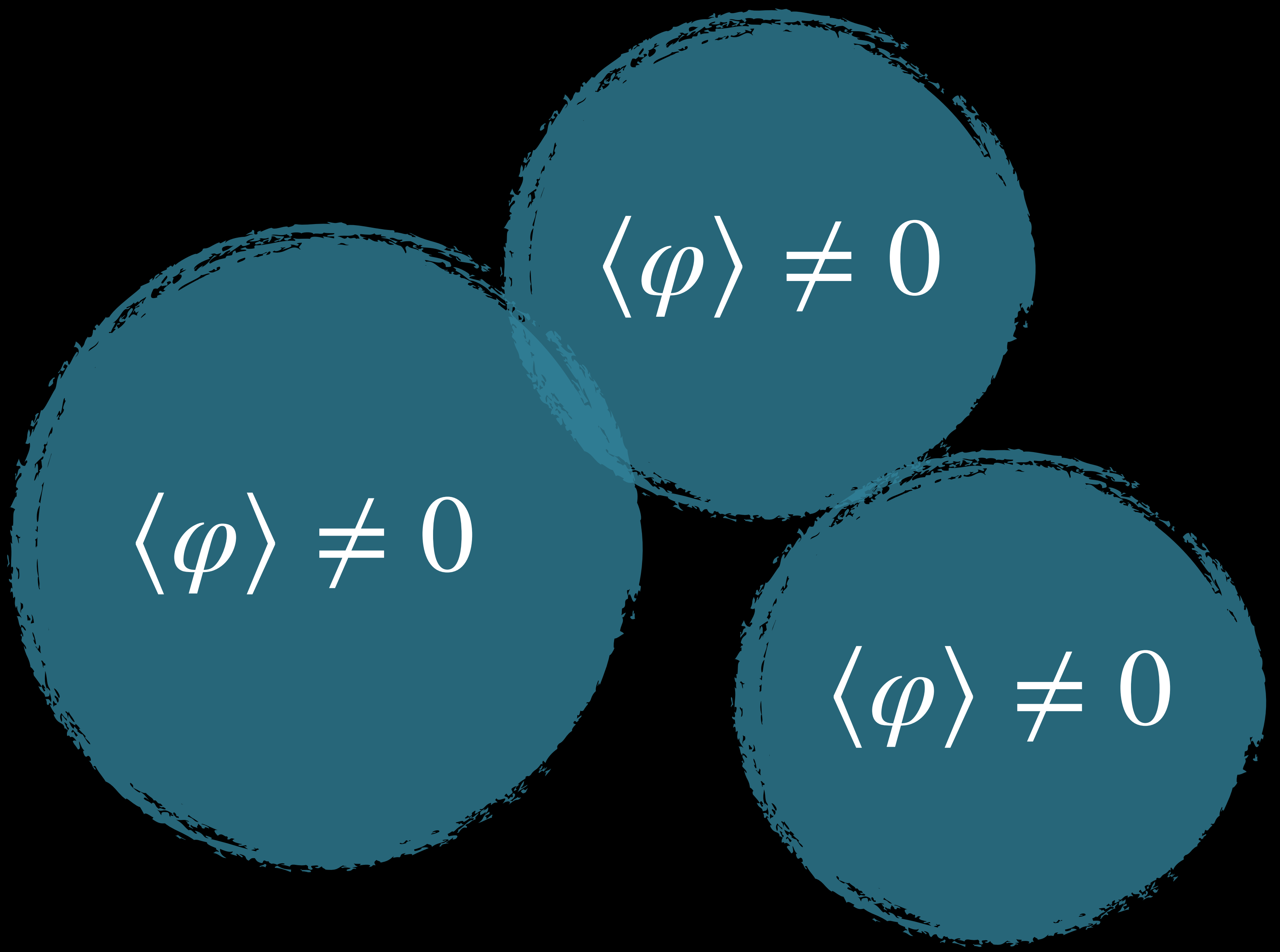
nucleation:
one bubble nucleates
per Hubble volume



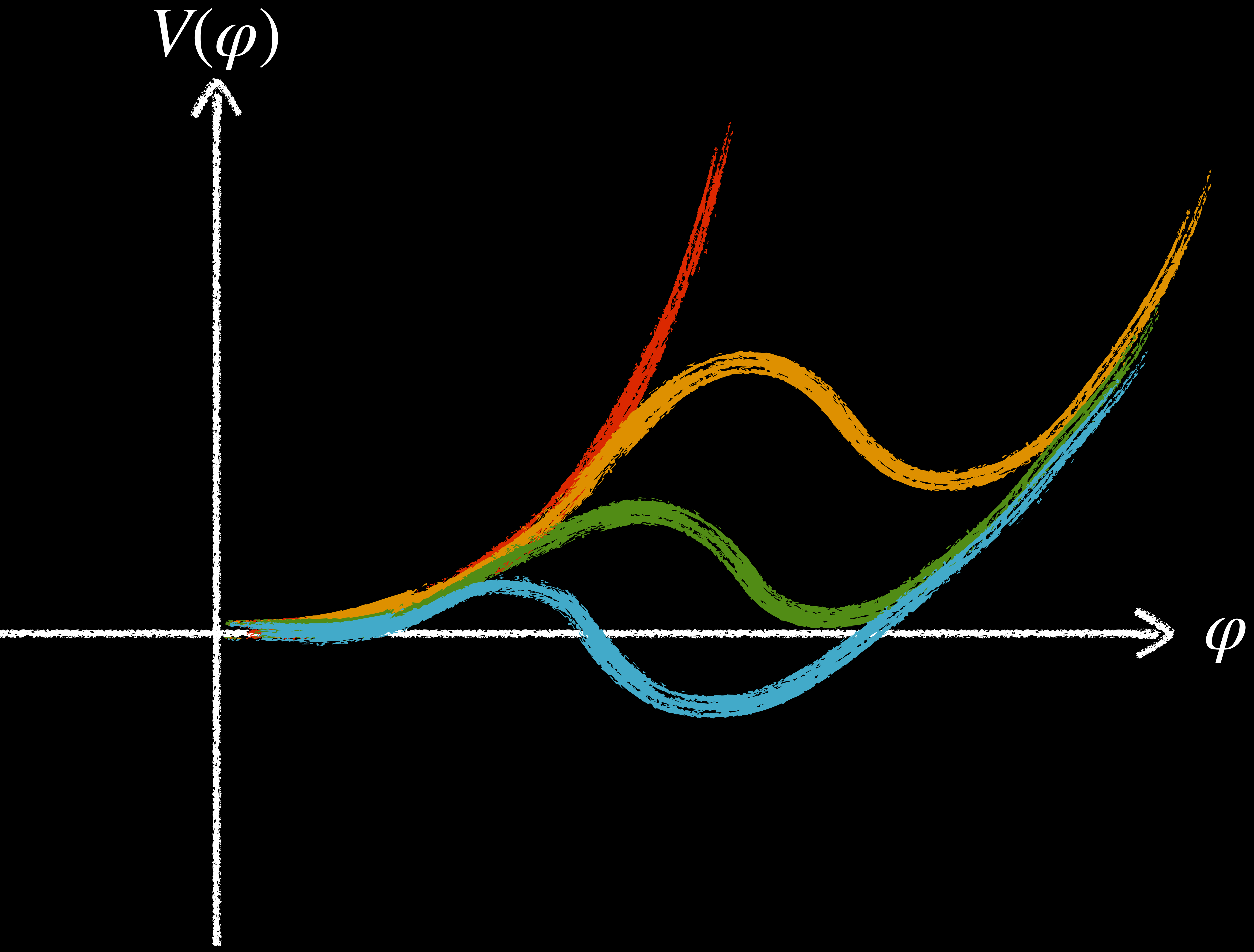
FIRST-ORDER PHASE TRANSITION



percolation: phase transition completes



SUPERCOOLED PHASE TRANSITION

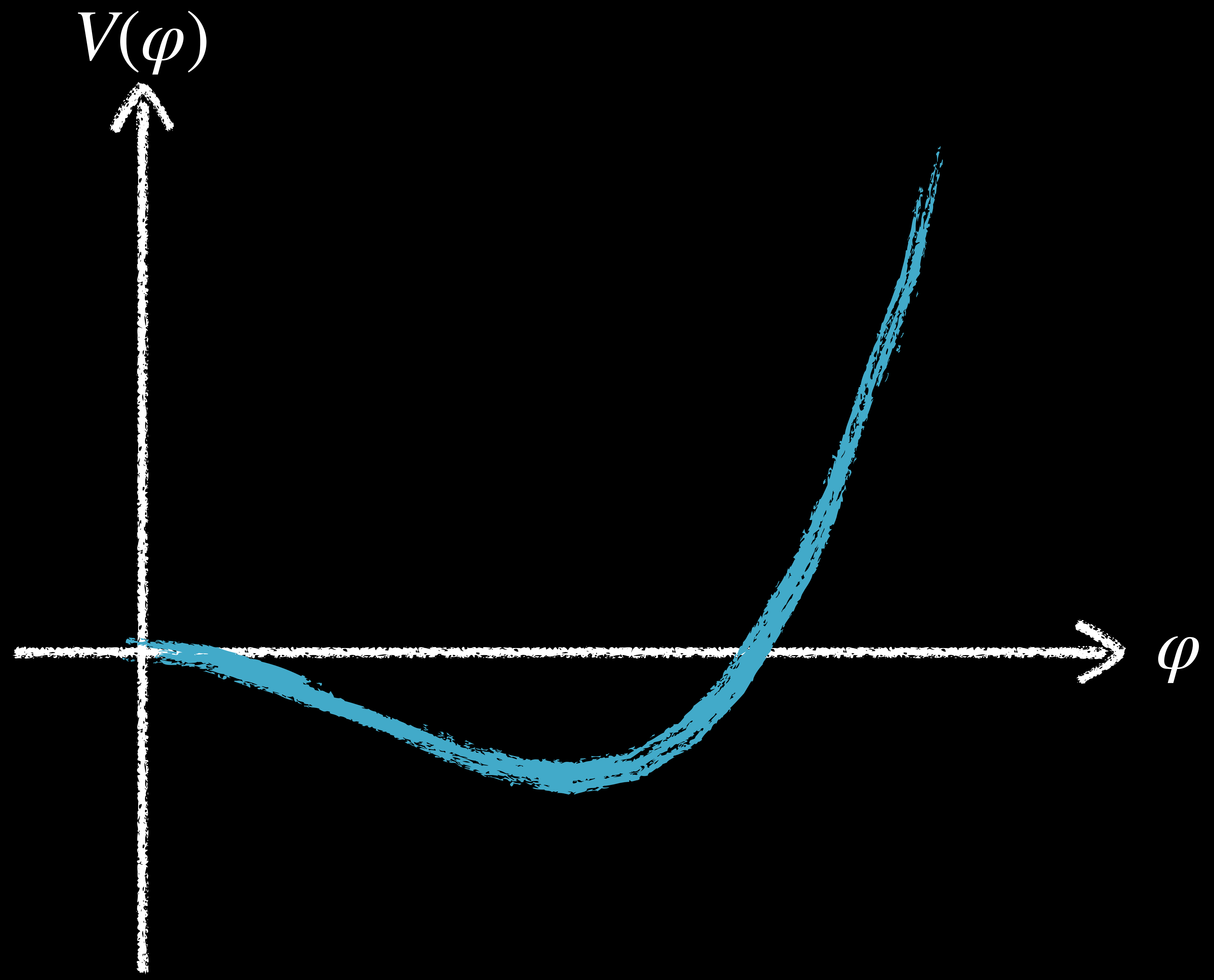


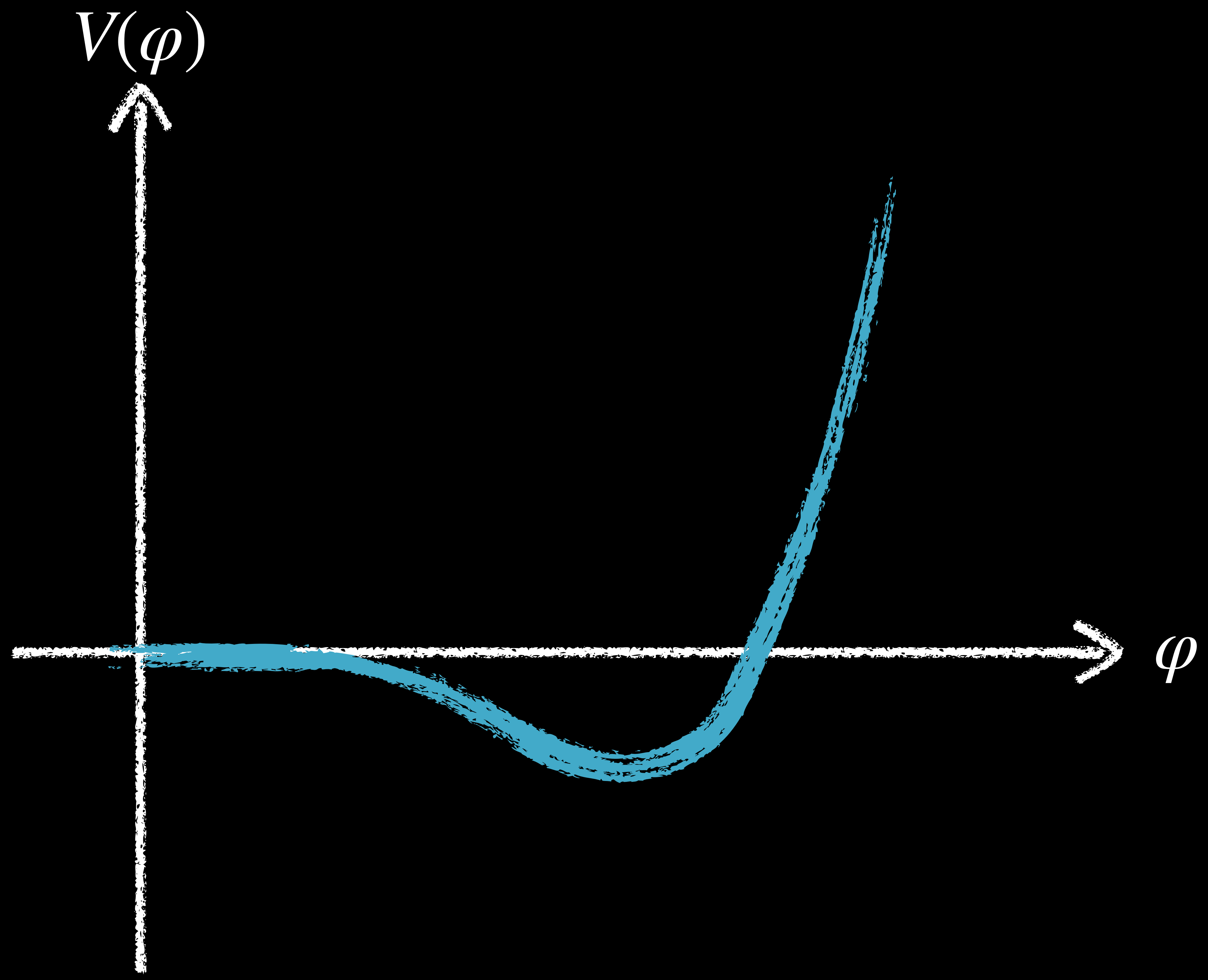
- Before nucleation: period of thermal inflation,
- Percolation temperature $\ll$ critical temperature,
- Huge energy release (compared to radiation energy), $\alpha \approx \dfrac{\Delta V}{\rho_{\rm rad}} \gg 1$,
- Percolation during inflation: make sure that bubbles percolate!
- Significant reheating after the PT

[L. Randall, G. Servant, JHEP 05 (2007) 054, T. Konstandin, G. Nardini, M. Quiros, PRD82 (2010) 083513, T. Konstandin, G. Servant, JCAP 1112 (2011) 009, J. Kubo, M. Yamada, JCAP 1612 (2016), T. Hambye, A. Strumia 88 (2013) 055022 and many more recent papers]

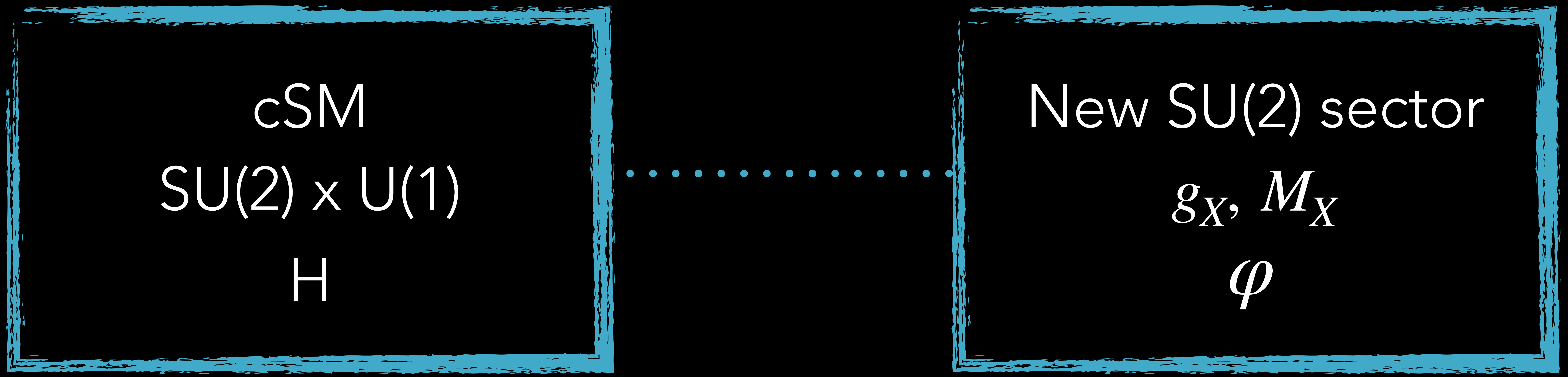
MODEL FOR SUPERCOOLING

ARCHETYPE: CLASSICAL SCALE INVARIANCE



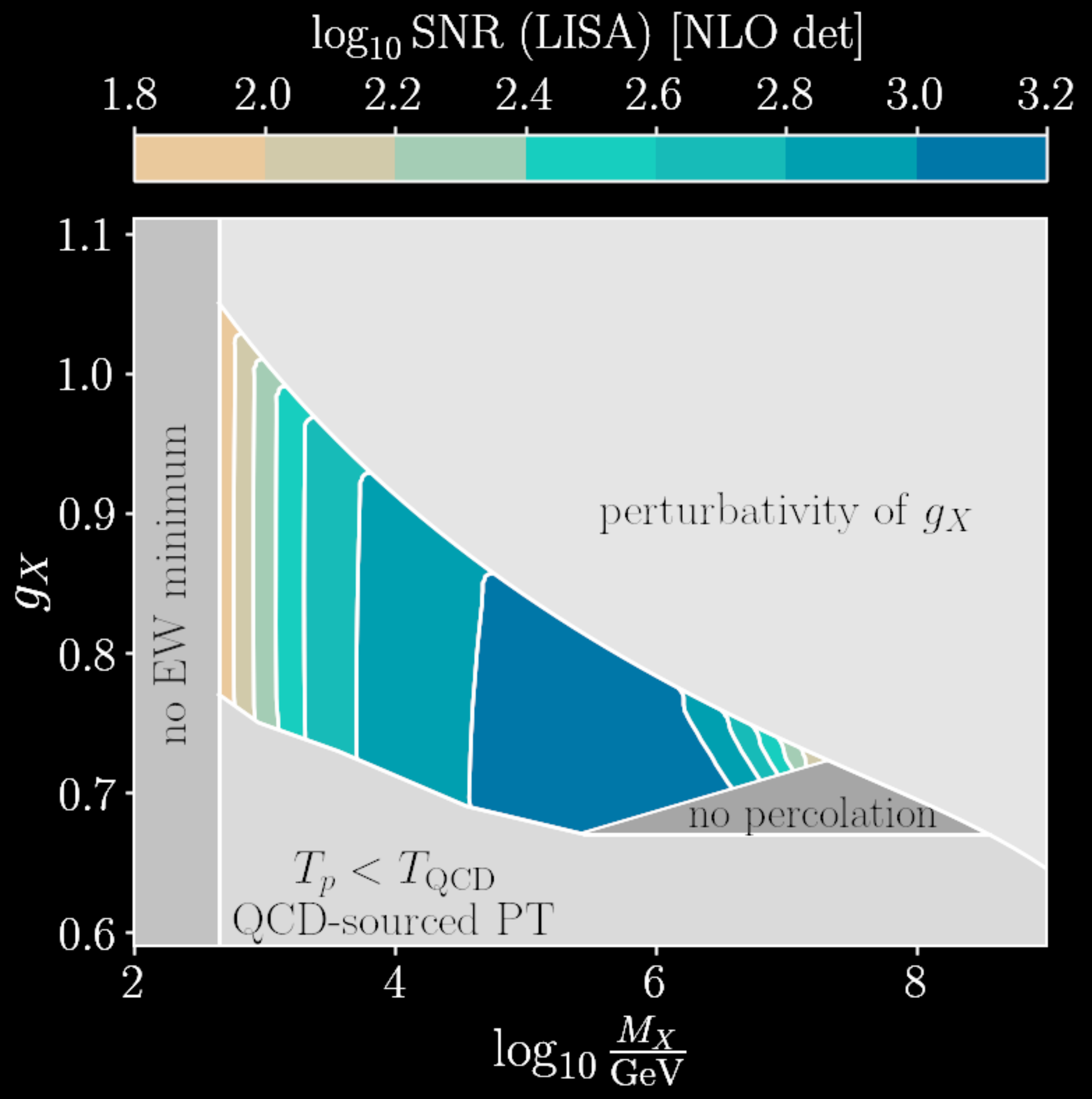


THE MODEL



[See also: T.Hambye, A.Strumia, PRD88 (2013) 055022, C.Carone, R.Ramos, PRD88 (2013) 055020, V.V.Khoze, C.McCabe, G.Ro, JHEP 08 (2014) 026, T. Hambye, A.Strumia, D.Teresi, JHEP 1808 (2018) 188, I.Baldes, C. Garcia-Cely, JHEP 05 (2019) 190, T.Prokopec, J.Rezacek, BS, JCAP 02(2019)009, D. Marfaria, P. Tseng, JHEP 02 (2021) 022]

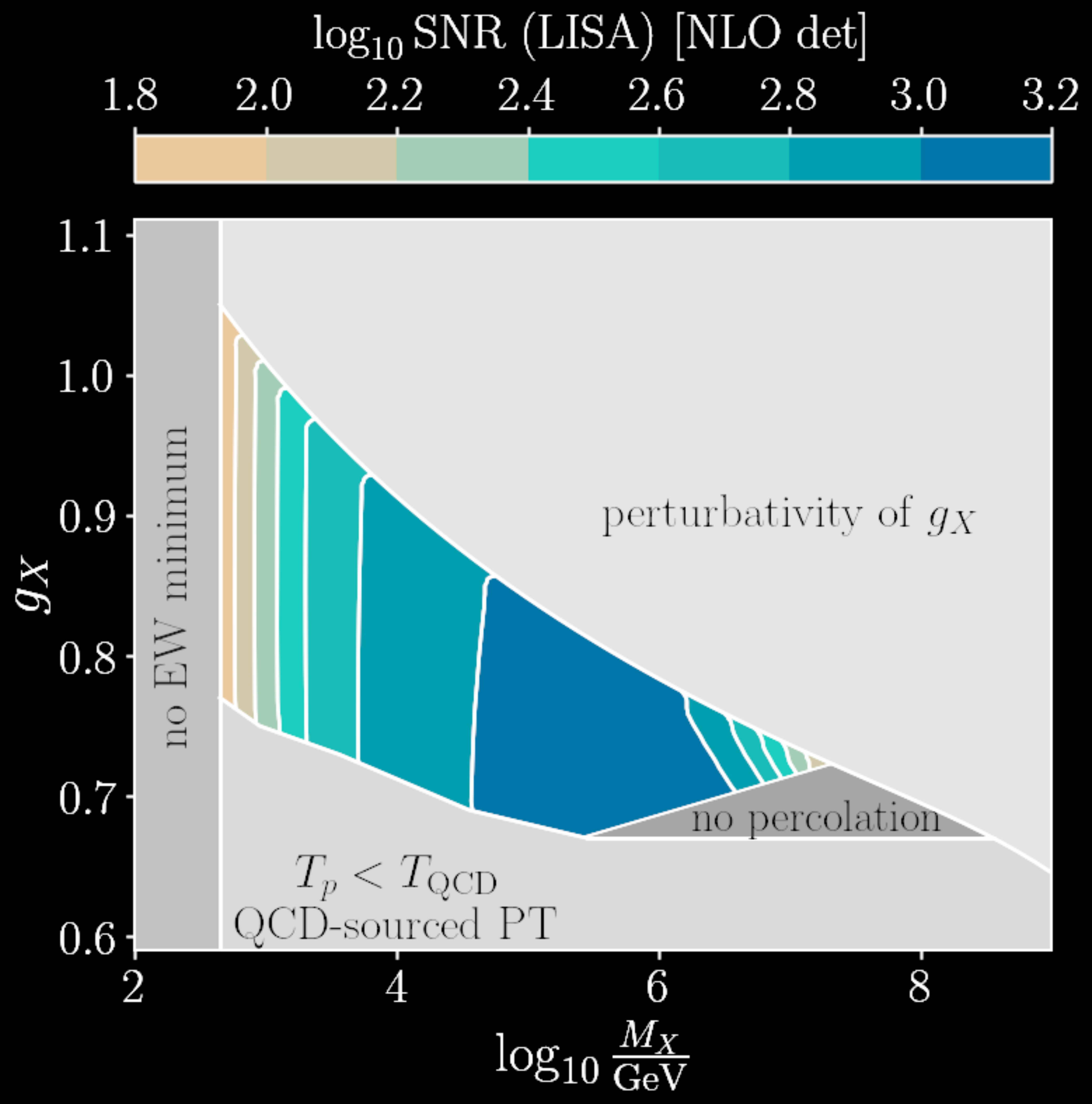
PROBING SU(2)CSM THROUGH GW



[Image adapted from: M. Kierkla, PhD thesis, University of Warsaw, 2025]

SU(2)cSM is falsifiable through GW

PROBING SU(2)CSM THROUGH GW



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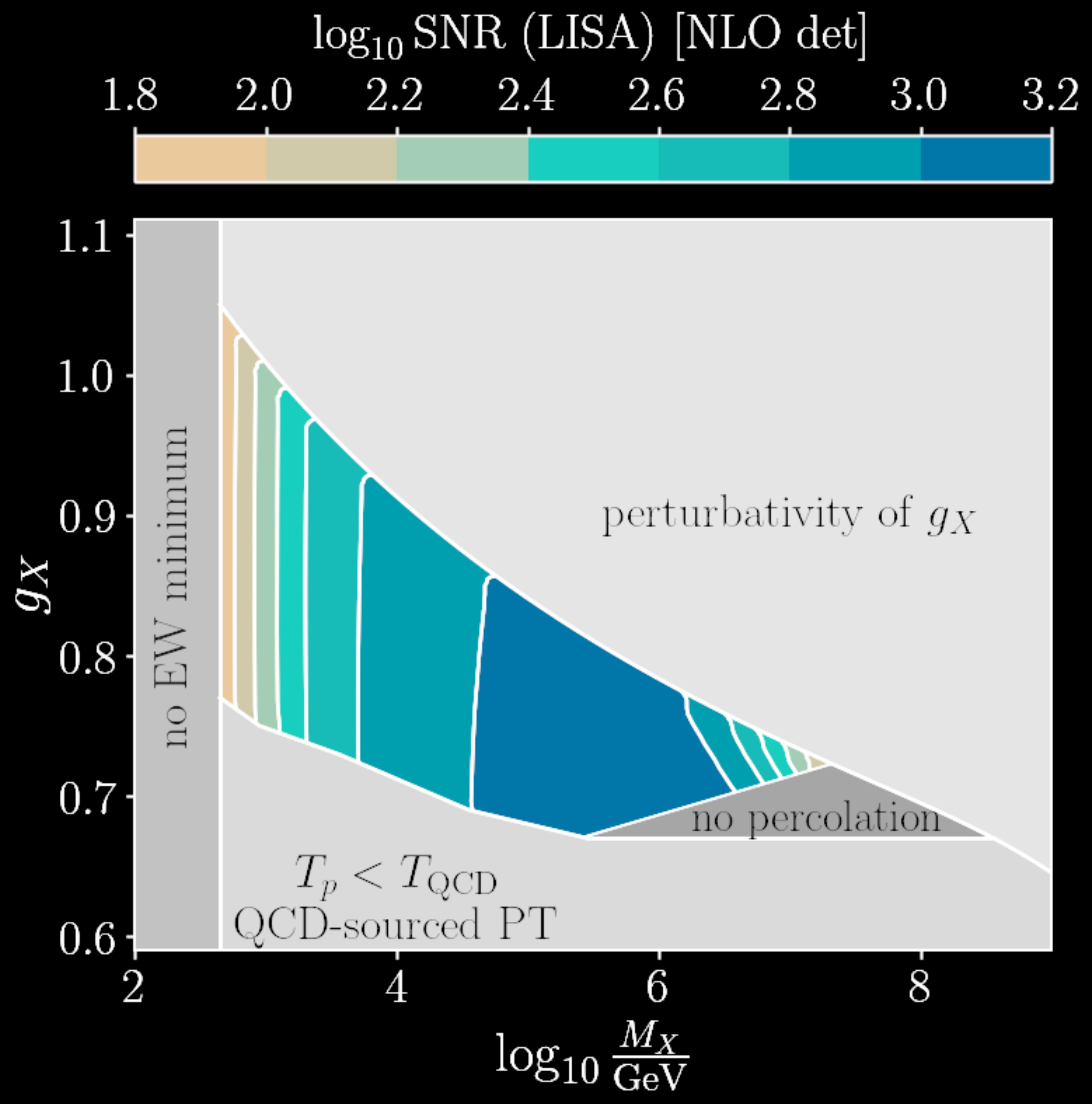
SU(2)cSM is falsifiable through GW

+ PBH
+ PTA signal

SUPERCOOLED PHASE TRANSITION

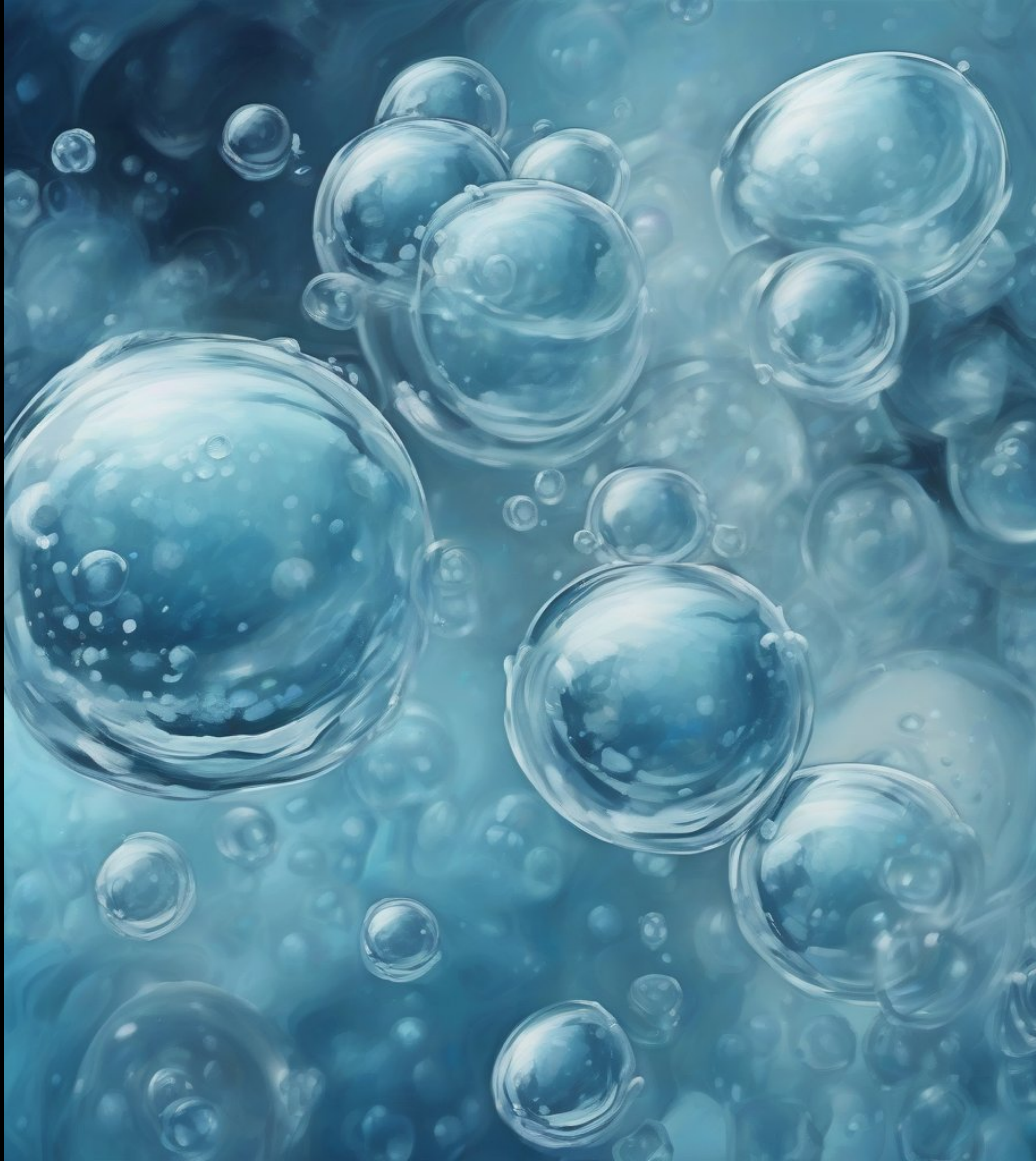


[Image adapted from: M. Kierkla, PhD thesis, University of Warsaw, 2025]

Large SNR →
good precision of
reconstruction.

ADVANCING

BUBBLE NUCLEATION RATE



BUBBLE NUCLEATION RATE

$$\Gamma = A_{\text{dyn}} \cdot A_{\text{stat}} = A_{\text{dyn}} \cdot A_{\text{det}} \cdot \exp(-S_{\text{eff}}[\varphi_b])$$

BUBBLE NUCLEATION RATE

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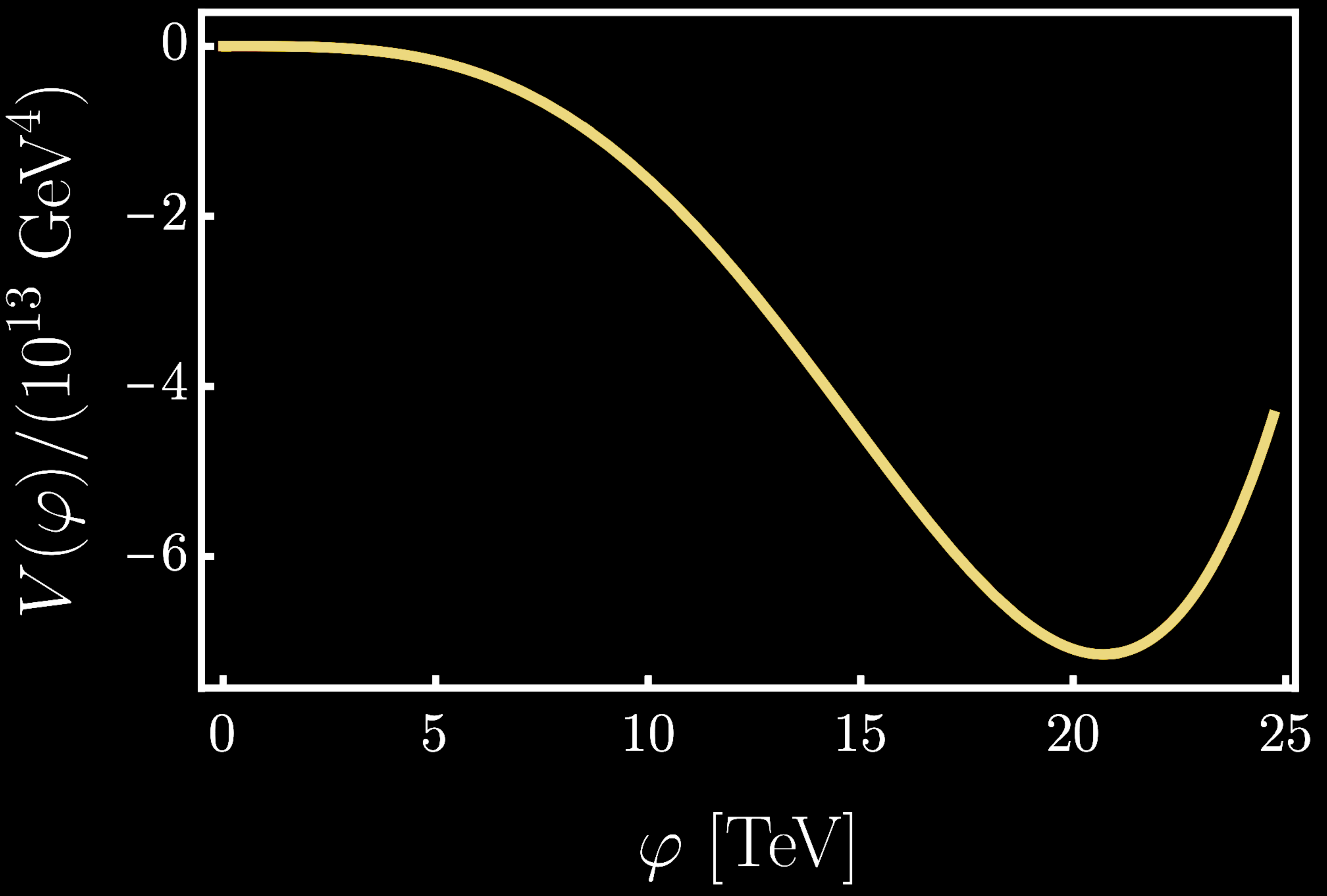
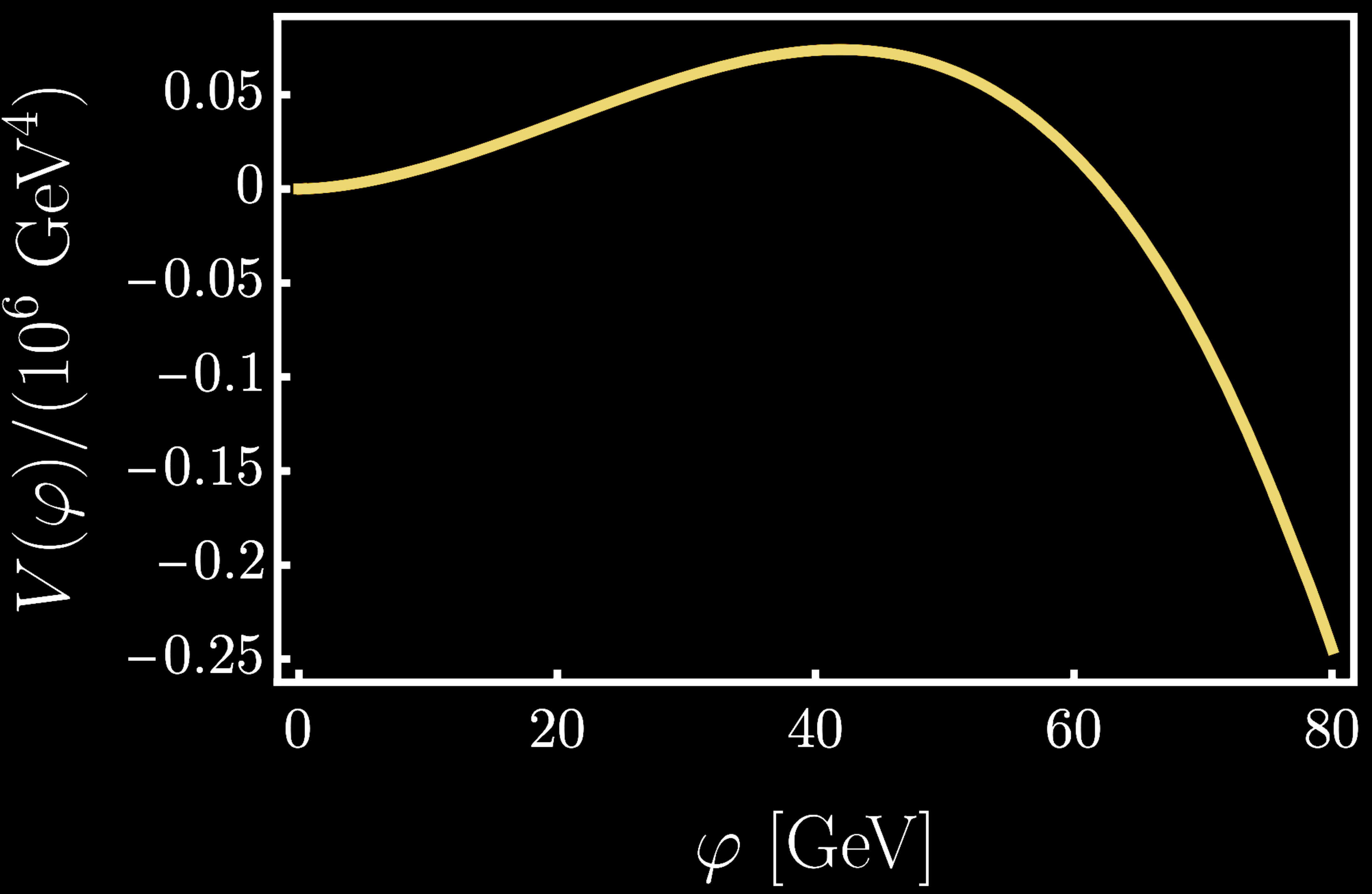
Proper treatment of:

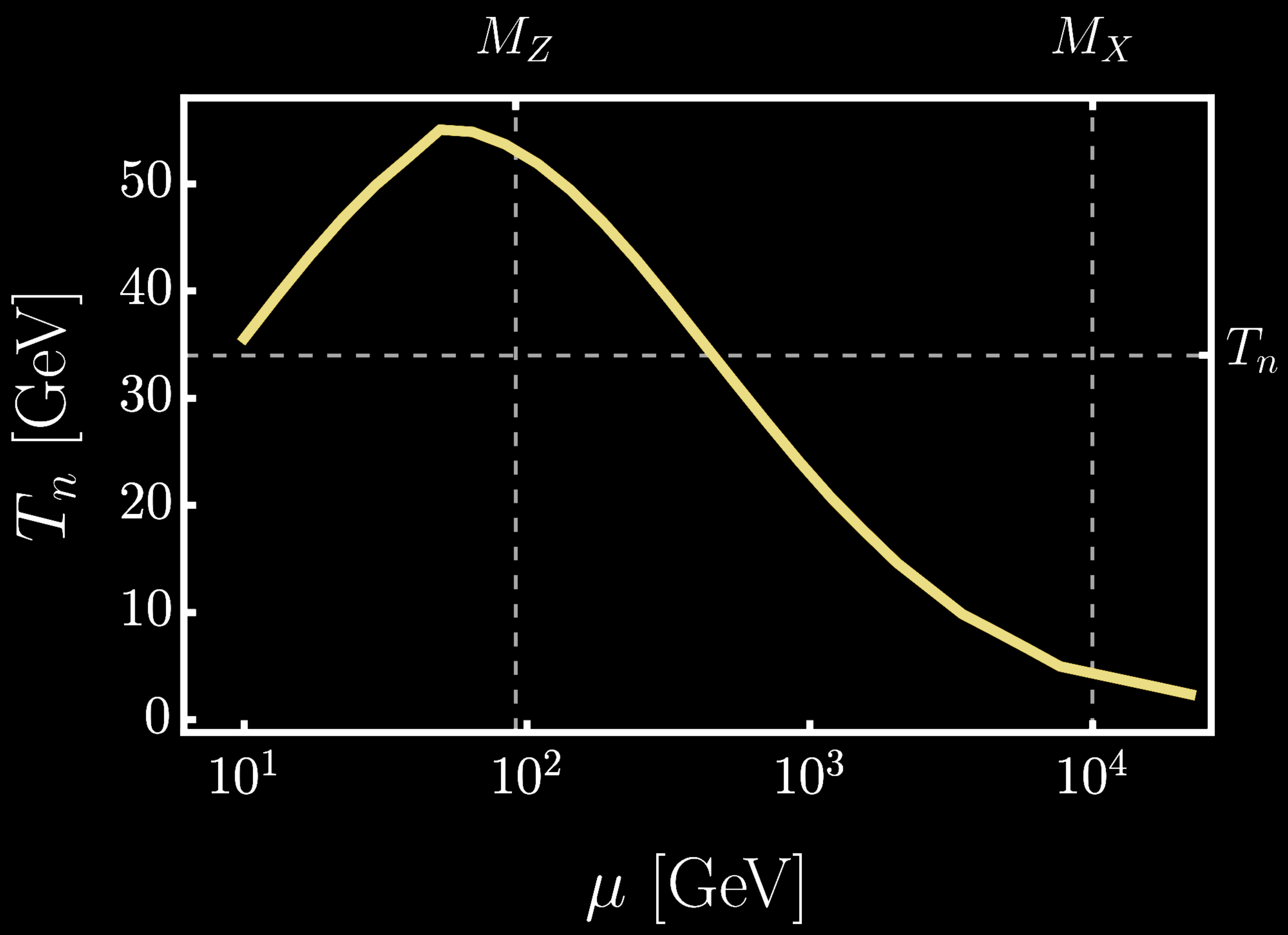
1. renormalisation-scale dependence
2. thermal resummations
3. exponential prefactors

HIGH-T VS LOW-T





SCALE DEPENDENCE - SOURCE OF UNCERTAINTY



[M. Kierkla, A. Karam, BŚ, JHEP 03 (2023) 007]

SUPERCOOLING AT HIGH TEMPERATURE??

High-temperature effective field theory

???

Supercooled phase transition

HIGH-TEMPERATURE EFT FOR NUCLEATION

High-temperature effective field theory

Supercooled phase transition

[M. Kierkla, BŚ, T.V.I. Tenkanen, J. van de Vis, JHEP 02 (2024) 234]

HIGH-TEMPERATURE EFT FOR NUCLEATION

High-temperature effective field theory

See the talk by Maciej Kierkla on Wednesday!

Supercooled phase transition

[M. Kierkla, BŚ, T.V.I. Tenkanen, J. van de Vis, JHEP 02 (2024) 234]

NUCLEATION RATE WITHOUT DERIVATIVE EXP.

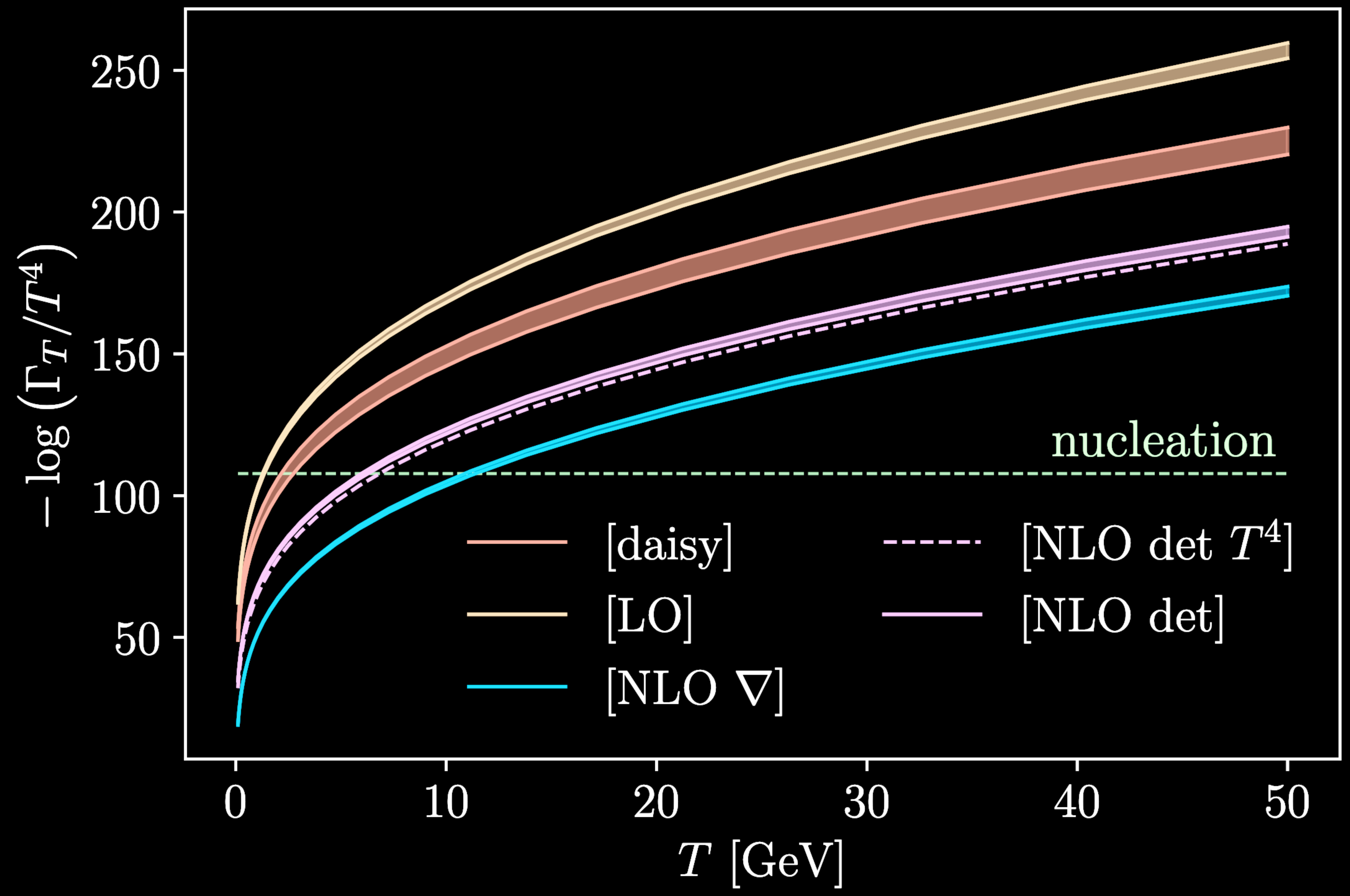
$$\Gamma = A_{\text{dyn}} \cdot A_{\text{stat}}$$

$$A_{\text{stat}} = \prod_a \mathcal{J}_a \mathcal{V}_a \sqrt{\frac{\det \mathcal{O}_a(\varphi_F)}{\det' \mathcal{O}_a(\varphi_b)}} \mathcal{J}_\phi \sqrt{\left| \frac{\det \mathcal{O}_\phi(\varphi_F)}{\det' \mathcal{O}_\phi(\varphi_b)} \right|} \, e^{-(S[\varphi_b]-S[\varphi_F])}$$

$$\mathcal{O}_a(\varphi) = -\partial^2 + m_a^2(\varphi)$$

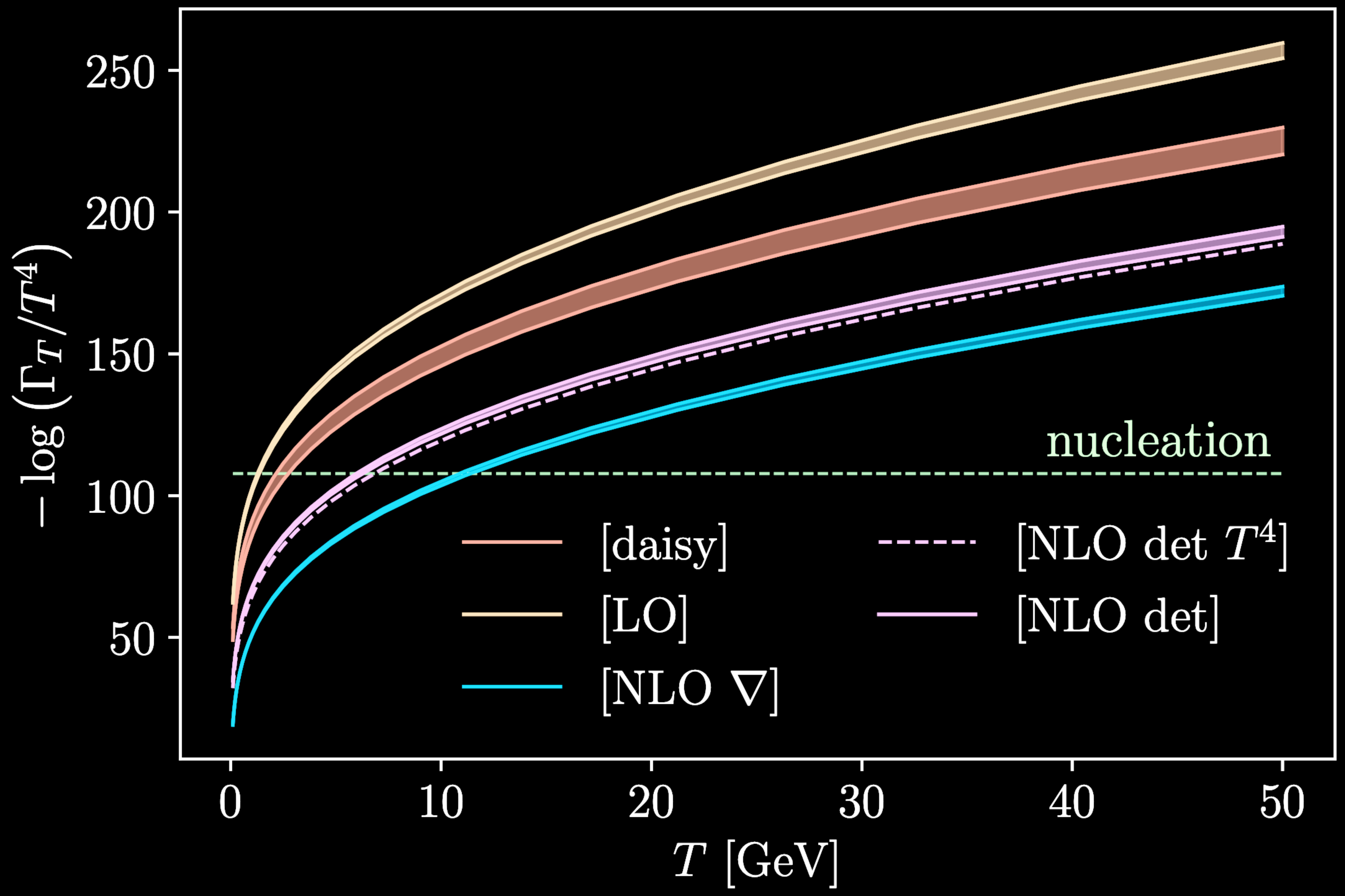
$$\mathcal{O}_\phi(\varphi) = -\partial^2 + (V^{\text{LO}})''(\varphi)$$

COMPARISON OF DIFFERENT APPROXIMATIONS



[M. Kierkla, P. Schicho, BŚ, T.V.I. Tenkanen, J. van de Vis, JHEP 07 (2025) 153]

COMPARISON OF DIFFERENT APPROXIMATIONS



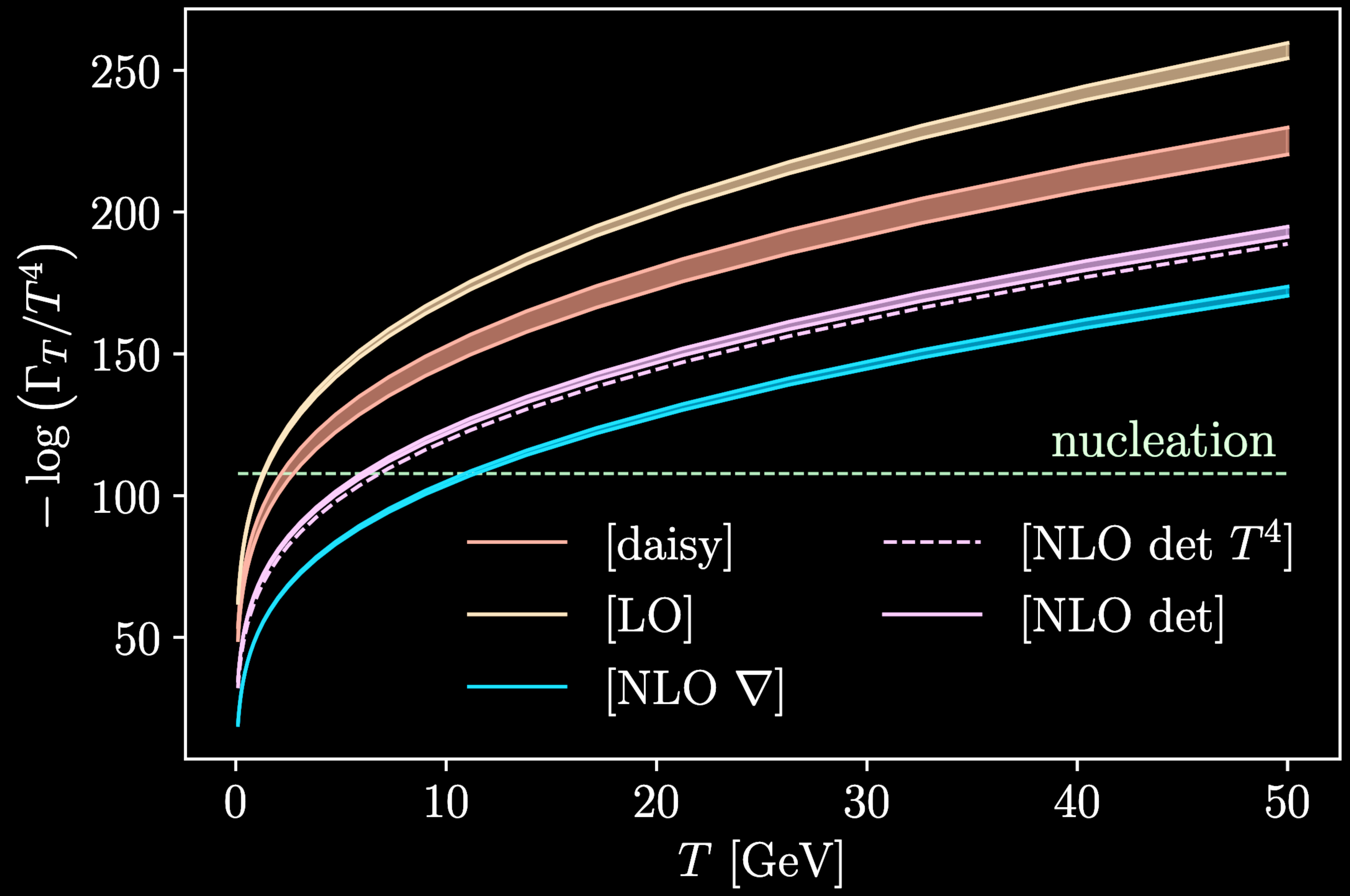
Relative changes:

Percolation temp.: up to 300%

Bubble radius: up to 25%

[M. Kierkla, P. Schicho, BŚ, T.V.I. Tenkanen, J. van de Vis, JHEP 07 (2025) 153]

COMPARISON OF DIFFERENT APPROXIMATIONS



Relative changes:

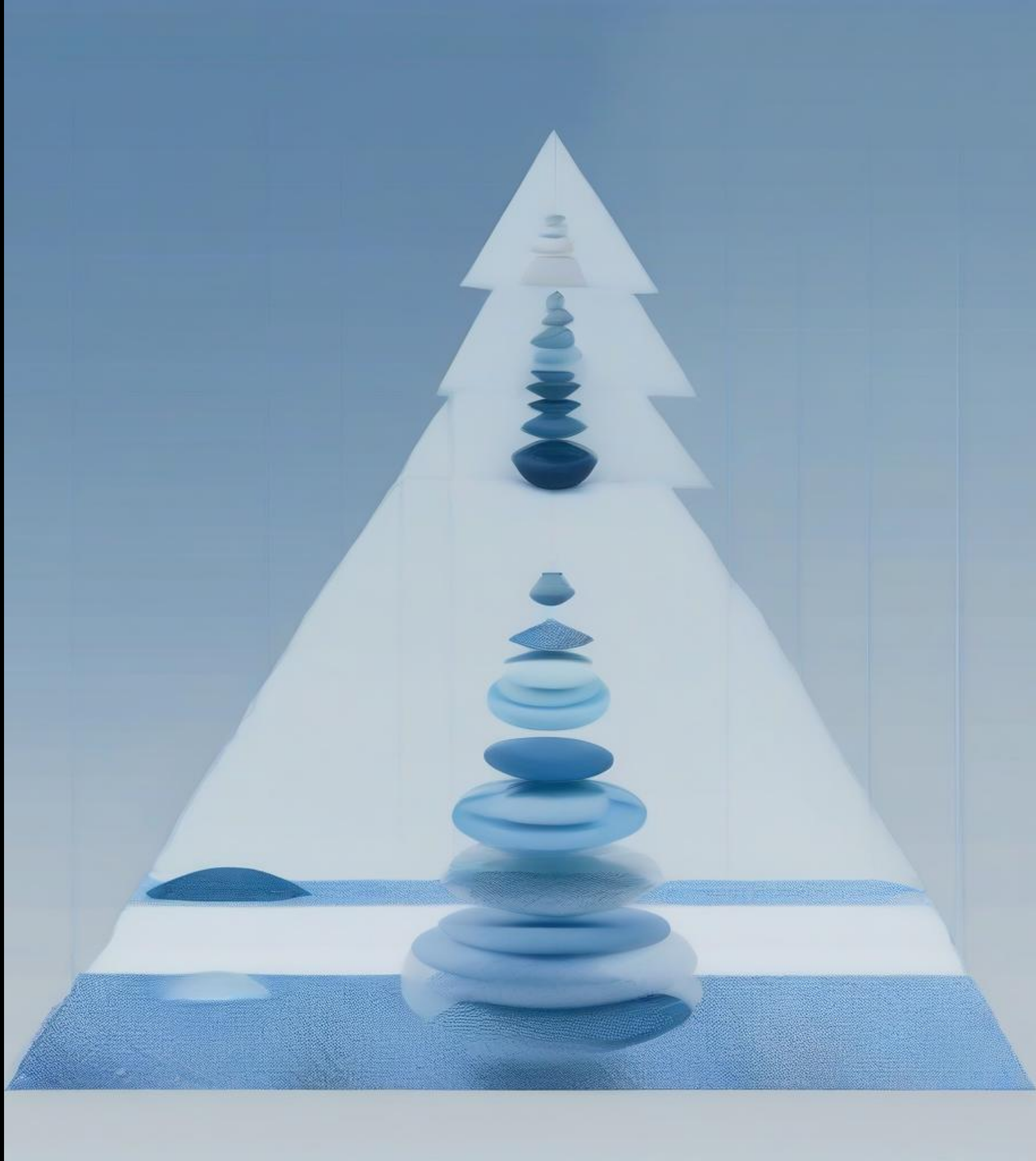
Percolation temp.: up to 300%

Bubb

Theoretical diligence is crucial!

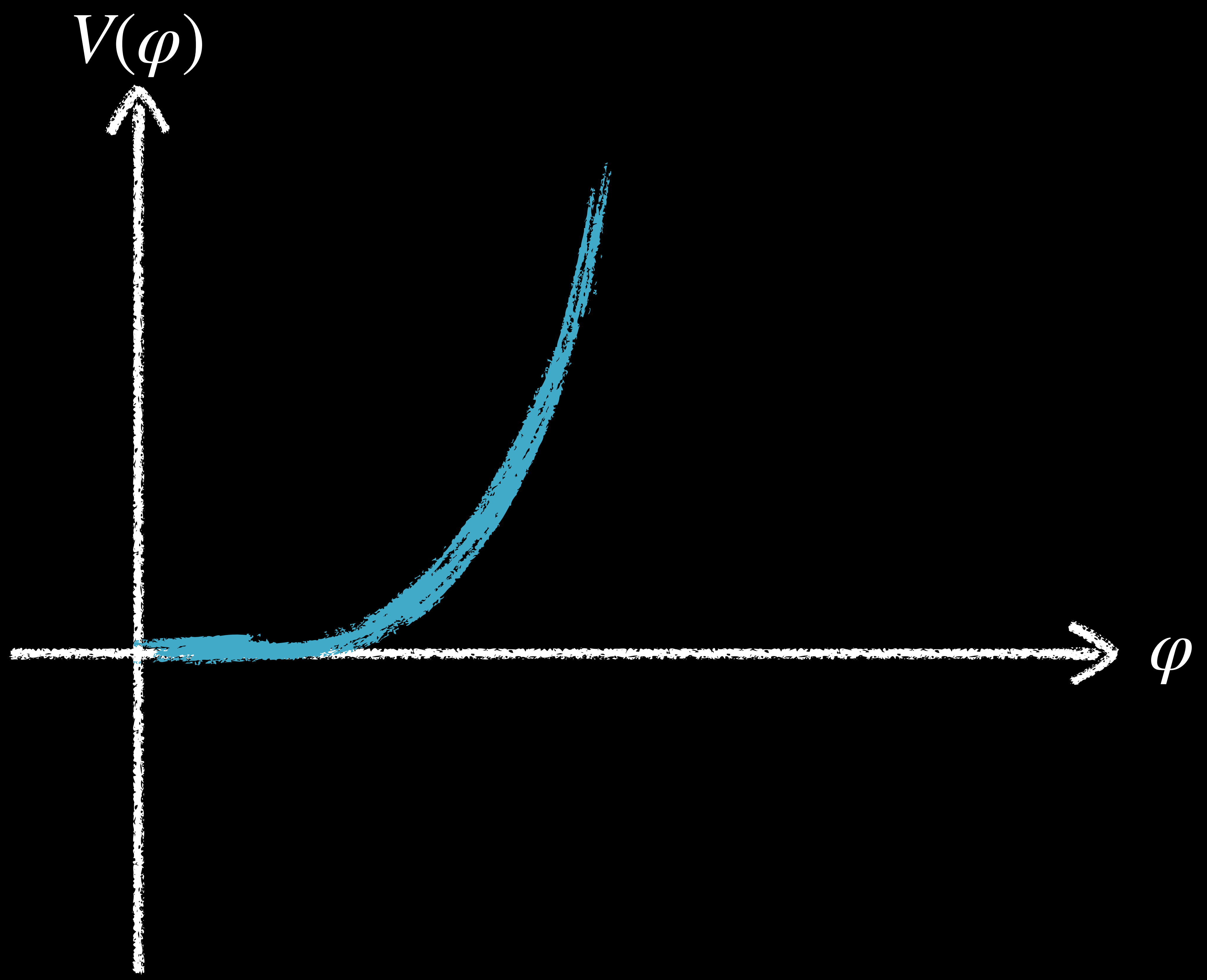
[M. Kierkla, P. Schicho, BŚ, T.V.I. Tenkanen, J. van de Vis, JHEP 07 (2025) 153]

RENORMALISATION



RADIATIVE SYMMETRY BREAKING

ABELIAN HIGGS MODEL



$$V^{(0)}(\varphi) = \frac{1}{4}\lambda\varphi^4$$

[S.Coleman, E. Weinberg, PRD 7 (1973) 1888]

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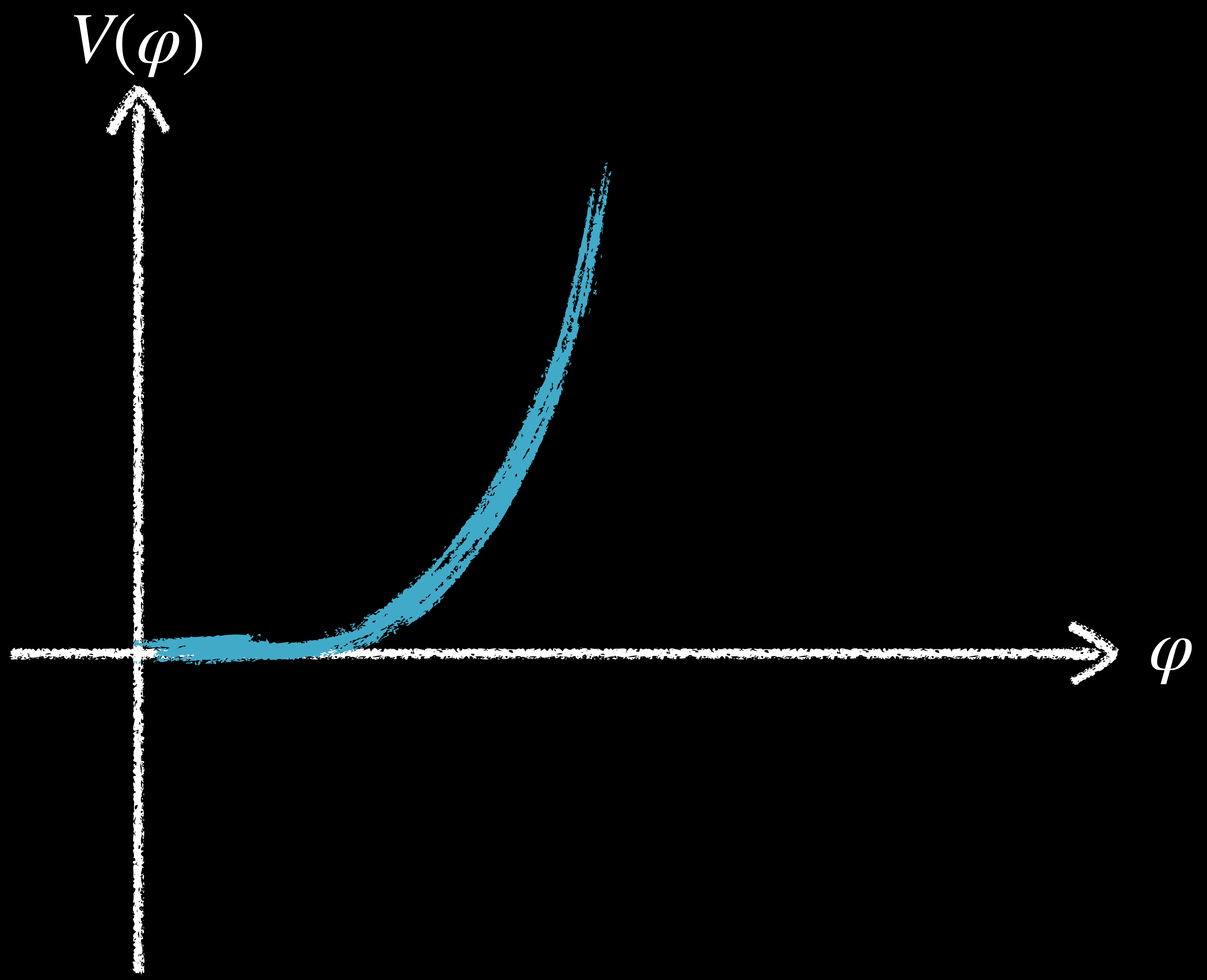

$$V^{(0)}(\varphi) = \frac{1}{4}\lambda\varphi^4$$

$$V^{(1)}(\varphi) = \frac{9\lambda^2\varphi^4}{64\pi^2}\left(\log\frac{3\lambda\varphi^2}{\mu^2} - \frac{3}{2}\right)$$

[S.Coleman, E. Weinberg, PRD 7 (1973) 1888]

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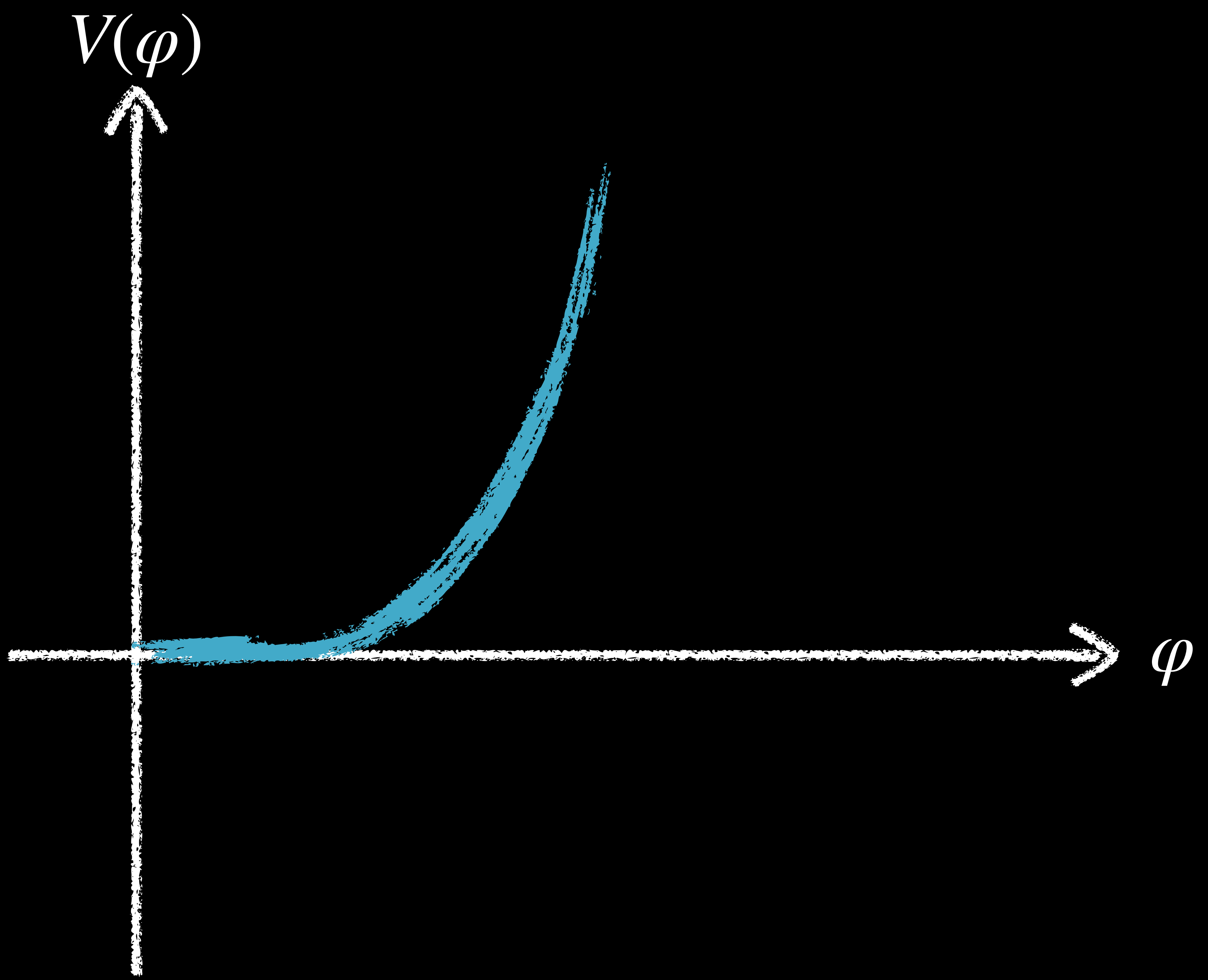

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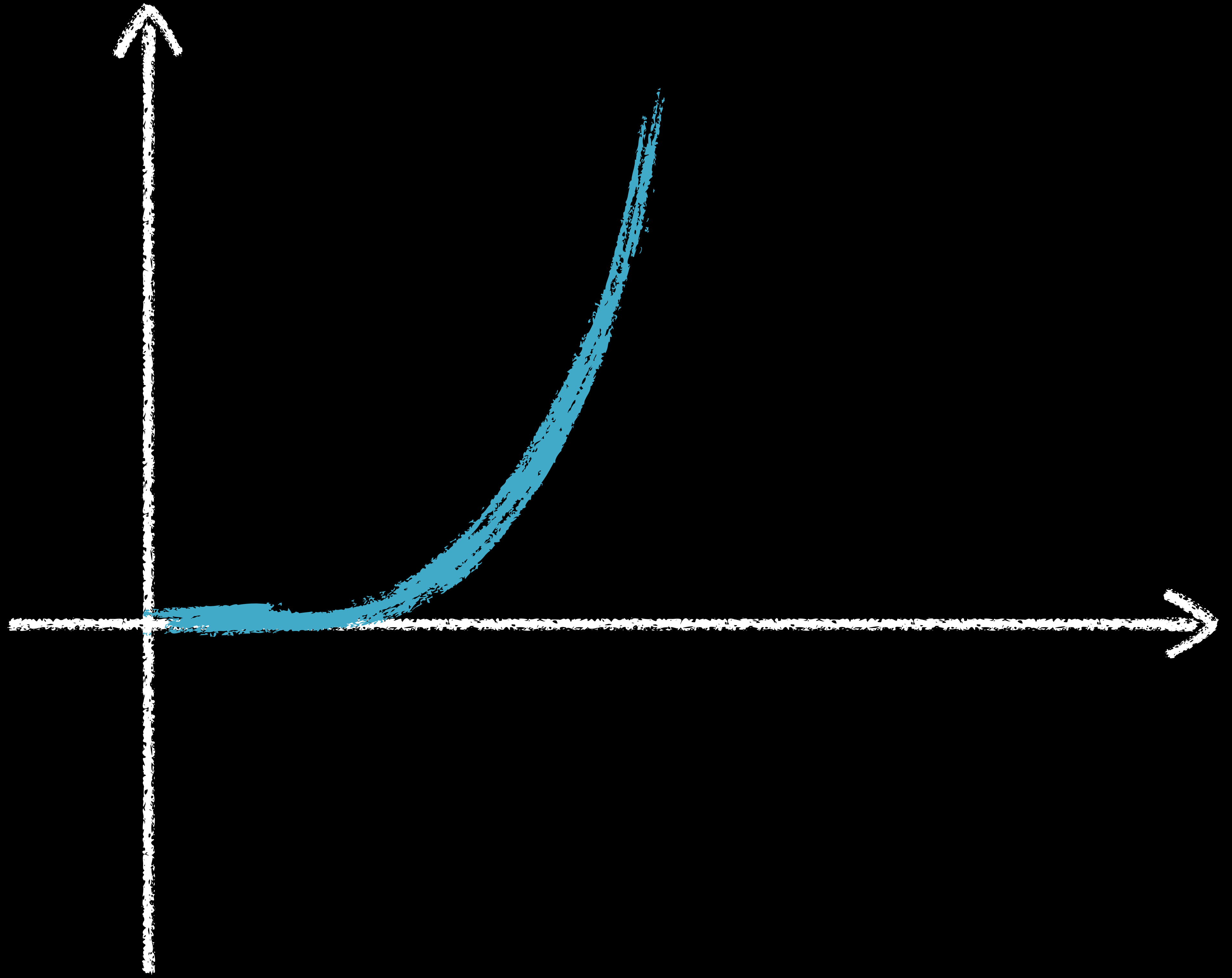

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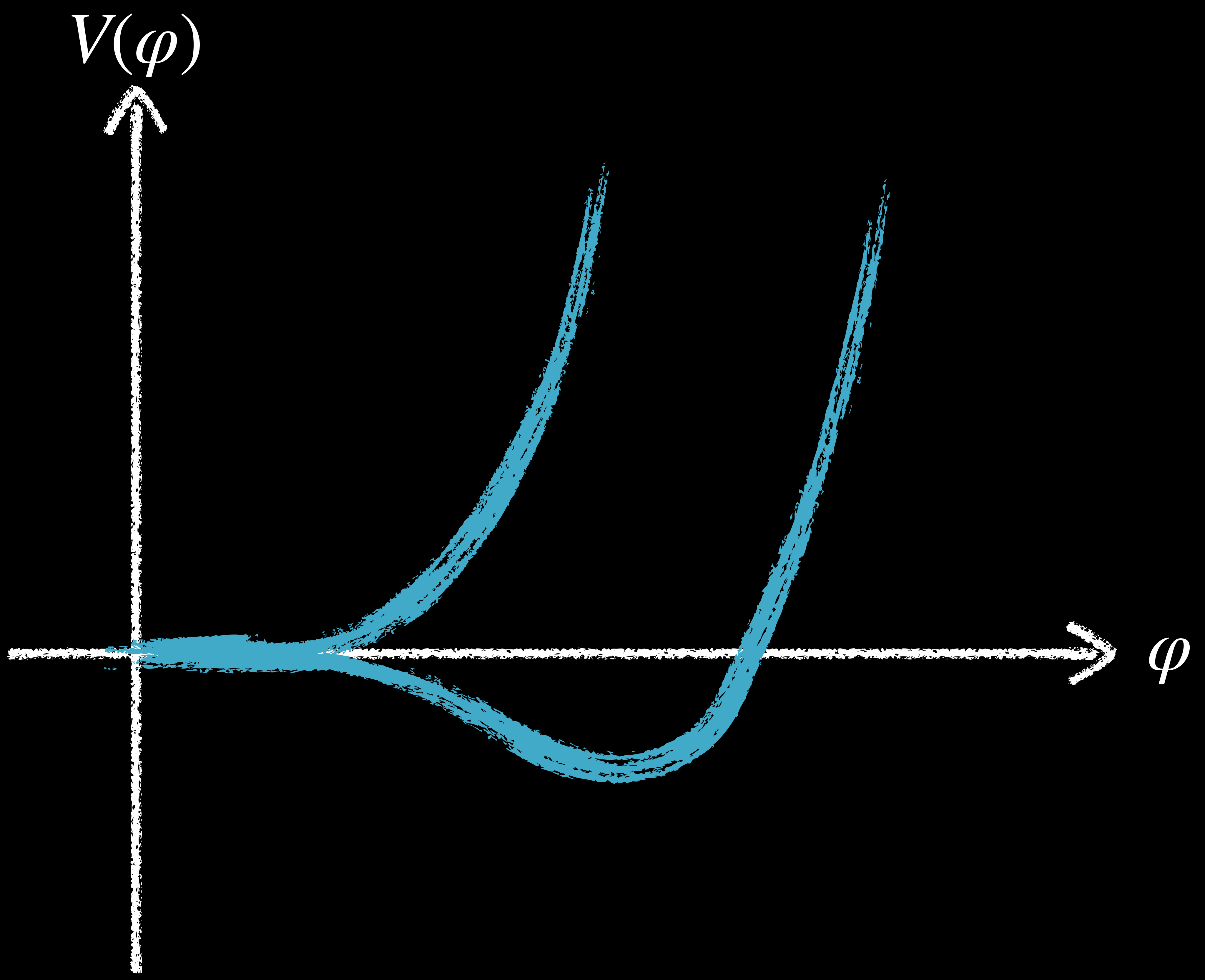
$$+ \frac{3e^4\varphi^4}{64\pi^2}\left(\log\frac{e^2\varphi^2}{\mu^2} - \frac{5}{6}\right)$$

[S.Coleman, E. Weinberg, PRD 7 (1973) 1888]

RADIATIVE SYMMETRY BREAKING

ABELIAN HIGGS MODEL



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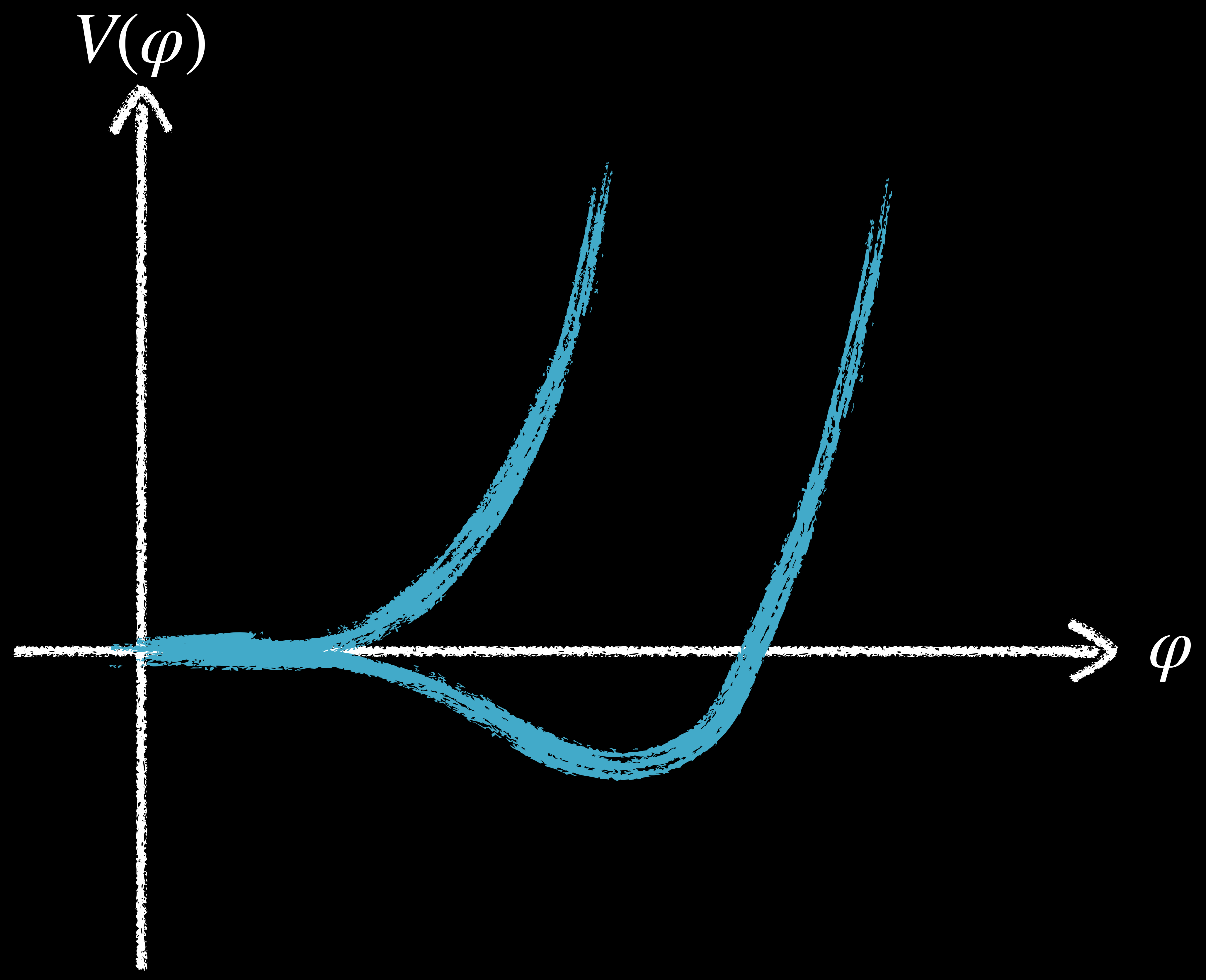

$$V^{(0)}(\varphi) = \frac{1}{4}\lambda\varphi^4 + \frac{3e^4\varphi^4}{64\pi^2}\left(\log\frac{e^2\varphi^2}{\mu^2} - \frac{5}{6}\right)$$

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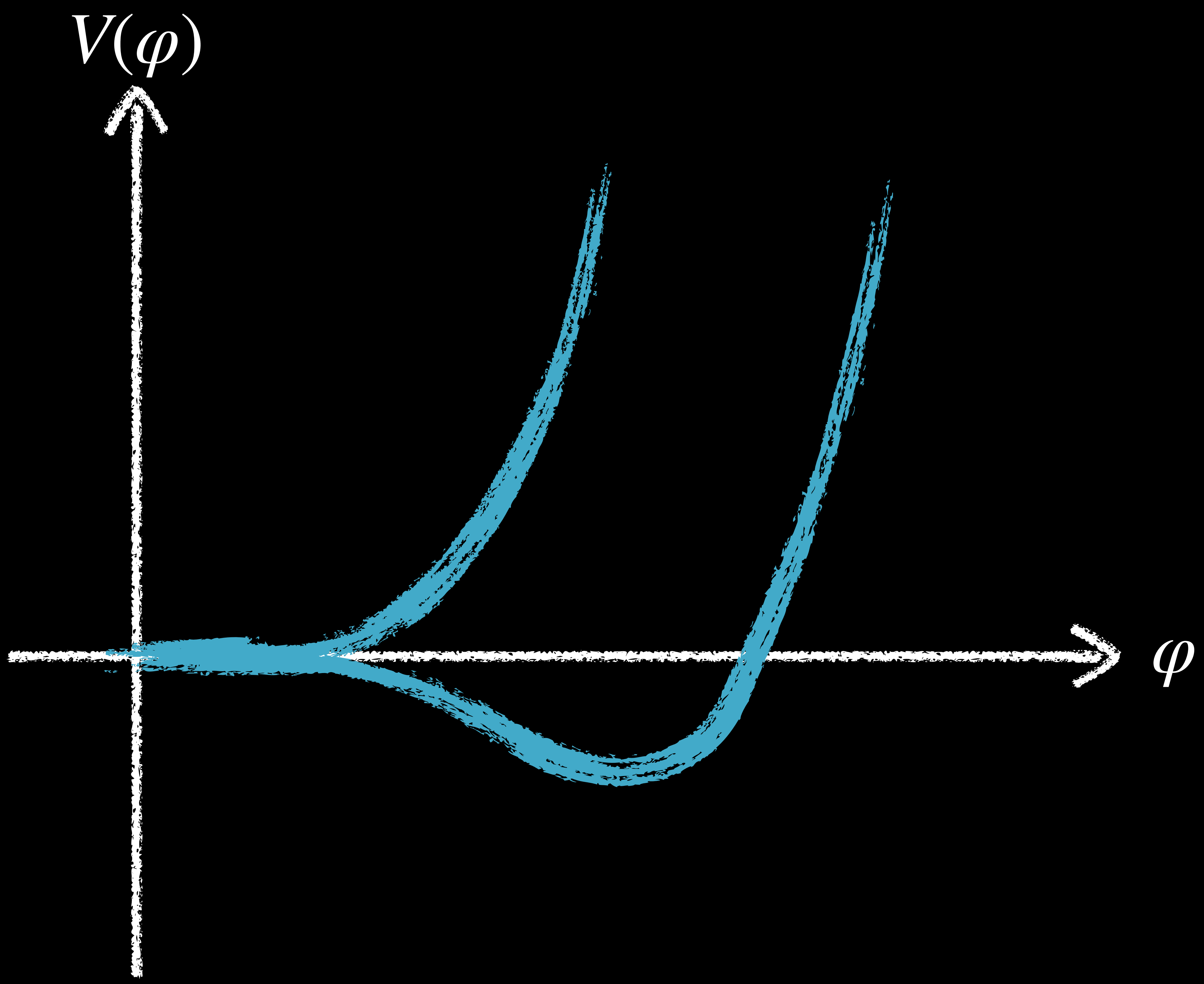

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From minimisation

$$\lambda_{\overline{\mathrm{MS}}} = \frac{4}{3}\kappa e^4 = \frac{e^4}{16\pi^2}$$

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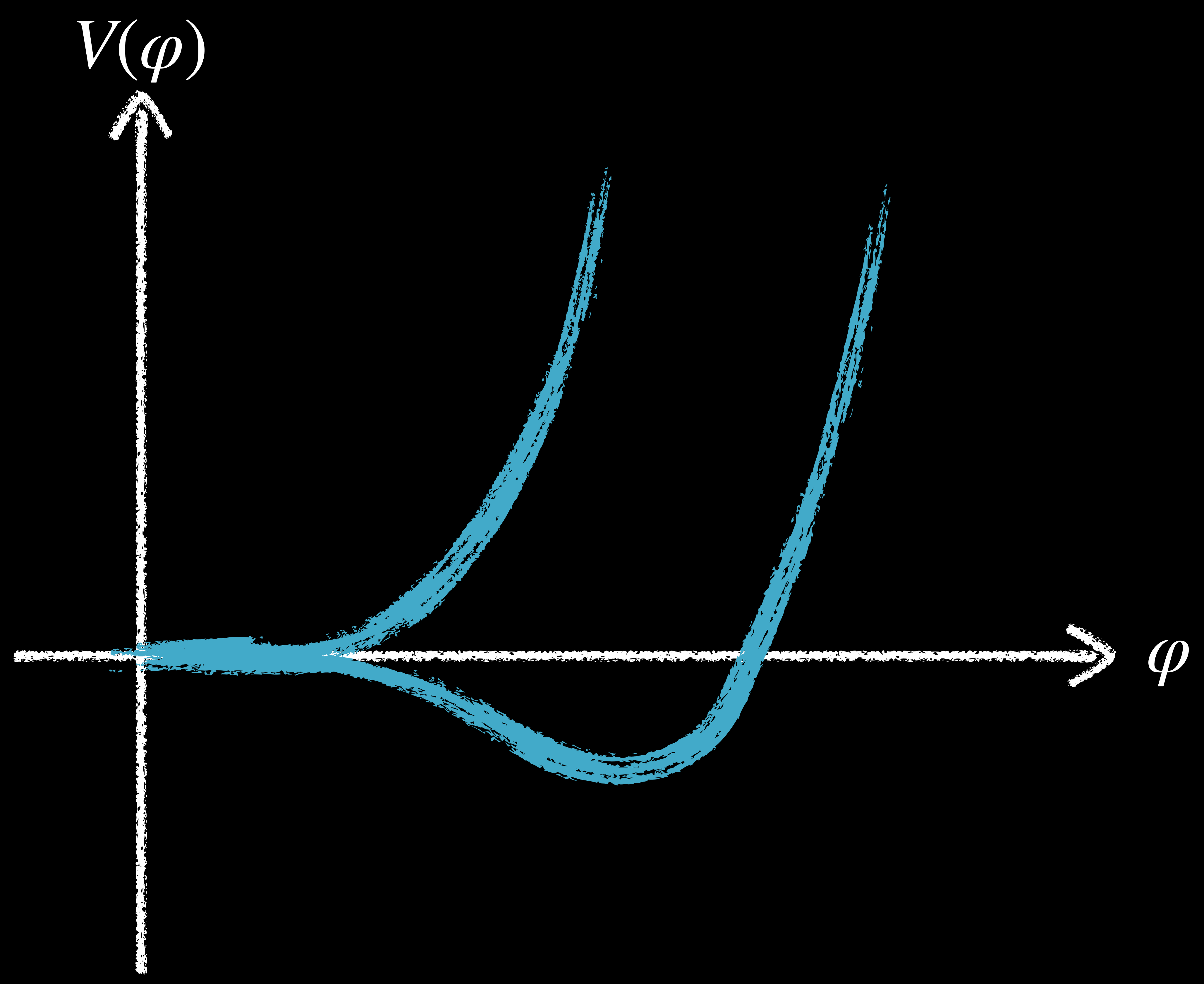
ABELIAN HIGGS MODEL



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From minimisation

$$\lambda_{\overline{\rm MS}} = \frac{4}{3}\kappa e^4 = \frac{e^4}{16\pi^2}$$

Predicted ratio of masses

$$\frac{\overline{M}_S^2}{\overline{M}_V^2} = 8\kappa e^2 = \frac{3e^2}{8\pi^2}$$

[S.Coleman, E. Weinberg, PRD 7 (1973) 1888]

$$V(\varphi) = \frac{1}{4}\lambda\varphi^4 + \overbrace{\frac{3}{64\pi^2}}^{\kappa} e^4\varphi^4\left(\log\frac{e^2\varphi^2}{\mu^2} - \frac{5}{6} + \eta\right) + \frac{1}{2}\delta m^2\varphi^2 + \frac{1}{4}\delta\lambda\varphi^4$$

MS

Counterterms

$$\begin{cases} \delta m^2_{\mathrm{MS}} = 0 \\ \delta\lambda_{\mathrm{MS}} = -4\kappa e^4\eta, \end{cases}$$

Mass ratio

$$\frac{\overline{M}_S^2}{\overline{M}_V^2} = 8\kappa e^2 \equiv r_{\mathrm{CW}}$$

OS

$$V(\varphi) = \frac{1}{4}\lambda\varphi^4 + \overset{\kappa}{\frac{3}{64\pi^2}} e^4\varphi^4\left(\log\frac{e^2\varphi^2}{\mu^2} - \frac{5}{6} + \eta\right) + \frac{1}{2}\delta m^2\varphi^2 + \frac{1}{4}\delta\lambda\varphi^4$$

MS

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OS

Renormalisation conditions:

$$\left.\frac{\mathrm{d}V}{\mathrm{d}\varphi}\right|_{\varphi=M_V/e} = 0$$

$$\left.\frac{\mathrm{d}^2V}{\mathrm{d}\varphi^2}\right|_{\varphi=M_V/e} = M_S^2$$

$$V(\varphi) = \frac{1}{4}\lambda\varphi^4 + \overset{\kappa}{\frac{3}{64\pi^2}} e^4\varphi^4\left(\log\frac{e^2\varphi^2}{\mu^2} - \frac{5}{6} + \eta\right) + \frac{1}{2}\delta m^2\varphi^2 + \frac{1}{4}\delta\lambda\varphi^4$$

MS

Counterterms

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OS

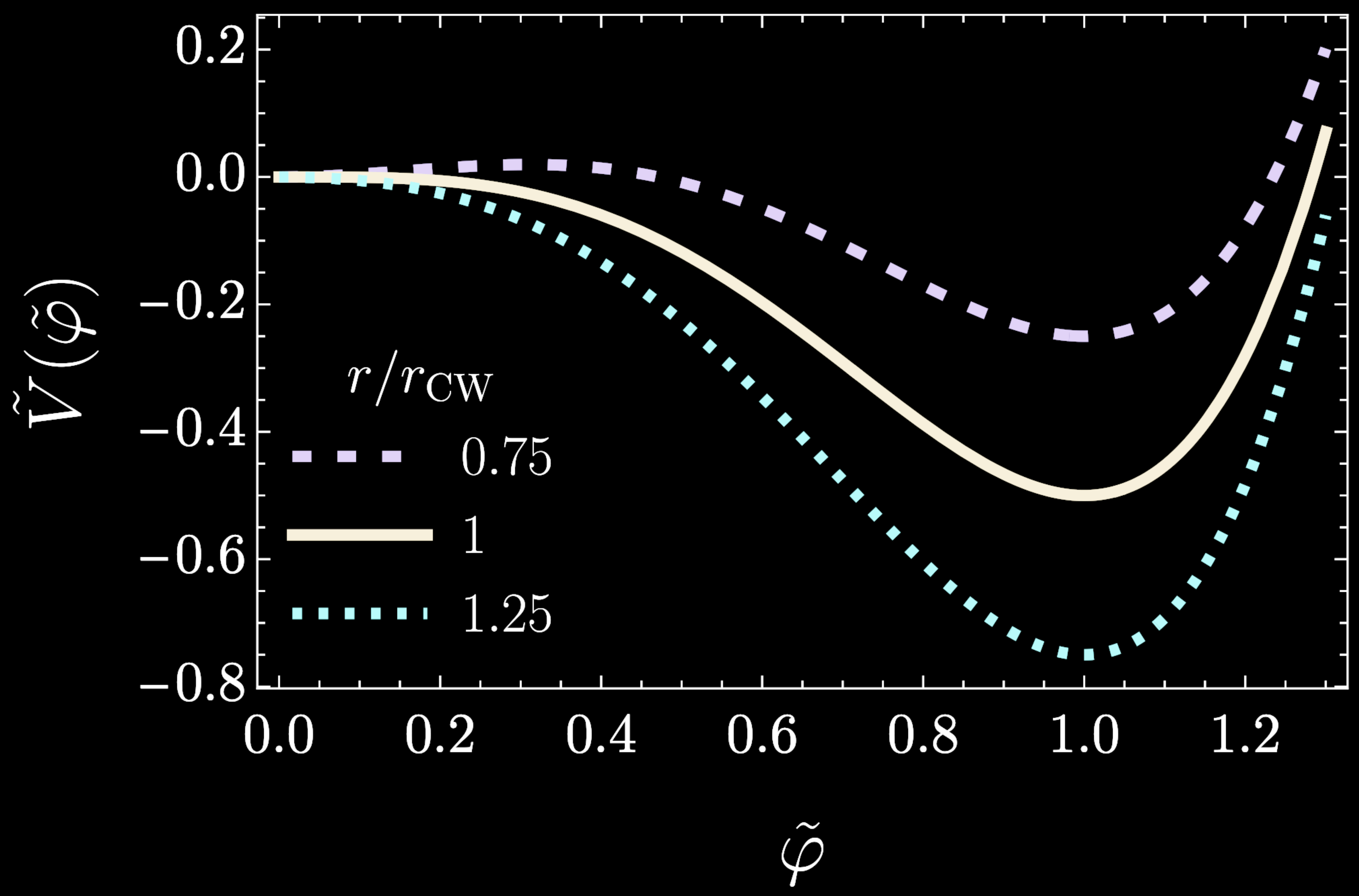
Counterterms

$$\begin{cases} \lambda_{\mathrm{OS}} + \delta\lambda_{\mathrm{OS}} = \dfrac{e^2}{2}\dfrac{M_S^2}{M_V^2} \\ \qquad -4\kappa e^4\left(\log\dfrac{M_V^2}{\mu^2} + \dfrac{2}{3} + \eta\right) \\ \delta m^2_{\mathrm{OS}} = -\dfrac{1}{2}M_S^2 + 4\kappa e^2 M_V^2 \end{cases}$$

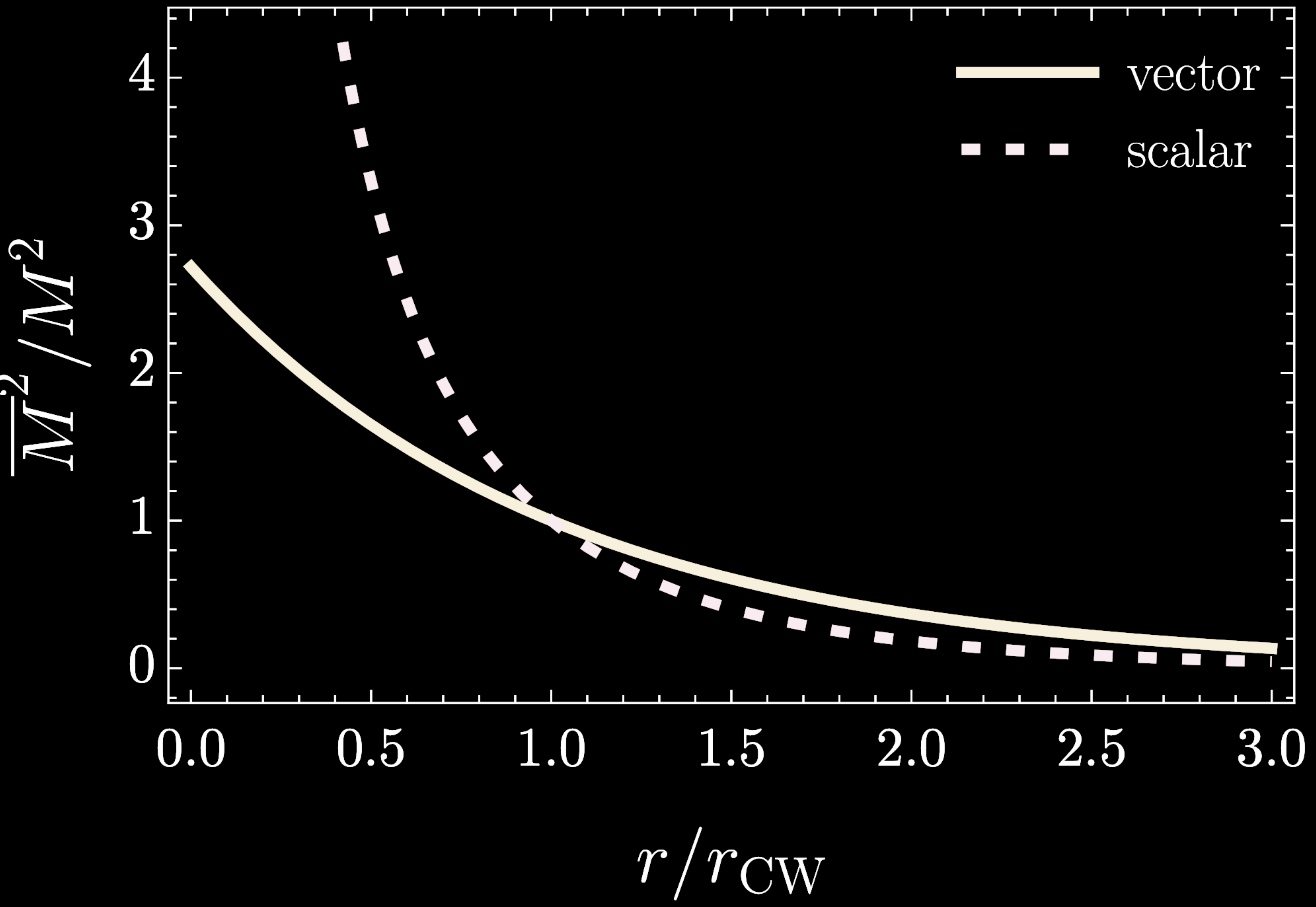
Mass ratio

$$r = \frac{M_S^2}{M_V^2}$$

THE ON-SHELL POTENTIAL



OS VS MS



vector
scalar
4
3
2
1
0
0.0
0.5
1.0
1.5
2.0
2.5
3.0
$\overline{M}^2/M^2$
r/r_{CW}

$$V(\varphi) = \frac{1}{4}\lambda\varphi^4 + \overset{\kappa}{\frac{3}{64\pi^2}} e^4\varphi^4\left(\log\frac{e^2\varphi^2}{\mu^2} - \frac{5}{6} + \eta\right) + \frac{1}{2}\delta m^2\varphi^2 + \frac{1}{4}\delta\lambda\varphi^4$$

MASSIVE CASE

Minimisation

$$\begin{cases} \lambda = \dfrac{e^2}{2}\dfrac{M_S^2}{M_V^2} \\ m^2 = -\dfrac{1}{2}M_S^2 \end{cases}$$

Counterterms

$$\begin{cases} \delta\lambda = -4\kappa e^4\left(\log\dfrac{M_V^2}{\mu^2} + \eta + \dfrac{2}{3}\right) \\ \delta m^2 = 4\kappa e^2 M_V^2 \end{cases}$$

OS

Counterterms

$$\begin{cases} \lambda_{\mathrm{OS}} + \delta\lambda_{\mathrm{OS}} = \dfrac{e^2}{2}\dfrac{M_S^2}{M_V^2} \\ \qquad\qquad -4\kappa e^4\left(\log\dfrac{M_V^2}{\mu^2} + \dfrac{2}{3} + \eta\right) \\ \delta m_{\mathrm{OS}}^2 = -\dfrac{1}{2}M_S^2 + 4\kappa e^2 M_V^2 \end{cases}$$

CONSEQUENCES

GW predictions are safe. If $r = r_{\mathrm{CW}}$, OS and MS are equivalent and no mass terms are generated.

With RSB a hierarchy of masses is introduced. Is it a good solution of the hierarchy problem?

SUMMARY

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Supercooled PTs are experimentally testable.

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Supercooled PTs are experimentally testable.

The experimental error of reconstruction potentially sma than the theoret uncertainty.

Theoretical diligence is crucial!

Radiative symmetry breaking

$\Longleftrightarrow$

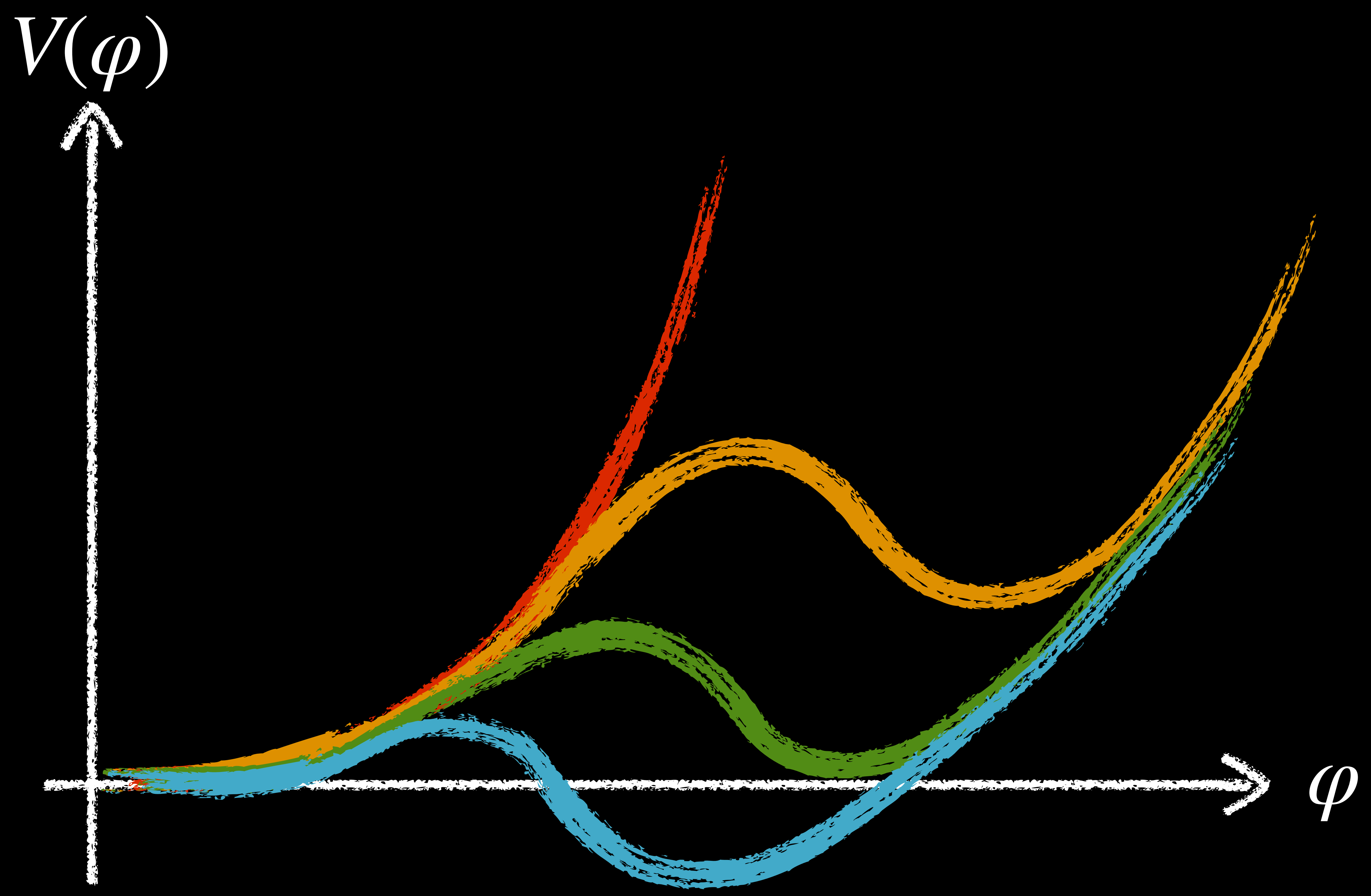
Hierarchical mass ordering

THANK YOU!

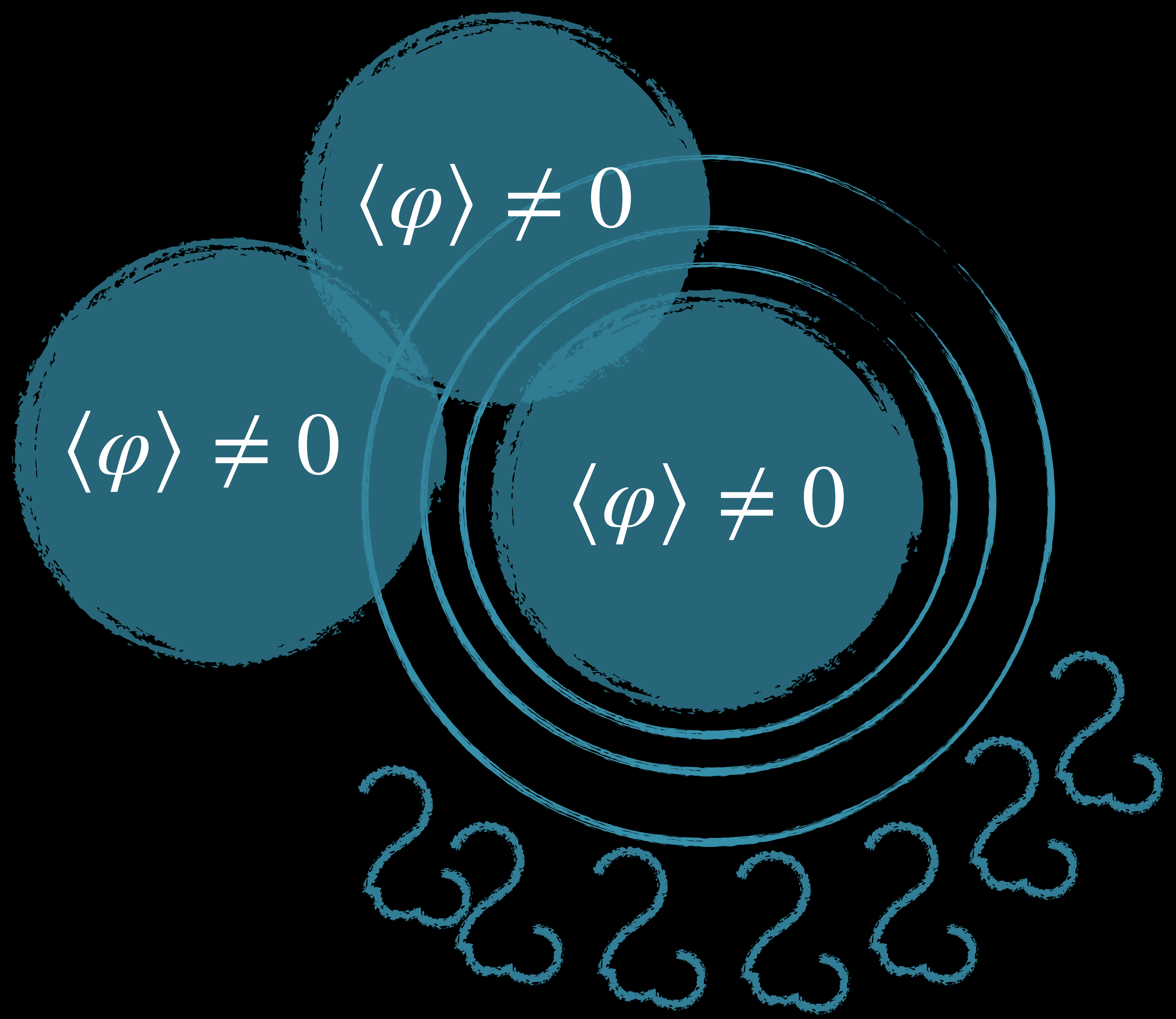
NATIONAL SCIENCE CENTRE
POLAND

BACKUP SLIDES

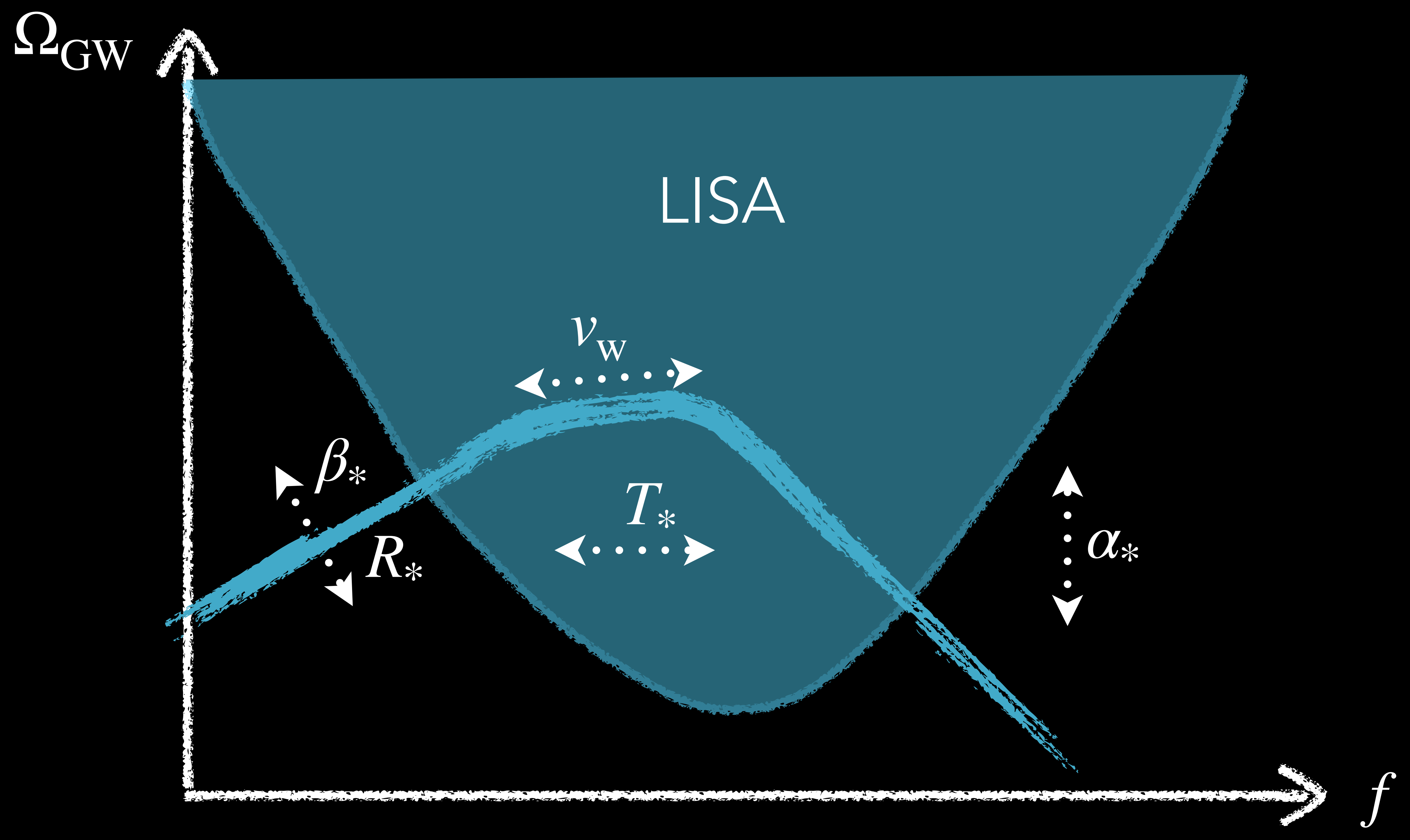
PHASE TRANSITION — MICROSCOPIC SCALES



PHASE TRANSITION — INTERMEDIATE SCALES

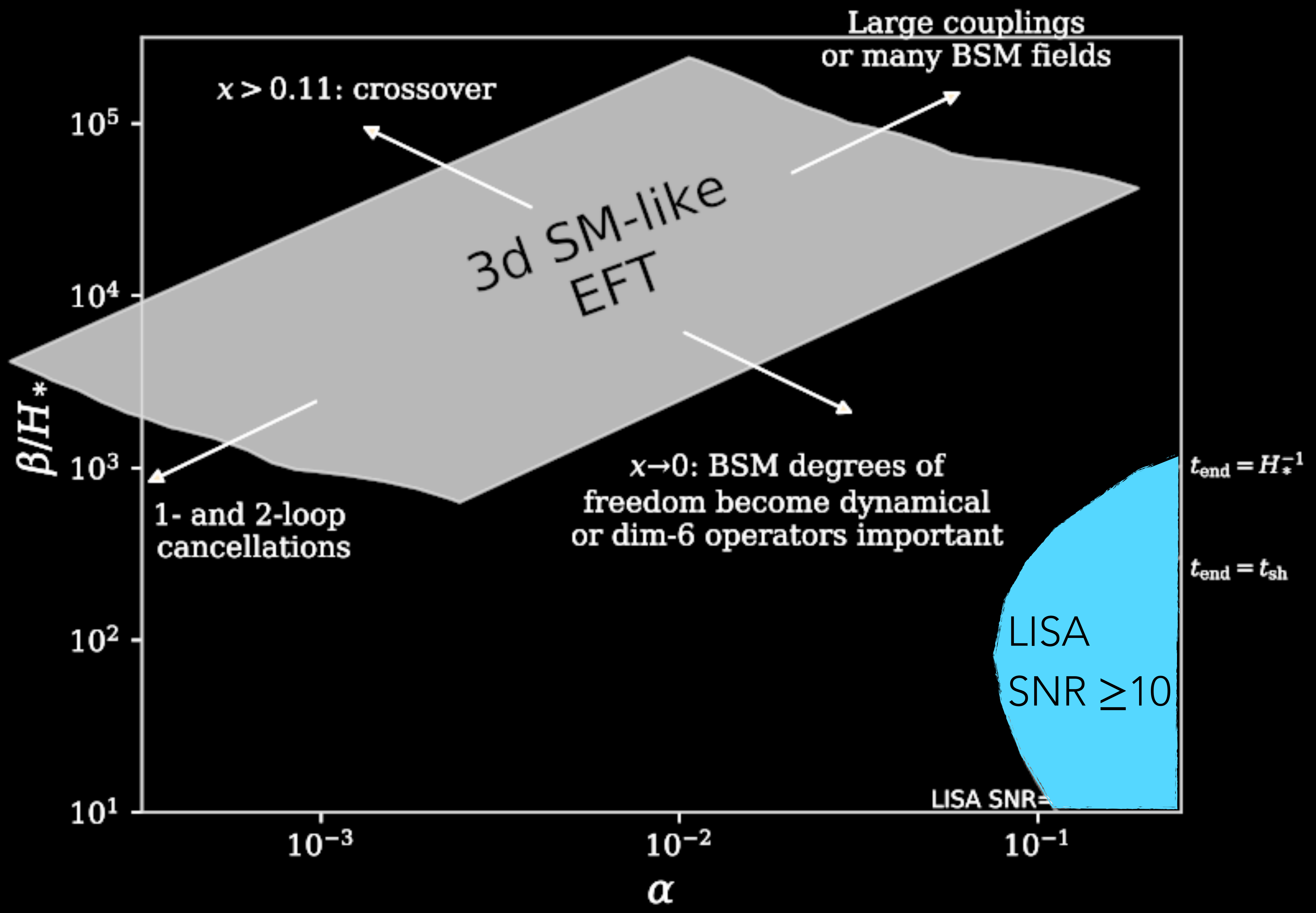


PHASE TRANSITION — MACROSCOPIC SCALES



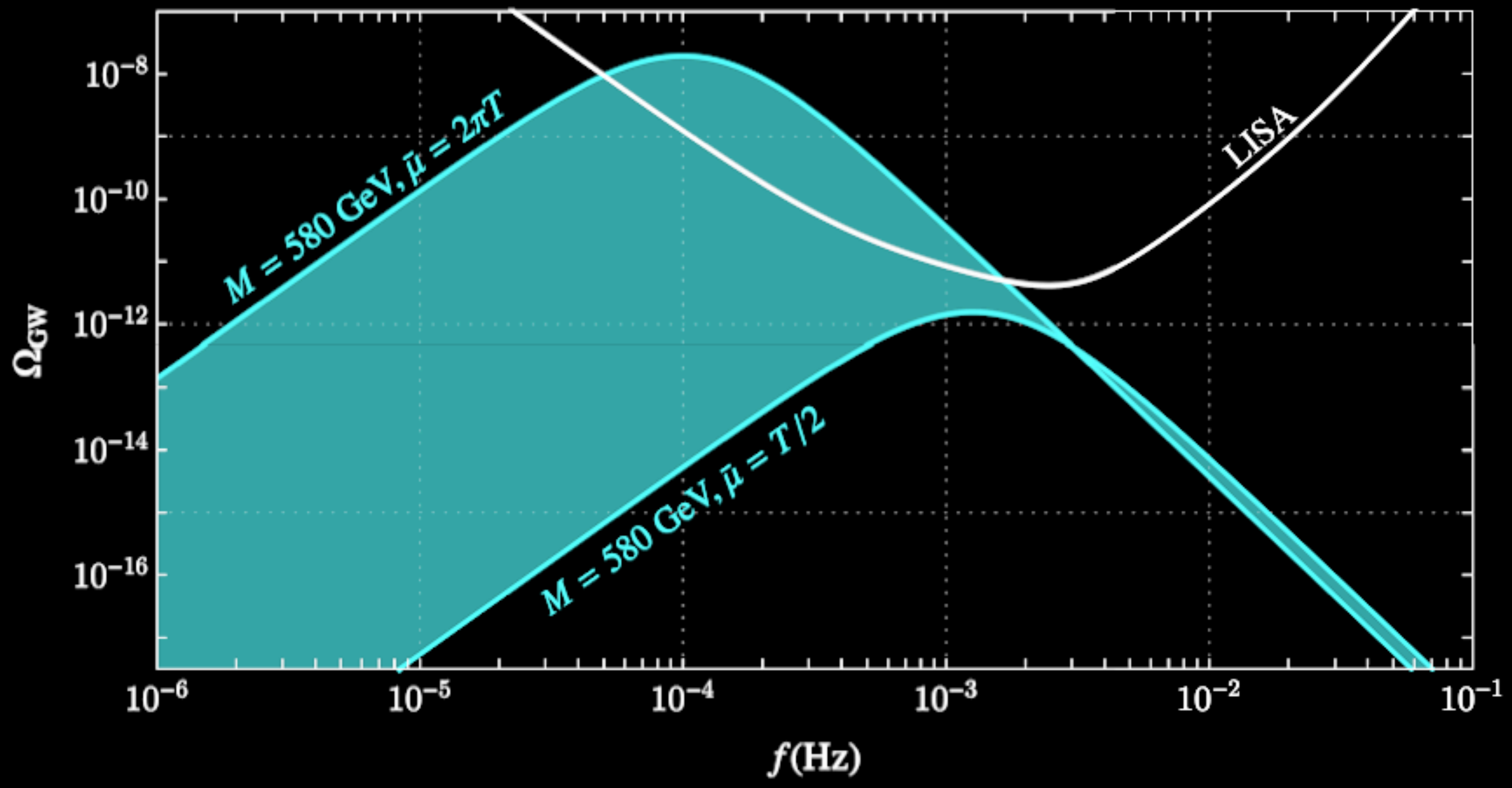
WHY DO WE NEED TO ADVANCE THE MICROPHYSICAL DESCRIPTION?

WHAT KIND OF PT CAN BE OBSERVABLE?



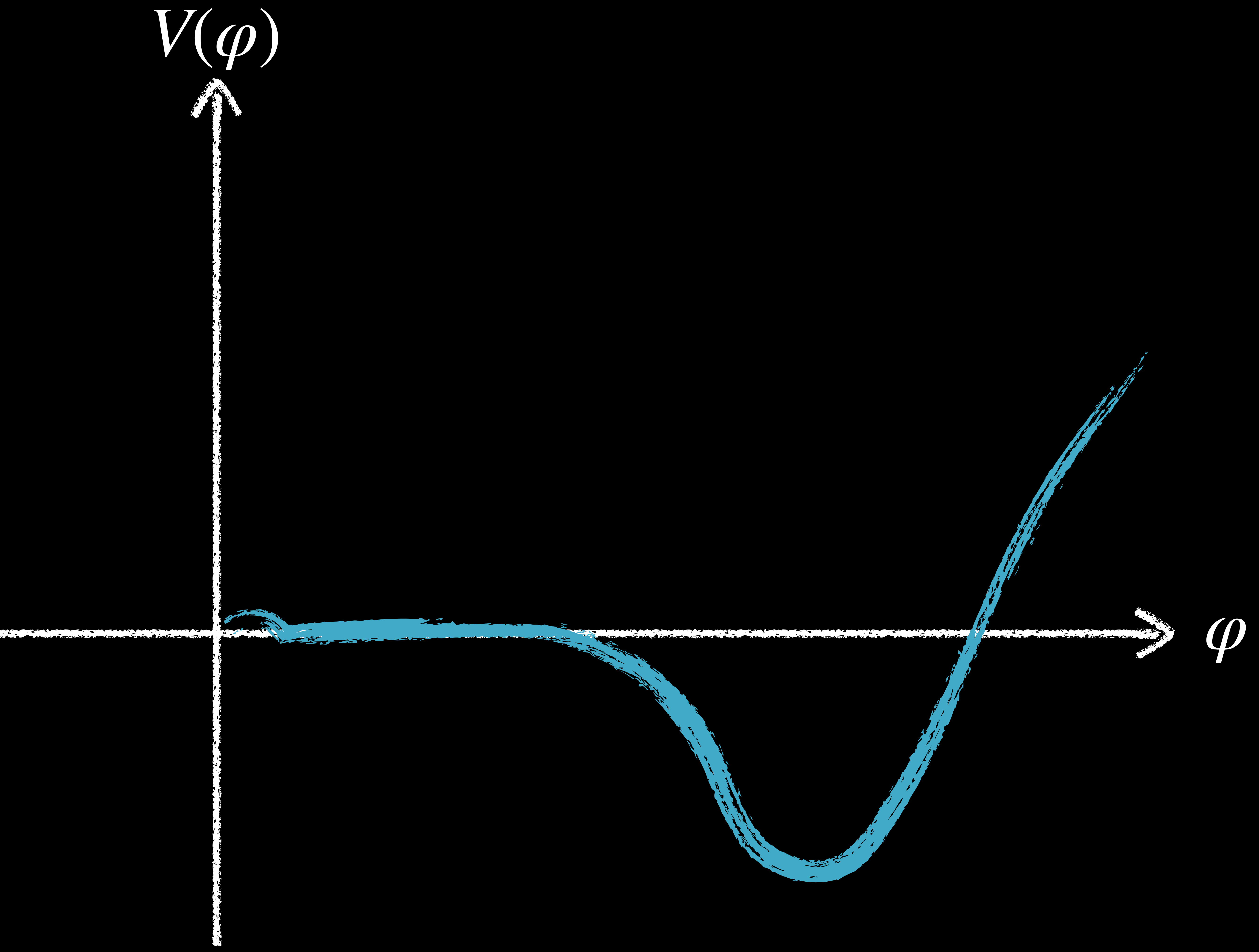
[Figure adapted from: Phys.Rev.D 100 (2019) 11, 115024, O. Gould, J. Kozaczuk, L. Niemi, M. J. Ramsey-Musolf, T. V.I. Tenkanen, D. J. Weir]

RG SCALE DEPENDENCE



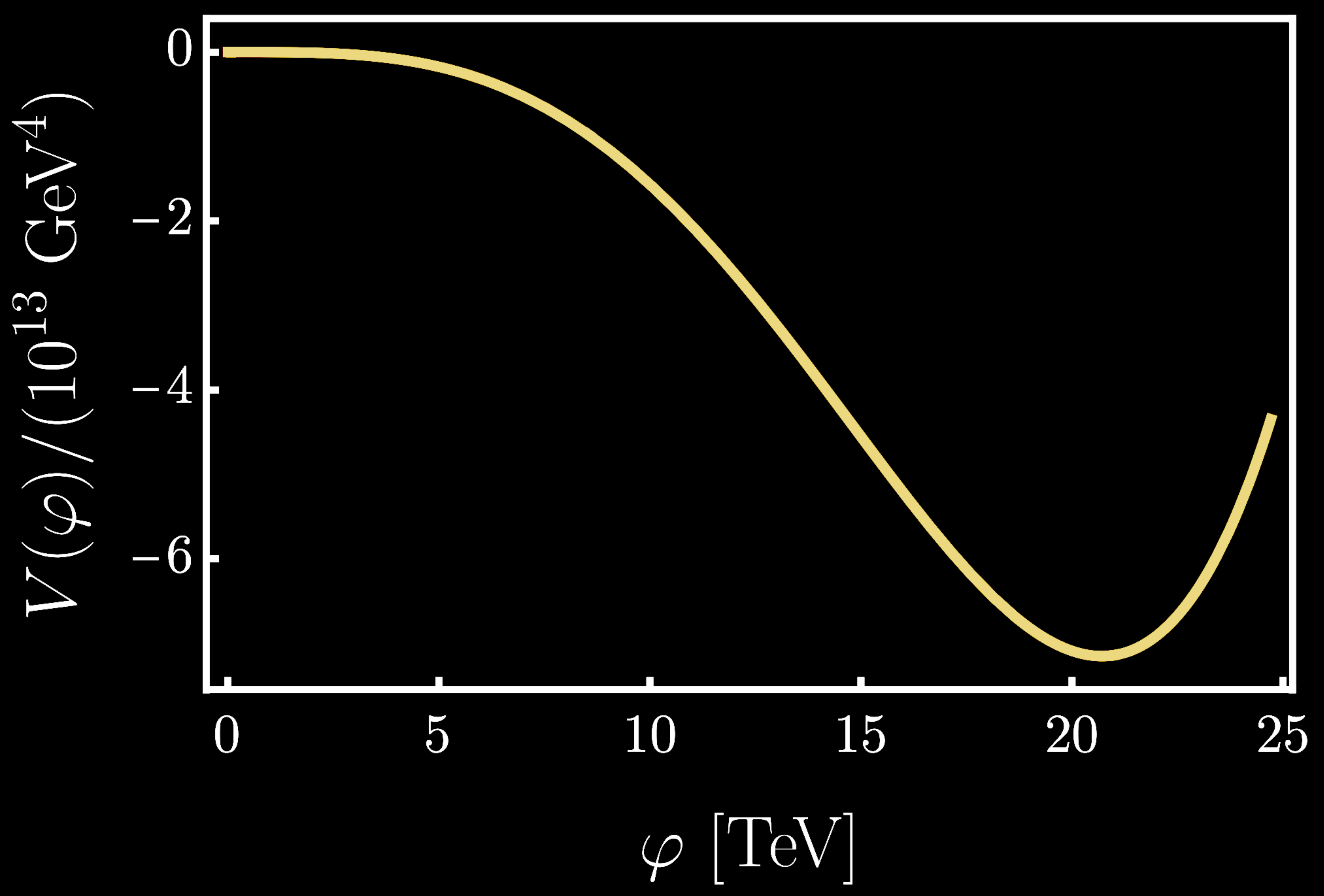
[plot adapted from: D. Croon, O. Gould, P. Schicho, T. Tenkanen, G. White, JHEP 04 (2021) 055]

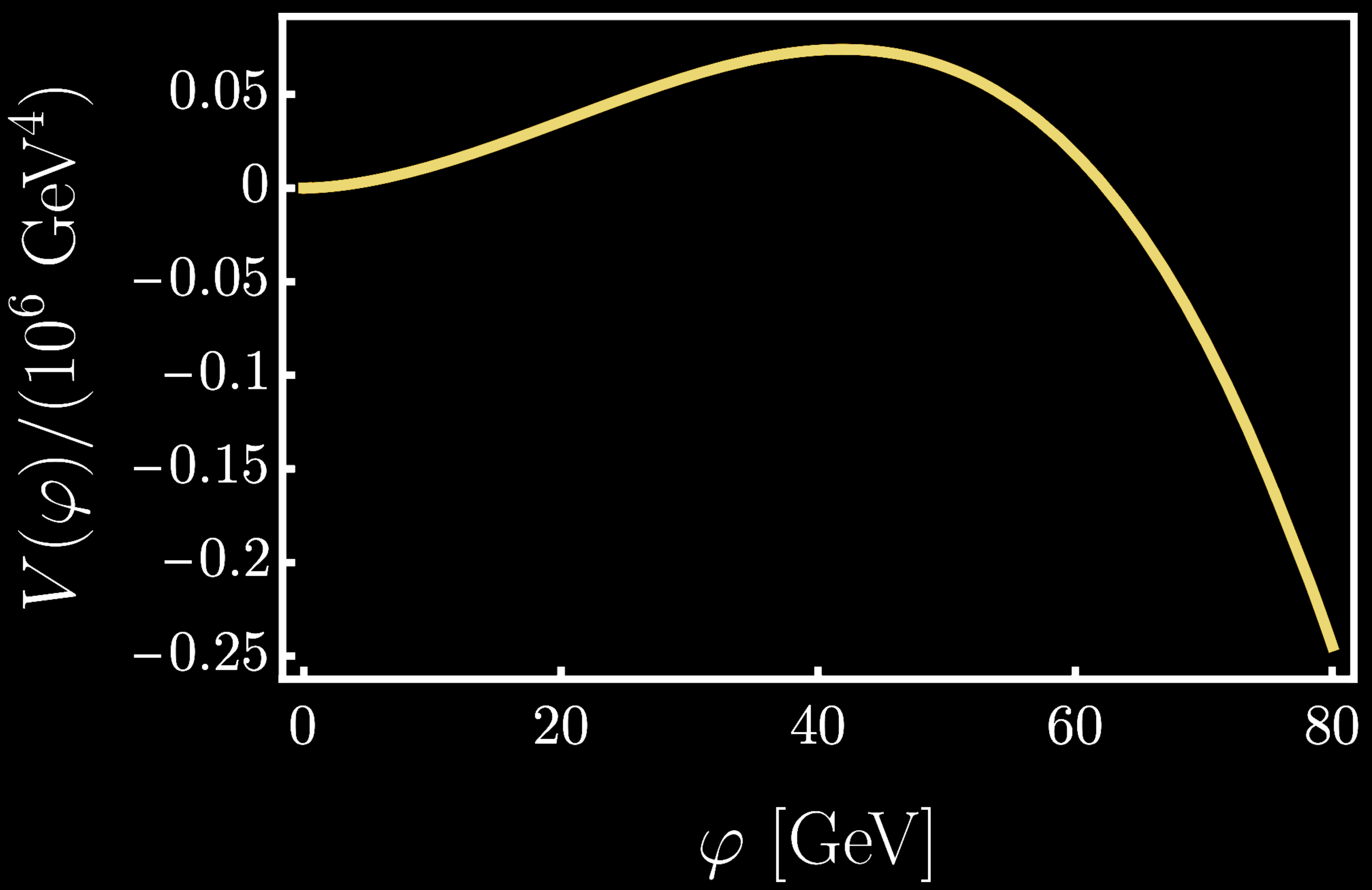
SUPERCOOLED PHASE TRANSITIONS



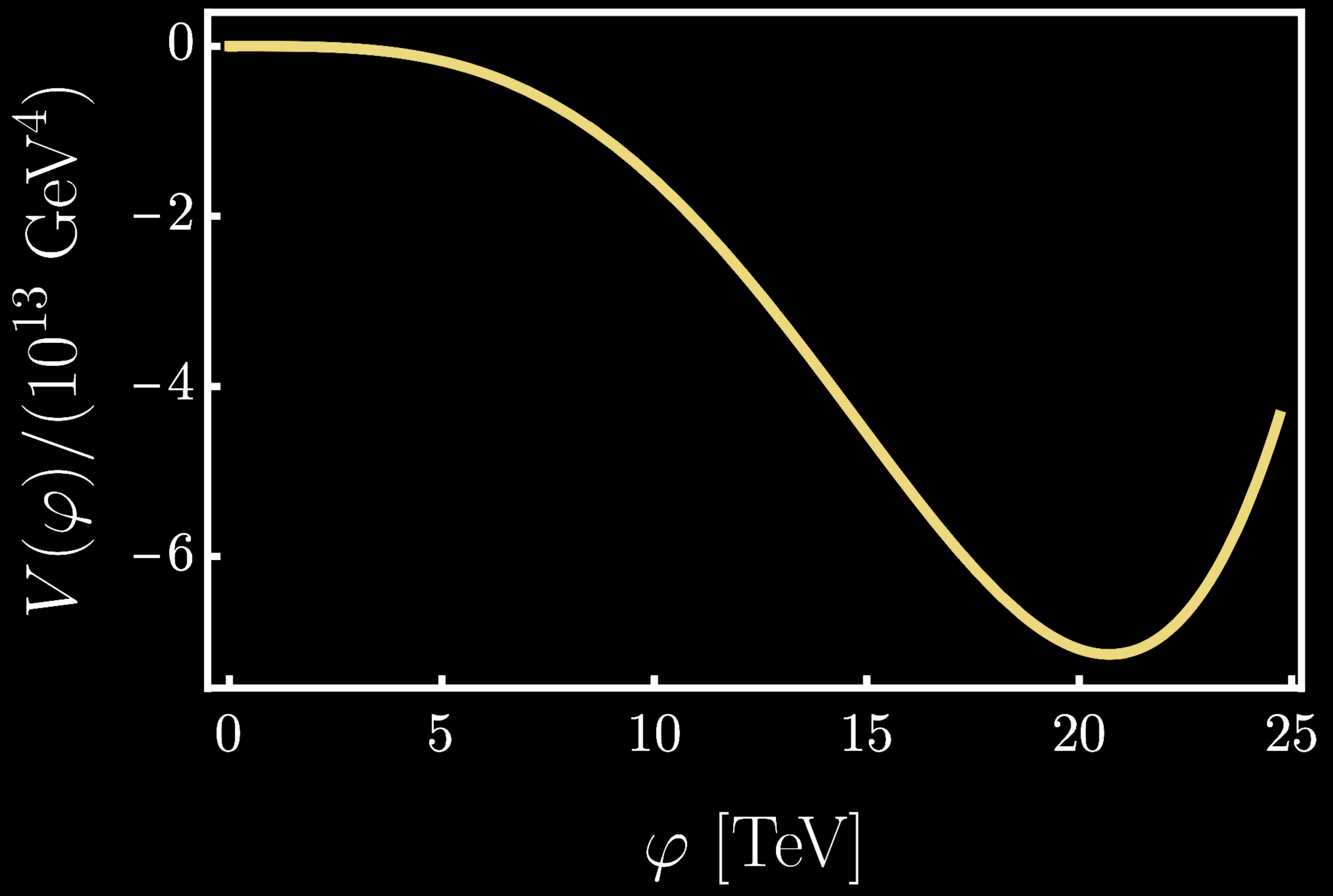
- Percolation temperature $\ll$ critical temperature,
- Before nucleation: period of thermal inflation,
- Huge energy release (compared to radiation energy), $\alpha \approx \dfrac{\Delta V}{\rho_{\rm rad}} \gg 1$,
- Significant reheating after the PT
- Typically realised in models with classical scale invariance

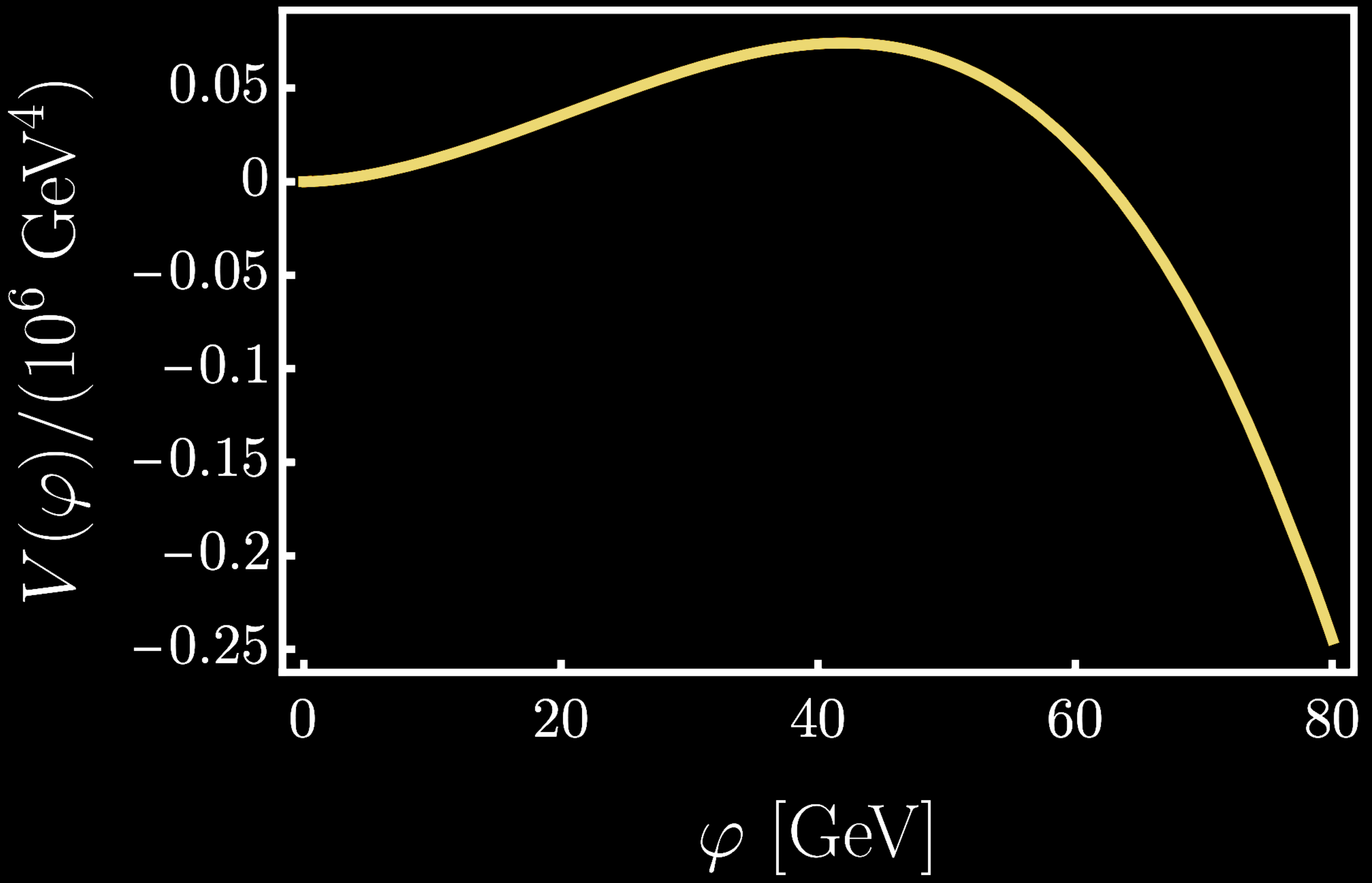
HIGH-T VS LOW-T





HIGH-T VS LOW-T





At large fields $M_X(\varphi)/T \gg 1$,
use LT approximation to compute T_{reh}, ΔV

At small fields $M_X(\varphi)/T \ll 1$,
use HT approximation to compute T_p, $R_* H_*$

WHAT'S NEW AT NLO?

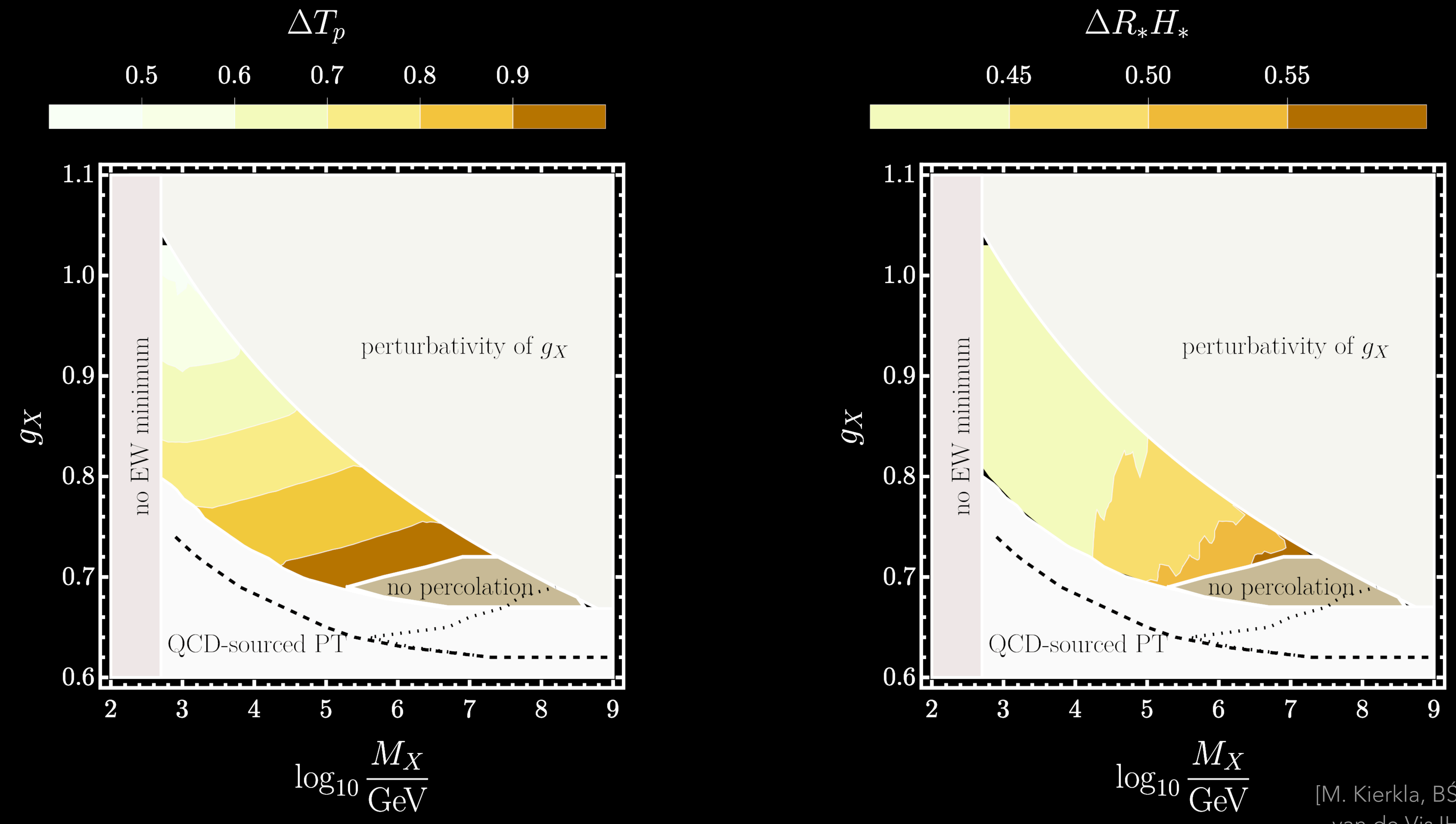
$$S_3^{\mathrm{EFT,\,NLO}} = 4\pi \int \mathrm{d}r\, r^2 \left(\frac{1}{2} Z_3^{\mathrm{NLO}}(v_3)(\partial_i v_3)^2 + V_3^{\mathrm{EFT,\,NLO}}(v_3) \right)$$

- New effective operator in the kinetic term
- Behaves badly for $\varphi_3 \to 0$

- two-loop matching
- NLO potential
- Includes missing (4d) RG scale dependence
- 3d scale invariant

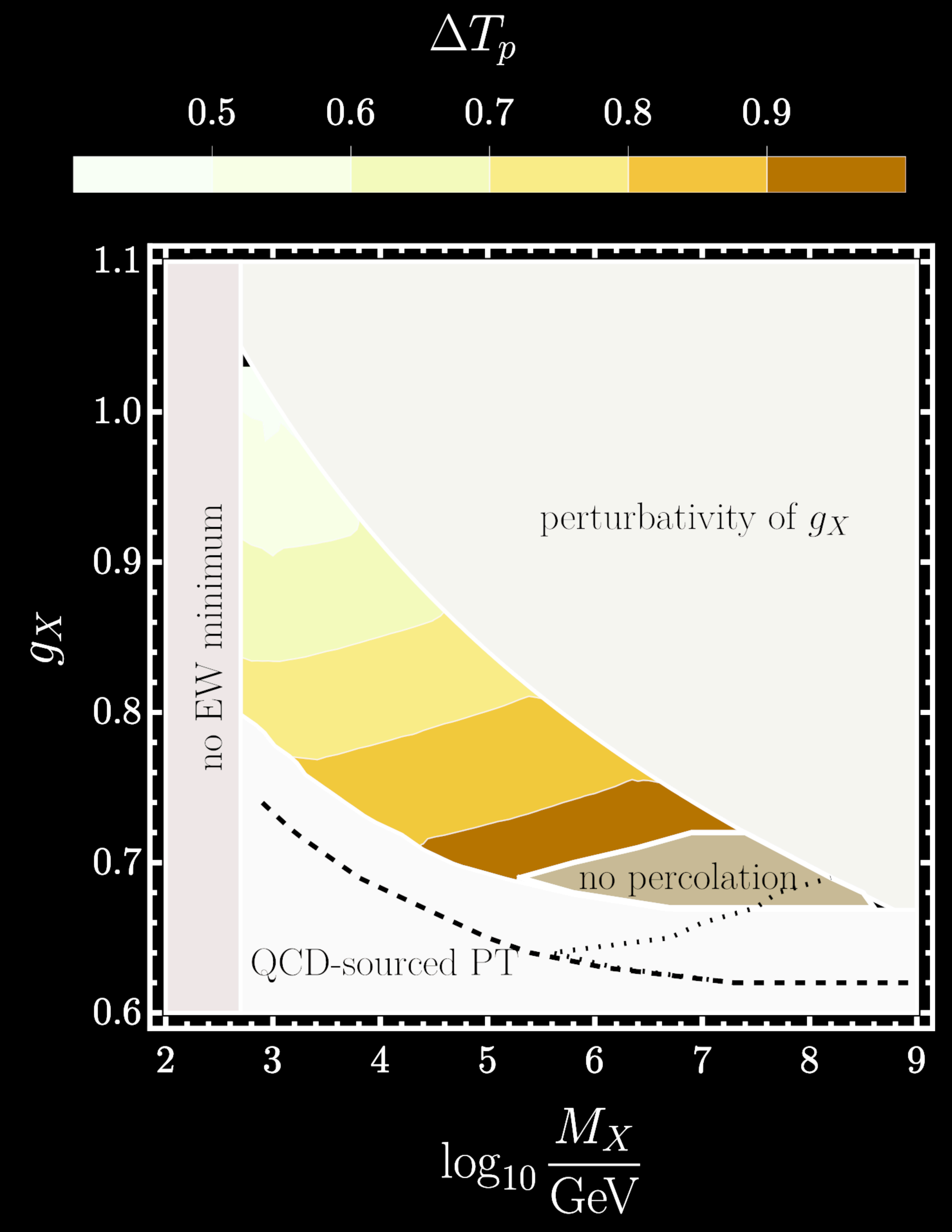
[M. Kierkla, BŚ, T.V.I. Tenkanen, J. van de VisJHEP 02 (2024) 234]

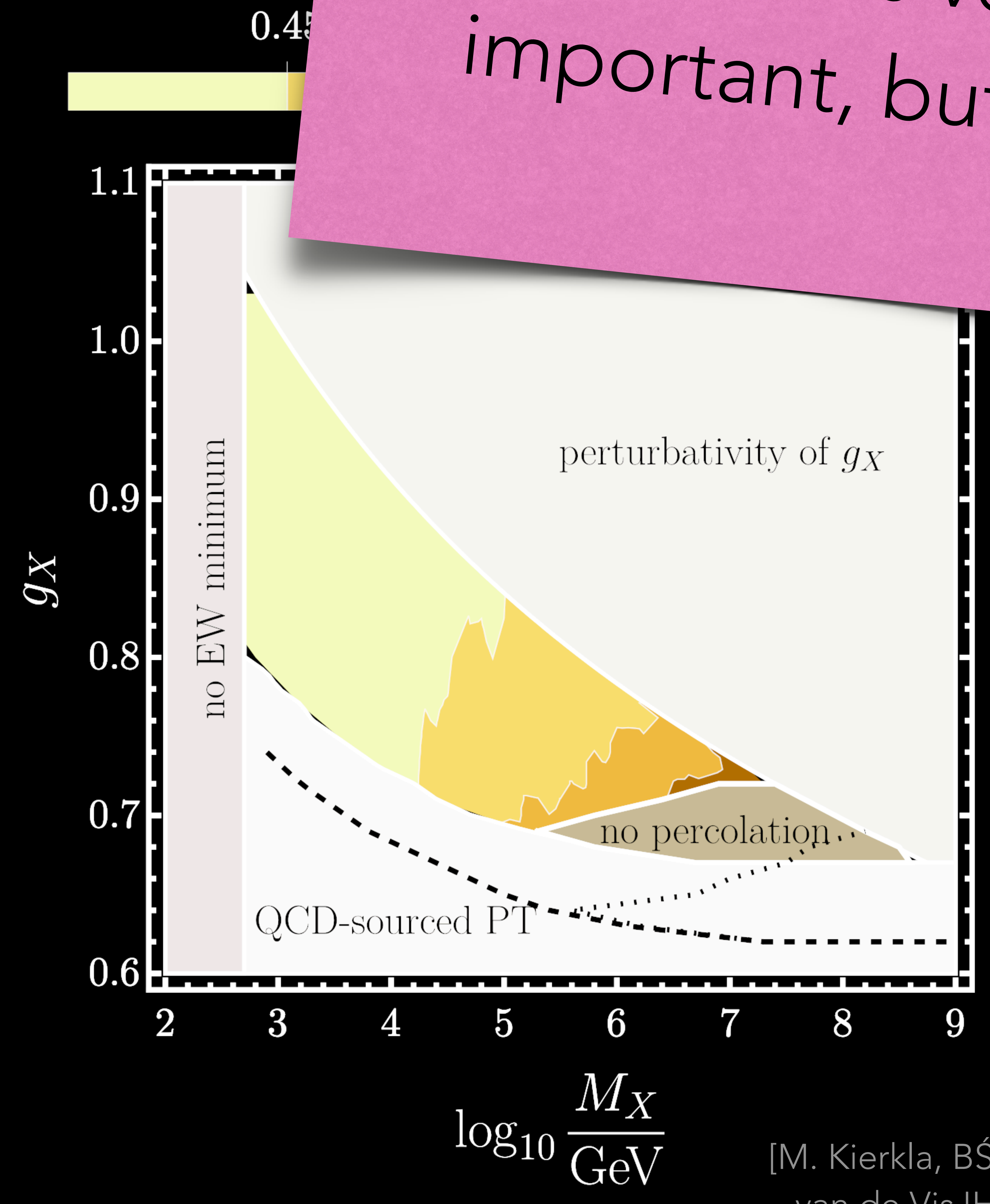
IMPROVED PRECISION CHANGES PREDICTIONS



[M. Kierkla, BŚ, T.V.I. Tenkanen, J. van de VisJHEP 02 (2024) 234]

IMPROVED PRECISION CHANGES

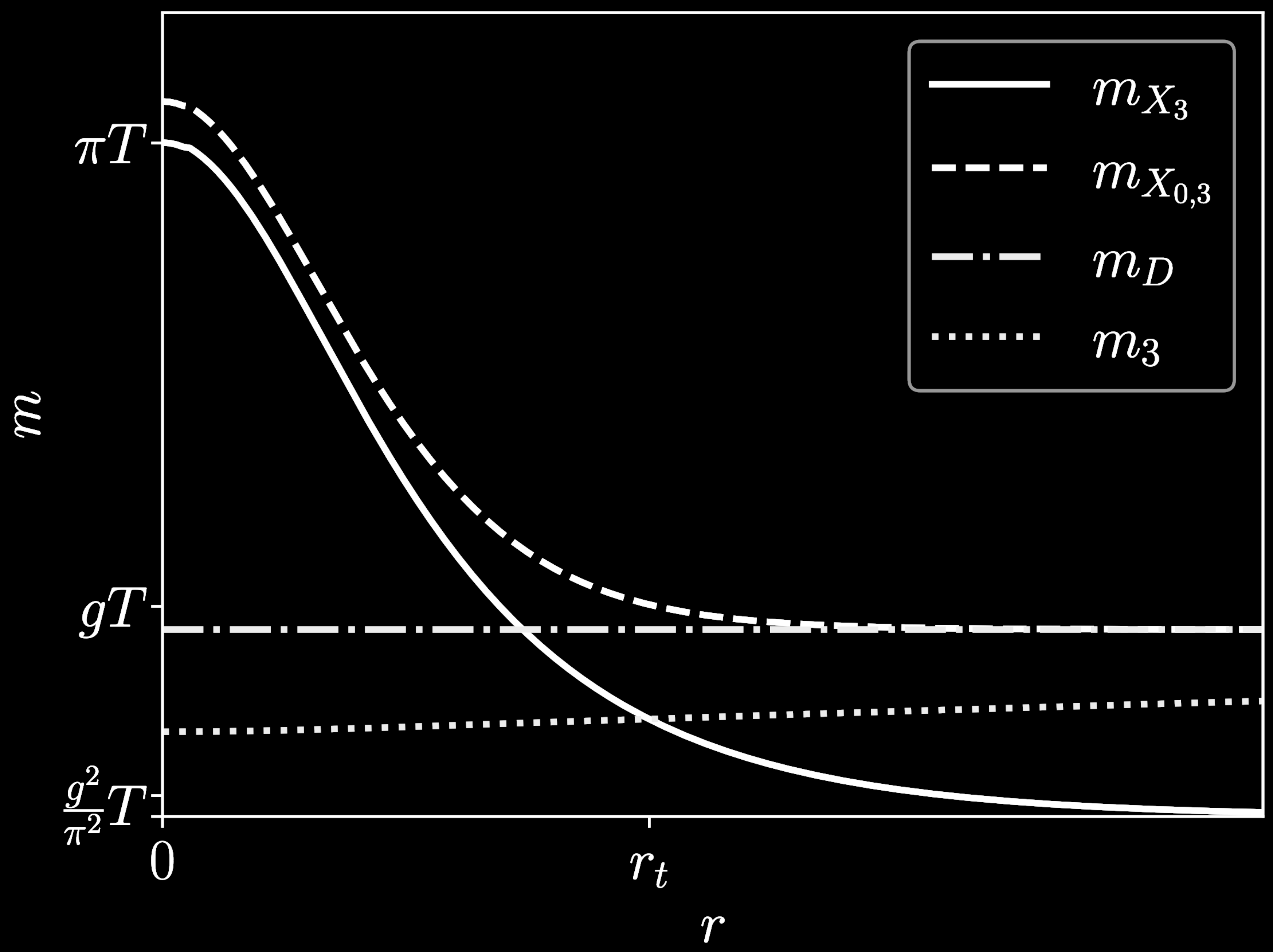




Higher-order corrections very important, but…

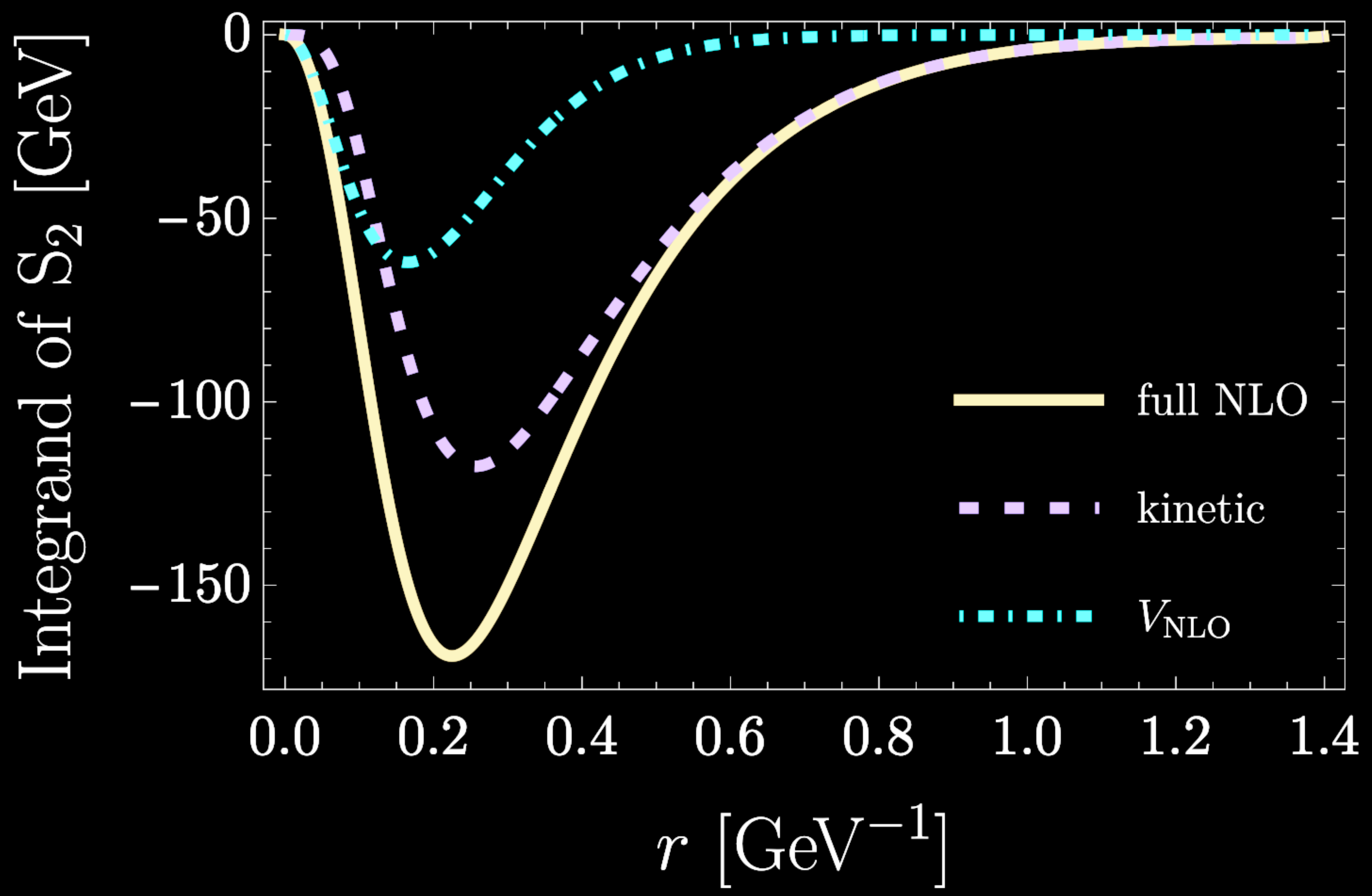
[M. Kierkla, BŚ, T.V.I. Tenkanen, J. van de VisJHEP 02 (2024) 234]

VALIDITY OF EFT — SCALE-SHIFTERS



[M. Kierkla, BŚ, T.V.I. Tenkanen, J. van de Vis, JHEP 02 (2024) 234]

DERIVATIVE EXPANSION



[M. Kierkla, BŚ, T.V.I. Tenkanen, J. van de Vis, JHEP 02 (2024) 234]

NUCLEATION RATE BEYOND LEADING ORDER

$$A_{\text{stat}} = \prod_a \mathcal{J}_a \mathcal{V}_a \sqrt{\frac{\det \mathcal{O}_a(\varphi_F)}{\det' \mathcal{O}_a(\varphi_b)}} \mathcal{J}_\phi \sqrt{\left| \frac{\det \mathcal{O}_\phi(\varphi_F)}{\det' \mathcal{O}_\phi(\varphi_b)} \right|} e^{-(S[\varphi_b]-S[\varphi_F])}$$

1. Construct the soft EFT (resum the gauge contributions)
2. Compute the LO bounce solution
3. Remove the gauge contributions from the action
4. Compute the one-loop effective action without the derivative expansion
5. Add the two-loop contribution (at NLO is the soft expansion) to the effective potential

[Procedure proposed in: O. Gould, J. Hirvonen, PRD 104 (2021) 9, 096015]

NUCLEATION RATE BEYOND LEADING ORDER

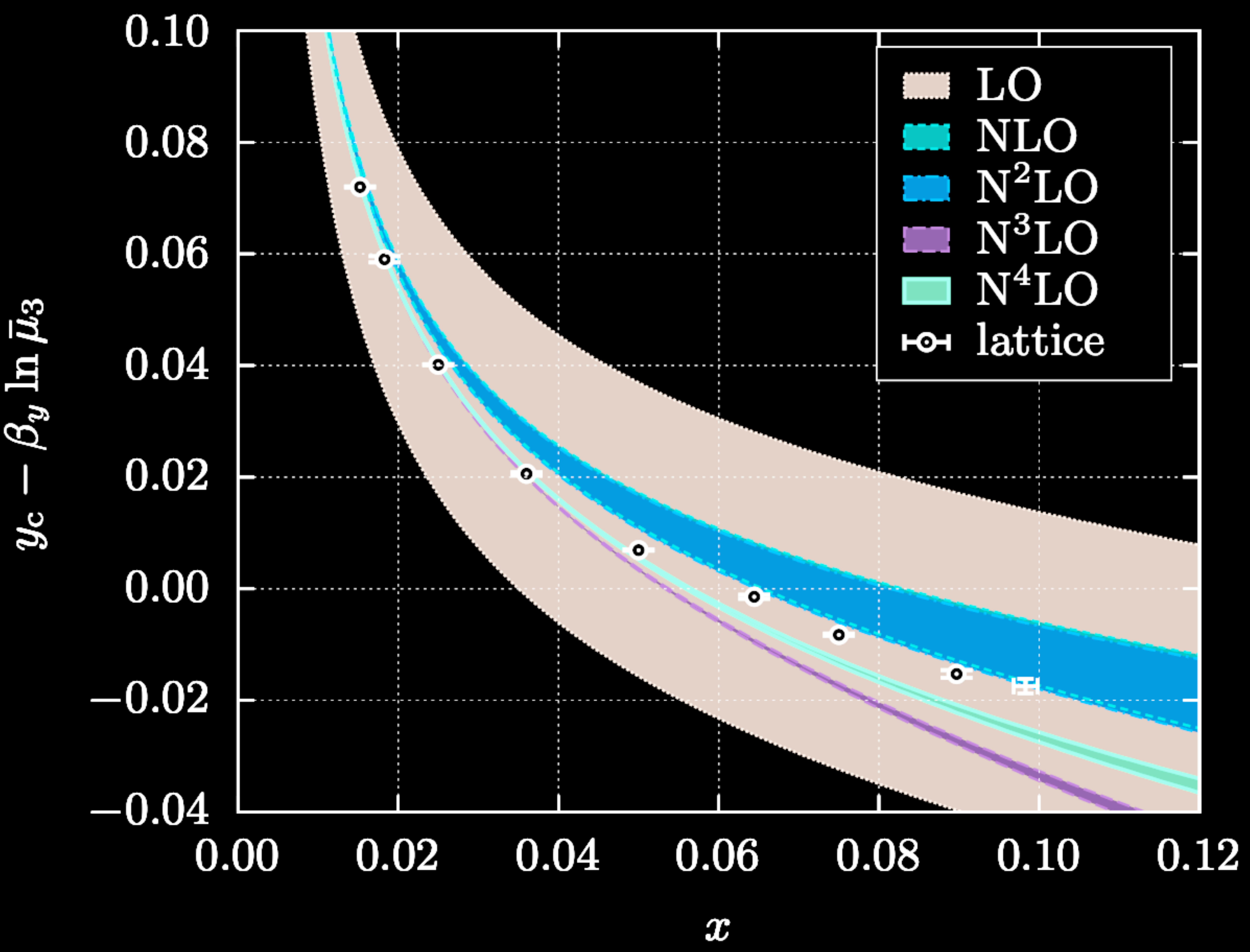
$$A_{\text{stat}} = \prod_a \mathcal{I}_a \mathcal{V}_a \sqrt{\frac{\det \mathcal{O}_a(\varphi_F)}{\det' \mathcal{O}_a(\varphi_b)}} \mathcal{I}_\phi \sqrt{\left| \frac{\det \mathcal{O}_\phi(\varphi_F)}{\det' \mathcal{O}_\phi(\varphi_b)} \right|} \, e^{-(S[\varphi_b]-S[\varphi_F])}$$

Inclusion of fluctuations from the scale-shifting fields

Test of validity of the momentum expansion

Test of the approximations for scalar prefactor

IS THE PERTURBATIVE APPROACH RELIABLE?

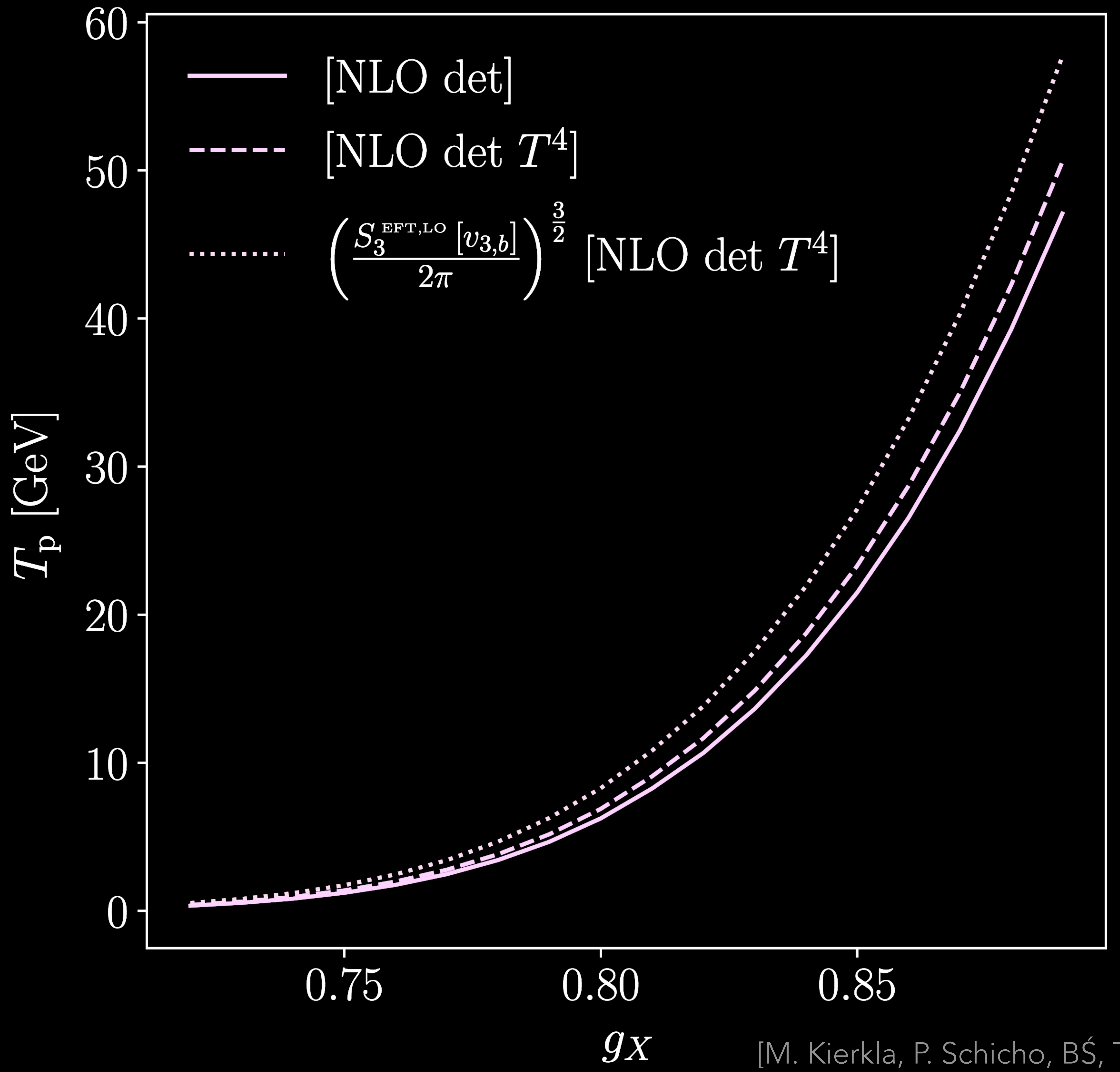


- Convergence in thermodynamical studies
- No lattice computation with $\lambda < 0$!

[figure adapted from: A. Ekstedt, P. Schicho, T. V. I. Tenkanen, Phys.Rev.D 110 (2024) 9, 096006]

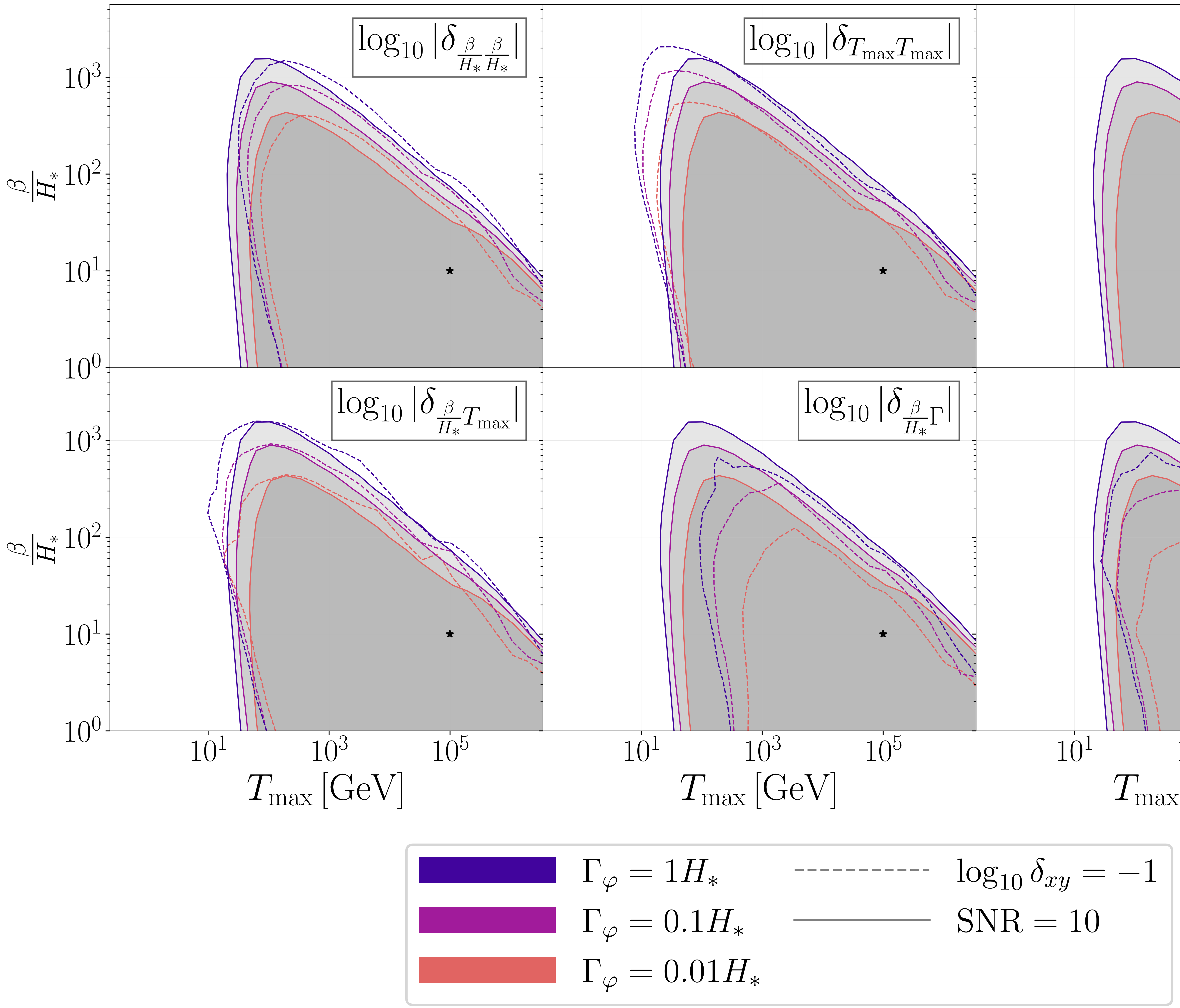
[See also: O. Gould and T. V. I. Tenkanen, JHEP 01 (2024) 048, L. Niemi et al. PRL 126 (2021) 171802, PRD 110 (2024) 115016]

SCALAR DETERMINANT



[M. Kierkla, P. Schicho, BŚ, T.V.I. Tenkanen, J. van de Vis, JHEP 07 (2025) 153]

WHAT CAN WE LEARN FROM THE SIGNAL?



[A. Gonstal, M. Lewicki, B. Świeżewska, 2502.18436]

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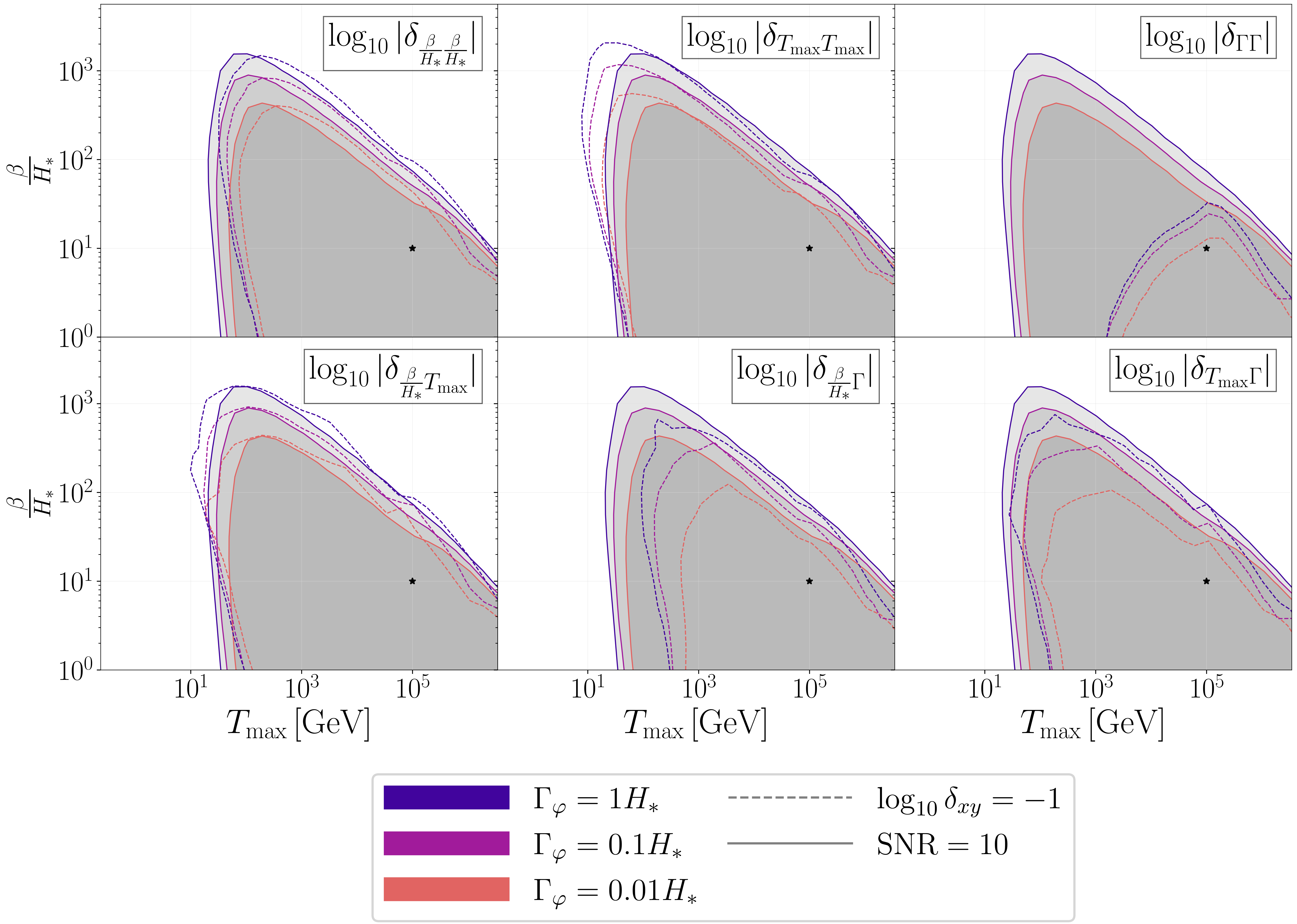
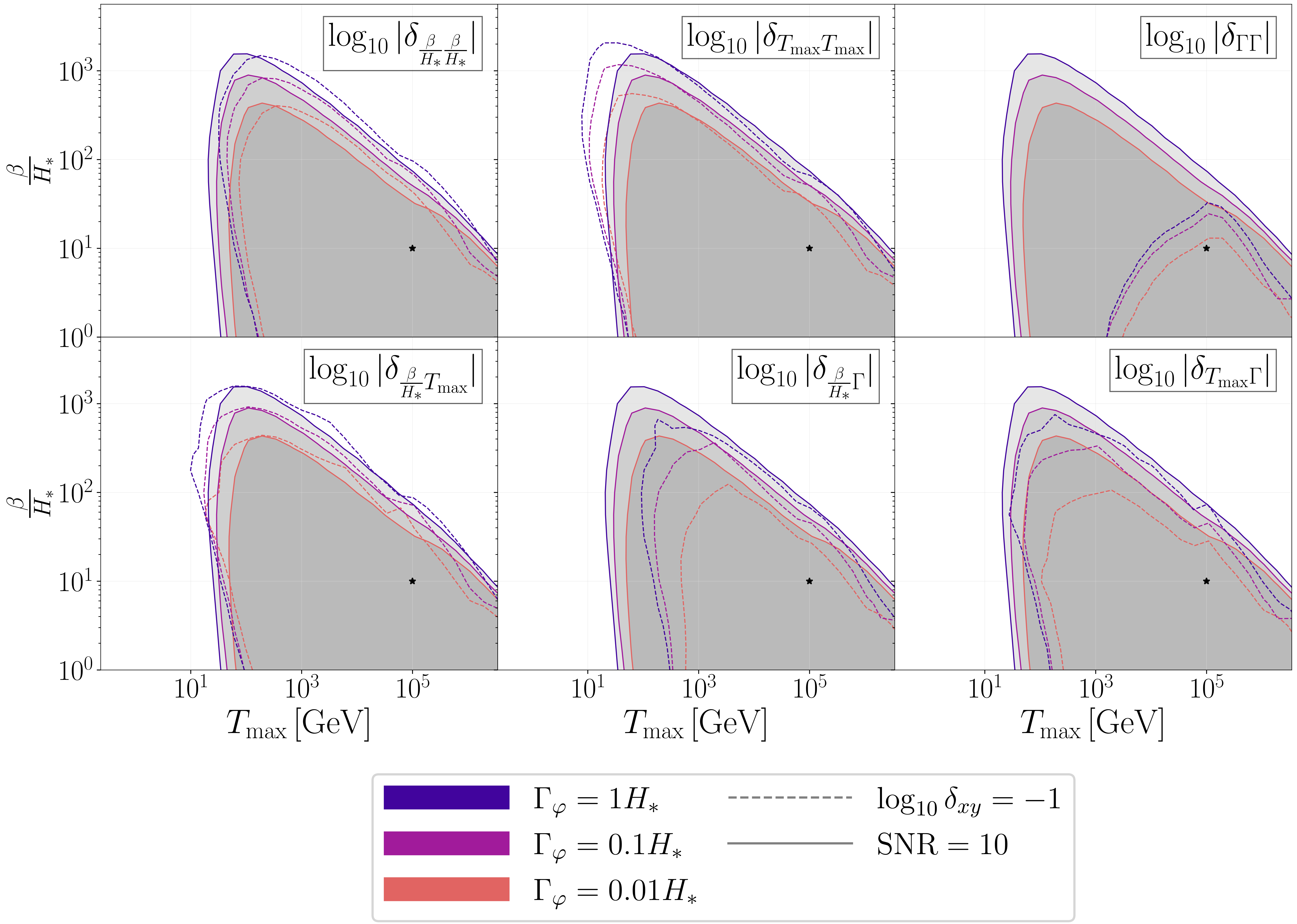


[A. Gonstal, M. Lewicki, B. Świeżewska, 2502.18436]

- Decay rate of the scalar (inaccessible at colliders?) can be determined from the spectrum

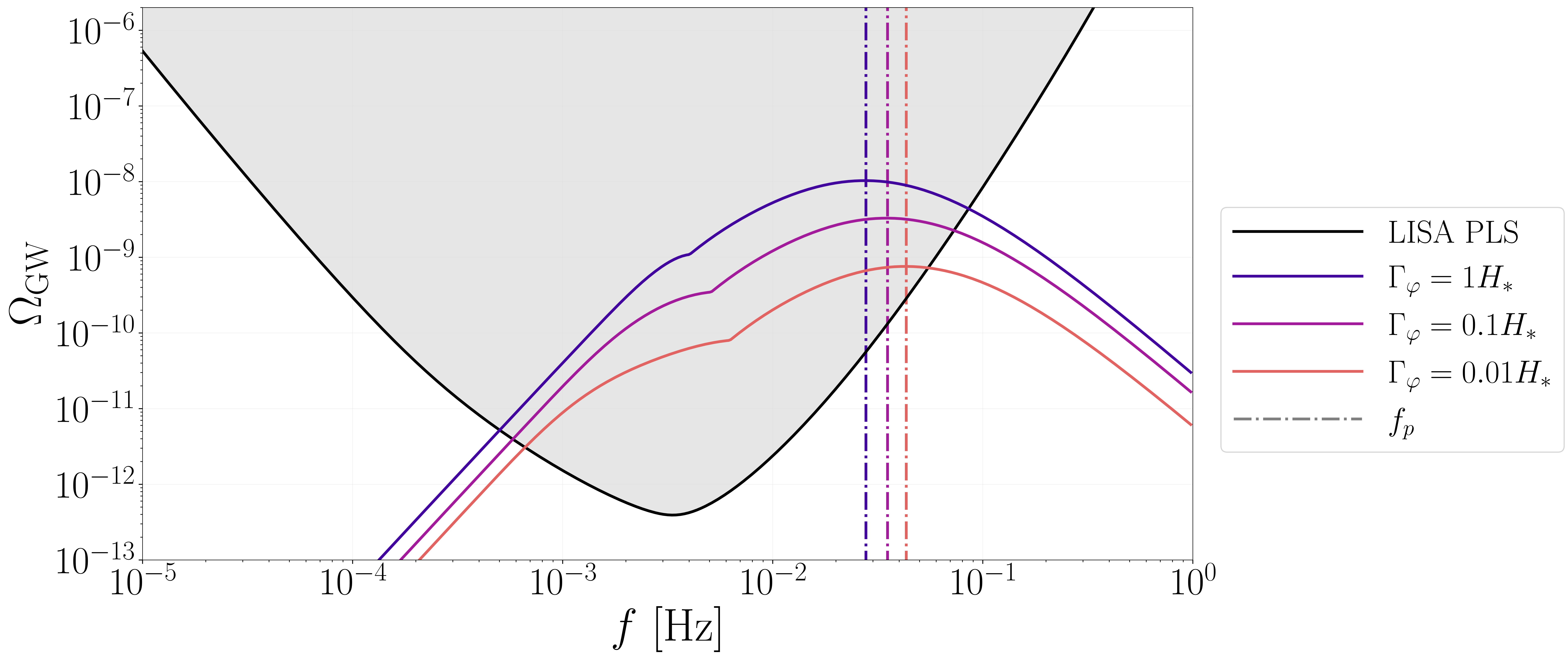
WHAT CAN WE LEARN FROM THE SIGNAL?



[A. Gonstal, M. Lewicki, B. Świeżewska, 2502.18436]

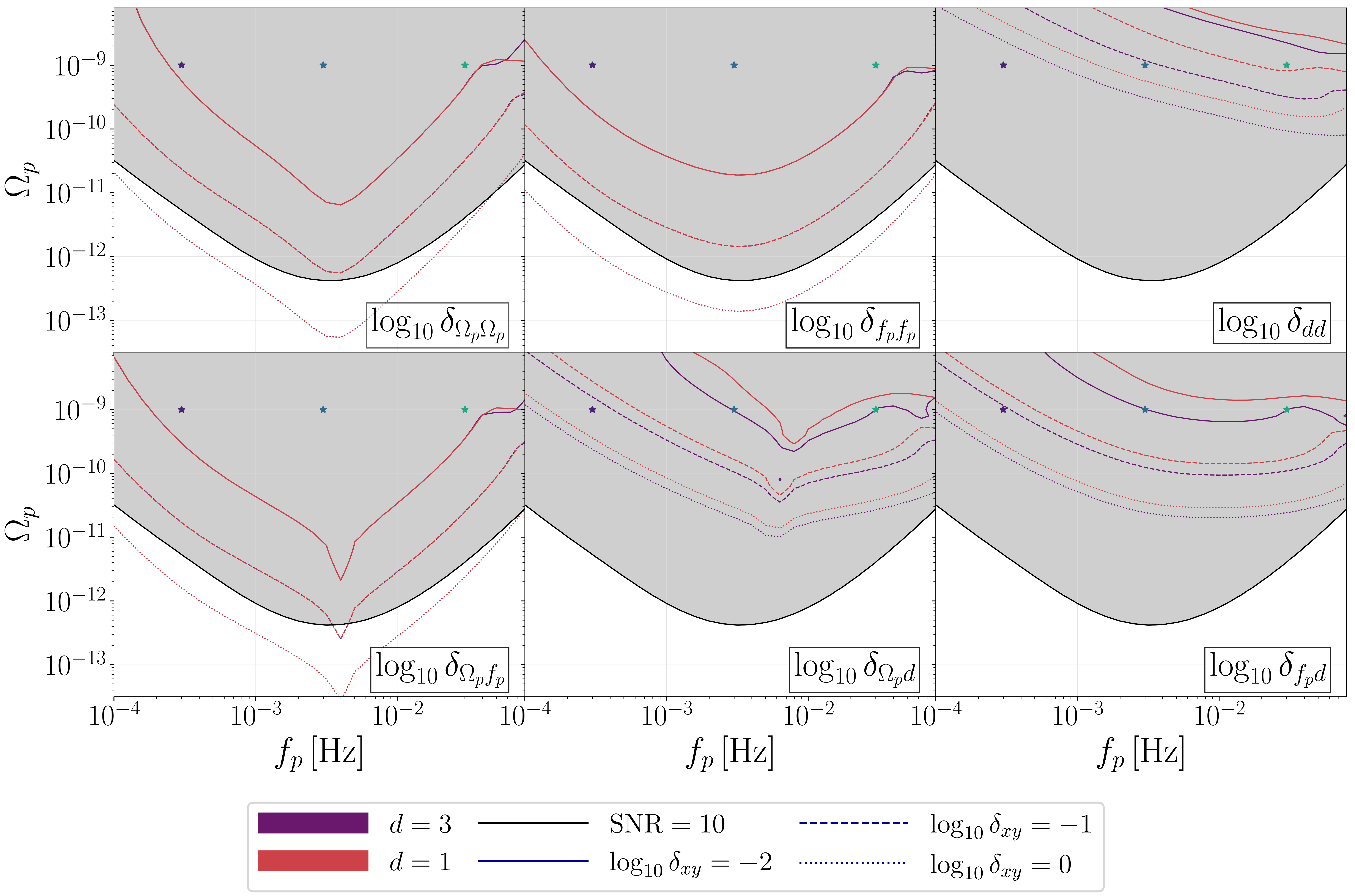
- Decay rate of the scalar (inaccessible at colliders?) can be determined from the spectrum
- Individual parameters can be determined with an accuracy better than 10%

MODIFIED EXPANSION AFFECTS THE SIGNAL



[A. Gonstal, M. Lewicki, B. Świeżewska, 2502.18436]

RECONSTRUCTION OF GEOMETRIC PARAMETERS



DETERMINANTS FROM THE EFFECTIVE ACTION

$$e^{-S_{\text{eff}}[\varphi]} = \int \mathscr{D}\chi \mathscr{D}\tilde{\varphi} e^{-S[\chi,\varphi+\tilde{\varphi}]}$$

DETERMINANTS FROM THE EFFECTIVE ACTION

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$$e^{-S_{\text{eff}}[\varphi]} = \int \mathcal{D}\chi e^{-S[\chi,\varphi]} \int \mathcal{D}\tilde{\varphi} e^{-\left(\frac{\delta S}{\delta\phi}[0,\varphi]\tilde{\varphi} + \frac{1}{2}\tilde{\varphi}\frac{\delta^2 S}{\delta\phi^2}[0,\varphi]\tilde{\varphi} + \dots\right)}$$

DETERMINANTS FROM THE EFFECTIVE ACTION

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For a scalar contribution and a constant background

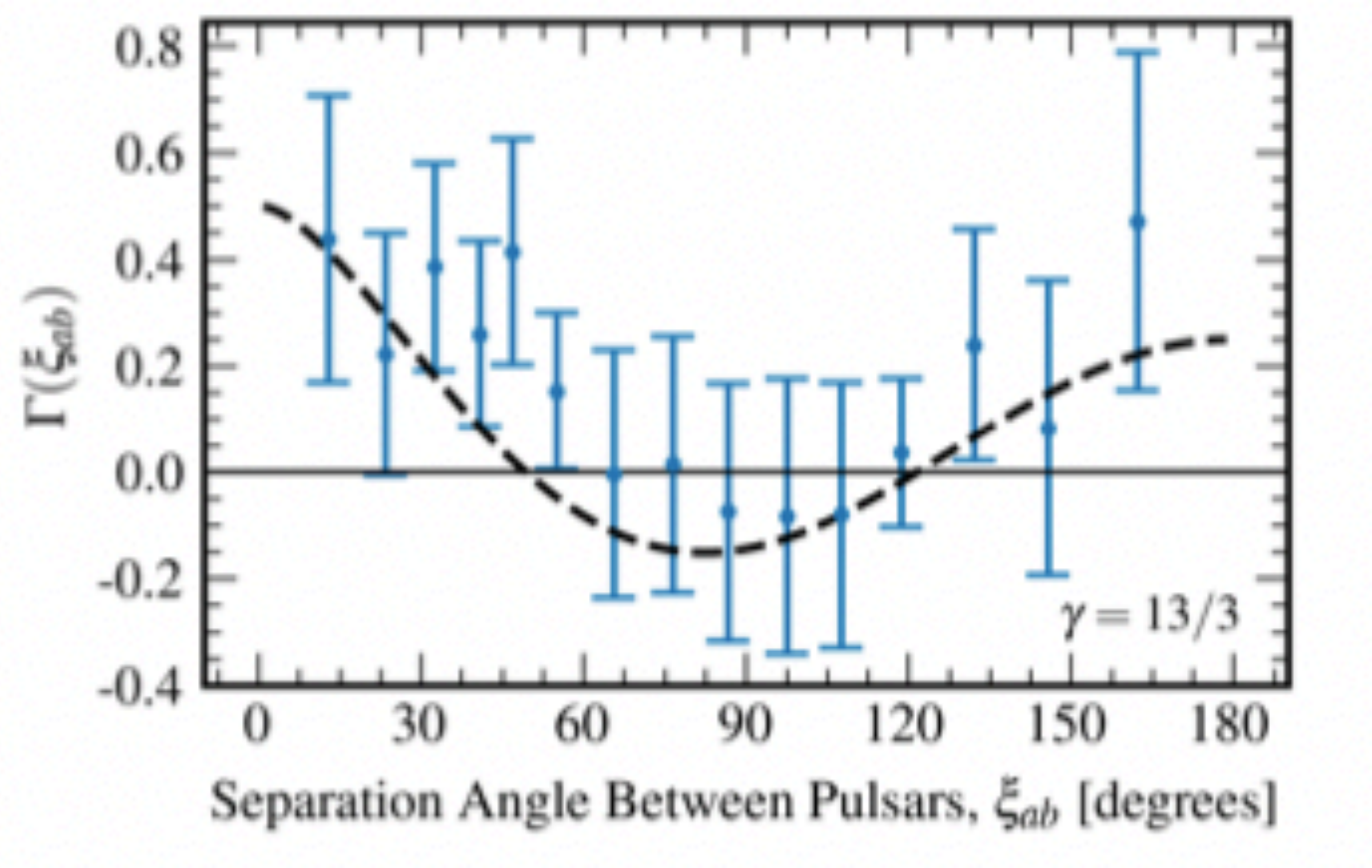
$$S^{(1)}_{\text{eff}}[\varphi_0] = \frac{1}{2}\text{tr}\log\left(-\partial^2 + \text{V}''(\varphi_0)\right) = \text{V}^{(1)}_{\text{eff}}(\varphi_0)$$

DETERMINANTS FROM THE EFFECTIVE ACTION

$$e^{-S_{\text{eff}}[\varphi]} = \prod_a \frac{1}{\sqrt{\det \mathcal{O}_a(\varphi)}} e^{-S[0,\varphi]}$$

$$A_{\text{stat}} = \prod_a \mathcal{J}_a \mathcal{V}_a \sqrt{\frac{\det \mathcal{O}_a(\varphi_F)}{\det' \mathcal{O}_a(\varphi_b)}} \mathcal{J}_\phi \sqrt{\left| \frac{\det \mathcal{O}_\phi(\varphi_F)}{\det' \mathcal{O}_\phi(\varphi_b)} \right|} \, e^{-(S[\varphi_b]-S[\varphi_F])}$$

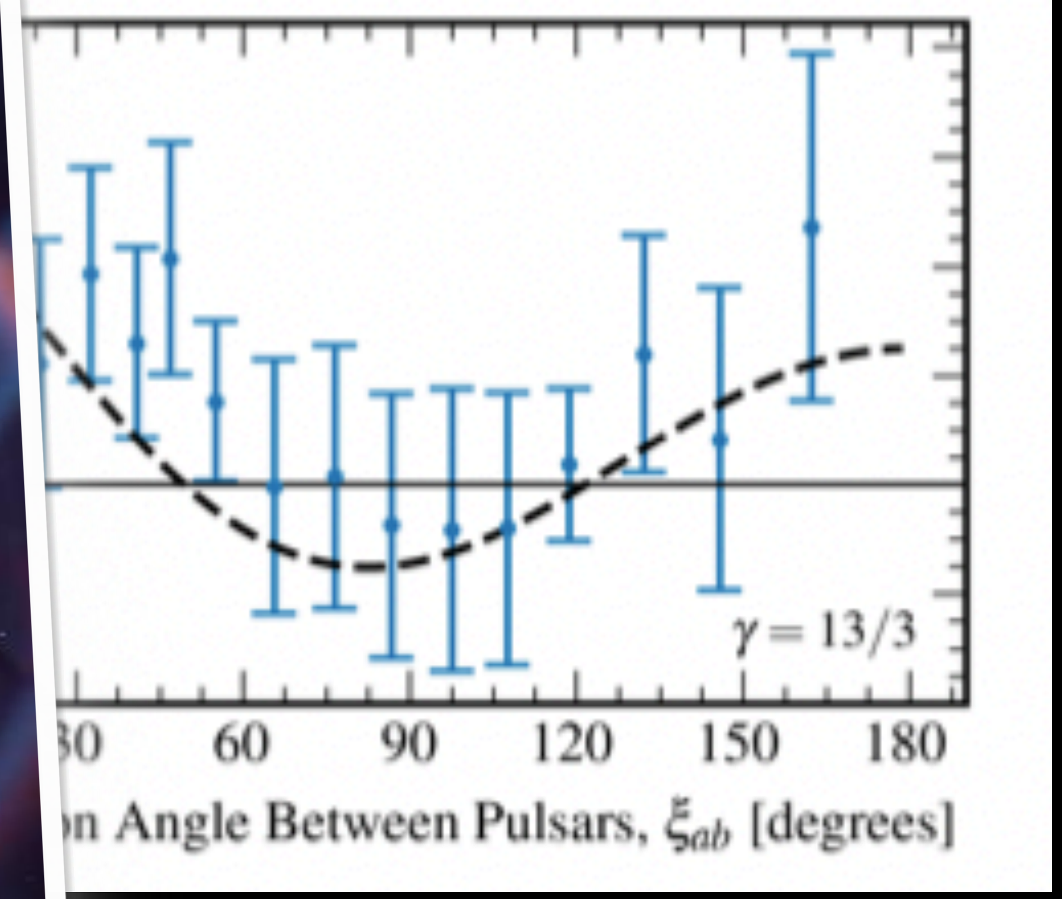
STOCHASTIC GW BACKGROUND



[G. Agazie et al. (NANOGrav Collaboration), Astrophys. J. Lett. 951, L8 (2023), arXiv:2306.16213]

STOCHASTIC GW BACKGROUND

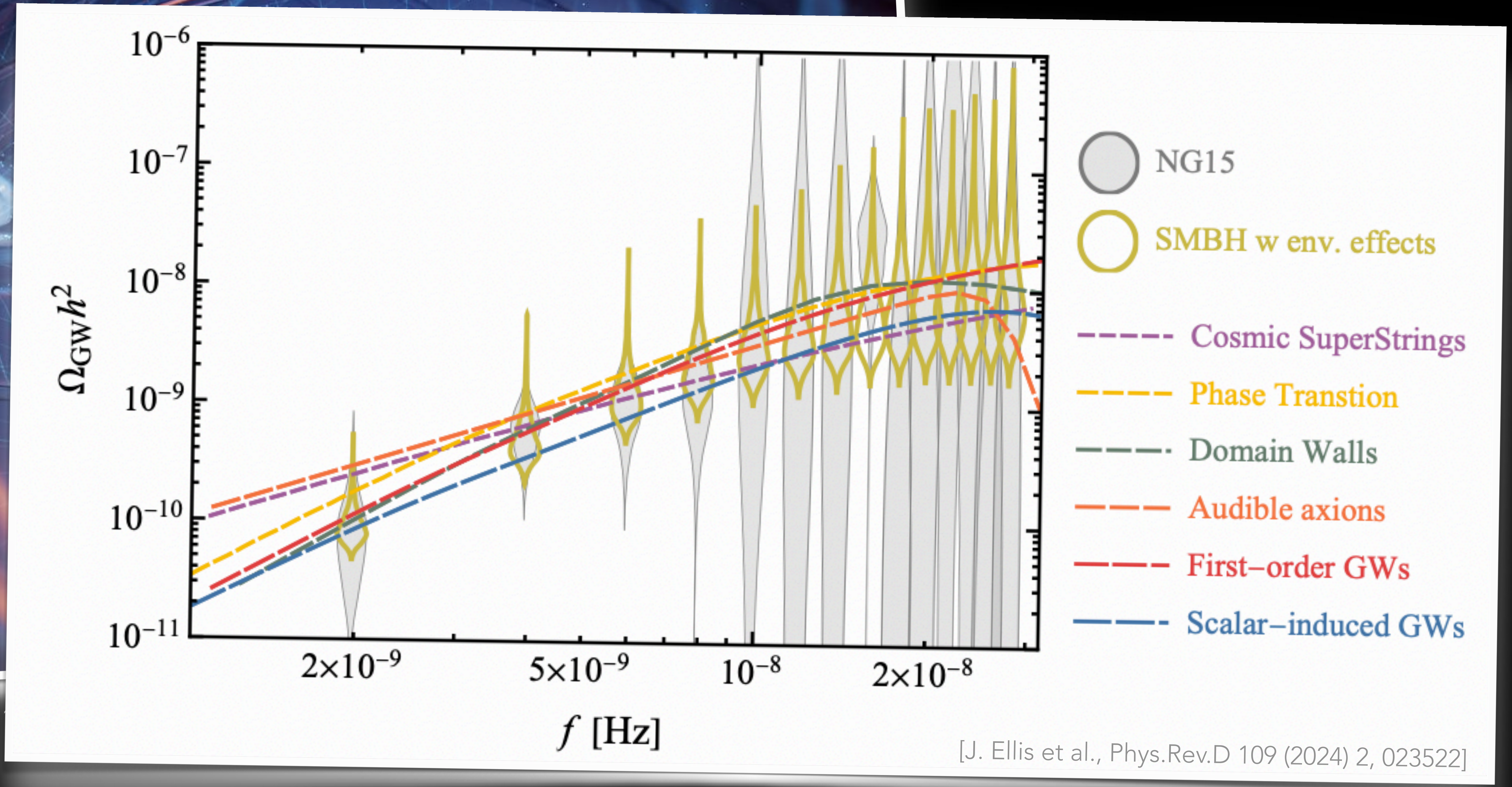
[źródło: nanograv.org, Olena Shmahalo]



et al. (NANOGrav Collaboration),
ett. 951, L8 (2023), arXiv:2306.16213]

STOCHASTIC GW BACKGROUND

[źród



[J. Ellis et al., Phys.Rev.D 109 (2024) 2, 023522]