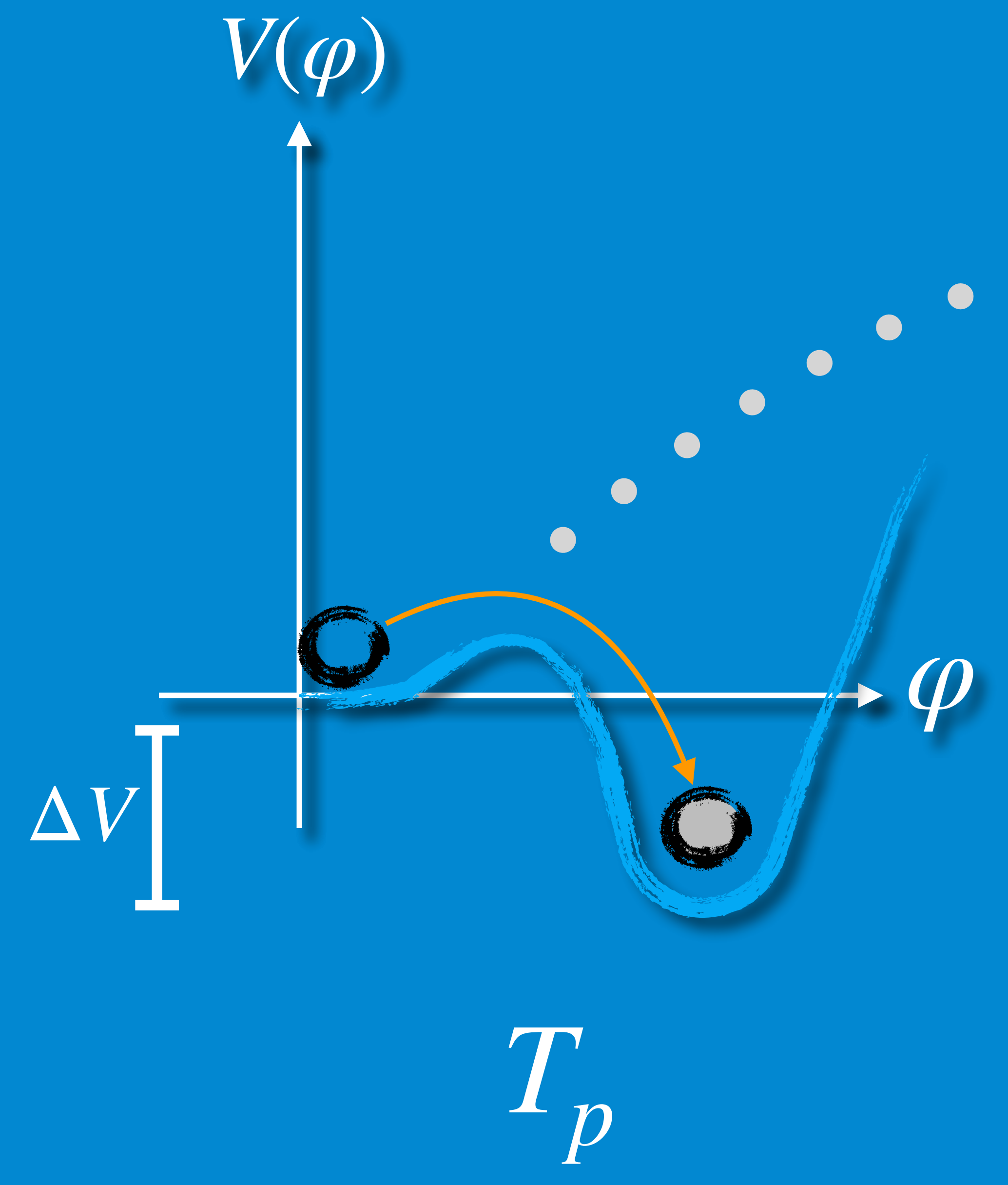
Finite-temperature bubble-nucleation with shifting scale hierarchies

Maciej Kierkla, University of Warsaw

Based on work with
- T. Tenkanen, J. van de Vis, P. Schicho, B. Świeżewska 2503.13597,
- N. Ramberg, P.Schicho, D. Schmitt, 2506.15496

This talk

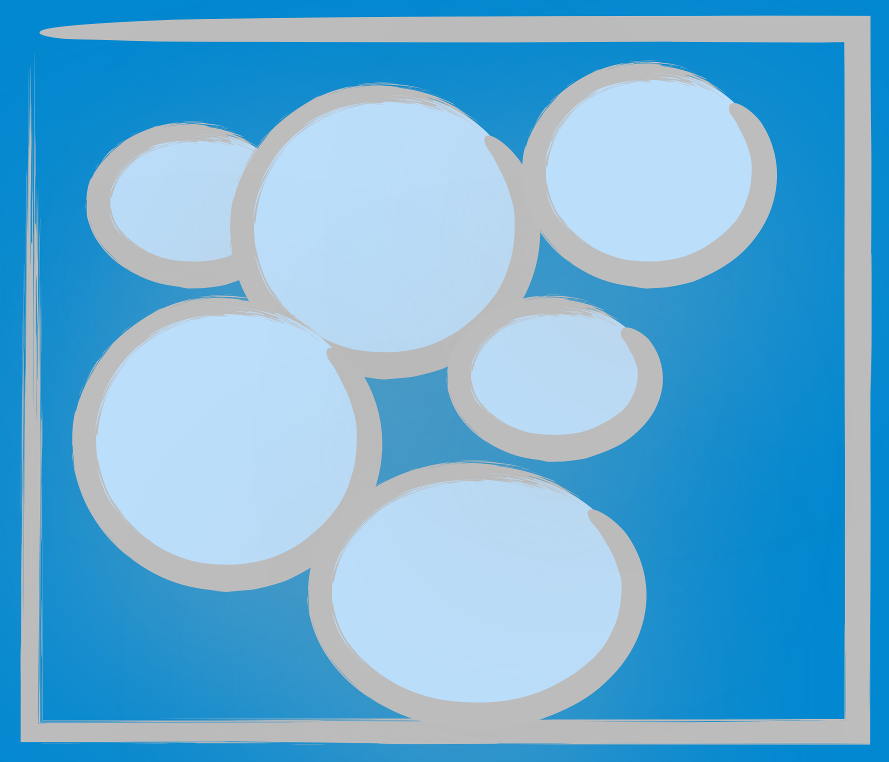
Field theory



Bubble nucleation rate

$$\frac{\Gamma_T}{H^4}$$

Cosmology



Bubble nucleation rate

$$\Gamma(T) = A_{\text{dyn}} \times A_{\text{stat}}$$

Non-equilibrium effects

Dimension $[T]$

(This talk)

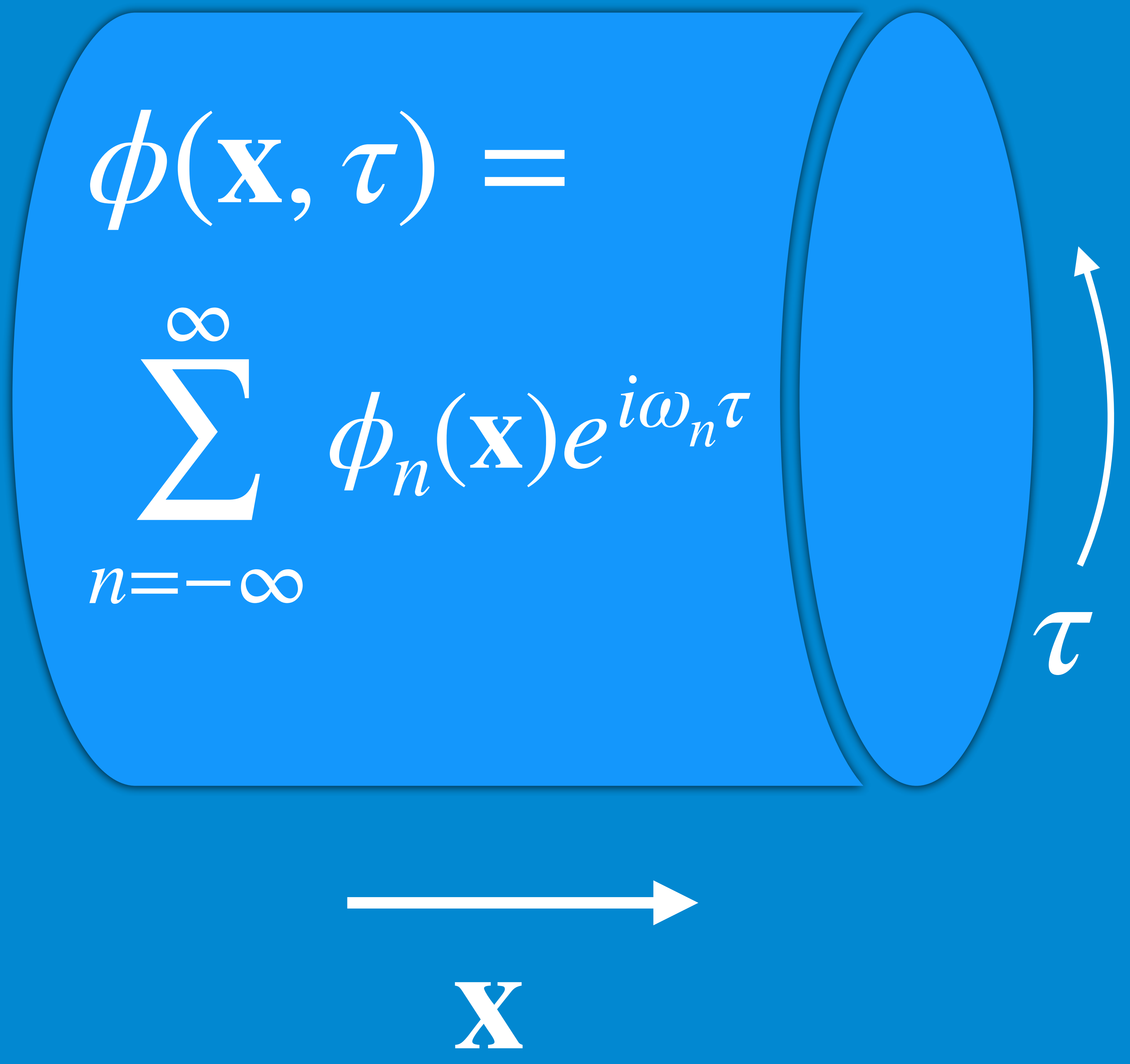
Equilibrium effects

Dimension $[T^3]$

See also: J. Hirvonen, 2403.07987, A. Ekstedt 2201.07331

Thermal field theory in imaginary time formalism

$$\phi(\mathbf{x},\tau) = \sum_{n=-\infty}^{\infty} \phi_n(\mathbf{x}) e^{i\omega_n \tau}$$

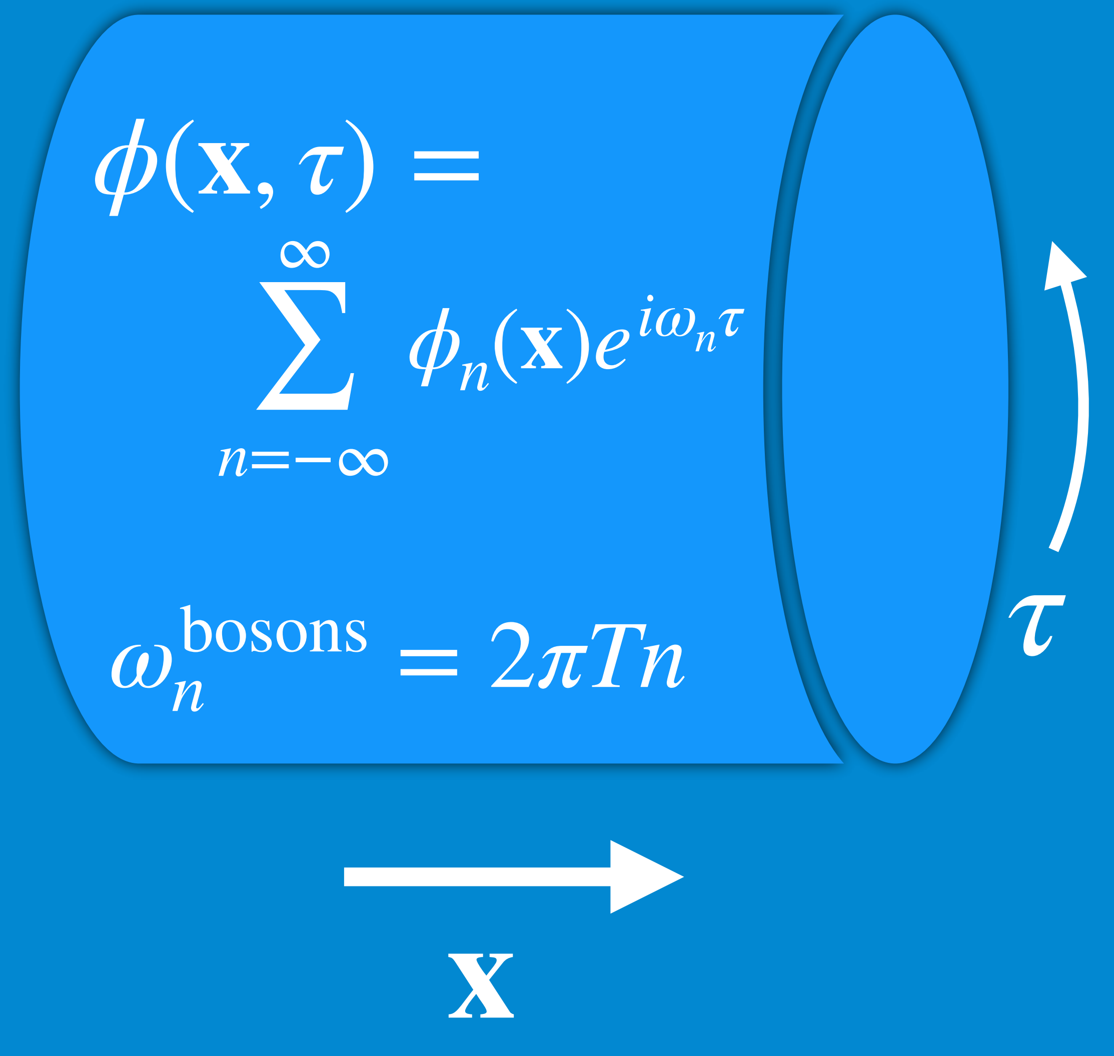


$$\omega_n^{\text{bosons}} = 2\pi T n$$

$$S = \frac{1}{T}\int d^3x \sum_{n=0}^{\infty}\left[\frac{1}{2}(\partial_i\phi_n)^2 + \frac{1}{2}(2\pi n T)^2\phi_n^2 + \frac{1}{2}m^2\phi_n^2\right]$$

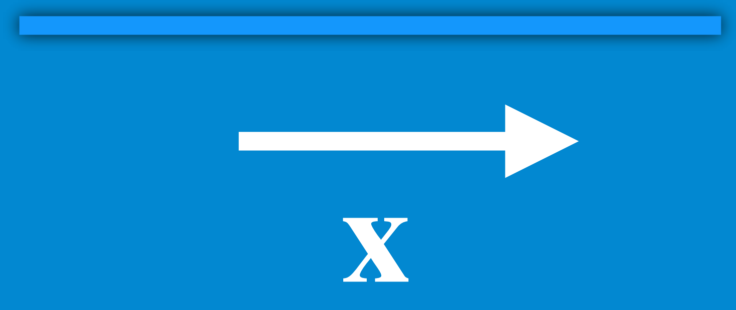
See also: M. Laine, A. Vuorinen, “Basics of thermal field theory”

High-T Dimensional Reduction (DR) = 3d EFT



Integrating out $\omega_{n\neq0}$

$\phi_3(\mathbf{x})$



$$Z = \int D\phi \exp\left(-\int_0^{\frac{1}{T}} \mathrm{d}\tau \int \mathrm{d}^3\mathbf{r}\ \mathscr{L}_E\right) \longrightarrow Z_3 = \int D\phi_{n=0} \exp\left(-\frac{1}{T}\int \mathrm{d}^3\mathbf{r}\ \mathscr{L}_3\right)$$

Fields: ϕ, X

Couplings: g, λ

Fields: $\phi_3, X_3, X_{0,3}$

Couplings: $m_3(T), g_3(T), \lambda_3(T)$

Statistical part

In the saddlepoint approximation

$$A_{\text{stat}} = Ae^{-(S_{\text{eff}}[\phi_b] - S_{\text{eff}}[\phi_F])}$$

ϕ_b *is the O(3) solution to EOMs,*

ϕ_F *is the false vacuum*

See also: J. Hirvonen, 2403.07987, A. Ekstedt 2201.07331

Effective action perturbatively

$$S_{\text{eff}}[\phi_b] = \underbrace{S^{\text{LO}}[\phi_b^{\text{LO}}]}_{\mathcal{O}(g^3)} + \underbrace{S^{\text{NLO}}[\phi_b^{\text{LO}}]}_{\mathcal{O}(g^4)} + \dots$$

I will refer to this as *soft expansion*

One can get to NLO just with the LO bounce solution!

=

See also: A. Ekstedt 2201.07331, 2104.11804

Nucleation rate in 3D SU(2)cSM at 1-loop

$$\Gamma = A_{\text{dyn}}\,\text{Det}_S\, e^{\underbrace{-\left(S_3\left[\phi_\text{b}\right] - S_3\left[\phi_\text{F}\right]\right) - \log \text{Det}_{X_0} - \log \text{Det}_{XG}}_{\equiv S_\text{eff}}}$$

overall prefactor
$\sim T^4$

(Effective) action in
the exponential

See also: A. Ekstedt, O. Gould, J. Hirvonen, 2308.15652; A. Ekstedt 2104.11804

Nucleation rate in 3D SU(2)cSM at 1-loop

$$\Gamma = A_{\text{dyn}}\,\text{Det}_S\, e^{-\left(S_3[\phi_\text{b}] - S_3[\phi_\text{F}]\right) - \log \text{Det}_{X_0} - \log \text{Det}_{XG}}$$

(Some) can be easily computed using

BubbleDet: A Python package to compute functional determinants for bubble nucleation

Andreas Ekstedt*[1], Oliver Gould†[2], and Joonas Hirvonen‡[3]

Vector-Goldstone determinant

(Not so easy to compute...)

See also: A. Ekstedt, O. Gould, J. Hirvonen, 2308.15652; A. Ekstedt 2104.11804

Vector-Goldstone determinant in derivative expansion

$$\frac{m_3}{m_X} \ll 1$$

$$\delta Z = -\frac{11}{16\pi}\frac{g_3}{\phi_b}$$

$$-\log \mathrm{Det}_{VG} \simeq \underbrace{\int_x \frac{-3}{48\pi} g^3 \phi_b^3}_{\text{soft LO}} + \underbrace{\int_x \frac{1}{2}\delta Z[\phi_b](\partial\phi_b)^2}_{\text{soft NLO}} + \ldots$$

Barrier generating term (contributes to V^{LO})

Terms with higher powers in derivatives

See also: A. Ekstedt, O. Gould, J. Hirvonen, 2308.15652; A. Ekstedt 2104.11804, J. Hirvonen et al 2112.08912, MK, B. Świeżewska, T.V.I Tenkanen, J. van de Vis, 2312.12413

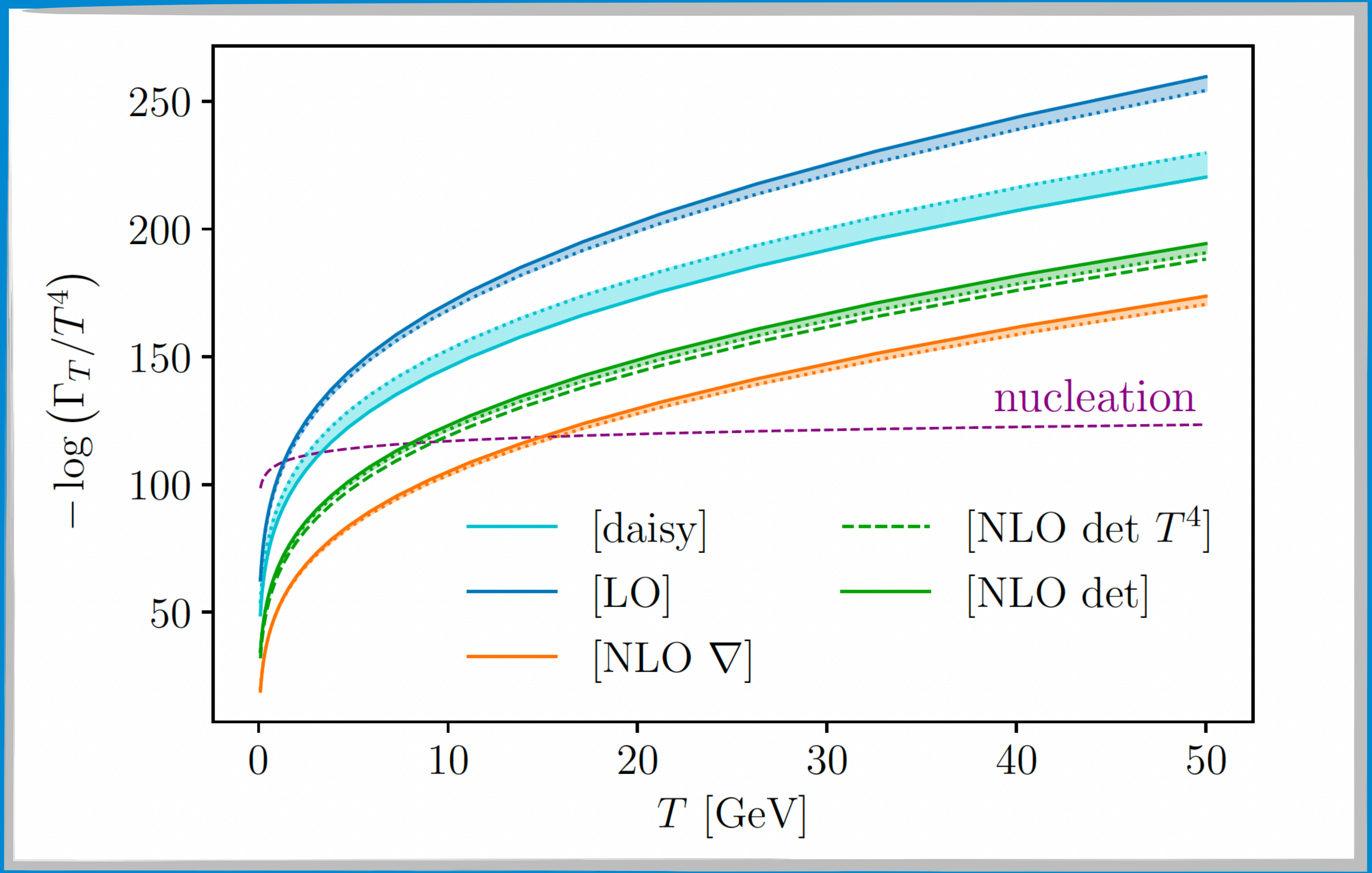
Q: Is derivative expansion always valid? 🤔

Q: Is derivative expansion always valid?

A: Nope 😢

Gauge modes **always** breake it!

Results: Nucleation rate from different "recipes"



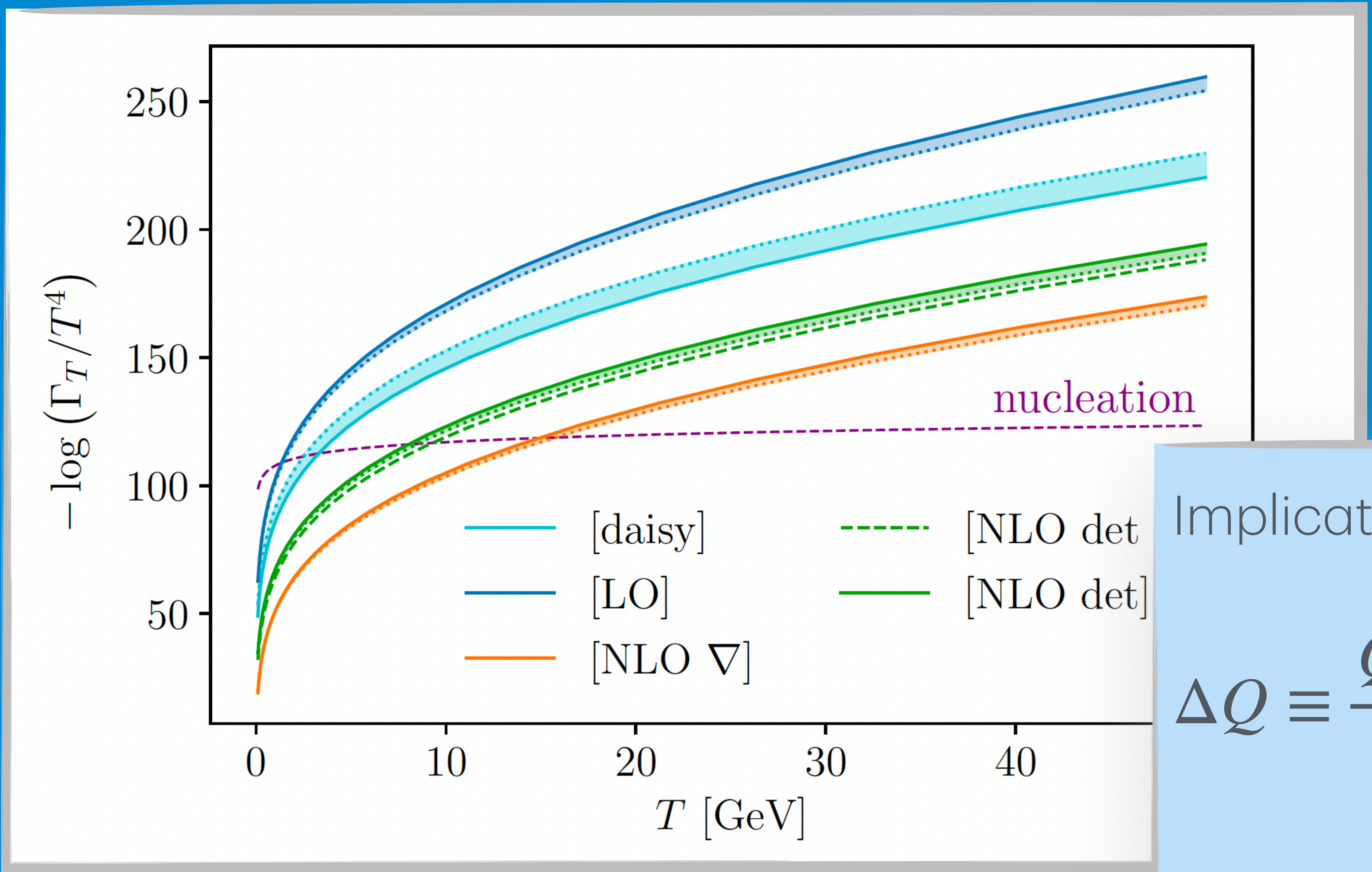
Note:

$A_{\rm dyn}\,\det{}_S \simeq T^4$

everywhere

except [NLO det]

Results: Nucleation rate from different "recipes"



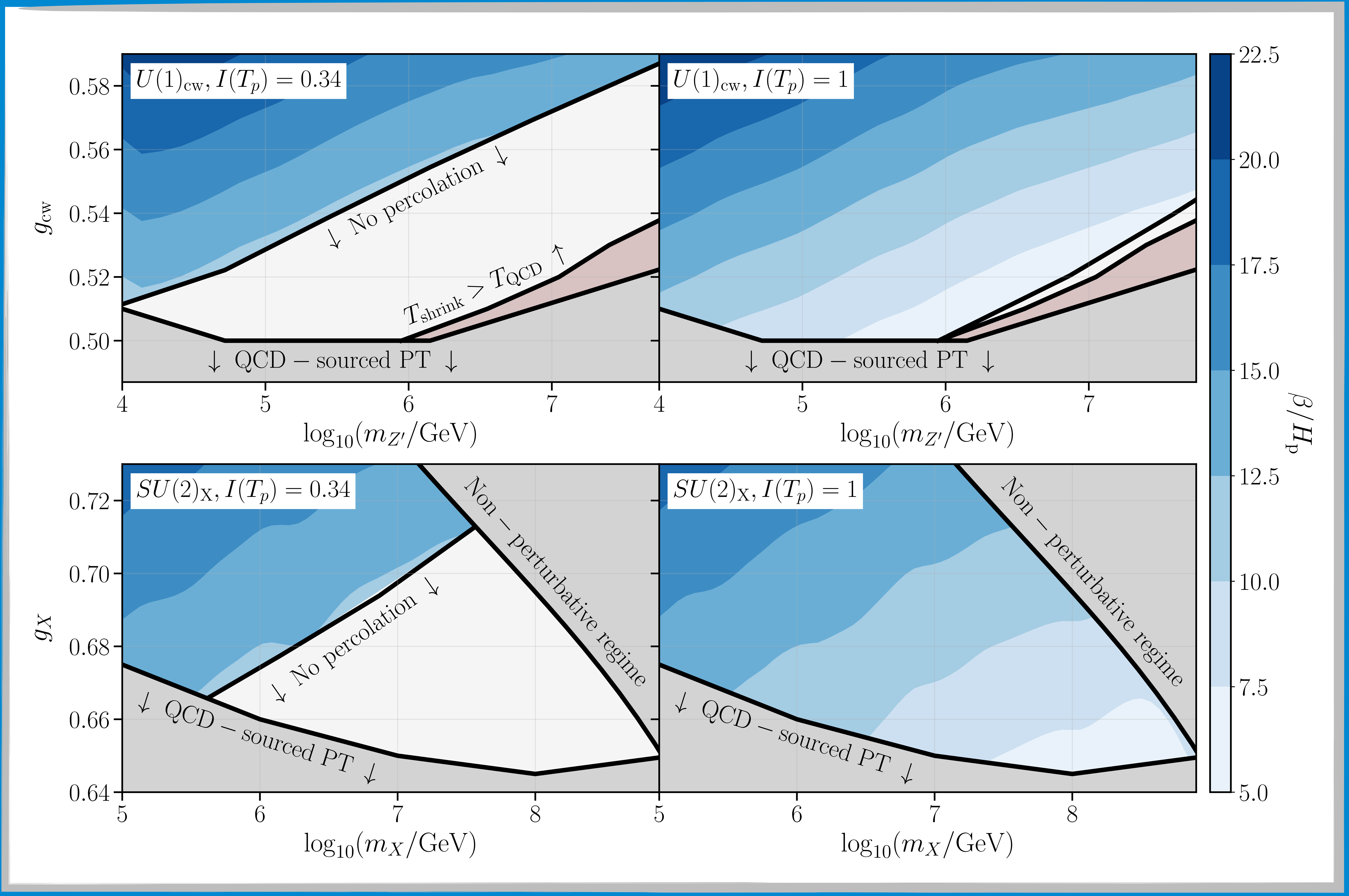
Implications for PT parameters:

$$\Delta Q \equiv \frac{Q^{[\text{NLO det}]} - Q^{[\text{NLO }\nabla]}}{Q^{[\text{NLO det}]}}$$

- $\Delta T_p \sim \mathcal{O}(40\%) - \mathcal{O}(300\%)$
- $\Delta R_* H_* \sim \mathcal{O}(15\%) - \mathcal{O}(22\%)$

Note:
$A_{\text{dyn}}\, \det_S \simeq T^4$
everywhere
except [NLO det]

You won't get slower EWPT than $\beta/H \sim 5$



MK, N. Ramberg, P. Schicho, D. Schmitt, 2506.15496

Summary

NLO corrections are **mandatory** for reliable pheno predictions

Derivative expansion for **gauge modes** introduces significant **errors**.

High-T EFT can be applied to **CSI models/supercooled PTs**

Min value of $\beta/H = 5$

Thank you!

4d theory

SU(2) parameters

g_X, M_X

$$V_{\text{tree}} = \frac{1}{4}\left(\lambda_1 h^4 + \lambda_2 h^2 \phi^2 + \lambda_3 \phi^4\right)$$

See also: C. D. Carone et al, 1307.8428, T. Hambye et al, 1306.2329, D. Marfatia et al, 2006.07313, I. Baldes et al, 1809.01198

(Soft scale) 3d EFT

$$S_3 = \int d^3\mathbf{x} \left\{ \frac{1}{4} F^a_{ij} F^a_{ij} + \left(D_i\phi\right)^\dagger \left(D_i\phi\right) + \frac{1}{2} \left(D_i X^a_0\right)^2 + V_3\left(\phi, X^a_0\right) \right\}$$

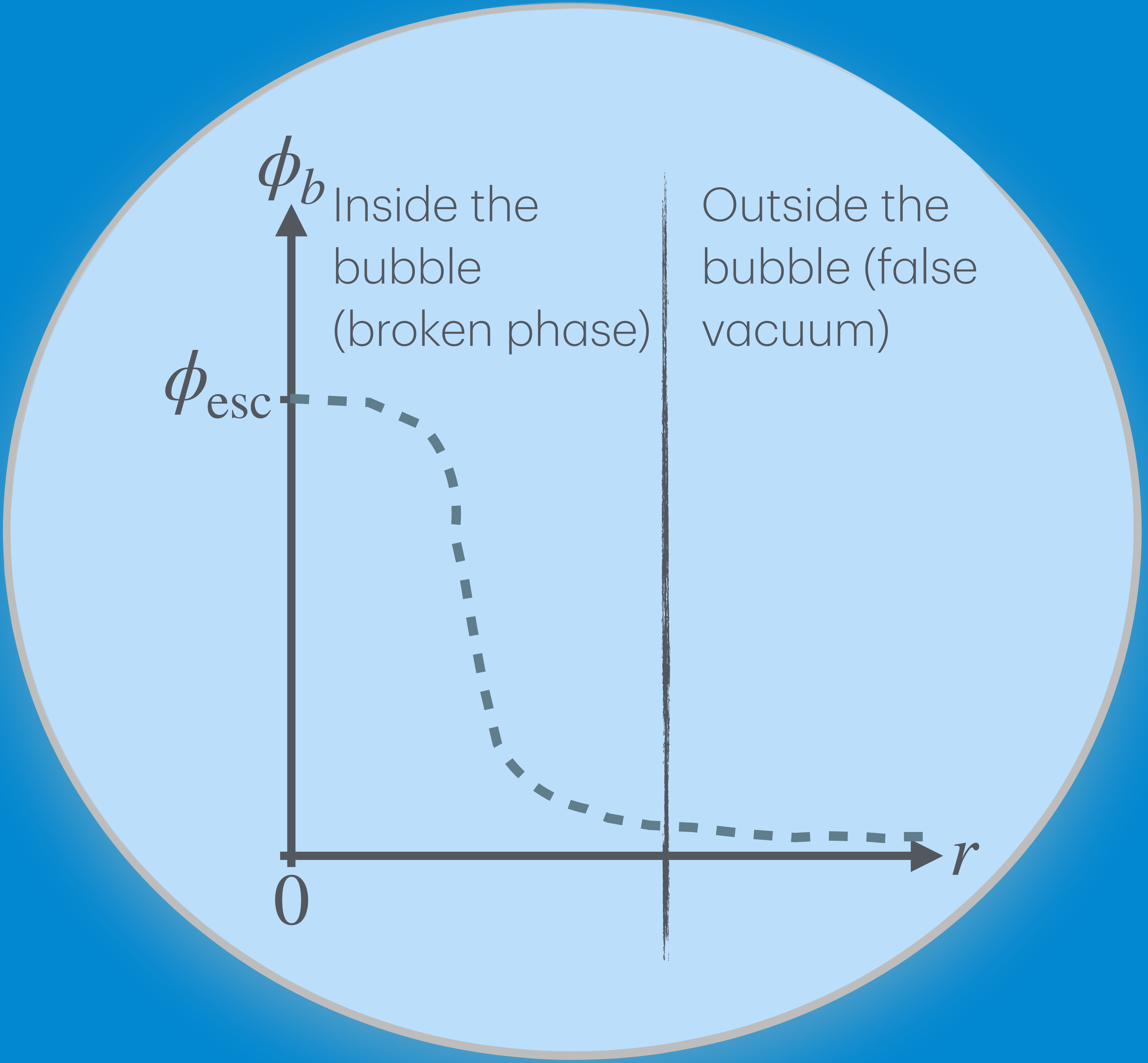
$$V_3(\phi_3) = \frac{1}{2} m_3^2 \phi_3^2 + \frac{1}{4} \lambda_3 \phi_3^4$$

$$m_3 \sim m_D = gT \neq 0$$

$$m^2_{X,3} = \frac{1}{4} g^2_{X,3} \phi_3^2, \quad m^2_{X_0,3} = m^2_{D,X} + \frac{1}{2} h_3 \phi_3^2 .$$

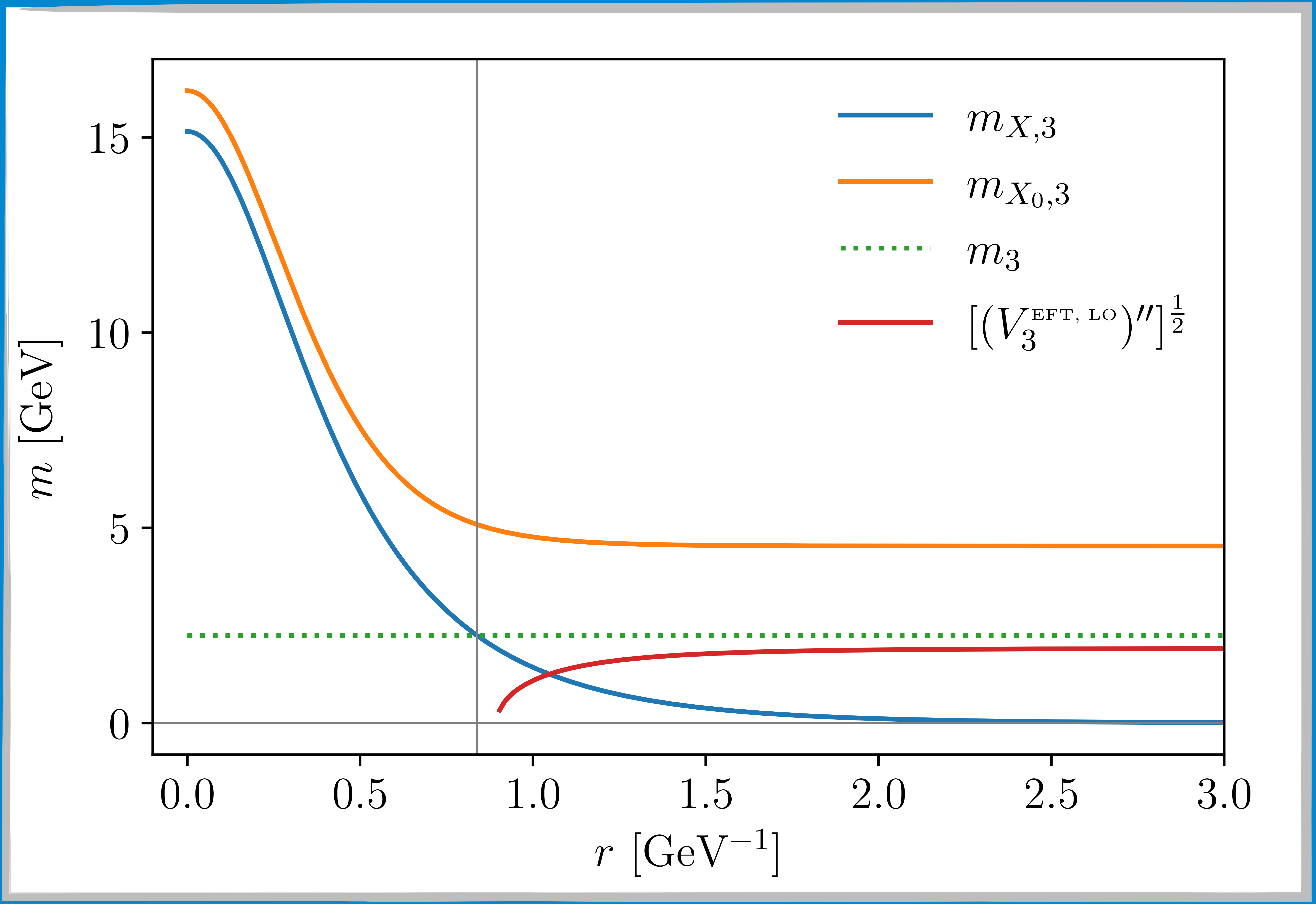
See also: MK, B. Świeżewska, T.V.I Tenkanen, J. van de Vis, 2312.12413

LO bounce solution



See also: S. Coleman Phys.Rev.D 15 (1977), A. Ekstedt 2201.07331, 2104.11804

Scale-shifters in SU(2)cSM



Reminder:

$$S_{\text{eff}}[\phi_b] = \underbrace{S^{\text{LO}}[\phi_b^{\text{LO}}]}_{\mathcal{O}(g^3)} + \underbrace{S^{\text{NLO}}[\phi_b^{\text{LO}}]}_{\mathcal{O}(g^4)} + \ldots$$

$$S^{\text{LO}}[\phi_b^{\text{LO}}] = 4\pi \int \mathrm{d}r\ r^2 \left[\frac{1}{2}\left(\partial_i \phi_b^{\text{LO}}\right)^2 + V^{\text{LO}}[\phi_b^{\text{LO}}]\right]$$

$$V^{\text{LO}}[\phi_b^{\text{LO}}] = \frac{1}{2}m_3^2\phi_3^2 + \frac{1}{4}\lambda_3\phi_3^4 - \frac{1}{12\pi}\left(6(m_{X,3}^2)^{\frac{3}{2}} + 3(m_{X_0,3}^2)^{\frac{3}{2}}\right)$$

$\mathcal{O}(\partial^0)$-terms in expansion of gauge determinants

See also: MK, B. Świeżewska, T.V.I Tenkanen, J. van de Vis, 2312.12413

Reminder:

$$S_{\text{eff}}[\phi_b] = \underbrace{S^{\text{LO}}[\phi_b^{\text{LO}}]}_{\mathcal{O}(g^3)} + \underbrace{S^{\text{NLO}}[\phi_b^{\text{LO}}]}_{\mathcal{O}(g^4)} + \ldots$$

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$$S^{\text{NLO}}[\phi_b^{\text{LO}}] = 4\pi \int \mathrm{d}r\ r^2 \left[\frac{1}{2}\delta Z[\phi_b](\partial\phi_b)^2 + V^{\text{NLO}}[\phi_b^{\text{LO}}]\right]$$

$$V^{\text{LO}}[\phi_b^{\text{LO}}] = \frac{1}{2}m_3^2\phi_3^2 + \frac{1}{4}\lambda_3\phi_3^4 - \frac{1}{12\pi}\left(6(m_{X,3}^2)^{\frac{3}{2}} + 3(m_{X_0,3}^2)^{\frac{3}{2}}\right)$$

$\mathcal{O}(\partial^2)$-terms in the expansion of gauge determinants

$\mathcal{O}(\partial^0)$-terms in expansion of gauge determinants

2-loop gauge contributions to the effective potential

See also: MK, B. Świeżewska, T.V.I Tenkanen, J. van de Vis, 2312.12413

S_{eff} at LO and NLO in derivative expansion

$$S^{\text{LO}}[\phi_b^{\text{LO}}] = 4\pi \int \mathrm{d}r\ r^2 \left[\frac{1}{2} \left(\partial_i \phi_b^{\text{LO}} \right)^2 + V^{\text{LO}}[\phi_b^{\text{LO}}] \right]$$

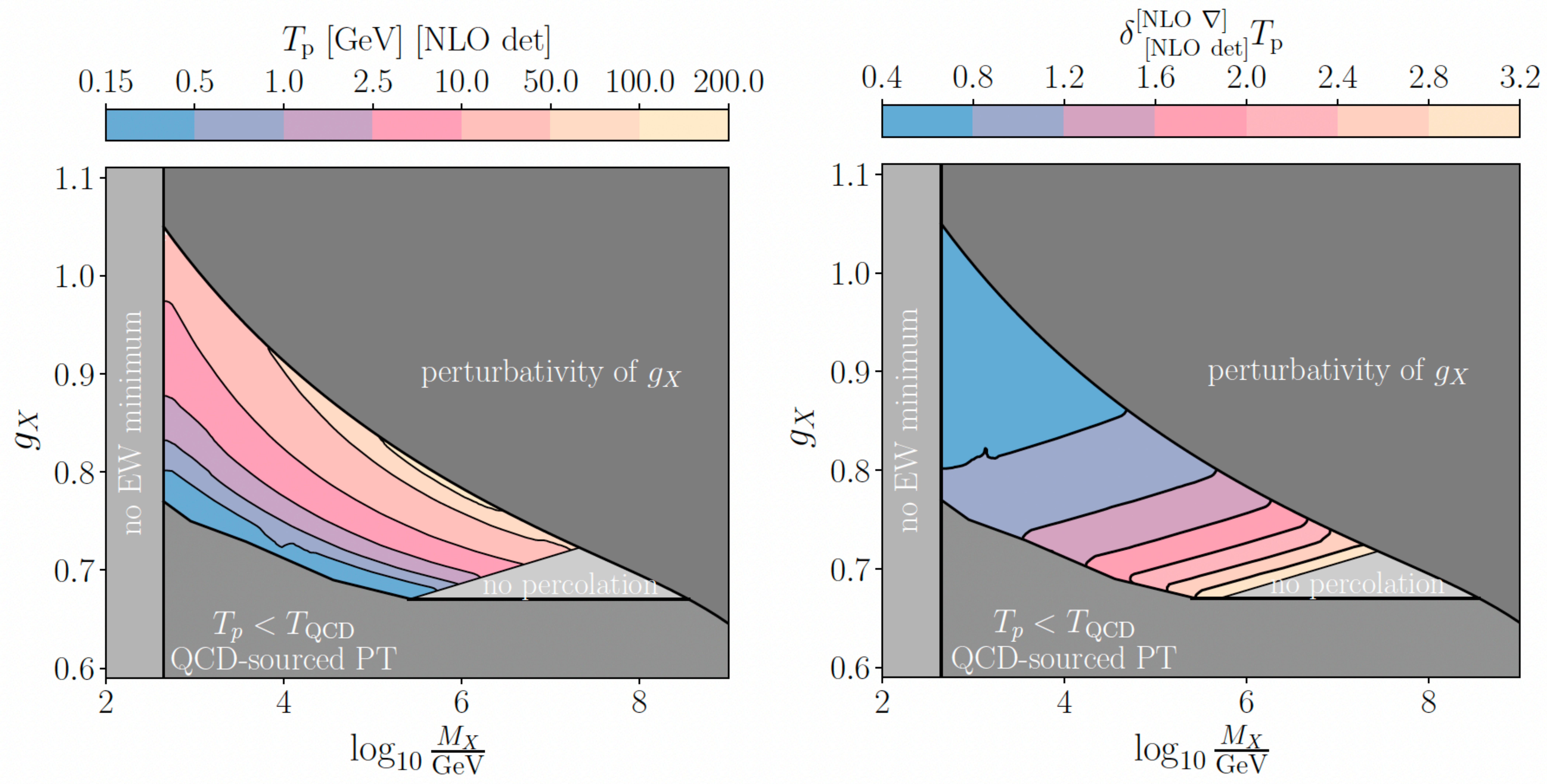
$$V^{\text{LO}}[\phi_b^{\text{LO}}] = \frac{1}{2} m_3^2 \phi_3^2 + \frac{1}{4} \lambda_3 \phi_3^4 - \frac{1}{12\pi} \left(6(m_{X,3}^2)^{\frac{3}{2}} + 3(m_{X_0,3}^2)^{\frac{3}{2}} \right)$$

$$S^{\text{NLO}}[\phi_b^{\text{LO}}] = 4\pi \int \mathrm{d}r\ r^2 \left[\frac{1}{2} \delta Z[\phi_b] (\partial \phi_b)^2 + V^{\text{NLO}}[\phi_b^{\text{LO}}] \right]$$

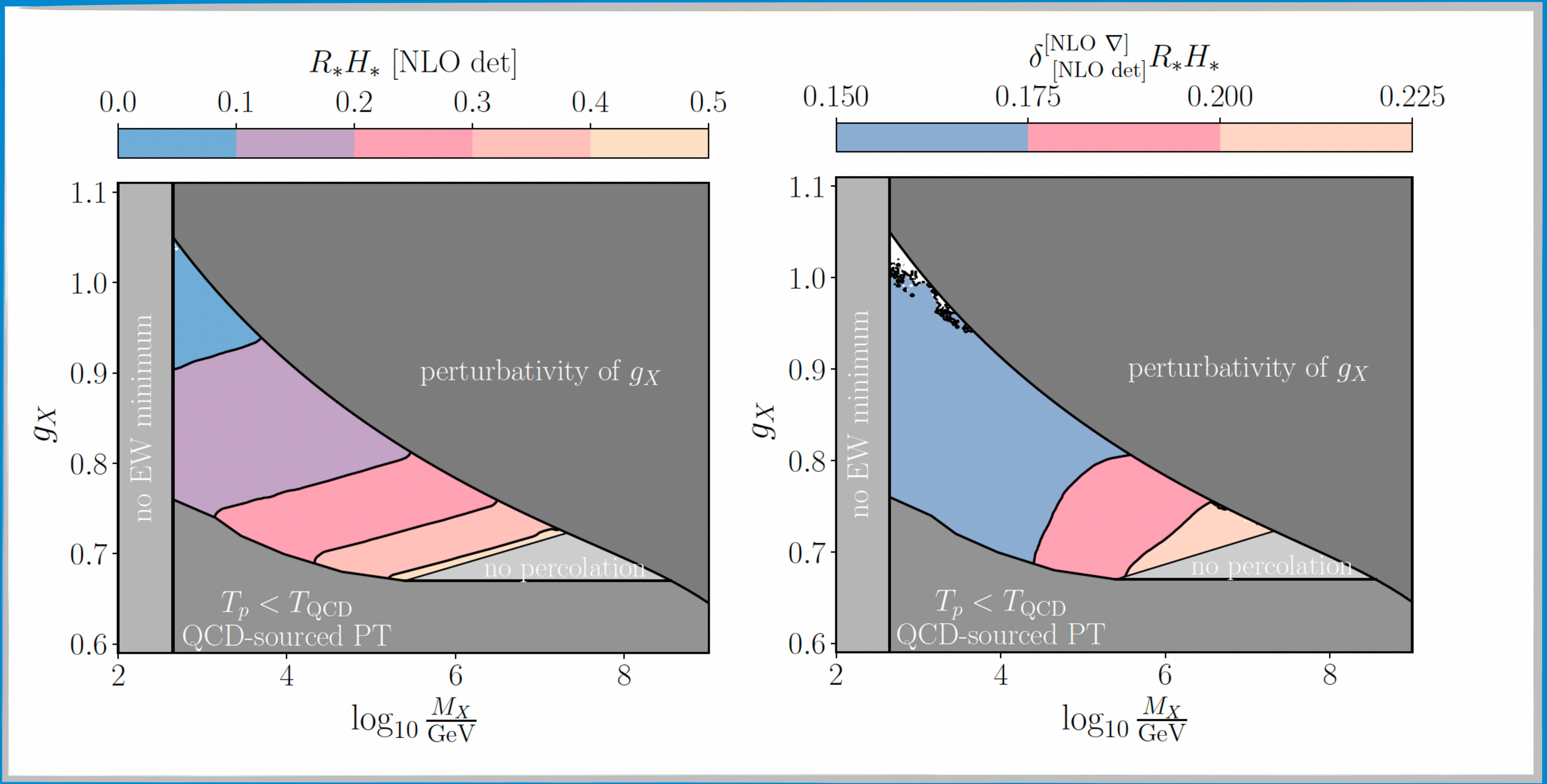
$$\begin{aligned} V^{\text{NLO}} = {} & \frac{1}{(4\pi)^2} \left\{ \frac{3}{64} g_{X,3}^2 \left(56 m_{X,3}^2 (1 - 3\ln(3)) + g_{X,3}^2 v_3^2 (2 - \ln(256)) \right. \right. \\ & \left. \left. + 2 \left(80 m_{X,3}^2 - 3 g_{X,3}^2 v_3^2 \right) \ln \left(\frac{\mu_3}{2 m_{X,3}} \right) \right) \right\} \\ & + \frac{1}{(4\pi)^2} \left\{ \frac{3}{4} g_{X,3}^2 \left(6 m_{X_0,3}^2 + 4 m_{X,3} m_{X_0,3} - m_{X,3}^2 \right) + \frac{15}{4} \kappa_3 m_{X_0,3}^2 \right. \\ & \left. - \frac{3}{8} h_3^2 v_3^2 \left(1 + 2 \ln \left(\frac{\mu_3}{2 m_{X_0,3}} \right) \right) - \frac{3}{2} g_{X,3}^2 \left(m_{X,3}^2 - 4 m_{X_0,3}^2 \right) \ln \left(\frac{\mu_3}{2 m_{X_0,3} + m_{X,3}} \right) \right\} \end{aligned}$$

See also: MK, B. Świeżewska, T.V.I Tenkanen, J. van de Vis, 2312.12413

NLO nucleation rate results: percolation temperature



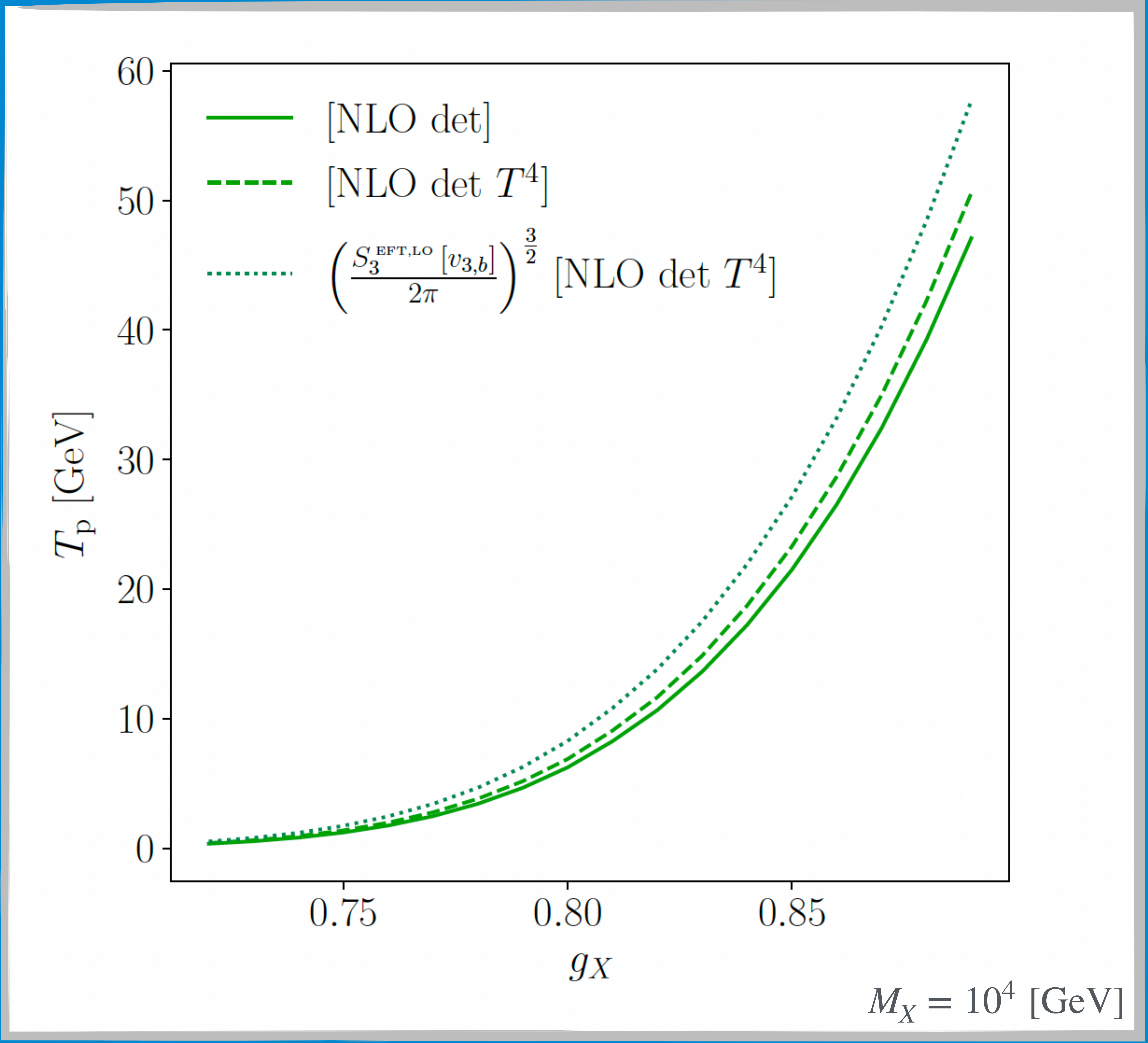
NLO nucleation rate results: bubble radius



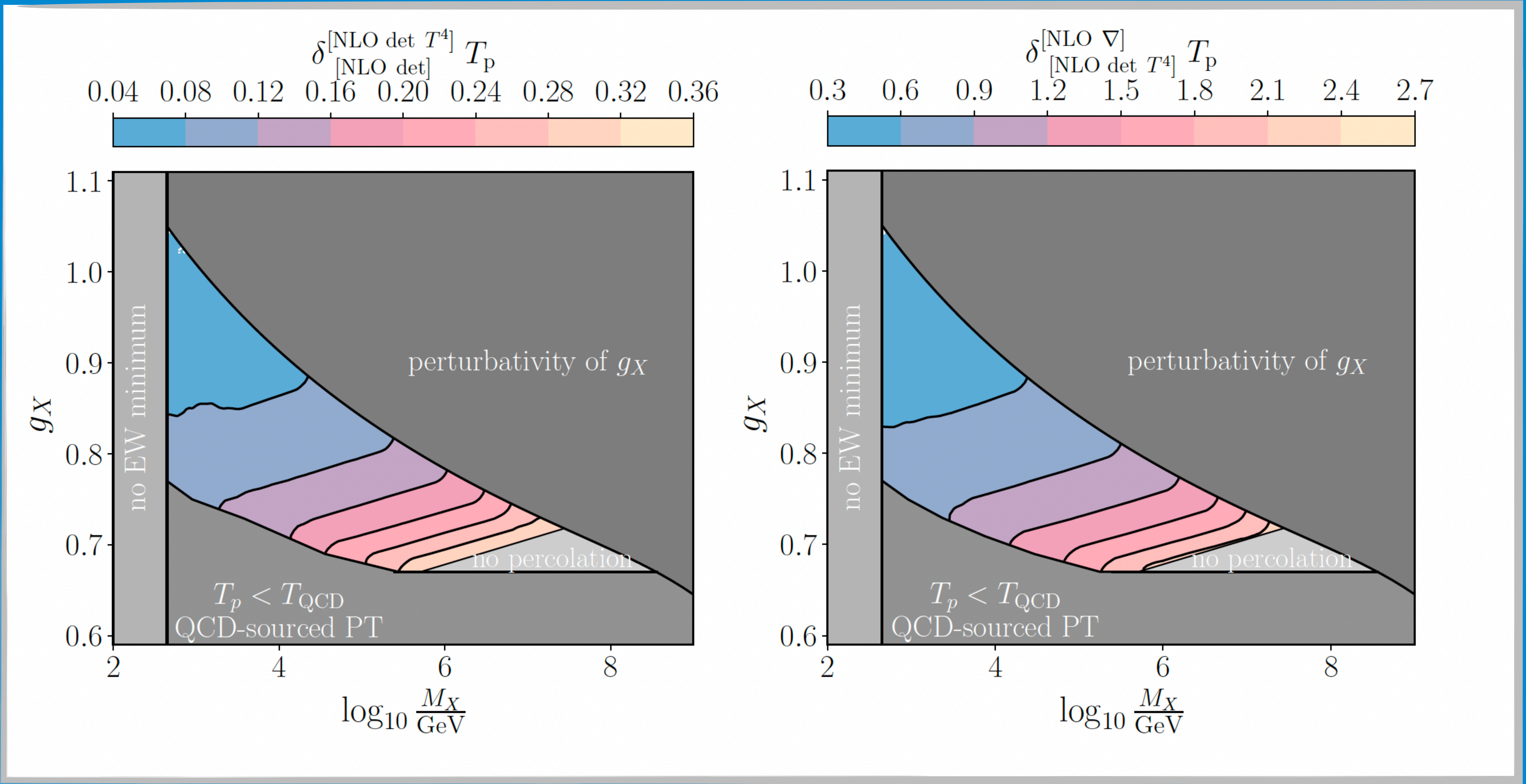
How to approximate the scalar prefactor?

Reminder:

$$\Gamma_T = A e^{-S}$$



Direct influence of scale shifters: percolation temperature



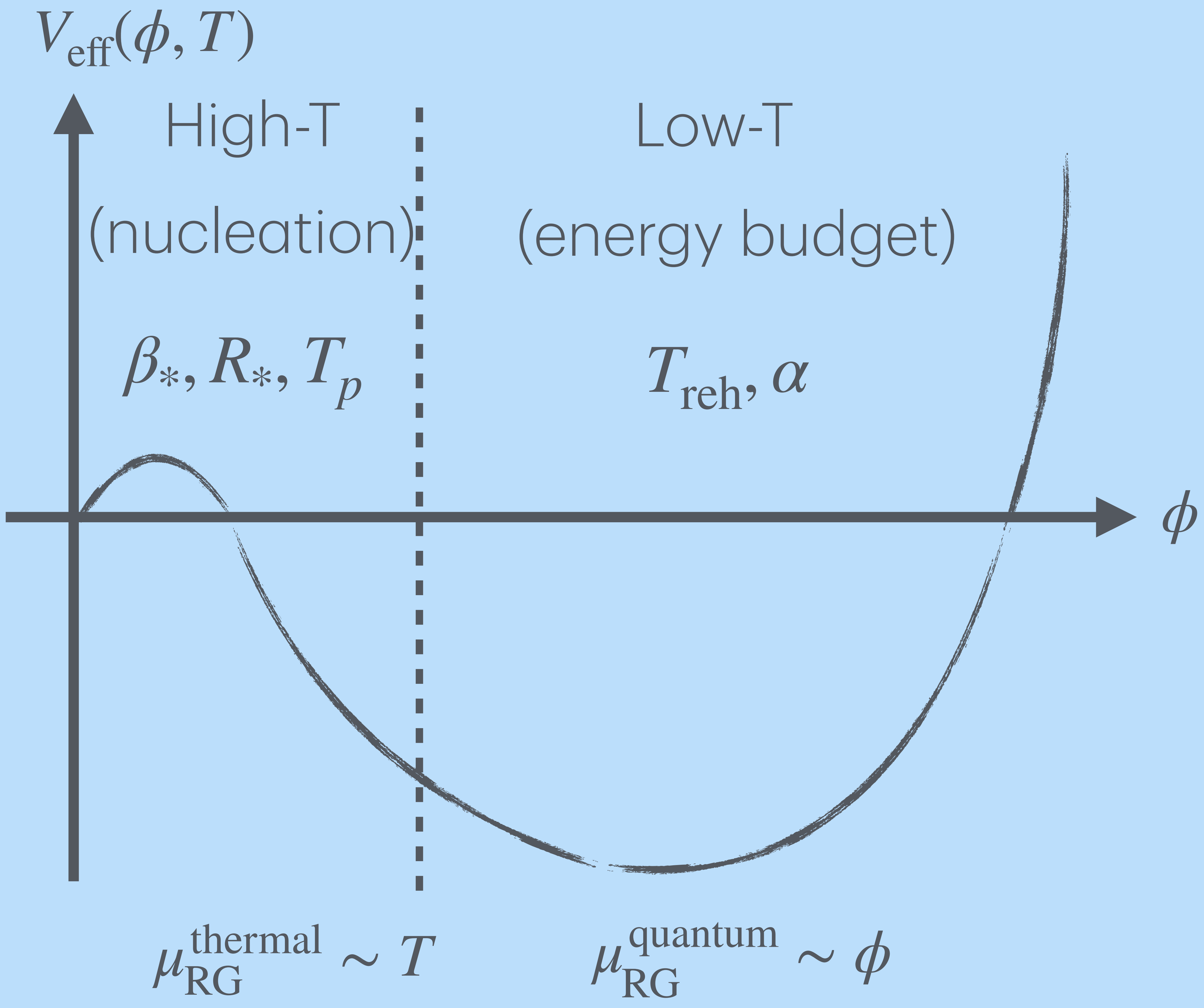
What happens at high-temperature?

We are interested in dark sector only i.e. scalar + SU(2) gauge boson

$$V_0 + V_{CW} + V_T^{HT} = \frac{9M_X(\varphi,t)^4}{64\pi^2}\left(\log\frac{M_X(\varphi,t)^2}{\mu(\varphi)^2} - \frac{5}{6}\right) - \frac{9M_X(\varphi)^4}{64\pi^2}\left(\log\frac{M_X^2(\varphi)}{16\pi^2T^2} - \frac{3}{2} + 2\gamma_E\right) + \dots$$

$$\log\frac{\mu^2}{16\pi T^2}$$

HT vs LT regimes



See also: MK, B. Świeżewska, T.V.I Tenkanen, J. van de Vis, 2312.12413

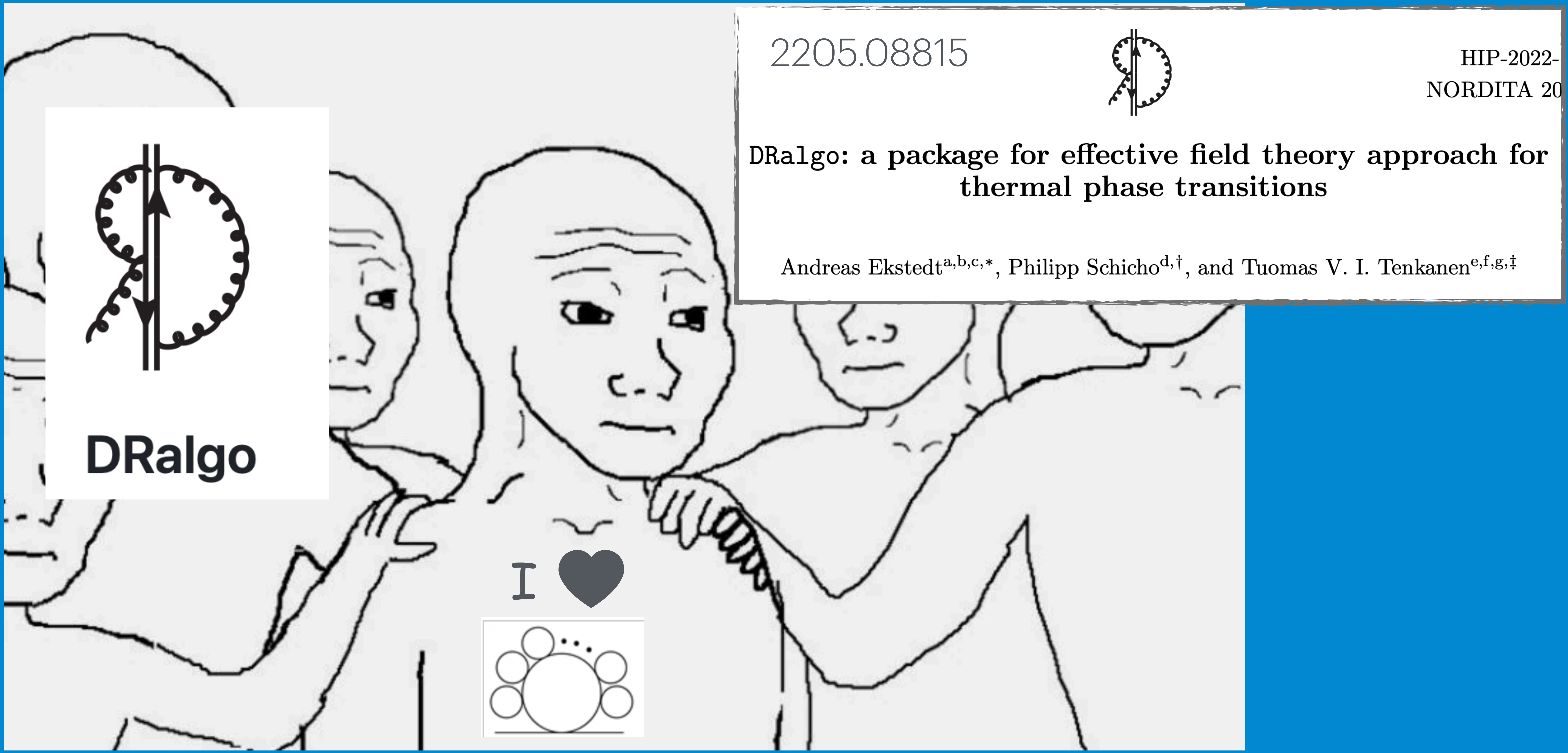
Daisy resummation

- Captures the "general" behaviour
- Troubled by RG-scale dependence
- Contains imaginary part in the effective potential
- Gauge-dependent

VS

Dimensional Reduction

- Captures *everything* at a given $\mathcal{O}(g)$
- Reduced RG-scale dependence
- No imaginary parts in the effective potential
- Gauge-invariant predictions
- One can increase precision consistently, order by order


DRalgo
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DRalgo: a package for effective field theory approach for thermal phase transitions
Andreas Ekstedt[a,b,c,*], Philipp Schicho[d,†], and Tuomas V. I. Tenkanen[e,f,g,‡]

Computing effective action - 3D EFT toy model

Consider a simple theory involving two fields Φ, χ

$$\mathscr{L}_3 = \frac{1}{2}(\partial_i \Phi)^2 + \frac{1}{2}(\partial_i \chi)^2 + \frac{1}{2} m^2 \Phi^2 + \frac{\lambda}{4}\Phi^4 + \kappa \chi^2 \Phi^2 + \frac{1}{2} M^2 \chi^2$$

In order to find nucleation rate we want to
compute effective action using the background field method

$$\Gamma \sim e^{-S_{\text{eff}}[\phi_b]} = \int \mathscr{D}\chi e^{-S_3[\phi_b, \chi]}$$

Integrate out fluctuations
on the background

Computing effective action - 3D toy model

$$\mathcal{L}_3 = \frac{1}{2}(\partial_i\Phi)^2 + \frac{1}{2}(\partial_i\chi)^2 + \frac{1}{2}m^2\Phi^2 + \frac{\lambda}{4}\Phi^4 + \kappa\chi^2\Phi^2 + \frac{1}{2}M^2\chi^2$$

Expand field on its background

$$\Phi \to \phi_b(r) + H(x)$$

At 1-loop we need quadratic terms only

$$\mathcal{L} \supset \chi \underbrace{\left(-\nabla^2 + M^2 + \kappa\phi_b^2\right)}_{\mathcal{M}_\chi} \chi$$

Performing the $\int \mathcal{D}\chi$

$$e^{iS_{\text{eff}}[\phi_b]} \sim \left[\det\left(-\nabla^2 + M^2 + \kappa\phi_b^2\right)\right]^{-\frac{1}{2}} e^{-S_3[\phi_b]}$$

Higher-order momentum terms

$$S_3^{\mathrm{EFT}}[v_{3,b}] \supset \int_{\mathbf{x}} \Big[V^{\mathrm{EFT}}(v_3) + \frac{1}{2} Z_{2,3} (\partial_i v_3)^2 + \frac{1}{2} Z_{4,3} (\partial^2 v_3)^2 + \frac{1}{2} Y_{3,3} (\partial_i v_3)^2 \partial^2 v_3 + \frac{1}{8} Y_{4,3} (\partial_i v_3)^2 (\partial_j v_3)^2 + \mathcal{O}(\partial^6) \Big] ,$$

See also: MK, P. Schicho B. Świeżewska, T.V.I Tenkanen, J. van de Vis, 2503.13597

Higher-order momentum terms

At 2-loop

$$S_3^{\mathrm{EFT}}[v_{3,b}] \supset \int_{\mathbf{x}} \Big[V^{\mathrm{EFT}}(v_3) + \frac{1}{2} Z_{2,3} (\partial_i v_3)^2 + \frac{1}{2} Z_{4,3} (\partial^2 v_3)^2 + \frac{1}{2} Y_{3,3} (\partial_i v_3)^2 \partial^2 v_3 + \frac{1}{8} Y_{4,3} (\partial_i v_3)^2 (\partial_j v_3)^2 + \mathcal{O}(\partial^6) \Big] ,$$

All of the red terms are divergent at large radius

See also: MK, P. Schicho B. Świeżewska, T.V.I Tenkanen, J. van de Vis, 2503.13597

Taming scale-shifters

Step 1. Obtain LO bounce solution $\phi_b^{\rm LO}$ using S^{LO}

Step 2. Evaluate determinants numerically $\text{Det}_{VG}[\phi_b^{\rm LO}]$

Step 2. Get the NLO bosons contribution $S^{\rm NLO} \sim V_3^{\rm NLO}$ on ϕ_b

Step 3. Obtain the action

$$S_{\rm eff} = (S^{\rm LO} - S^{\rm LO}_{\rm bosons}) - \log \text{Det}_{VG} - S^{\rm pot}_{\rm NLO}$$

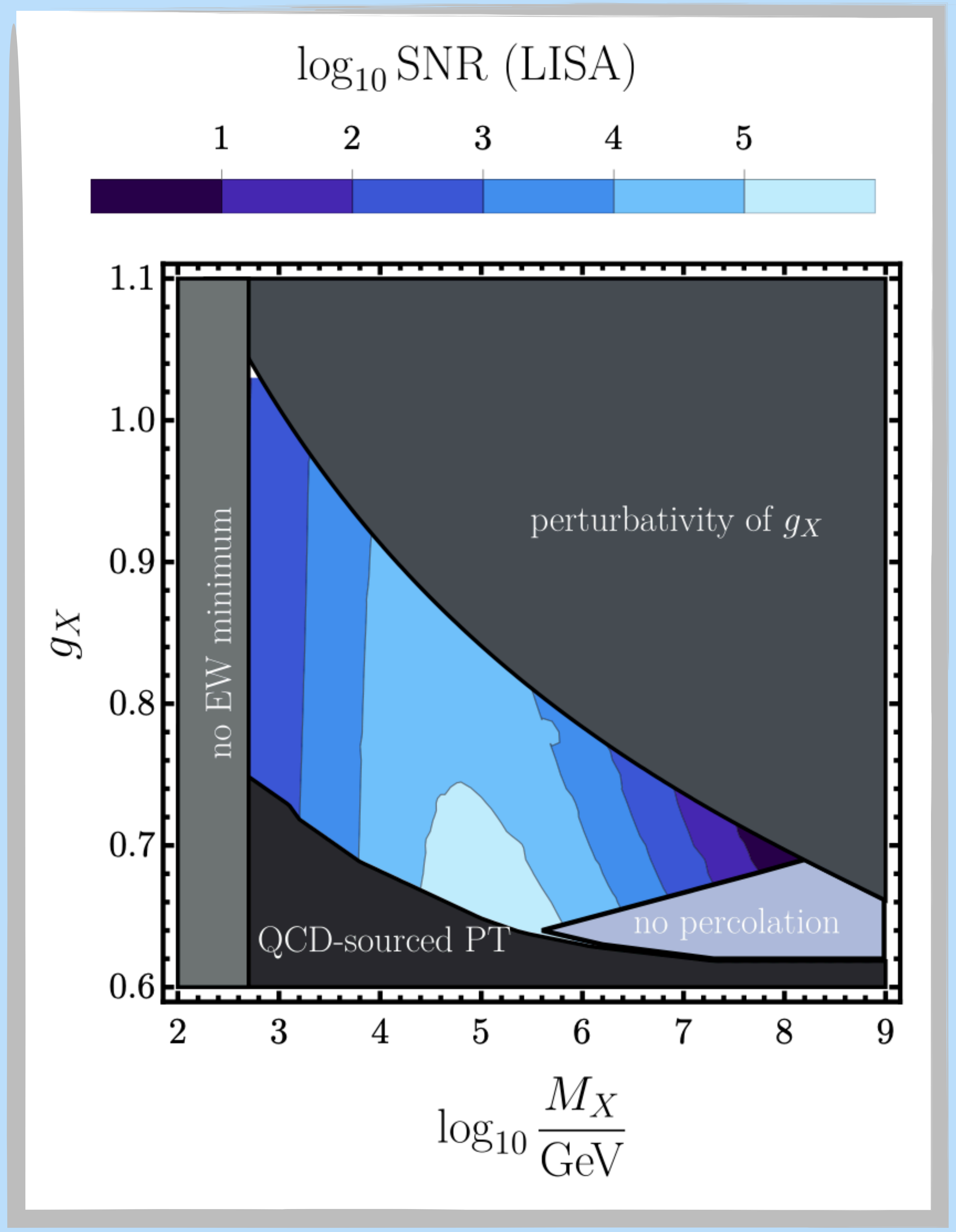
Step 4. Calculate nucleation rate

$$\Gamma_T = A_{\rm dyn}\, \text{Det}_H\, e^{-S_{\rm eff}[\phi_b]}$$

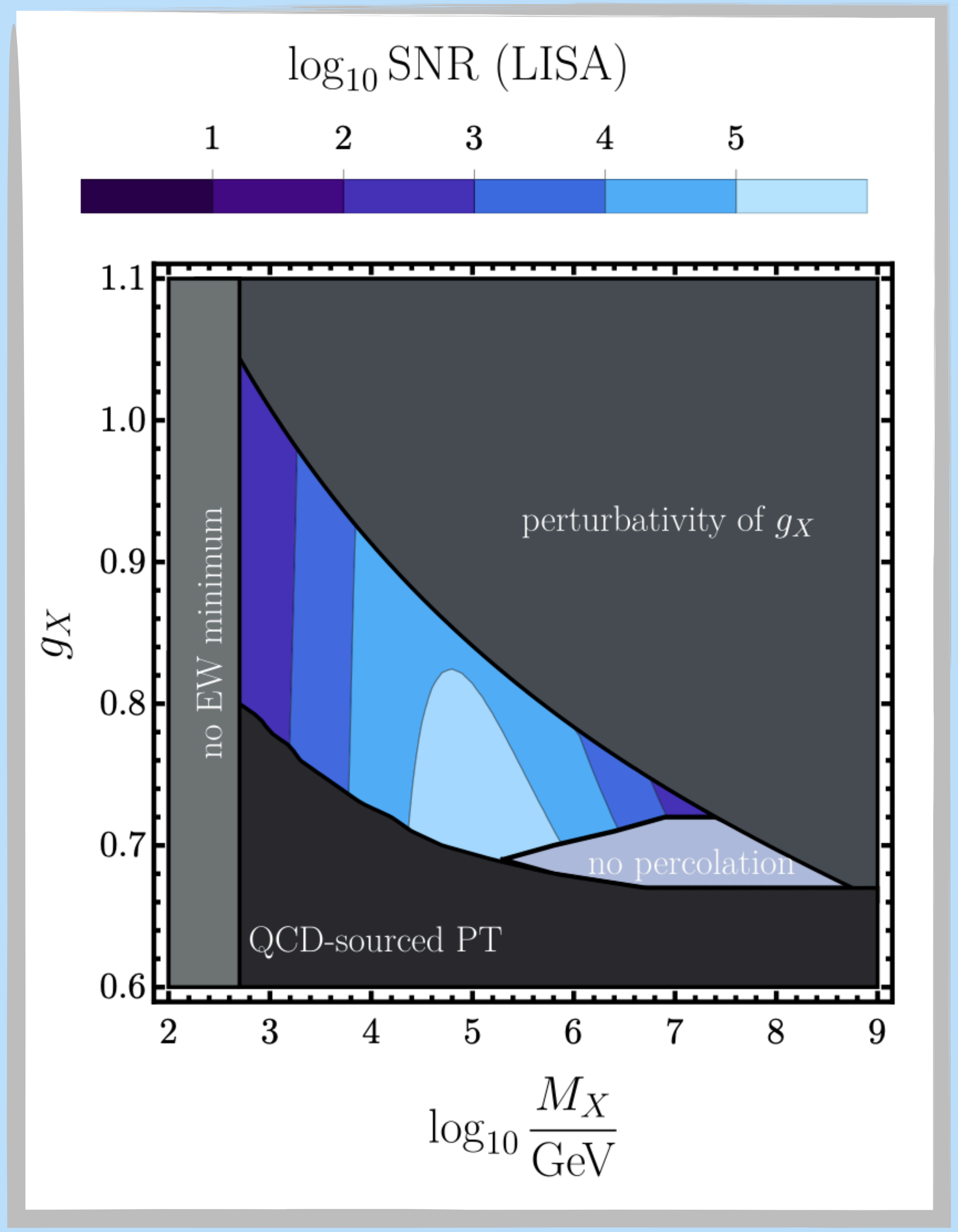
See also: O. Gould, J. Hirvonen, 2108.04377, MK, P. Schicho, B. Świeżewska, T. Tenkanen, J. Van de Vis 2502.xxxxx

LISA SNR ([NLO grad] vs [daisy])

[NLO grad]



[daisy]



See also: MK, B. Świeżewska, T.V.I Tenkanen, J. van de Vis, 2312.12413