Western Norway
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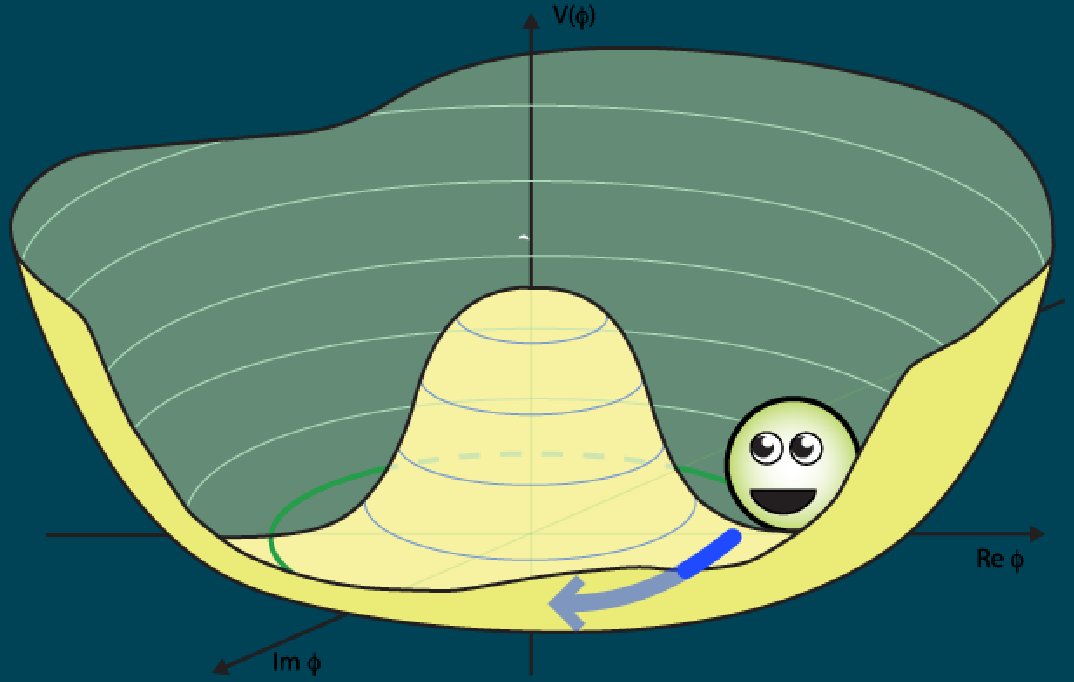
Imaginary scaling

Talk given at Scalars 2025, Warsaw, September 2025



Odd Magne Øgreid

Based on work with Pedro Ferreira, Bohdan Grzadkowski and Per Osland.

Eur. Phys. J. C **84**, 234 (2024) and arXiv:2506.21145

The 2HDM potential

$$
\begin{aligned}
V_{\rm tree} \quad = \quad & m_{11}^2 \Phi_1^\dagger \Phi_1 + m_{22}^2 \Phi_2^\dagger \Phi_2 - [m_{12}^2 \Phi_1^\dagger \Phi_2 + {\rm h.c.}] \\
& + \frac{1}{2}\lambda_1 (\Phi_1^\dagger \Phi_1)^2 + \frac{1}{2}\lambda_2 (\Phi_2^\dagger \Phi_2)^2 + \lambda_3 (\Phi_1^\dagger \Phi_1)(\Phi_2^\dagger \Phi_2) + \lambda_4 (\Phi_1^\dagger \Phi_2)(\Phi_2^\dagger \Phi_1) \\
& + \left\{ \frac{1}{2}\lambda_5 (\Phi_1^\dagger \Phi_2)^2 + \left[\lambda_6 (\Phi_1^\dagger \Phi_1) + \lambda_7 (\Phi_2^\dagger \Phi_2)\right] \Phi_1^\dagger \Phi_2 + {\rm h.c.} \right\}
\end{aligned}
$$

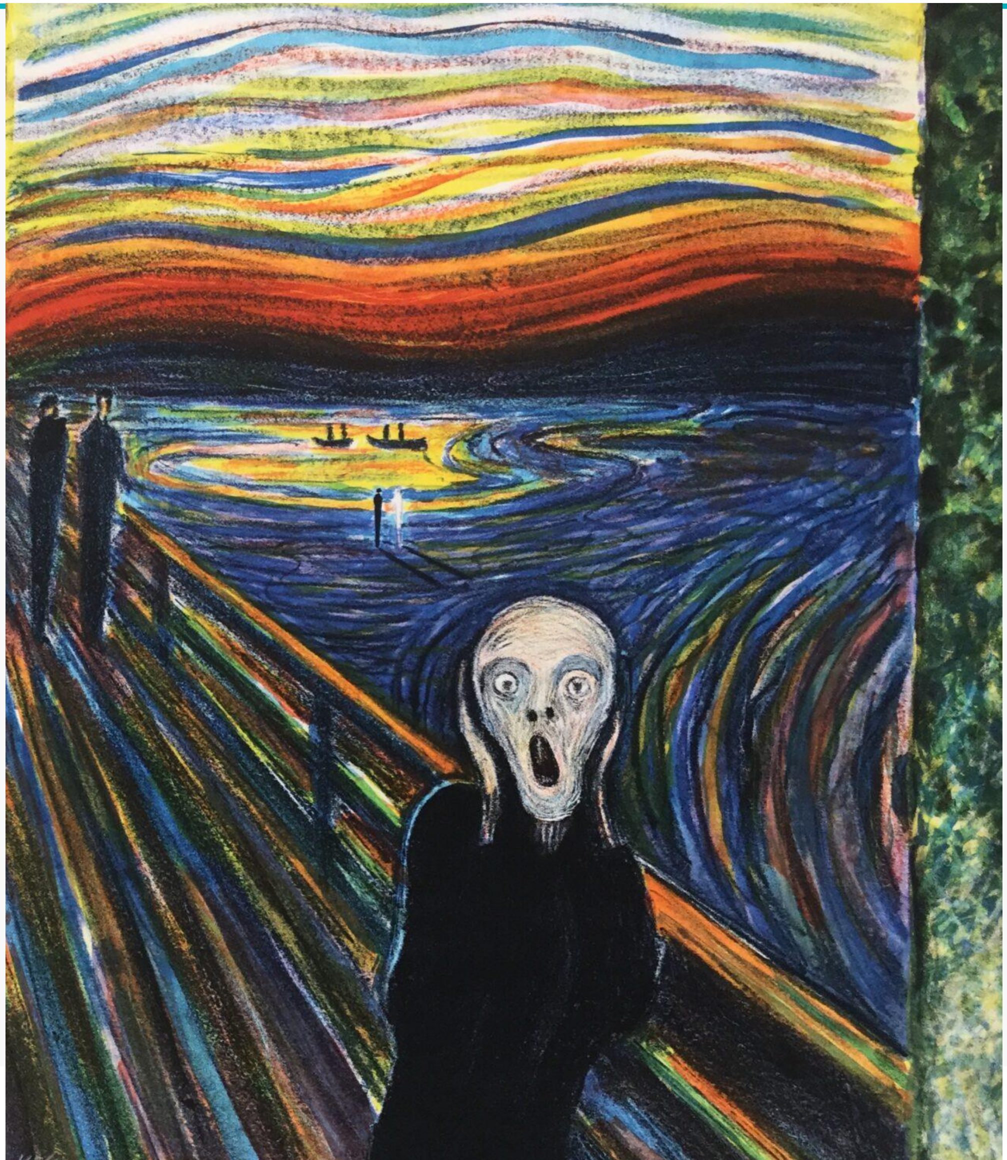
- 14 parameters (reducible to 11)
- 4 **complex** parameters
- Six transformations on the doublets are known that leave both the potential and kinetic terms unchanged. In addition, there are custodial symmetries.

Implications for the potential parameters in the symmetry basis

	V_2			V_4						
Symmetry	m_{11}^2	m_{22}^2	m_{12}^2	λ_1	λ_2	λ_3	λ_4	λ_5	λ_6	λ_7
CP1			real					real	real	real
Z_2			0						0	0
U(1)			0					0	0	0
CP2		m_{11}^2	0		λ_1					$-\lambda_6$
CP3		m_{11}^2	0		λ_1			$\lambda_1 - \lambda_3 - \lambda_4$ (real)	0	0
SO(3)		m_{11}^2	0		λ_1		$\lambda_1 - \lambda_3$	0	0	0

- Symmetries may or may not be spontaneously broken by the vacuum.
- These parameter relations are RGE-stable to all orders, indicating that these are indeed symmetries of the model.

An unexpected discovery



- During work on the physical properties of softly broken symmetries in the 2HDM we made a startling discovery:

- The set of constraints

$$\begin{aligned} \lambda_6 = \lambda_7 &= 0, \\ \lambda_5 &= \lambda_1 - \lambda_3 - \lambda_4 \,(\text{real}), \\ \lambda_2 &= \lambda_1, \\ m_{11}^2 + m_{22}^2 &= 0, \end{aligned}$$

  seems to be RGE-stable to one-loop order

- Are we just in a weird basis? No!

Excitement!

- OMO discovers using one-loop beta functions that these combined constraints are RGE-stable. Challenges PF to explain what is going on
- PF confirms, but suggests that this might be a one-loop «accident», and finds a simpler set of constraints that are one-loop RGE-stable, namely

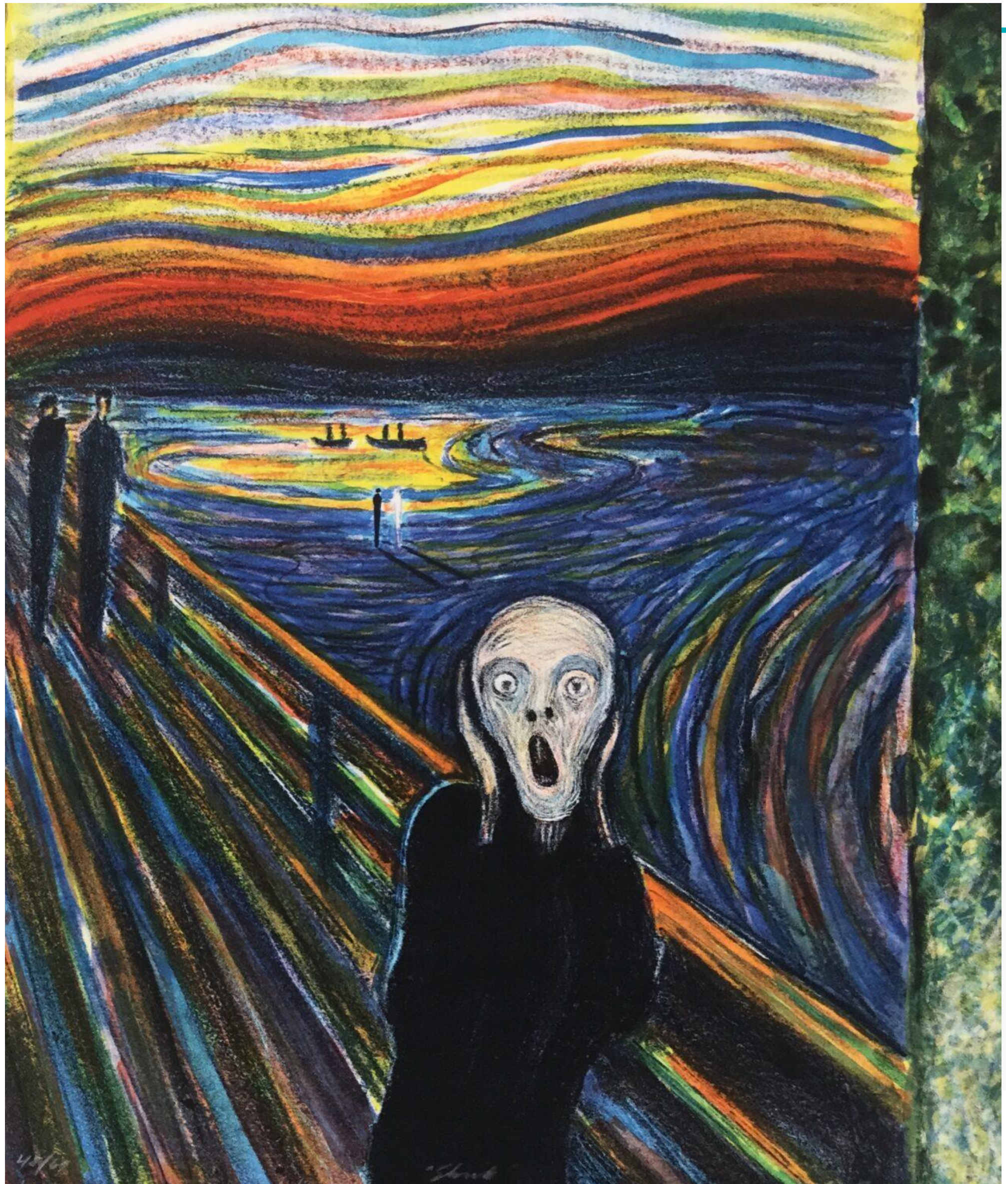
$$\begin{aligned} \lambda_6 = \lambda_7 &= 0, \\ \lambda_2 &= \lambda_1, \\ m_{11}^2 + m_{22}^2 &= 0, \end{aligned}$$

- OMO checks against two-loop beta functions and finds that these combined constraints are RGE-stable up to two loops.
- An excited PF confirms and also verifies RGE-stability up to three-loop order (Bednyakov) for the set of constraints

$$\begin{aligned} \lambda_6 + \lambda_7 &= 0, \\ \lambda_2 &= \lambda_1, \\ m_{11}^2 + m_{22}^2 &= 0, \end{aligned}$$

- PF finds that using Bednyakov's results one can argue that it holds to all orders.
- PF discovers that the bilinear form of the 2HDM suggests a simple transformation yielding the above constraints.
- OMO discovers a weird transformation of the fields that leaves the potential, scalar and gauge kinetic terms invariant
- Realistic Yukawas has been found for some cases

RGE stable to all orders



$$\begin{aligned} \lambda_6 + \lambda_7 &= 0, \\ \lambda_2 &= \lambda_1, \\ m_{11}^2 + m_{22}^2 &= 0, \end{aligned}$$

- We discovered another set of parameter relations, not known from before, that were shown to be RGE-stable to all orders.
- Did we discover a new symmetry of the 2HDM, or is there another explanation?

Bilinears and the r_0-symmetry

- Potential in bilinear notation

$$V = M_\mu r^\mu + \Lambda_{\mu\nu} r^\mu r^\nu$$

where

$$\begin{aligned} r^\mu &= (r_0, r_1, r_2, r_3) = (r_0, \vec{r}), \\ M^\mu &= (m_{11}^2 + m_{22}^2, 2\mathrm{Re}(m_{12}^2), -2\mathrm{Im}(m_{12}^2), m_{22}^2 - m_{11}^2) = (M_0, \vec{M}), \end{aligned}$$

$$\Lambda^{\mu\nu} = \begin{pmatrix} \Lambda_{00} & \vec{\Lambda} \\ \vec{\Lambda}^T & \Lambda \end{pmatrix} = \begin{pmatrix} \frac{1}{2}(\lambda_1+\lambda_2)+\lambda_3 & -\mathrm{Re}\,(\lambda_6+\lambda_7) & \mathrm{Im}\,(\lambda_6+\lambda_7) & \frac{1}{2}(\lambda_2-\lambda_1) \\ -\mathrm{Re}\,(\lambda_6+\lambda_7) & \lambda_4+\mathrm{Re}\,(\lambda_5) & -\mathrm{Im}\,(\lambda_5) & \mathrm{Re}\,(\lambda_6-\lambda_7) \\ \mathrm{Im}\,(\lambda_6+\lambda_7) & -\mathrm{Im}\,(\lambda_5) & \lambda_4-\mathrm{Re}\,(\lambda_5) & -\mathrm{Im}\,(\lambda_6-\lambda_7) \\ \frac{1}{2}(\lambda_2-\lambda_1) & \mathrm{Re}\,(\lambda_6-\lambda_7) & -\mathrm{Im}\,(\lambda_6-\lambda_7) & \frac{1}{2}(\lambda_1+\lambda_2)-\lambda_3 \end{pmatrix}.$$

$$\begin{aligned} r_0 &= \frac{1}{2}\left(\Phi_1^\dagger\Phi_1 + \Phi_2^\dagger\Phi_2\right), \\ r_1 &= \frac{1}{2}\left(\Phi_1^\dagger\Phi_2 + \Phi_2^\dagger\Phi_1\right) = \mathrm{Re}\left(\Phi_1^\dagger\Phi_2\right), \\ r_2 &= -\frac{i}{2}\left(\Phi_1^\dagger\Phi_2 - \Phi_2^\dagger\Phi_1\right) = \mathrm{Im}\left(\Phi_1^\dagger\Phi_2\right), \\ r_3 &= \frac{1}{2}\left(\Phi_1^\dagger\Phi_1 - \Phi_2^\dagger\Phi_2\right). \end{aligned}$$

- The six “classic” symmetries result from demanding invariance under bilinear transformations.
- Invariance under $r_0 \to -r_0$ implies

$$M_0 = 0 \quad \text{and} \quad \vec{\Lambda} = 0$$

equivalent to

$$\begin{aligned} \lambda_6 + \lambda_7 &= 0, \\ \lambda_2 &= \lambda_1, \\ m_{11}^2 + m_{22}^2 &= 0, \end{aligned}$$

- Hence the name r_0- symmetry
- Impossible to change sign of r_0 using HF- or CP-transformations

The r_0-symmetric potential

› Potential in bilinear notation

$$V = M_\mu r^\mu + \Lambda_{\mu\nu} r^\mu r^\nu$$

where

$$\begin{aligned} r^\mu &= (r_0\,, r_1\,, r_2\,, r_3) = (r_0\,, \vec{r})\,, \\ M^\mu &= (0\,, 2\mathrm{Re}(m_{12}^2)\,, -2\mathrm{Im}(m_{12}^2)\,, -2m_{11}^2) = (M_0\,, \vec{M})\,, \end{aligned}$$

$$\Lambda^{\mu\nu} = \begin{pmatrix} \Lambda_{00} & \vec{\Lambda} \\ \vec{\Lambda}^T & \Lambda \end{pmatrix} = \begin{pmatrix} \lambda_1+\lambda_3 & 0 & 0 & 0 \\ 0 & \lambda_4+\mathrm{Re}\,(\lambda_5) & -\mathrm{Im}\,(\lambda_5) & 2\mathrm{Re}\,(\lambda_6) \\ 0 & -\mathrm{Im}\,(\lambda_5) & \lambda_4-\mathrm{Re}\,(\lambda_5) & -2\mathrm{Im}\,(\lambda_6) \\ 0 & 2\mathrm{Re}\,(\lambda_6) & -2\mathrm{Im}\,(\lambda_6) & \lambda_1-\lambda_3 \end{pmatrix}.$$

$$\begin{aligned} r_0 &= \tfrac{1}{2}\left(\Phi_1^\dagger\Phi_1 + \Phi_2^\dagger\Phi_2\right), \\ r_1 &= \tfrac{1}{2}\left(\Phi_1^\dagger\Phi_2 + \Phi_2^\dagger\Phi_1\right) = \mathrm{Re}\left(\Phi_1^\dagger\Phi_2\right), \\ r_2 &= -\tfrac{i}{2}\left(\Phi_1^\dagger\Phi_2 - \Phi_2^\dagger\Phi_1\right) = \mathrm{Im}\left(\Phi_1^\dagger\Phi_2\right), \\ r_3 &= \tfrac{1}{2}\left(\Phi_1^\dagger\Phi_1 - \Phi_2^\dagger\Phi_2\right). \end{aligned}$$

› Potential in standard notation

$$\begin{aligned} V_{\mathrm{tree}} &= m_{11}^2\left(\Phi_1^\dagger\Phi_1 - \Phi_2^\dagger\Phi_2\right) - [m_{12}^2\Phi_1^\dagger\Phi_2 + \mathrm{h.c.}] \\ &+\frac{1}{2}\lambda_1\left((\Phi_1^\dagger\Phi_1)^2 + (\Phi_2^\dagger\Phi_2)^2\right) \\ &+\lambda_3(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2) + \lambda_4(\Phi_1^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1) \\ &+\left\{\frac{1}{2}\lambda_5(\Phi_1^\dagger\Phi_2)^2 + \lambda_6\left[(\Phi_1^\dagger\Phi_1) - (\Phi_2^\dagger\Phi_2)\right]\Phi_1^\dagger\Phi_2 + \mathrm{h.c.}\right\} \end{aligned}$$

RGE stability

- Using $M_0 = 0 \quad \text{and} \quad \vec{\Lambda} = 0$
- we were able to show, using results of A.V. Bednyakov (*"On three-loop RGE for the Higgs sector of 2HDM", JHEP 11 (2018) 154*) that

$$\begin{aligned} \beta_{m_{11}^2+m_{22}^2} &= 0, \\ \beta_{\lambda_1-\lambda_2} &= 0, \\ \beta_{\lambda_6+\lambda_7} &= 0. \end{aligned}$$

to all orders.

- Thus,

$$\begin{aligned} \lambda_6 + \lambda_7 &= 0, \\ \lambda_2 &= \lambda_1, \\ m_{11}^2 + m_{22}^2 &= 0, \end{aligned}$$

is a fixed point under the running of the RGE to all orders.

- Same behavior as the fixed points of the HF/CP symmetries.
- Can also include fermions (at least up to two-loop order).

More on the r_0 - symmetry

- Parameterise the two doublets as

$$\Phi_1 = \begin{pmatrix} \phi_1 + i\phi_2 \\ \phi_3 + i\phi_4 \end{pmatrix}, \quad \Phi_2 = \begin{pmatrix} \phi_5 + i\phi_6 \\ \phi_7 + i\phi_8 \end{pmatrix},$$

then

$$\begin{aligned} r_0 &= \frac{1}{2}(\phi_1^2 + \phi_2^2 + \phi_3^2 + \phi_4^2 + \phi_5^2 + \phi_6^2 + \phi_7^2 + \phi_8^2), \\ r_1 &= \phi_1\phi_5 + \phi_2\phi_6 + \phi_3\phi_7 + \phi_4\phi_8, \\ r_2 &= -\phi_2\phi_5 + \phi_1\phi_6 - \phi_4\phi_7 + \phi_3\phi_8, \\ r_3 &= \frac{1}{2}(\phi_1^2 + \phi_2^2 + \phi_3^2 + \phi_4^2 - \phi_5^2 - \phi_6^2 - \phi_7^2 - \phi_8^2). \end{aligned}$$

- Want to change sign of r_0 while r_1, r_2 , r_3 are unchanged

$$\begin{pmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \\ \phi_4 \\ \phi_5 \\ \phi_6 \\ \phi_7 \\ \phi_8 \end{pmatrix} \to \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & i & 0 & 0 \\ 0 & 0 & 0 & 0 & i & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & i \\ 0 & 0 & 0 & 0 & 0 & 0 & i & 0 \\ 0 & -i & 0 & 0 & 0 & 0 & 0 & 0 \\ -i & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -i & 0 & 0 & 0 & 0 \\ 0 & 0 & -i & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \\ \phi_4 \\ \phi_5 \\ \phi_6 \\ \phi_7 \\ \phi_8 \end{pmatrix}$$

works!!!

- What about the kinetic terms? Define

$$D^\mu = \partial^\mu + \frac{ig}{2}\sigma_i W_i^\mu + i\frac{g'}{2}B^\mu,$$

and scalar kinetic terms

$$\mathcal{L}_k = (D_\mu \Phi_1)^\dagger (D^\mu \Phi_1) + (D_\mu \Phi_2)^\dagger (D^\mu \Phi_2)$$

- Turns out to be invariant under r_0 provided also

$$x_\mu \to i x_\mu, \quad B_\mu \to i B_\mu,$$

$$W_{1\mu} \to i W_{1\mu}, \quad W_{2\mu} \to -i W_{2\mu}, \quad W_{3\mu} \to i W_{3\mu}.$$

- Combined transformation of fields and spacetime coordinates, all scaled by imaginary unit $\pm i$
- Imaginary scaling !!!

More on the r_0-symmetry

- Gauge kinetic terms

$$\mathcal{L}^B = -\frac{1}{4}B_{\mu\nu}B^{\mu\nu} - \frac{1}{4}W_{i\mu\nu}W_i^{\mu\nu}$$

where

$$\begin{aligned} B^{\mu\nu} &= \partial^\nu B^\mu - \partial^\mu B^\nu, \\ W_i^{\mu\nu} &= \partial^\nu W_i^\mu - \partial^\mu W_i^\nu + g\epsilon_{ijk}W_j^\mu W_k^\nu, \end{aligned}$$

- Also invariant under imaginary scaling.

Symmetry or anomaly?

Consider one-loop effective potential (Coleman-Weinberg)

$$\begin{aligned} V_{\rm eff}^{(S)} &= \frac{1}{2}\int \frac{d^4 p_E}{(2\pi)^4}\mathbf{Tr}\left[\ln(p_E^2 + M_S^2)\right] \\ &= -\frac{1}{2}\int \frac{d^4 p_E}{(2\pi)^4}\left[\mathbf{Tr}\sum_{n=1}^{\infty}\frac{(-1)^n}{n}\left(\frac{M_S^2}{p_E^2}\right)^n\right] \end{aligned}$$

where $\left(M_S^2\right)_{ij} \equiv \partial^2 V_{\rm tree}/(\partial\phi_i\partial\phi_j)$

is a scalar mass-squared matrix calculated for a given tree-level potential at a constant classical field.

n odd: Terms change sign under r_0

n even: Terms invariant under r_0

Momentum behavior under imaginary scaling transformation

$$V_{\rm eff}^{(S)} = \frac{1}{2}\int \frac{d^4 p_E}{(2\pi)^4} \mathbf{Tr}\left[\ln(p_E^2 + M_S^2)\right]$$

$$= -\frac{1}{2}\int \frac{d^4 p_E}{(2\pi)^4} \left[\mathbf{Tr}\sum_{n=1}^{\infty}\frac{(-1)^n}{n}\left(\frac{M_S^2}{p_E^2}\right)^n\right]$$

- Terms with *n* odd would be invariant if

$$p_E^2 \to -p_E^2$$

under r_0 transformation.

- Recall that $x_\mu \to i x_\mu$ implies

$$x^2 \to -x^2$$

- What does this imply for momentum?

- In QM momentum operator is

$$\hat{p}_\mu \equiv i\partial_\mu \xrightarrow{r_0} -i\,\hat{p}_\mu$$

- In QFT, $p_\mu \equiv \int d^3x\, \Theta_{0\mu}$

where $\Theta_{\nu\mu} = \sum_i \frac{\partial \mathcal{L}}{\partial(\partial^\nu \phi_i)} \partial_\mu \phi_i - \eta_{\mu\nu}\mathcal{L}$

- Implying $p_\mu \xrightarrow{r_0} -i\,p_\mu$
- Also, Fourier transforms between coordinate space and momentum space would be nonsensical if *x* were imaginary and *p* were real. $e^{-ipx}, \quad e^{ipx}$

The minimal model

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi_1 \partial^\mu \phi_1 + \partial_\mu \phi_2 \partial^\mu \phi_2) - V(\phi_1, \phi_2)$$

- with

$$V(\phi_1,\, \phi_2) = \frac{1}{2} m_1^2 (\phi_1^2 - \phi_2^2) + m_{12}^2 \phi_1 \phi_2 + \frac{1}{2}\lambda_1(\phi_1^4 + \phi_2^4) + \lambda_3(\phi_1\phi_2)^2 + \lambda_6(\phi_1^2 - \phi_2^2)\phi_1\phi_2$$

- Can rotate into basis where $\lambda_6 = 0$

The minimal model

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi_1 \partial^\mu \phi_1 + \partial_\mu \phi_2 \partial^\mu \phi_2) - V(\phi_1, \phi_2)$$

- with

$$V(\phi_1,\, \phi_2) = \frac{1}{2} m_1^2 (\phi_1^2 - \phi_2^2) + m_{12}^2 \phi_1 \phi_2 + \frac{1}{2} \lambda_1 (\phi_1^4 + \phi_2^4) + \lambda_3 (\phi_1 \phi_2)^2$$

- Can rotate into basis where $\lambda_6 = 0$
- Invariant under the r_0 like transformation

$$x^\mu \to i x^\mu, \quad \phi_1 \to i\phi_2, \quad \phi_2 \to -i\phi_1$$

- Field dependent squared mass matrix:

$$(M_S^2)_{ij} = \begin{pmatrix} m_1^2 + 6\lambda_1 \phi_1^2 + 2\lambda_3 \phi_2^2 & m_{12}^2 + 4\lambda_3 \phi_1 \phi_2 \\ m_{12}^2 + 4\lambda_3 \phi_1 \phi_2 & -m_1^2 + 6\lambda_1 \phi_2^2 + 2\lambda_3 \phi_1^2 \end{pmatrix}$$

The minimal model – bilinear formalism

$$V(r^\mu) = -M_\mu r^\mu + \Lambda_{\mu\nu} r^\mu r^\nu$$

$$r_0^2 - r_1^2 - r_2^2 = 0$$

- with $r_0 \equiv \frac{1}{2}(\phi_1^2 + \phi_2^2)\,, \quad r_1 \equiv \phi_1\phi_2\,, \quad r_2 \equiv \frac{1}{2}(\phi_1^2 - \phi_2^2)$

- Invariant under the r_0 like transformation $(r_0, r_1, r_2) \xrightarrow{r_0} (-r_0, r_1, r_2)$

$$M^\mu \equiv (0, m_{12}^2, m_1^2)$$

$$\Lambda^{\mu\nu} \equiv \begin{pmatrix} \Lambda_{00} & 0 & 0 \\ 0 & \Lambda_{11} & \Lambda_{12} \\ 0 & \Lambda_{21} & \Lambda_{22} \end{pmatrix} = \begin{pmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_3 & 0 \\ 0 & 0 & \lambda_1 \end{pmatrix}$$

The potential of the minimal model

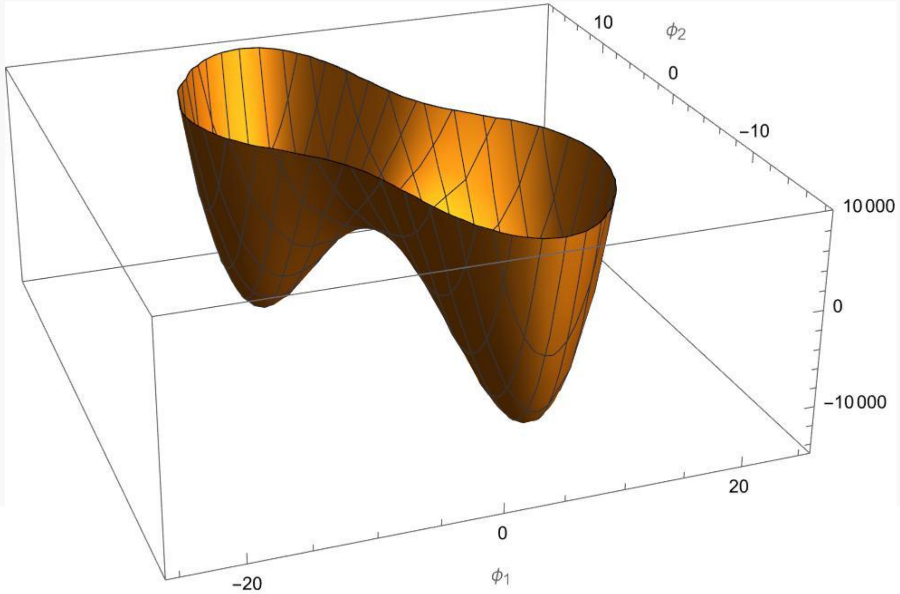
$$V(\phi_1, \phi_2) = \frac{1}{2}m_1^2(\phi_1^2 - \phi_2^2) + m_{12}^2\phi_1\phi_2 + \frac{1}{2}\lambda_1(\phi_1^4 + \phi_2^4) + \lambda_3(\phi_1\phi_2)^2$$

BFB:

$$\begin{aligned} \lambda_1 &> 0 \\ \lambda_1 + \lambda_3 &> 0 \end{aligned}$$

- Saddle point at the origin $\phi_1 = \phi_2 = 0$
- Two degenerate minima in opposite directions determined from stationary point equations

$$\begin{aligned} m_1^2 &= 2\lambda_1(\phi_2^2 - \phi_1^2) \\ m_{12}^2 &= -2(\lambda_1 + \lambda_3)\phi_1\phi_2 \end{aligned}$$



$$\begin{aligned} m_1^2 &= 10^2 \\ m_{12}^2 &= 20^2 \\ \lambda_1 &= 1 \\ \lambda_3 &= 2 \end{aligned}$$

The minimal model – one-loop effective potential

- Adopt cut-off regularization

$$
\begin{aligned}
V_{\rm eff}^{(S)} &= \frac{1}{2}\int \frac{d^4p_E}{(2\pi)^4}\mathbf{Tr}\left[\ln(p_E^2+M_S^2)\right] \\
&= -\frac{1}{2}\int \frac{d^4p_E}{(2\pi)^4}\left[\mathbf{Tr}\sum_{n=1}^{\infty}\frac{(-1)^n}{n}\left(\frac{M_S^2}{p_E^2}\right)^n\right] \\
&= -\frac{1}{64\pi^2}\sum_{n=1}^{\infty}\left[\frac{(-1)^n}{n}\int_0^{\Lambda_{UV}^2} d\rho\rho\,\mathbf{Tr}\left(\frac{M_S^2}{\rho}\right)^n\right]
\end{aligned}
$$

- Where we have introduced new variable of integration $\rho \equiv p_E^2$
- Possible to give explicit expression for minimal model (mass squared matrix is 2x2)
- Note that replacing $\Lambda_{UV}^2 \to -\Lambda_{UV}^2$ is equivalent to substituting $p_E^2 \to -p_E^2$

The minimal model – mass matrix

- Field dependent mass matrix

$$(M_S^2)_{ij} = \begin{pmatrix} m_1^2 + 6\lambda_1\phi_1^2 + 2\lambda_3\phi_2^2 & m_{12}^2 + 4\lambda_3\phi_1\phi_2 \\ m_{12}^2 + 4\lambda_3\phi_1\phi_2 & -m_1^2 + 6\lambda_1\phi_2^2 + 2\lambda_3\phi_1^2 \end{pmatrix}$$

- Eigenvalues given by $M_{1,2}^2(r^\mu) = 2(3\lambda_1 + \lambda_3)r_0 \pm \sqrt{\Delta}$
- Transformation of eigenvalues under r_0:

$$M_1^2 \xrightarrow{r_0} -M_2^2 \quad \text{and} \quad M_2^2 \xrightarrow{r_0} -M_1^2$$

- They transform into each other - along with a change of sign
- We shall also assume that $\Lambda_{UV}^2 \xrightarrow{r_0} -\Lambda_{UV}^2$

$$\Delta = (m_1^2)^2 + (m_{12}^2)^2 + 4m_1^2(3\lambda_1 - \lambda_3)r_2 + 8m_{12}^2\lambda_3 r_1 + 16\lambda_3^2 r_0^2 + 12(3\lambda_1 + \lambda_3)(\lambda_1 - \lambda_3)r_2^2$$

The minimal model – one-loop effective potential

- Perform integration to get

$$V_{\text{eff}}^{\text{1-loop}}(r^\mu) = \frac{\Lambda_{\text{UV}}^2}{32\pi^2} \sum_{i=1,2} M_i^2(r^\mu) + \frac{1}{64\pi^2} \sum_{i=1,2} M_i^4(r^\mu) \left[\log \frac{M_i^2(r^\mu)}{\Lambda_{\text{UV}}^2} - \frac{1}{2}\right]$$

+ irrelevant terms

Λ_{UV}^2	sign change
$\sum_{i=1,2} M_i^2(r^\mu)$	sign change
$\sum_{i=1,2} M_i^4(r^\mu)$	invariant
$\sum_{i=1,2} M_i^4(r^\mu) \log \frac{M_i^2(r^\mu)}{\Lambda_{\text{UV}}^2}$	invariant

We conclude that under the r_0 transformation, the one-loop effective potential of the minimal model is invariant provided

$$\Lambda_{UV}^2 \xrightarrow{r_0} -\Lambda_{UV}^2$$

We argued before that this is equivalent to

$$p_E^2 \rightarrow -p_E^2$$

The minimal model – one-loop effective potential

- Path integral formalism does involve momentum integration

$$V_{\text{eff}}(\phi_{cl}) \propto \int \prod_k \mathcal{D}(\phi_k) \exp\left\{-i \int d^4x \; \phi_i(x) \left[\Box_x \, \delta_{ij} + \left(M_S^2(\phi_{cl})\right)_{ij}\right] \phi_j(x)\right\}$$

$\prod_k \mathcal{D}(\phi_k)$ **invariant**

d^4x **invariant**

- $\Box_x \equiv \partial_\mu \partial^\mu \rightarrow -\Box_x$

$\phi_i^2 \rightarrow -\phi_i^2$

$\left(M_S^2\right)_{ij} \equiv \partial^2 V_{\text{tree}} / (\partial\phi_i \partial\phi_j)$

We conclude that under the r_0 transformation, the one-loop effective potential of the minimal model is invariant provided $x_\mu \rightarrow i x_\mu$

Invariance requires a combination of field-transformations and transformation of space-time coordinates.

r_0 invariant 2HDM – one-loop effective potential

- Path integral formalism

$$V_{\text{eff}}(\phi_{cl}) \propto \int \prod_k \mathcal{D}(\phi_k) \exp\left\{-i\int d^4x\, \phi_i(x)\left[\Box_x\, \delta_{ij} + \left(M_S^2(\phi_{cl})\right)_{ij}\right]\phi_j(x)\right\}$$

- $\prod_k \mathcal{D}(\phi_k)$ **invariant**

  d^4x **invariant**

  $\phi_i \Box_x \delta_{ij} \phi_j$ **invariant**

  $\phi_i \left(M_S^2\right)_{ij} \phi_j$ **invariant**

  $\left(M_S^2\right)_{ij} \equiv \partial^2 V_{\text{tree}}/(\partial\phi_i \partial\phi_j)$

We conclude that under the r_0 transformation, the one-loop effective potential of the minimal model is invariant provided $x_\mu \to i x_\mu$

Invariance requires a combination of field-transformations and transformation of space-time coordinates.

This is unlike HF and CP transformations.

Physical implications of the r_0 invariant 2HDM

- General 2HDM potential has 11 independent physical parameters
- Instead pick 11 masses/couplings to describe model
- - 4 squared masses
  - 3 gauge couplings
  - 4 scalar couplings

$$e_1^2 + e_2^2 + e_3^2 = v^2$$

$$\mathcal{P} \equiv \{M_{H^\pm}^2, M_1^2, M_2^2, M_3^2, e_1, e_2, e_3, q_1, q_2, q_3, q\}$$

$$\begin{aligned} e_i &\equiv \frac{2}{g^2}\text{Coefficient}(\mathcal{L}, H_i W^- W^+) \\ q_i &\equiv \text{Coefficient}(V, H_i H^- H^+) \\ q &\equiv \text{Coefficient}(V, H^- H^- H^+ H^+). \end{aligned}$$

- All observables arising from the potential expressible through these 11 parameters.
- All other trilinear and quadrilinear scalar couplings expressible through these 11 parameters.

First introduced in:

Grzadkowski, Ogreid & Osland: JHEP 11 (2014) 084 and Phys. Rev. D 94, 115002

Description of translation process:

Ogreid: PoS CORFU2017 (2018) 065

Remaining scalar couplings expressible in terms of $\mathcal{P}$*:*

Grzadkowski, Haber, Ogreid & Osland: JHEP 12 (2018) 056

Physical implications of the r_0 invariant 2HDM

- The four(!) parameter constraints

$$\lambda_6 + \lambda_7 = 0,$$
$$\lambda_2 = \lambda_1,$$
$$m_{11}^2 + m_{22}^2 = 0,$$

- Translate into four(!) physical constraints

$$\begin{cases} v^2(e_1 q_2 - e_2 q_1) + e_1 e_2 (M_2^2 - M_1^2) = 0, \\ v^2(e_1 q_3 - e_3 q_1) + e_1 e_3 (M_3^2 - M_1^2) = 0, \\ v^2(e_2 q_3 - e_3 q_2) + e_2 e_3 (M_3^2 - M_2^2) = 0, \end{cases}$$

$$q = \frac{1}{2v^4}(e_1^2 M_1^2 + e_2^2 M_2^2 + e_3^2 M_3^2),$$

$$M_{H^\pm}^2 = \frac{1}{2}(e_1 q_1 + e_2 q_2 + e_3 q_3) + \frac{1}{2v^2}(e_1^2 M_1^2 + e_2^2 M_2^2 + e_3^2 M_3^2),$$

- Fixed points of the potential to all orders under running of RGE.

Summary

- Formulation of softly broken symmetries in terms of physical parameter set led to the discovery of a new «symmetry».
- Cannot be formulated in terms of transformation on the doublets, but in terms of transformation on the bilinears ***or*** on the components of the doublets.
- Rotation to imaginary spacetime must be accompanied by rotation into imaginary momentum space.
- Leaves potential, scalar kinetic and gauge kinetic terms invariant. Yukawas?

- One-loop effective Coleman-Weinberg potential shown to be invariant under imaginary scaling. Needs $x_\mu \to i x_\mu$
- New 2HDM-models that provides new phenomenology with new physical implications
- Groups are looking into extending imaginary scaling to fermionic sector.
- Applications to hierarchy problem?
- 3HDMs next?