2HDM Higgs Decays $h \to f\bar{f}$ in the Decoupling Scheme

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FlexibleDecay

- extension of spectrum-generator generator FlexibleSUSY
- enables automatic calculation of scalar decay widths in large variety of BSM models
- current public version: [Athron et al.'22]
  - in general: decays calculated at LO
  - for Higgs bosons: some higher-order SM corrections known from literature added
  - well-suited renormalization scheme used: “decoupling scheme”
- **goal**: implement full NLO calculation in decoupling scheme for $h \to f\bar{f}$

Decoupling Scheme

- definition of the decoupling scheme: [Kotlarski, Lang'25]
  - SM-like parameters in decoupling scheme numerically equal to $\overline{\mathrm{MS}}$ parameters in SM:
    $$p^{\mathrm{dec}}(Q) = p^{\overline{\mathrm{MS}},\mathrm{SM}}(Q)$$
    with Q usually equal to mass of decaying particle
  - bare parameter independent of scheme choice:
    $$p_B = p^{\mathrm{dec}} + \delta p^{\mathrm{dec}} = p^{\mathrm{OS}} + \delta p^{\mathrm{OS}}$$
    $$p_B^{\mathrm{SM}} = p^{\overline{\mathrm{MS}},\mathrm{SM}} + \delta p^{\overline{\mathrm{MS}},\mathrm{SM}} = p^{\mathrm{OS},\mathrm{SM}} + \delta p^{\mathrm{OS},\mathrm{SM}}$$
    for any SM-like parameter p
  - equality $p^{\mathrm{OS}} = p^{\mathrm{OS},\mathrm{SM}}$ implies decoupling prescription:
    $$\boxed{\delta p^{\mathrm{dec}} = \delta p^{\overline{\mathrm{MS}},\mathrm{SM}} + \delta p^{\mathrm{OS}} - \delta p^{\mathrm{OS},\mathrm{SM}}}$$
  - $\overline{\mathrm{MS}}$/$\overline{\mathrm{DR}}$ renormalization applied to BSM parameters
  - on-shell field renormalization adopted

2HDM (Type II)

- Higgs potential (with softly broken $\mathbb{Z}_2$-symmetry):

$$
\begin{aligned}
V_H = {} & m_{11}^2 \Phi_1^\dagger \Phi_1 + m_{22}^2 \Phi_2^\dagger \Phi_2 - m_{12}^2 \left(\Phi_1^\dagger \Phi_2 + \Phi_2^\dagger \Phi_1 \right) \\
& + \frac{\lambda_1}{2} \left(\Phi_1^\dagger \Phi_1 \right)^2 + \frac{\lambda_2}{2} \left(\Phi_2^\dagger \Phi_2 \right)^2 + \lambda_3 \left(\Phi_1^\dagger \Phi_1 \right) \left(\Phi_2^\dagger \Phi_2 \right) + \lambda_4 \left(\Phi_1^\dagger \Phi_2 \right) \left(\Phi_2^\dagger \Phi_1 \right) \\
& + \frac{\lambda_5}{2} \left[\left(\Phi_1^\dagger \Phi_2 \right)^2 + \left(\Phi_2^\dagger \Phi_1 \right)^2 \right]
\end{aligned}
$$

with $m_{12}^2, \lambda_5 \in \mathbb{R}$

- Higgs-fermion coupling structure in type-II 2HDM:

$$
\mathcal{L}_{\text{Yuk}} = -y_u \overline{Q} \Phi_2^c u_R - y_d \overline{Q} \Phi_1 d_R - y_e \overline{L} \Phi_1 e_R + \text{h.c.}
$$

- tree-level mixing in Higgs sector:

$$
\begin{pmatrix} H \\ h \end{pmatrix} = R(\alpha) \begin{pmatrix} \phi_1^0 \\ \phi_2^0 \end{pmatrix} \qquad \begin{pmatrix} G_0 \\ A \end{pmatrix} = R(\beta) \begin{pmatrix} \chi_1^0 \\ \chi_2^0 \end{pmatrix} \qquad \begin{pmatrix} G^\pm \\ H^\pm \end{pmatrix} = R(\beta) \begin{pmatrix} \phi_1^\pm \\ \phi_2^\pm \end{pmatrix}
$$

2HDM (Type II)

- relations for tree-level Higgs masses:

$$m_h^2 = \frac{m_{12}^2}{s_\beta c_\beta} c_{\alpha-\beta}^2 + \left(\lambda_1 s_\alpha^2 c_\beta^2 + \lambda_2 c_\alpha^2 s_\beta^2 - \tfrac{1}{2}\lambda_{345} s_{2\alpha} s_{2\beta}\right) v^2$$

$$m_H^2 = \frac{m_{12}^2}{s_\beta c_\beta} s_{\alpha-\beta}^2 + \left(\lambda_1 c_\alpha^2 c_\beta^2 + \lambda_2 s_\alpha^2 s_\beta^2 + \tfrac{1}{2}\lambda_{345} s_{2\alpha} s_{2\beta}\right) v^2$$

$$m_A^2 = \frac{m_{12}^2}{s_\beta c_\beta} - \lambda_5 v^2$$

$$m_{H^\pm}^2 = \frac{m_{12}^2}{s_\beta c_\beta} - \frac{\lambda_4 + \lambda_5}{2} v^2$$

  with $\lambda_{345} = \lambda_3 + \lambda_4 + \lambda_5$
- choice of mostly physical parametrization:

$$P_{\text{phys}} = \underbrace{\{e, g_s, m_{f_i}, m_W^2, m_Z^2, m_h^2\}}_{=:P_{\text{phys}}^{\text{SM}}} \cup \underbrace{\{m_H^2, m_A^2, m_{H^\pm}^2, \lambda_5, \alpha, \beta\}}_{=:P_{\text{phys}}^{\text{BSM}}}$$

Decoupling Limit in the 2HDM

- result for decay shall correspond to SM result if:

h light, while H, A and $H^{\pm}$ heavy $\iff$ **decoupling limit**

- decoupling limit can be realized via:
  - Let Higgs mass parameters diverge while keeping them equal:

$$\boxed{m_H = m_A = m_{H^{\pm}} =: \zeta \to \infty}$$

  - At same time, let $\cos(\alpha - \beta)$ vanish as:

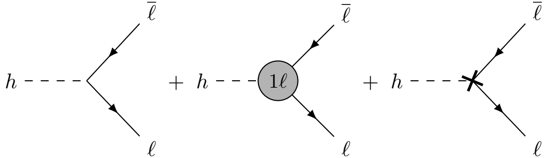
$$\boxed{\cos(\alpha - \beta) \sim \frac{1}{\zeta} \to 0}$$

$\Rightarrow \lambda_i$ remain small and thus perturbative!

Higgs Decay $h \to \ell\ell$ in 2HDM Type II

Renormalized 1-loop vertex function:

$\hat{\Gamma}^{1\ell}_{h\ell\ell} =$



$$= \frac{m_\ell \sin\alpha}{v\cos\beta} + \text{genuine 1-loop diagrams}$$

$$+ \frac{m_\ell \sin\alpha}{v\cos\beta}\left[\frac{\delta m_\ell}{m_\ell} - \delta Z_v + \cot\alpha\,\delta\alpha + \tan\beta\,\delta\beta + \frac{1}{2}\left(\delta Z_{hh} - \cot\alpha\,\delta Z_{Hh} + \delta Z^L_\ell + \delta Z^R_\ell\right)\right]$$

$$= F_L \mathbb{P}_L + F_R \mathbb{P}_R$$

with counterterm containing:

- SM-like mass and coupling RCs $\to$ decoupling prescription
- field RCs $\to$ OS-renormalized
- mixing angle RCs $\to$ $\overline{\text{MS}}$-renormalized **?**

Mixing Angle Renormalization

- Higgs field renormalization:

$$\begin{pmatrix} H_{\text{bare}} \\ h_{\text{bare}} \end{pmatrix} = \underbrace{\begin{pmatrix} 1+\frac{1}{2}\delta Z_{HH} & \frac{1}{2}\delta Z_{Hh} \\ \frac{1}{2}\delta Z_{hH} & 1+\frac{1}{2}\delta Z_{hh} \end{pmatrix}}_{=:Z_H^{1/2}} \begin{pmatrix} H \\ h \end{pmatrix} \qquad \begin{pmatrix} \phi^0_{1,\text{bare}} \\ \phi^0_{2,\text{bare}} \end{pmatrix} = \underbrace{\begin{pmatrix} 1+\frac{1}{2}\delta Z_{11} & \frac{1}{2}\delta Z_{12} \\ \frac{1}{2}\delta Z_{21} & 1+\frac{1}{2}\delta Z_{22} \end{pmatrix}}_{=:Z_\phi^{1/2}} \begin{pmatrix} \phi^0_1 \\ \phi^0_2 \end{pmatrix}$$

- mixing angles as independent model parameters $\rightarrow$ need to be renormalized:

$$\alpha_{\text{bare}} = \alpha + \delta\alpha$$

- use relation between bare quantities in gauge and mass bases:

$$\begin{pmatrix} H_{\text{bare}} \\ h_{\text{bare}} \end{pmatrix} = R(\alpha_{\text{bare}}) \begin{pmatrix} \phi^0_{1,\text{bare}} \\ \phi^0_{2,\text{bare}} \end{pmatrix} \quad \Rightarrow \quad \sqrt{Z_H} = R(\delta\alpha)R(\alpha)\sqrt{Z_{\phi^0}}R(\alpha)^\dagger$$

$$\Rightarrow \quad \delta\alpha = \frac{1}{4}\left(\delta Z_{Hh} - \delta Z_{hH}\right) + \frac{1}{4}\left(\delta Z_{21} - \delta Z_{12}\right)$$

Mixing Angle Renormalization

- $\overline{\text{MS}}$-scheme:

  $\rightarrow$ part of "standard recipe" for decoupling scheme

  – for UV-finiteness, diagonal $\sqrt{Z_\phi}$ sufficient:

$$\sqrt{Z_\phi} = \begin{pmatrix} 1 + \frac{1}{2}\delta Z_{11} & 0 \\ 0 & 1 + \frac{1}{2}\delta Z_{22} \end{pmatrix} \quad \Longleftrightarrow \quad \delta Z_{12} = \delta Z_{21} = 0$$

$$\Rightarrow \quad \boxed{\delta\alpha^{\overline{\text{MS}}} = \frac{1}{4}\left(\delta Z_{Hh}^{\overline{\text{MS}}} - \delta Z_{hH}^{\overline{\text{MS}}}\right) = \left.\frac{\Sigma_{Hh}(m_h^2) + \Sigma_{Hh}(m_H^2) - 2\delta t_{Hh}}{2(m_H^2 - m_h^2)}\right|_{\text{div}}}$$

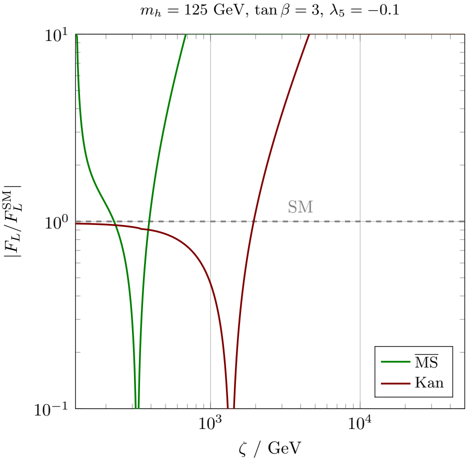
- Kanemura scheme: [Kanemura et al.'04]

  – idea: use symmetric $\sqrt{Z_\phi} \quad \Longleftrightarrow \quad \delta Z_{12} = \delta Z_{21}$

$$\Rightarrow \quad \boxed{\delta\alpha^{\text{Kan}} = \frac{1}{4}\left(\delta Z_{Hh}^{\text{OS}} - \delta Z_{hH}^{\text{OS}}\right) = \frac{\Sigma_{Hh}(m_h^2) + \Sigma_{Hh}(m_H^2) - 2\delta t_{Hh}}{2(m_H^2 - m_h^2)}}$$

Study of Decoupling Behaviour

- behaviour of form factor F_L for $\cos(\alpha - \beta) = 0$ (alignment limit):



Study of Decoupling Behaviour

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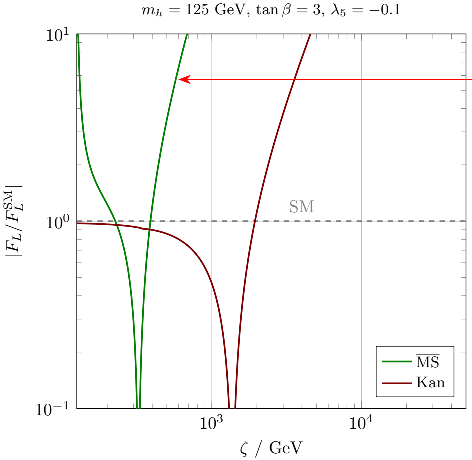
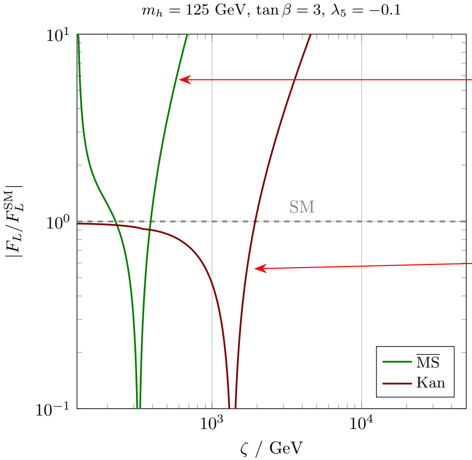


$\overline{\text{MS}}$-scheme:
Strong non-decoupling effect + pole at $\zeta = m_h$ introduced via δZ_{Hh}!

$$\delta Z_{Hh} = \frac{2\text{Re}[\Sigma_{Hh}(m_h^2) - \delta t_{hH}]}{m_H^2 - m_h^2}$$

Study of Decoupling Behaviour

- behaviour of form factor F_L for $\cos(\alpha-\beta)=0$ (alignment limit):



$\overline{\mathrm{MS}}$-scheme:
Strong non-decoupling effect + pole at $\zeta = m_h$ introduced via δZ_{Hh}!

$$\delta Z_{Hh} = \frac{2\mathrm{Re}[\Sigma_{Hh}(m_h^2) - \delta t_{hH}]}{m_H^2 - m_h^2}$$

Kanemura scheme:
more stable, but still non-decoupling due to incomplete cancellation of δZ_{Hh}

$$\delta\alpha - \frac{1}{2}\delta Z_{Hh} = \frac{\Sigma_{Hh}(m_H^2) - \Sigma_{Hh}(m_h^2)}{2(m_H^2 - m_h^2)}$$

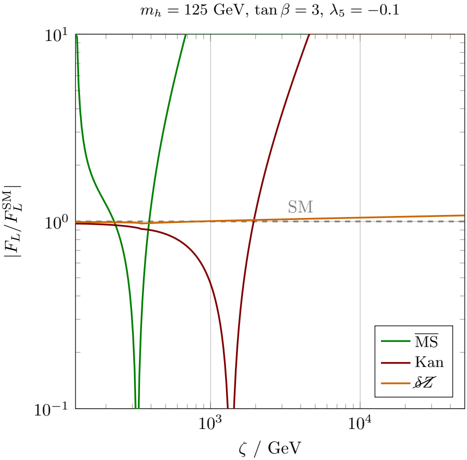
Mixing Angle Renormalization

- δZ-cancelling scheme:
  - idea: modify $\delta\alpha$ such that finite parts of δZ_{Hh} and δZ_{hh} are cancelled
  - this implies:

$$\boxed{\delta\alpha^{\not{\delta Z}} := \delta\alpha^{\overline{\text{MS}}} + \frac{1}{2}\delta Z^{\text{OS}}_{Hh}\Big|_{\text{fin}} - \frac{\tan\alpha}{2}\,\delta Z^{\text{OS}}_{hh}\Big|_{\text{fin}}}$$

Study of Decoupling Behaviour

- behaviour of form factor F_L for $\cos(\alpha - \beta) = 0$ (alignment limit):



Study of Decoupling Behaviour

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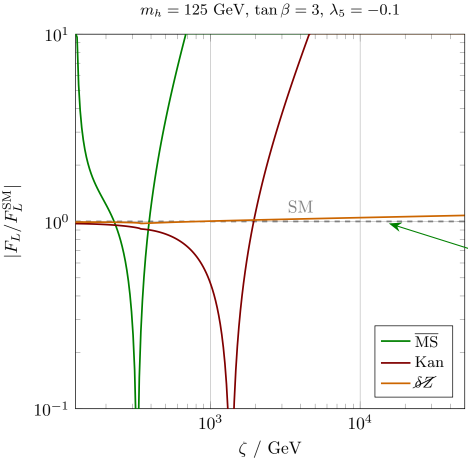

$\not{\delta Z}$ scheme:
Decoupling almost achieved!

Conclusion

- $\mathrm{2HDM}$ as testing ground for automated scalar decay calculation in models with extended Higgs sector by `FlexibleSUSY/FlexibleDecay`
- behaviour for $\zeta \to \infty$ strongly scheme dependent $\Rightarrow$ importance of cancellation between δZ_{hh}, δZ_{Hh} and $\delta\alpha$
- problem: how can alignment/decoupling limit be defined at loop-level?
- **idea**: renormalize α such that

$$\cos(\alpha - \beta) = 0 \quad \Longrightarrow \quad h \text{ is fully SM-like}$$

  or at least

$$\cos(\alpha - \beta) = 0 \quad \Longrightarrow \quad \hat{\Gamma}^{\mathrm{2HDM}}_{h\ell\ell} = \hat{\Gamma}^{\mathrm{SM},\overline{\mathrm{MS}}}_{h\ell\ell}$$