CP Violation in the Weinberg 3HDM Potential

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Motivation for three Higgs doublets

New sources of CP violation in the scalar sector

Possibility of having a discrete symmetry and still have CP violation, explicit or spontaneous (in 2HDM imposing symmetry excludes CP violation in scalar sector)

Rich phenomenology, including possible DM candidates

Motivation for imposing discrete symmetries

As the number of Higgs doublets increases so does the number of free parameters

Symmetries reduce the number of free parameters leading to (testable) predictions

Symmetries help control HFCNC (e.g. NFC or MFV suppression in BGL models)

Symmetries are needed to stabilise DM

Weinberg 3HDM (1976)

- **to explain experimental observation of CP violation**
- **only four quarks were known, no complex CKM**
- **Z_2 X Z_2 symmetry to prevent FCNC**

  (each quark sector, up and down, only couples to one doublet - NFC)
- **complex couplings introduced in the scalar potential**
- **despite Z_2 X Z_2 symmetry CP can be violated explicitly in scalar sector**

Branco 1980: Weinberg model with real couplings

- **Possibility of Spontaneous CP violation**
- **NFC and six quarks leads to real CKM**

Our Framework: Weinberg 3HDM with real coefficients

Notation and Definitions:

Parametrisation of three Higgs doublets after spontaneous symmetry breaking

$$\phi_i = e^{i\theta_i} \begin{pmatrix} \varphi_i^+ \\ \frac{1}{\sqrt{2}}(v_i + \eta_i + i\chi_i) \end{pmatrix}, \quad i = 1, 2, 3.$$

the v_i are real

Imposing two $\mathbb{Z}_2$ symmetries, we automatically get a third one

doublets may then be assigned the $\mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_2$ parities

$$\phi_1 : (+1, +1, -1) \qquad \phi_2 : (-1, +1, +1) \qquad \phi_3 : (+1, -1, +1)$$

Most general $Z_2 \times Z_2$ symmetric 3HDM potential (Notation of Ivanov and Nishi):

$$V = V_2 + V_4, \quad \text{with} \quad V_4 = V_0 + V_{\rm ph},$$

$$V_2 = -[m_{11}(\phi_1^\dagger\phi_1) + m_{22}(\phi_2^\dagger\phi_2) + m_{33}(\phi_3^\dagger\phi_3)],$$

$$\begin{aligned} V_0 = {} & \lambda_{11}(\phi_1^\dagger\phi_1)^2 + \lambda_{22}(\phi_2^\dagger\phi_2)^2 + \lambda_{33}(\phi_3^\dagger\phi_3)^2 \\ & + \lambda_{12}(\phi_1^\dagger\phi_1)(\phi_2^\dagger\phi_2) + \lambda_{13}(\phi_1^\dagger\phi_1)(\phi_3^\dagger\phi_3) + \lambda_{23}(\phi_2^\dagger\phi_2)(\phi_3^\dagger\phi_3) \\ & + \lambda'_{12}(\phi_1^\dagger\phi_2)(\phi_2^\dagger\phi_1) + \lambda'_{13}(\phi_1^\dagger\phi_3)(\phi_3^\dagger\phi_1) + \lambda'_{23}(\phi_2^\dagger\phi_3)(\phi_3^\dagger\phi_2), \end{aligned}$$

$$V_{\rm ph} = \lambda_1(\phi_2^\dagger\phi_3)^2 + \lambda_2(\phi_3^\dagger\phi_1)^2 + \lambda_3(\phi_1^\dagger\phi_2)^2 + \text{ h.c.}$$

This basis is called the symmetry basis. Notice the reduced number of independent parameters

In the absence of V_{ph} the potential acquires a U(1) X U(1) symmetry

General case for the Weinberg model with real coefficients:

Minimization conditions: 3 moduli and 2 phases

May express m_{ii} in terms of λs (3 conditions)

May relate λ_2 and λ_3 to λ_1 (2 conditions)

Consequence:

Mass-squared matrices are homogeneous in λs,
i.e., masses are bounded
by the perturbativity constraint on λs

List of solutions of stationary-point equations

Let {i, j, k} be any permutation of {1, 2, 3}

Here, for symmetry reasons, we assume the most general form for the vevs:

$$\phi_i = \frac{e^{i\theta_i}}{\sqrt{2}} \begin{pmatrix} 0 \\ v_i \end{pmatrix}, \quad i = 1, 2, 3.$$

The list that follows was obtained for the real potential

CPC indicates that the solution is CP conserving

CPV indicates that the solution is CP violating

Solutions are ordered by the number of zero vevs, first two zero vevs,
next one zero vev finally no zero vevs

Solution 1 (CPC):

$$v_i = 0, \quad v_j = 0, \quad m_{kk} = \lambda_{kk} v_k^2.$$

Solution 2 (CPV):

$$v_i = 0, \quad \lambda_i = 0, \quad \text{(for a single i)}$$
$$m_{jj} = \frac{1}{2} v_k^2 \left(\lambda'_{jk} + \lambda_{jk}\right) + \lambda_{jj} v_j^2, \quad m_{kk} = \frac{1}{2} v_j^2 \left(\lambda'_{jk} + \lambda_{jk}\right) + \lambda_{kk} v_k^2.$$

This solution has a $\mathbb{Z}_2$ symmetry preserved by the vacuum.

Solution 3 (CPC):

$$v_i = 0, \quad \sin(\theta_k - \theta_j) = 0,$$
$$m_{jj} = \frac{1}{2} v_k^2 \left(\lambda'_{jk} + \lambda_{jk} + 2\lambda_i\right) + \lambda_{jj} v_j^2, \quad m_{kk} = \frac{1}{2} v_j^2 \left(\lambda'_{jk} + \lambda_{jk} + 2\lambda_i\right) + \lambda_{kk}$$

Solution 4 (CPC):

$$v_i = 0, \quad \cos(\theta_k - \theta_j) = 0,$$
$$m_{jj} = \frac{1}{2} v_k^2 \left(\lambda'_{jk} + \lambda_{jk} - 2\lambda_i\right) + \lambda_{jj} v_j^2, \quad m_{kk} = \frac{1}{2} v_j^2 \left(\lambda'_{jk} + \lambda_{jk} - 2\lambda_i\right) + \lambda_{kk}$$

Solution 5 (CPC):

$$\sin(\theta_3 - \theta_1) = \sin(\theta_2 - \theta_1) = 0,$$
$$m_{11} = \frac{1}{2} \left(v_2^2 \left(\lambda'_{12} + 2\lambda_3 + \lambda_{12}\right) + v_3^2 \left(\lambda'_{13} + 2\lambda_2 + \lambda_{13}\right) + 2\lambda_{11} v_1^2\right),$$
$$m_{22} = \frac{1}{2} \left(v_1^2 \left(\lambda'_{12} + 2\lambda_3 + \lambda_{12}\right) + v_3^2 \left(\lambda'_{23} + 2\lambda_1 + \lambda_{23}\right) + 2\lambda_{22} v_2^2\right),$$
$$m_{33} = \frac{1}{2} \left(v_1^2 \left(\lambda'_{13} + 2\lambda_2 + \lambda_{13}\right) + v_2^2 \left(\lambda'_{23} + 2\lambda_1 + \lambda_{23}\right) + 2\lambda_{33} v_3^2\right).$$

Solution 6 (CPC):

$$\cos(\theta_3 - \theta_1) = \sin(\theta_2 - \theta_1) = 0,$$
$$m_{11} = \frac{1}{2} \left(v_2^2 \left(\lambda'_{12} + 2\lambda_3 + \lambda_{12}\right) + v_3^2 \left(\lambda'_{13} - 2\lambda_2 + \lambda_{13}\right) + 2\lambda_{11} v_1^2\right),$$
$$m_{22} = \frac{1}{2} \left(v_1^2 \left(\lambda'_{12} + 2\lambda_3 + \lambda_{12}\right) + v_3^2 \left(\lambda'_{23} - 2\lambda_1 + \lambda_{23}\right) + 2\lambda_{22} v_2^2\right),$$
$$m_{33} = \frac{1}{2} \left(v_1^2 \left(\lambda'_{13} - 2\lambda_2 + \lambda_{13}\right) + v_2^2 \left(\lambda'_{23} - 2\lambda_1 + \lambda_{23}\right) + 2\lambda_{33} v_3^2\right).$$

Solution 7 (CPC):

$$\sin(\theta_3 - \theta_1) = \cos(\theta_2 - \theta_1) = 0,$$
$$m_{11} = \frac{1}{2} \left(v_2^2 \left(\lambda'_{12} - 2\lambda_3 + \lambda_{12}\right) + v_3^2 \left(\lambda'_{13} + 2\lambda_2 + \lambda_{13}\right) + 2\lambda_{11} v_1^2\right),$$
$$m_{22} = \frac{1}{2} \left(v_1^2 \left(\lambda'_{12} - 2\lambda_3 + \lambda_{12}\right) + v_3^2 \left(\lambda'_{23} - 2\lambda_1 + \lambda_{23}\right) + 2\lambda_{22} v_2^2\right),$$
$$m_{33} = \frac{1}{2} \left(v_1^2 \left(\lambda'_{13} + 2\lambda_2 + \lambda_{13}\right) + v_2^2 \left(\lambda'_{23} - 2\lambda_1 + \lambda_{23}\right) + 2\lambda_{33} v_3^2\right).$$

Solution 8 (CPC):

$$\cos(\theta_3 - \theta_1) = \cos(\theta_2 - \theta_1) = 0,$$
$$m_{11} = \frac{1}{2} \left(v_2^2 \left(\lambda'_{12} - 2\lambda_3 + \lambda_{12}\right) + v_3^2 \left(\lambda'_{13} - 2\lambda_2 + \lambda_{13}\right) + 2\lambda_{11} v_1^2\right),$$
$$m_{22} = \frac{1}{2} \left(v_1^2 \left(\lambda'_{12} - 2\lambda_3 + \lambda_{12}\right) + v_3^2 \left(\lambda'_{23} + 2\lambda_1 + \lambda_{23}\right) + 2\lambda_{22} v_2^2\right),$$
$$m_{33} = \frac{1}{2} \left(v_1^2 \left(\lambda'_{13} - 2\lambda_2 + \lambda_{13}\right) + v_2^2 \left(\lambda'_{23} + 2\lambda_1 + \lambda_{23}\right) + 2\lambda_{33} v_3^2\right).$$

Solution 9 (CPC):

$$\sin(\theta_j - \theta_i) = 0, \quad \lambda_i = \lambda_j = 0,$$
$$m_{ii} = \tfrac{1}{2}\left(v_j^2\left(\lambda'_{ij} + \lambda_{ij} + 2\lambda_k\right) + v_k^2\left(\lambda'_{ik} + \lambda_{ik}\right) + 2\lambda_{ii}v_i^2\right),$$
$$m_{jj} = \tfrac{1}{2}\left(v_i^2\left(\lambda'_{ij} + \lambda_{ij} + 2\lambda_k\right) + v_k^2\left(\lambda'_{jk} + \lambda_{jk}\right) + 2\lambda_{jj}v_j^2\right),$$
$$m_{kk} = \tfrac{1}{2}\left(v_i^2\left(\lambda'_{ik} + \lambda_{ik}\right) + v_j^2\left(\lambda'_{jk} + \lambda_{jk}\right) + 2\lambda_{kk}v_k^2\right).$$

Solution 10 (CPC):

$$\cos(\theta_j - \theta_i) = 0, \quad \lambda_i = \lambda_j = 0,$$
$$m_{ii} = \tfrac{1}{2}\left(v_j^2\left(\lambda'_{ij} + \lambda_{ij} - 2\lambda_k\right) + v_k^2\left(\lambda'_{ik} + \lambda_{ik}\right) + 2\lambda_{ii}v_i^2\right),$$
$$m_{jj} = \tfrac{1}{2}\left(v_i^2\left(\lambda'_{ij} + \lambda_{ij} - 2\lambda_k\right) + v_k^2\left(\lambda'_{jk} + \lambda_{jk}\right) + 2\lambda_{jj}v_j^2\right),$$
$$m_{kk} = \tfrac{1}{2}\left(v_i^2\left(\lambda'_{ik} + \lambda_{ik}\right) + v_j^2\left(\lambda'_{jk} + \lambda_{jk}\right) + 2\lambda_{kk}v_k^2\right).$$

Solution 11 (CPV):

$$m_{11} = \tfrac{1}{2}\left(v_2^2\left(\lambda'_{12} + \lambda_{12}\right) + v_3^2\left(\lambda'_{13} + \lambda_{13}\right) + \frac{2\lambda_1 v_2^2 v_3^2 \sin^2 2(\theta_2-\theta_3)}{v_1^2 \sin 2(\theta_1-\theta_2)\sin 2(\theta_1-\theta_3)} + 2\lambda_{11}v_1^2\right),$$
$$m_{22} = \tfrac{1}{2}\left(v_1^2\left(\lambda'_{12} + \lambda_{12}\right) + v_3^2\left(\lambda'_{23} + \lambda_{23}\right) + 2\lambda_{22}v_2^2\right) + \lambda_1 v_3^2 \frac{\sin 2(\theta_1-\theta_3)}{\sin 2(\theta_1-\theta_2)},$$
$$m_{33} = \tfrac{1}{2}\left(v_1^2\left(\lambda'_{13} + \lambda_{13}\right) + v_2^2\left(\lambda'_{23} + \lambda_{23}\right) + 2\lambda_{33}v_3^2\right) + \lambda_1 v_2^2 \frac{\sin 2(\theta_1-\theta_2)}{\sin 2(\theta_1-\theta_3)},$$
$$\lambda_3 = \frac{\lambda_1 v_3^2 \sin 2(\theta_2-\theta_3)}{v_1^2 \sin 2(\theta_1-\theta_2)}, \quad \lambda_2 = -\frac{\lambda_1 v_2^2 \sin 2(\theta_2-\theta_3)}{v_1^2 \sin 2(\theta_1-\theta_3)}.$$

Solution 11 is Branco's spontaneous CP violating solution (1980), it is a more general solution than Solution 2. In solution 11 all lambda_i are taken to be different from zero

Solution 2 has only two vevs different from zero, one CP violating phase survives. Five of the CP-odd invariants given in what follows are non-vanishing containing the factor (for i=1):

$$J_0 \equiv v_2 v_3 \lambda_2 \lambda_3 \sin 2(\theta_2 \quad \theta_3)$$

Conditions for CP Conservation

$$V = Y_{ab}(\phi_a^\dagger \phi_b) + \tfrac{1}{2} Z_{abcd}(\phi_a^\dagger \phi_b)(\phi_c^\dagger \phi_d)$$

$$\hat{V}_{ab} = \frac{v_a e^{i\theta_a}}{v} \frac{v_b e^{\;\; i\theta_b}}{v}$$

Here, the indices a, b, $c, \ldots$ can take the values 1,2,3, identifying the three fields

Whenever the stationary point equations are satisfied, the real Z_2x Z_2 symmetric three-Higgs-doublet potential conserves CP if and only if all of the 15 CP-odd invariants given here vanish

$$
\begin{aligned}
J_1 &= \mathrm{Im}\,\{\hat{V}_{ac}\hat{V}_{be}Z_{cadf}Z_{edfg}Z_{gbhh}\},\\
J_2 &= \mathrm{Im}\,\{\hat{V}_{ac}\hat{V}_{be}Z_{cadf}Z_{edfg}Z_{ghhb}\},\\
J_3 &= \mathrm{Im}\,\{\hat{V}_{ac}\hat{V}_{be}Z_{cadf}Z_{egfd}Z_{gbhh}\},\\
J_4 &= \mathrm{Im}\,\{\hat{V}_{ac}\hat{V}_{bd}Z_{cedg}Z_{eafh}Z_{gbhf}\},\\
J_5 &= \mathrm{Im}\,\{\hat{V}_{ac}\hat{V}_{bd}Z_{cedg}Z_{ehfa}Z_{gfhb}\},\\
J_6 &= \mathrm{Im}\,\{\hat{V}_{ac}\hat{V}_{bd}Z_{cedf}Z_{eafg}Z_{gbhh}\},\\
J_7 &= \mathrm{Im}\,\{\hat{V}_{ad}\hat{V}_{be}\hat{V}_{cf}Z_{daeh}Z_{fbgi}Z_{hcig}\},\\
J_8 &= \mathrm{Im}\,\{\hat{V}_{ad}\hat{V}_{be}\hat{V}_{cf}Z_{daeh}Z_{figb}Z_{hgic}\},\\
J_9 &= \mathrm{Im}\,\{\hat{V}_{ad}\hat{V}_{be}\hat{V}_{cf}Z_{daeg}Z_{fbgh}Z_{hcii}\},\\
J_{10} &= \mathrm{Im}\,\{\hat{V}_{ad}\hat{V}_{be}\hat{V}_{cf}Z_{daeg}Z_{fhgi}Z_{hbic}\},\\
J_{11} &= \mathrm{Im}\,\{\hat{V}_{ac}\hat{V}_{be}Z_{cadg}Z_{edff}Z_{gihh}Z_{ibjj}\},\\
J_{12} &= \mathrm{Im}\,\{\hat{V}_{ac}\hat{V}_{be}Z_{cadg}Z_{effd}Z_{ghhi}Z_{ijjb}\},\\
J_{13} &= \mathrm{Im}\,\{\hat{V}_{ac}\hat{V}_{be}Z_{cadf}Z_{edfg}Z_{gihj}Z_{ibjh}\},\\
J_{14} &= \mathrm{Im}\,\{\hat{V}_{ac}\hat{V}_{bd}Z_{cedf}Z_{eafg}Z_{gihj}Z_{ibjh}\},\\
v^6 J_{15} &= \mathrm{Im}\,\{\hat{V}_{ac}\hat{V}_{bd}Y_{cf}Y_{dg}Y_{ea}Z_{fbge}\}.
\end{aligned}
$$

Concerning the construction CP-odd invariants involving scalars there is a long literature on this subject dating from long ago. I put here the examples cited in our paper

[12] L. Lavoura and J. P. Silva, *Fundamental CP violating quantities in a SU(2) × U(1) model with many Higgs doublets*, *Phys. Rev. D* **50** (1994) 4619–4624, [hep-ph/9404276].

[13] F. J. Botella and J. P. Silva, *Jarlskog - like invariants for theories with scalars and fermions*, *Phys. Rev. D* **51** (1995) 3870–3875, [hep-ph/9411288].

[14] G. C. Branco, M. N. Rebelo and J. I. Silva-Marcos, *CP-odd invariants in models with several Higgs doublets*, *Phys. Lett. B* **614** (2005) 187–194, [hep-ph/0502118].

[15] J. F. Gunion and H. E. Haber, *Conditions for CP-violation in the general two-Higgs-doublet model*, *Phys. Rev. D* **72** (2005) 095002, [hep-ph/0506227].

[16] S. Davidson and H. E. Haber, *Basis-independent methods for the two-Higgs-doublet model*, *Phys. Rev. D* **72** (2005) 035004, [hep-ph/0504050].

[17] H. E. Haber and D. O'Neil, *Basis-independent methods for the two-Higgs-doublet model. II. The Significance of tanβ*, *Phys. Rev. D* **74** (2006) 015018, [hep-ph/0602242].

Conditions for CP Conservation (cont)

This result is valid for all possible different solutions to the stationary-point equations given before

There are no redundant invariants in this set of 15

We do not know if there exists a smaller set of different CP-odd invariants implying CP conservation

Not all CP-odd quantities can be written as linear combinations of these fifteen CP-odd invariants

For 2HDM see works by Grzadkowski, Ogreid, Osland, 2014, 2016

All invariants involve at least two factors V_hat

This results from the limited form of allowed Z tensors, therefore only with two V_hat factors can the invariant be sensitive to CP violating phases appearing in the vevs

Only J_{15} involves the Y tensors

$$
\begin{aligned}
J_1 &= \mathrm{Im}\,\{\hat{V}_{ac}\hat{V}_{be}Z_{cadf}Z_{edfg}Z_{gbhh}\},\\
J_2 &= \mathrm{Im}\,\{\hat{V}_{ac}\hat{V}_{be}Z_{cadf}Z_{edfg}Z_{ghhb}\},\\
J_3 &= \mathrm{Im}\,\{\hat{V}_{ac}\hat{V}_{be}Z_{cadf}Z_{egfd}Z_{gbhh}\},\\
J_4 &= \mathrm{Im}\,\{\hat{V}_{ac}\hat{V}_{bd}Z_{cedg}Z_{eafh}Z_{gbhf}\},\\
J_5 &= \mathrm{Im}\,\{\hat{V}_{ac}\hat{V}_{bd}Z_{cedg}Z_{ehfa}Z_{gfhb}\},\\
J_6 &= \mathrm{Im}\,\{\hat{V}_{ac}\hat{V}_{bd}Z_{cedf}Z_{eafg}Z_{gbhh}\},\\
J_7 &= \mathrm{Im}\,\{\hat{V}_{ad}\hat{V}_{be}\hat{V}_{cf}Z_{daeh}Z_{fbgi}Z_{hcig}\},\\
J_8 &= \mathrm{Im}\,\{\hat{V}_{ad}\hat{V}_{be}\hat{V}_{cf}Z_{daeh}Z_{figb}Z_{hgic}\},\\
J_9 &= \mathrm{Im}\,\{\hat{V}_{ad}\hat{V}_{be}\hat{V}_{cf}Z_{daeg}Z_{fbgh}Z_{hcii}\},\\
J_{10} &= \mathrm{Im}\,\{\hat{V}_{ad}\hat{V}_{be}\hat{V}_{cf}Z_{daeg}Z_{fhgi}Z_{hbic}\},\\
J_{11} &= \mathrm{Im}\,\{\hat{V}_{ac}\hat{V}_{be}Z_{cadg}Z_{edff}Z_{gihh}Z_{ibjj}\},\\
J_{12} &= \mathrm{Im}\,\{\hat{V}_{ac}\hat{V}_{be}Z_{cadg}Z_{effd}Z_{ghhi}Z_{ijjb}\},\\
J_{13} &= \mathrm{Im}\,\{\hat{V}_{ac}\hat{V}_{be}Z_{cadf}Z_{edfg}Z_{gihj}Z_{ibjh}\},\\
J_{14} &= \mathrm{Im}\,\{\hat{V}_{ac}\hat{V}_{bd}Z_{cedf}Z_{eafg}Z_{gihj}Z_{ibjh}\},\\
v^6 J_{15} &= \mathrm{Im}\,\{\hat{V}_{ac}\hat{V}_{bd}Y_{cf}Y_{dg}Y_{ea}Z_{fbge}\}.
\end{aligned}
$$

Global Measures of CP violation in Weinberg 3HDM with Spontaneous CP Violation

Global measures of CP violation, choice based on the 15 invariants

$$A_{\text{sum}} = \log_{10} \sum_{i=1}^{15} J_i^2, \quad A_{\text{max}} = \log_{10}(\max_i J_i^2). \tag{4.7}$$

Lower points are associated with lower masses of the lightest neutral scalar

Yellow points are obtained by imposing lower bound of 45 Gev on the lightest neutral scalar

Plots suggest that suppressed CP violation may require one or more light scalars

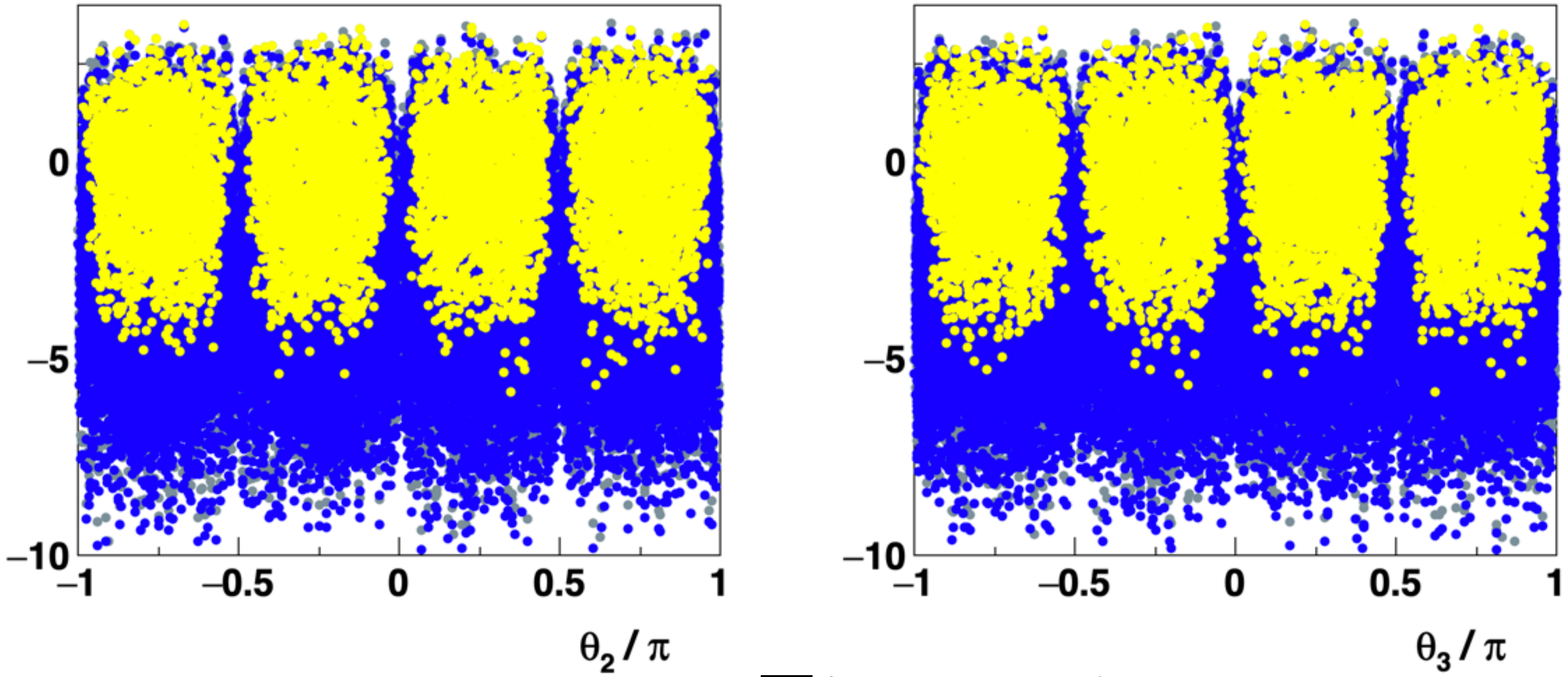


Figure 1

We show, in Fig. 1, scatter plots of the measures of CP violation, A_{sum} (grey, bottom layer, mostly covered), A_{max} (blue, over grey), defined by Eq. (4.7), vs θ_2/π and θ_3/π, based on the scan performed in Ref. [2]. The cut-off at high values is obviously caused by the upper bounds on the λs (from perturbativity, since the invariants are polynomials in the lambdas). Ref.[2] refers to the second reference in cover page

Experimental Constraints

Yukawa Sector

	u	d	e	Inert doublets
Type I-like	ϕ_1	ϕ_1	ϕ_1	at most 2
Type II-like	ϕ_1	ϕ_2	ϕ_2	at most 1
Lepton specific-like	ϕ_1	ϕ_1	ϕ_2	at most 1
Flipped-like	ϕ_1	ϕ_2	ϕ_1	at most 1
Type Z	ϕ_1	ϕ_2	ϕ_3	none

Table 1: The different NFC-respecting Yukawa structures for the ${}_2 \times {}_2$-symmetric 3HDM.

$\mathcal{L}_Y = \bar{Q}_L^0 Y^u \tilde{\phi}_a u_R^0 + \bar{Q}_L^0 Y^d \phi_b d_R^0 + \bar{E}_L^0 Y^e \phi_c e_R^0 + \text{h.c.},$ the indices a, b, c may not be different

Our study focused on the Type Z Yukawa sector because it is the least constrained scenario

In our previous work cited on the front page we had imposed theoretical constraints (perturbativity, unitarity and boundedness from below) as well as compatibility with the measured WWh_{SM} coupling and the CP constraint on the $h_{\text{SM}} \to \bar{\tau}\tau$ coupling

Now, we also impose the constraint from electroweak precision observables, S, T and U from the digamma signal strength $(h_{\text{SM}} \to \gamma\gamma)$, from $\bar{B} \to X_s\gamma$ and from the electron EDM.

Scalar mass ranges

CP violation effects

The Electron EDM

Barr-Zee (1990), Pilaftsis (2002)

The experimental upper bound has recently been tightened from $1.1\times 10^{29}\ e\cdot\text{cm}$ (2018)

to $4.1 \times 10^{30}\ e \cdot \text{cm}$ (2022)

The SM contribution is of the order $10^{38}\ e \cdot \text{cm}$ which is much smaller than the scalar contribution of the present model

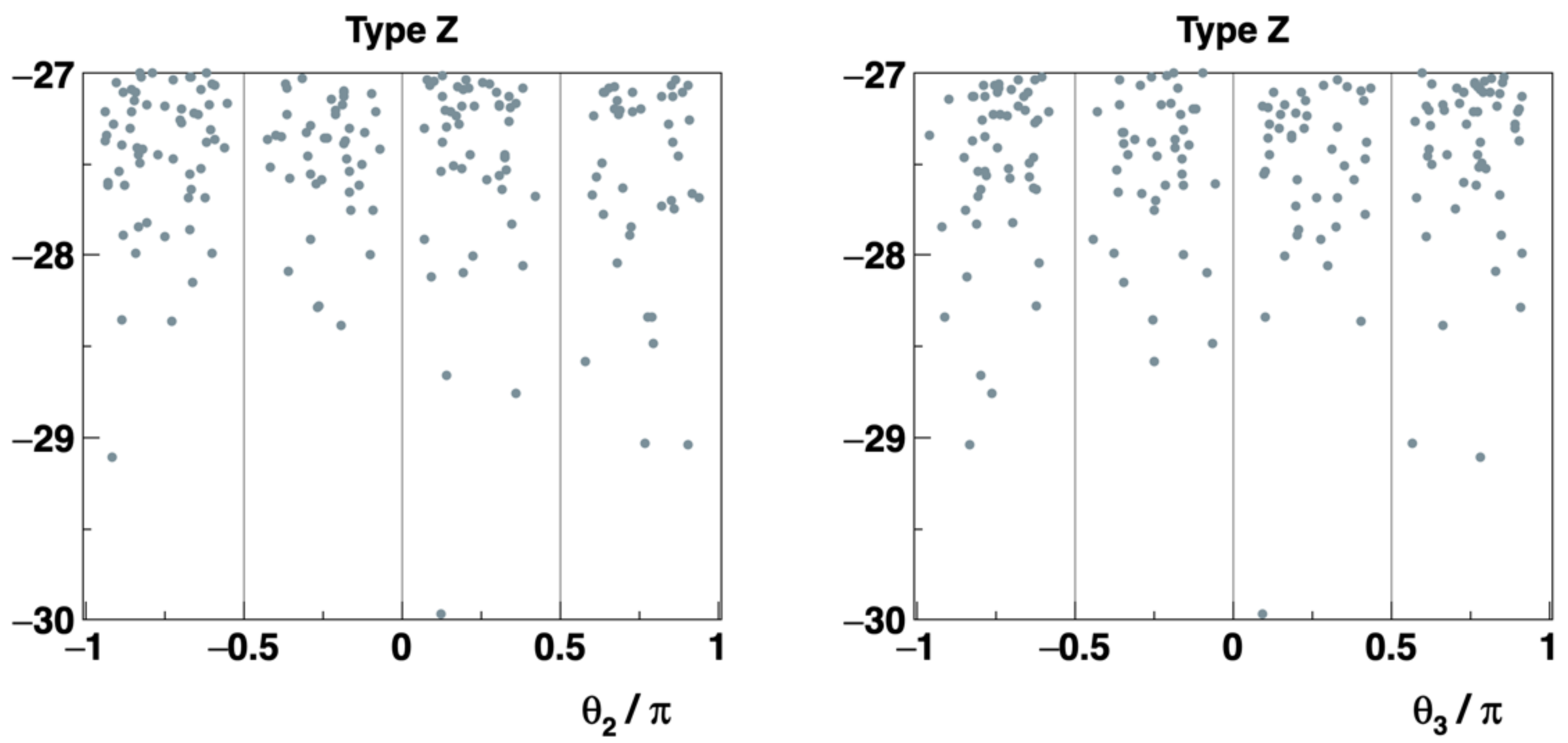
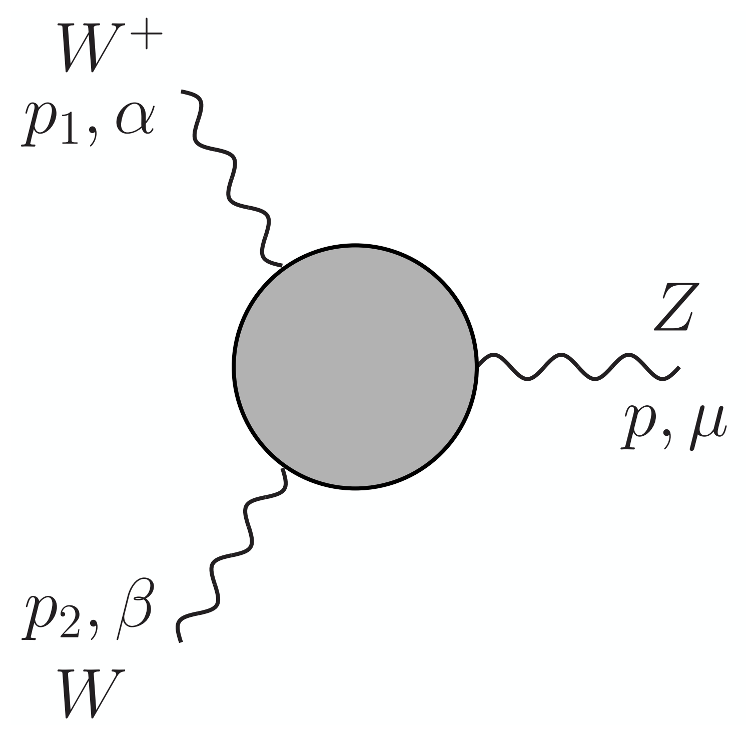


Figure 6: Scatter plots of logarithms of the electron EDM vs the angles θ_2 and θ_3. These points correspond to random scans over the scalar potential parameters subject to theoretical and collider bounds specified above. Few parameter points that survive the previous constraints survive the experimental EDM bound allowing only the lower region of $|d_e/e|$.

CP violation effects (cont)

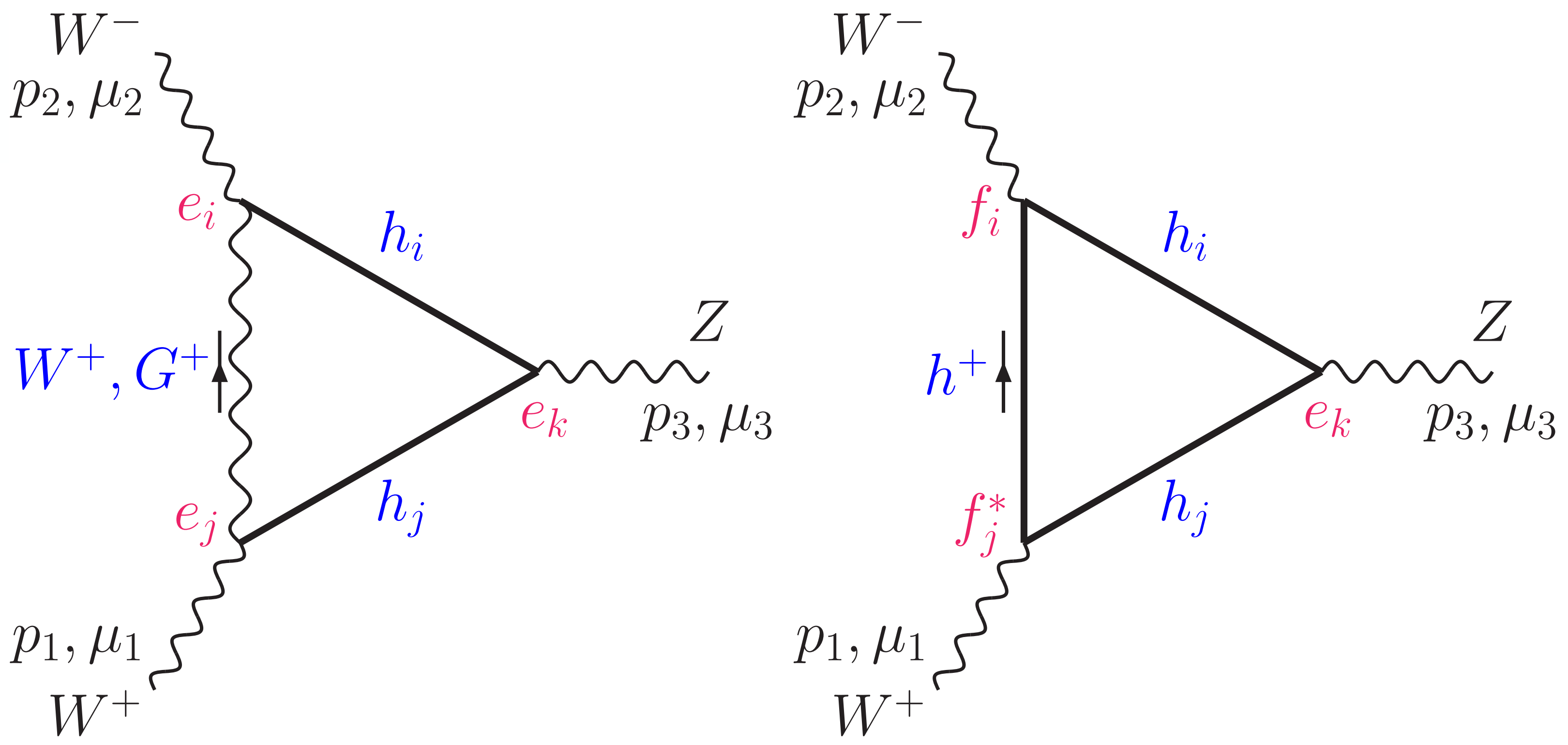
$$W^+W \quad \to Z$$



The vertex is present at the tree level, with a well-known, CP-conserving structure

Triangle diagrams contribute to CP violation in this vertex

Loop contributions to the CP-violating $W^+W^- \to Z$ amplitude in the 2HDM,

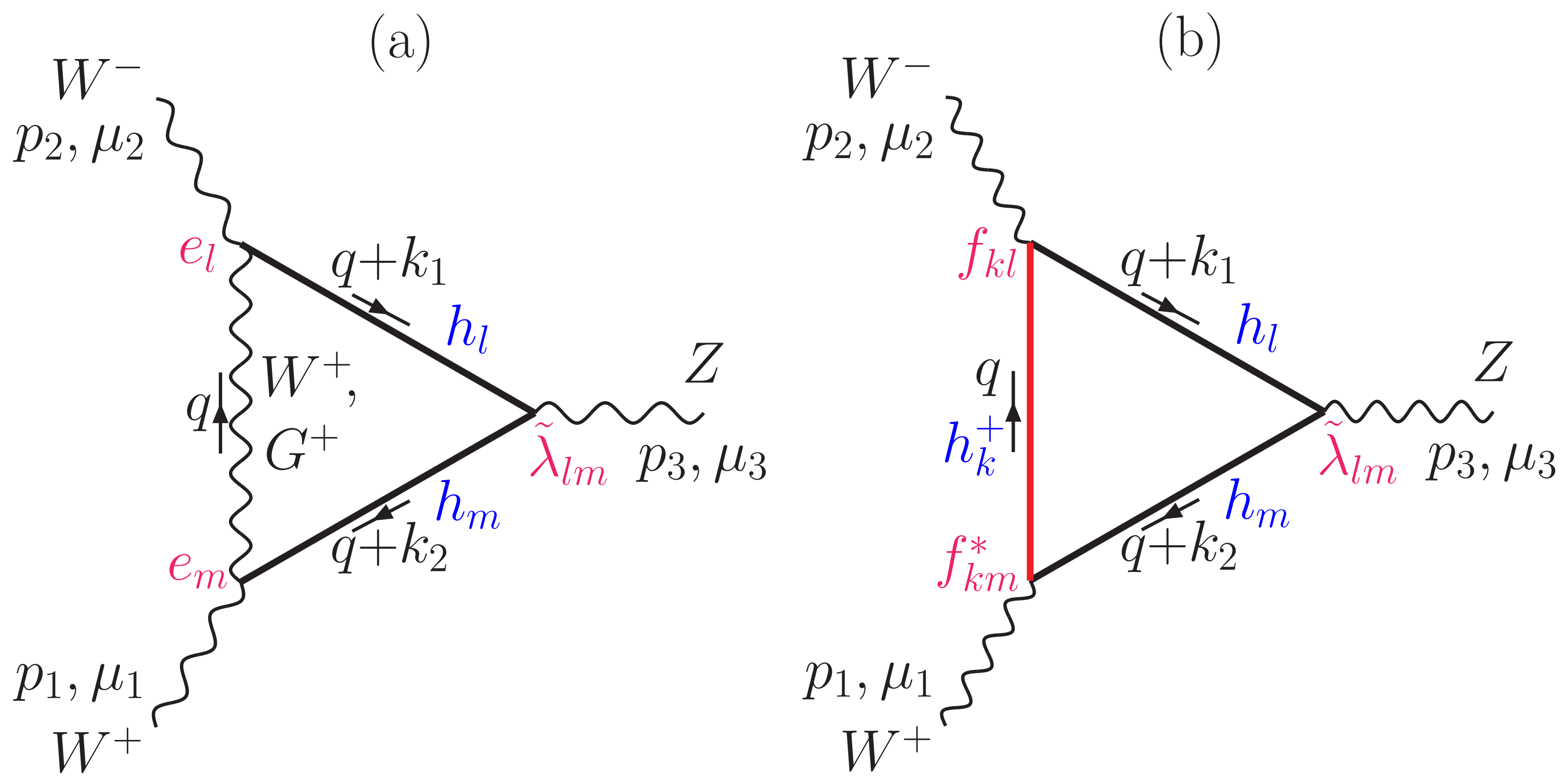


Here, $i, j \in \{1, 2, 3\}$ label the neutral scalars

proportional to ImJ_2

Loop contributions to the CP-violating $W^+W^- \to Z$ amplitude. 3HDM

CP-odd invariants that appear in these contributions are much more complicated than in the case of 2HDM and may require J's not included in those chosen before



Here, $k, l, m \in \{1, 2, 3, 4, 5\}$ label the neutral scalars

CP violation effects (cont.)

$$h_i^+ h_i^- \to h_1^\pm h_2^\mp$$

Involves only scalar particles in the initial and final states

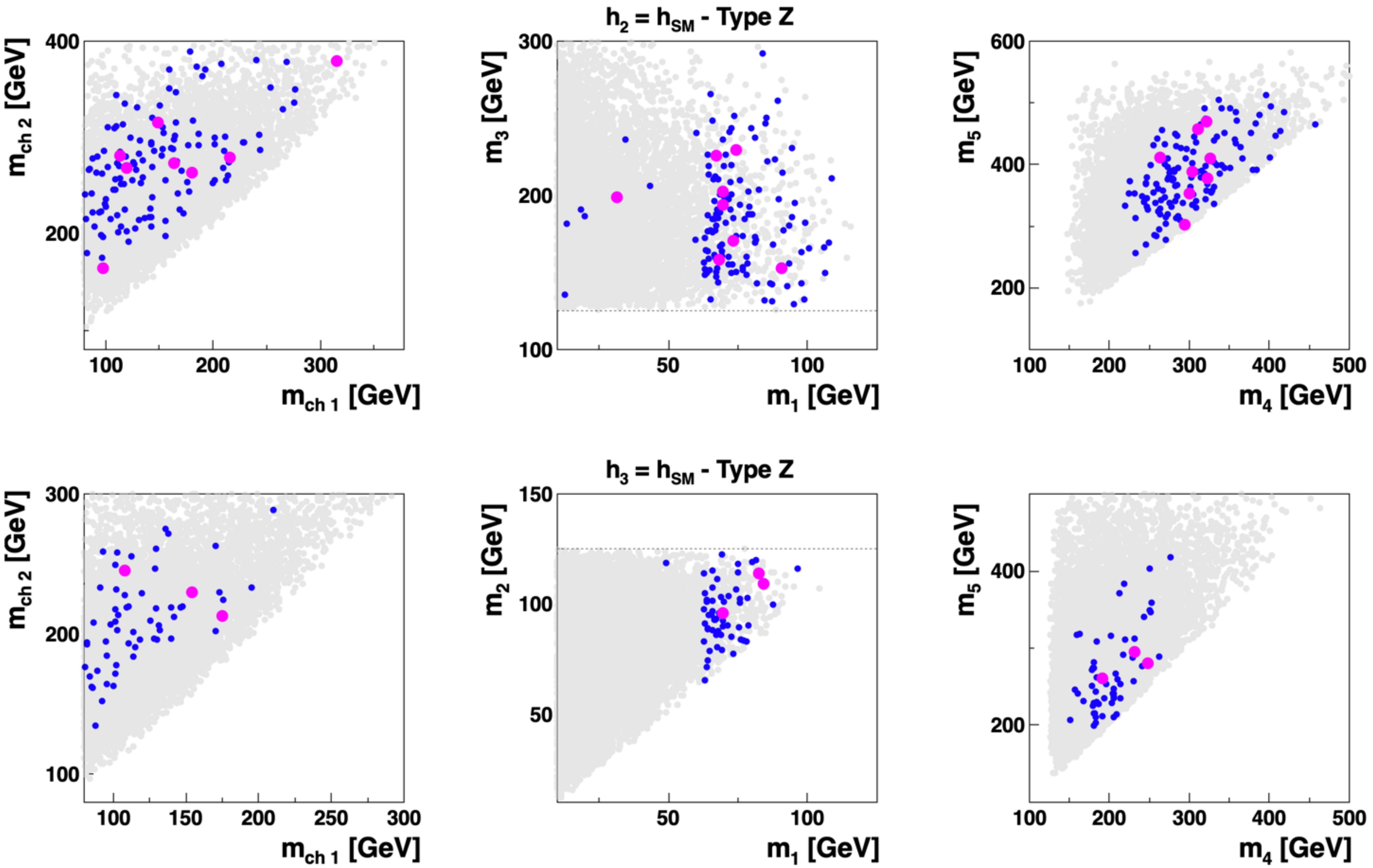
At loop level an asymmetry :

$$\mathcal{A}_{\rm ch} = \frac{\sigma(h_i^+ h_i^- \to h_2^+ h_1^-) - \sigma(h_i^+ h_i^- \to h_2^- h_1^+)}{\sigma(h_i^+ h_i^- \to h_2^+ h_1^-) + \sigma(h_i^+ h_i^- \to h_2^- h_1^+)}$$

can be generated

H. Haber, V. Keus, R. Santos 2022

Scalar mass distributions



Blue points

$$|d_e| < 10^{-27} e \cdot \mathrm{cm}$$

Purple points

$$|d_e| < 5 \cdot 10^{-29} e \cdot \mathrm{cm}$$

$$m_1 < m_2 < m_3 < m_4 < m_5$$

Conclusions

We have provided a full set of CP-odd invariants for the real Weinberg potential with complex vevs

The real Weinberg potencial allows for spontaneous CP violation (known in 1980). The full set of solutions for the vacuum are given.

Even with sizeable CP violating phases in the scalar sector the model can remain consistent with the current electron EDM bound

In our analysis, light scalars and sizeable $Zh_{\mathrm{SM}}h_i$ couplings appear

This potential tends to yield two light CP mixed neutral states as well as at least one pair of light charged scalars