Critical Unstable Qubits in Particle Physics and Beyond

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Based on:

- D. Karamitros, T. McKelvey, AP, PRD111 (2025) 096020 [MITP-25-020]
- D. Karamitros, T. McKelvey, AP, PRD108 (2023) 016006
- AP, NPB504 (1997) 61

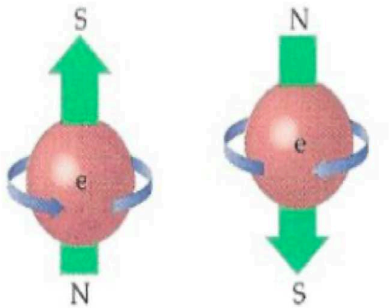
Outline:

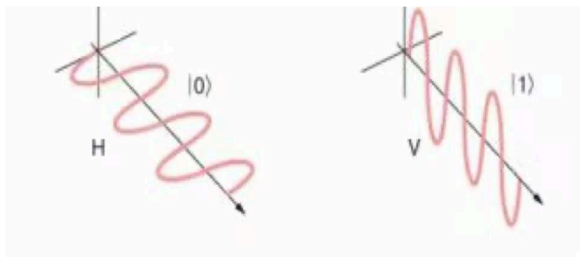
- UN celebrates 100 years of Quantum Science since '1925'
- Qubits in the Bloch Sphere Formalism
- Unstable Qubits in the Co-Decaying Frame
- Critical Unstable Qubits (CUQs)
- $B^0\bar{B}^0$-Meson System as a CUQ
- CUQs Beyond Particle Physics
- Summary

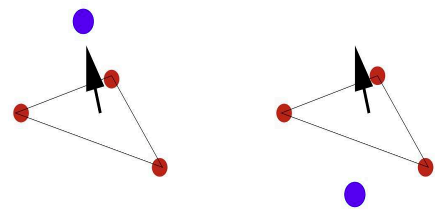
• Qubits in the Bloch Sphere Formalism

– **Qubit**: two-level quantum system that can be in a superposition of two *distinct* quantum states *or* bits, e.g. $|0\rangle$ and $|1\rangle$.

Examples: spin-states of e^- linearly polarized light Ammonia molecule







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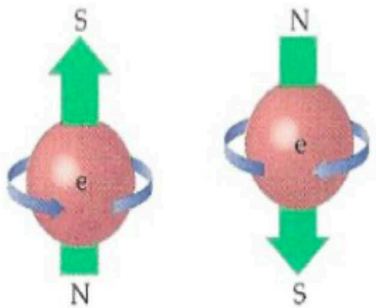
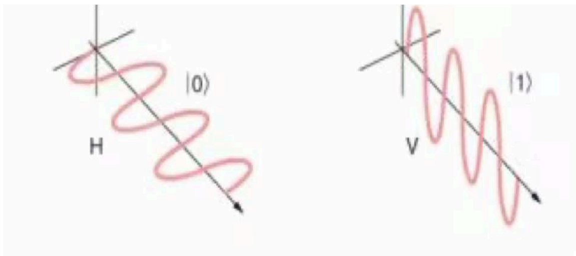
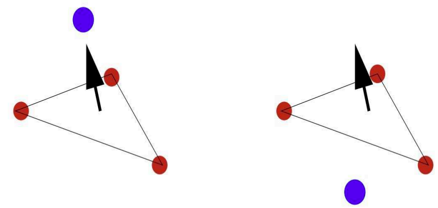
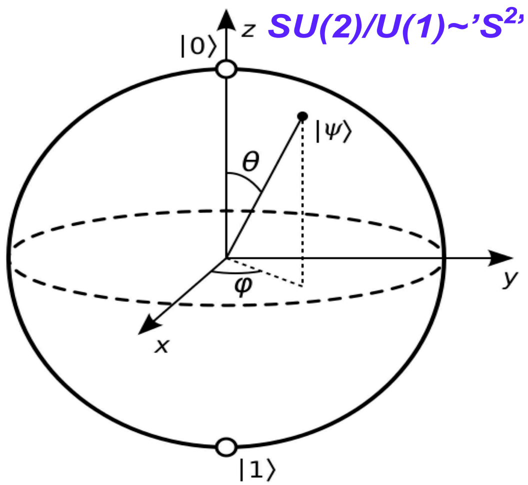






– **Bloch sphere rep:** $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$, [F. Bloch '46]

with $\alpha = \cos\left(\frac{\theta}{2}\right)$ & $\beta = e^{i\varphi}\sin\left(\frac{\theta}{2}\right)$



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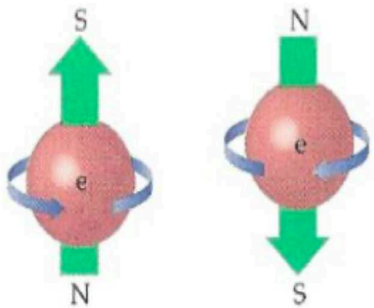

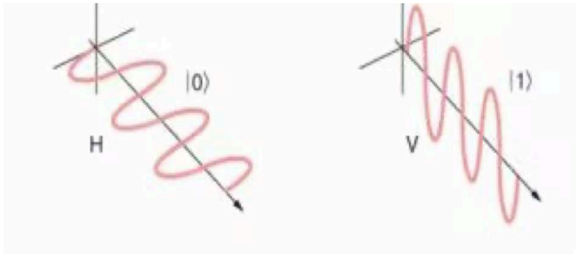

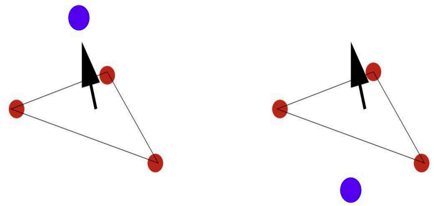

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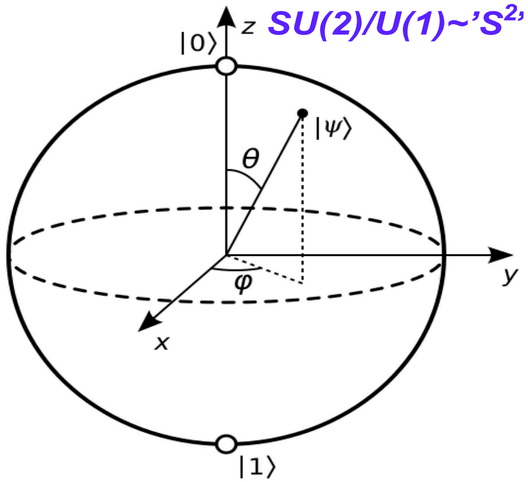

– **Time evolution of qubit:** [E. Schrödinger '1925]

$$i\,\frac{\mathrm{d}}{\mathrm{d}t}|\psi\rangle = \mathsf{H}\,|\psi\rangle\,,$$

with H is a 2D Hamiltonian written in the qubit basis: $\{|0\rangle\,,|1\rangle\}$.

– **Density Matrix:** an operator ρ that encodes all the information of a quantum system, for both *pure* and *mixed* states,

$$\rho = \sum_{k=1}^{N} w_k \, |\psi^k\rangle \, \langle\psi^k| \,, \quad \text{with } \rho^\dagger = \rho \;\; \& \;\; \mathrm{Tr}\,\rho = 1$$

Pure states are idempodent: $\rho^2 = \rho$, whereas $\rho^2 \neq \rho$ for **mixed states**

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– **Time evolution of qubit reformulated:** [J. von Neumann '27, I.I. Rabi '37]

$$\frac{\mathrm{d}}{\mathrm{d}t}\rho \;=\; -i\,[\mathsf{H}\,,\rho] \quad\Rightarrow\quad \frac{\mathrm{d}}{\mathrm{d}t}\mathbf{b} \;=\; -2\,\mathbf{E}\times\mathbf{b}\,,$$

where $\mathsf{H} \;\equiv\; E_\mu\sigma^\mu = E^0\mathbf{1}_2 - \mathbf{E}\cdot\boldsymbol{\sigma}$ and $\mathsf{H} = \mathsf{H}^\dagger$

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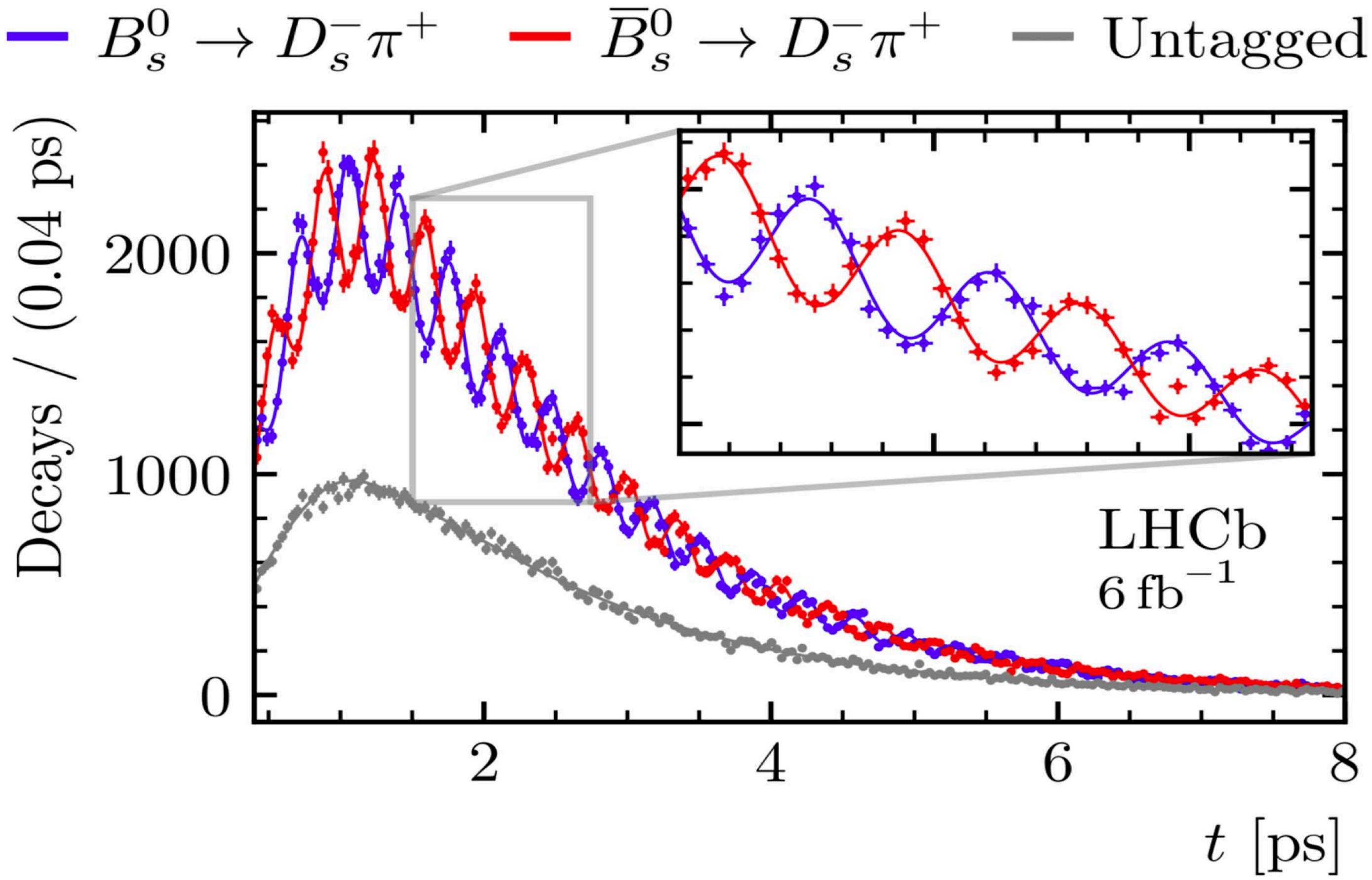

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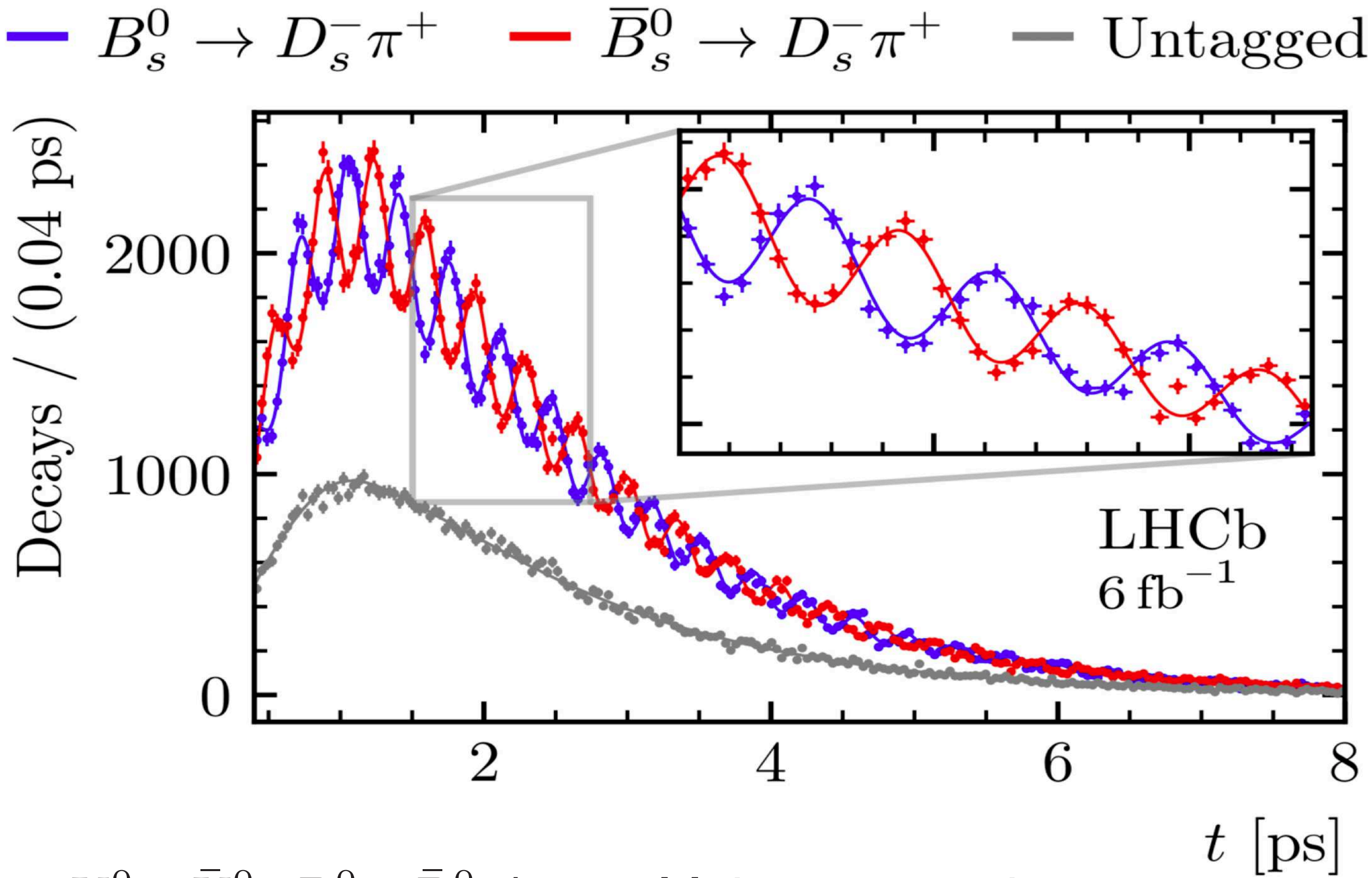

Other systems: $K^0-\bar{K}^0$, $D^0-\bar{D}^0$, heavy Majorana neutrinos . . .
At large t, they approach a pure state || the long-lived state of the system.

– Time evolution of unstable qubit:

[V. Weisskopf, E. Wigner '30;
T. D. Lee, R. Oehme, C.-N. Yang '57
. . . AP '97]

$$i\,\frac{\mathrm{d}}{\mathrm{d}t}\,|\Psi\rangle \;=\; \mathsf{H}_{\mathsf{eff}}\,|\Psi\rangle \quad\Rightarrow\quad \frac{\mathrm{d}}{\mathrm{d}t}\,\rho \;=\; -i\,[\mathsf{E}\,,\rho] \;-\; \frac{1}{2}\{\Gamma\,,\rho\}\,,$$

where $\mathsf{H}_{\mathsf{eff}} \;\equiv\; \mathsf{E} - \frac{i}{2}\Gamma \;\neq\; \mathsf{H}^{\dagger}_{\mathsf{eff}}$, with $\mathsf{E} \equiv E_\mu \sigma^\mu = E^0\mathbf{1}_2 - \mathbf{E}\cdot\boldsymbol{\sigma}$
and $\Gamma \equiv \Gamma_\mu \sigma^\mu = \Gamma^0\mathbf{1}_2 - \boldsymbol{\Gamma}\cdot\boldsymbol{\sigma}$.

Probability leakage from the qubit subspace:

$$\frac{\mathrm{d}}{\mathrm{d}t}\,\mathsf{Tr}\,\rho \;=\; -\mathsf{Tr}\,(\Gamma\rho) \;\neq\; 0 \quad\Rightarrow\quad \mathsf{Tr}\,\rho(t) \;\neq\; 1\,, \quad \text{for } t > 0.$$

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$\widehat{\rho}$: co-decaying density matrix $\longrightarrow$ $\mathbf{b}$: co-decaying Bloch vector.

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$$\frac{\mathrm{d}}{\mathrm{d}t}\,\widehat{\rho} \;=\; -i\,[\mathsf{E}\,,\widehat{\rho}] \;-\; \frac{1}{2}\{\Gamma\,,\widehat{\rho}\} \;+\; \mathsf{Tr}\,(\Gamma\widehat{\rho})\,\widehat{\rho}\,,$$

[Karamitros, McKelvey, AP '23, '25]

$$\Rightarrow\quad \boxed{\frac{\mathrm{d}\mathbf{b}}{\mathrm{d}t} \;=\; -2\,\mathbf{E}\times\mathbf{b} \;+\; \boldsymbol{\Gamma} \;-\; (\boldsymbol{\Gamma}\cdot\mathbf{b})\,\mathbf{b}}$$

Note: differential equation non-linear in $\mathbf{b}$, but first-order in t.

– Master evolution equation of unstable qubit (co-decaying frame):

$$\boxed{\frac{\mathrm{d}\mathbf{b}}{\mathrm{d}\tau} \;=\; -\frac{1}{r}\,\mathbf{e}\times\mathbf{b} + \boldsymbol{\gamma} - (\boldsymbol{\gamma}\cdot\mathbf{b})\,\mathbf{b}}$$

with $\tau \equiv |\boldsymbol{\Gamma}|\,t$, $r \equiv |\boldsymbol{\Gamma}|/(2|\mathbf{E}|)$, and unit vectors: $\mathbf{e} \equiv \mathbf{E}/|\mathbf{E}|$, $\boldsymbol{\gamma} \equiv \boldsymbol{\Gamma}/|\boldsymbol{\Gamma}|$.

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– **Energy- and decay-width differences of the eigenstates of $\mathbf{H}_{\mathrm{eff}}$:**

$$\Delta E \;=\; 2|\mathbf{E}|\,\mathrm{Re}\Big(\sqrt{1 - r^2 - 2ir\cos\theta_{\mathbf{e}\boldsymbol{\gamma}}}\Big)\,,$$

$$\Delta\Gamma \;=\; -4|\mathbf{E}|\,\mathrm{Im}\Big(\sqrt{1 - r^2 - 2ir\cos\theta_{\mathbf{e}\boldsymbol{\gamma}}}\Big)\,,$$

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with $\theta_{\mathbf{e}\boldsymbol{\gamma}} = \angle(\mathbf{e}\,,\boldsymbol{\gamma})$.

– **Large-time behaviour of b when $\mathrm{e}\perp\gamma$:**

$$\mathbf{b}(\tau\to\infty) \;=\; \frac{\sqrt{r^2-1}}{r}\,\boldsymbol{\gamma} - \frac{1}{r}\,\mathbf{e}\times\boldsymbol{\gamma}$$

approaches the long-lived state of $\mathrm{H}_{\mathrm{eff}}$, <u>but only possible for $r > 1$</u>.

• **Critical Unstable Qubits (CUQs)**

[Karamitros, McKelvey, AP '23, '25]

– **CUQ:** unstable qubit having (i) $\mathbf{E} \perp \mathbf{\Gamma}$ and (ii) $r \equiv |\mathbf{\Gamma}|/(2|\mathbf{E}|) < 1$.

$\Rightarrow$ the two eigenstates of $\mathrm{H}_{\mathrm{eff}}$ have equal lifetimes: $\Delta\Gamma = 0$.

$\Rightarrow$ Oscillations in the *co-decaying* frame continue indefinitely.

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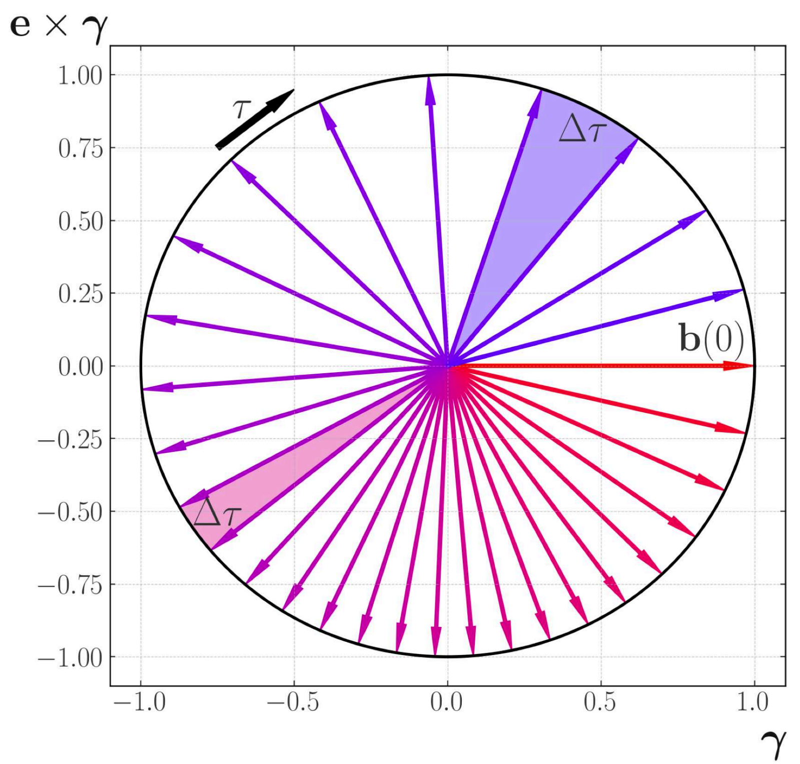
– **The angular speed of b:**

$$\dot{\varphi} \;=\; -\frac{1}{r} \;-\; \sin\varphi\,.$$

Inhomogeneous oscillation frequency rotating *slower* in the lower half plane:

$$\tan\frac{\theta}{2} \;=\; -\sqrt{\frac{1+r}{1-r}}\,\tan\frac{\omega t}{2}\,,$$

with $\theta = \varphi + \frac{\pi}{2}$ & $\omega = 2|\mathbf{E}|\sqrt{1-r^2}$.



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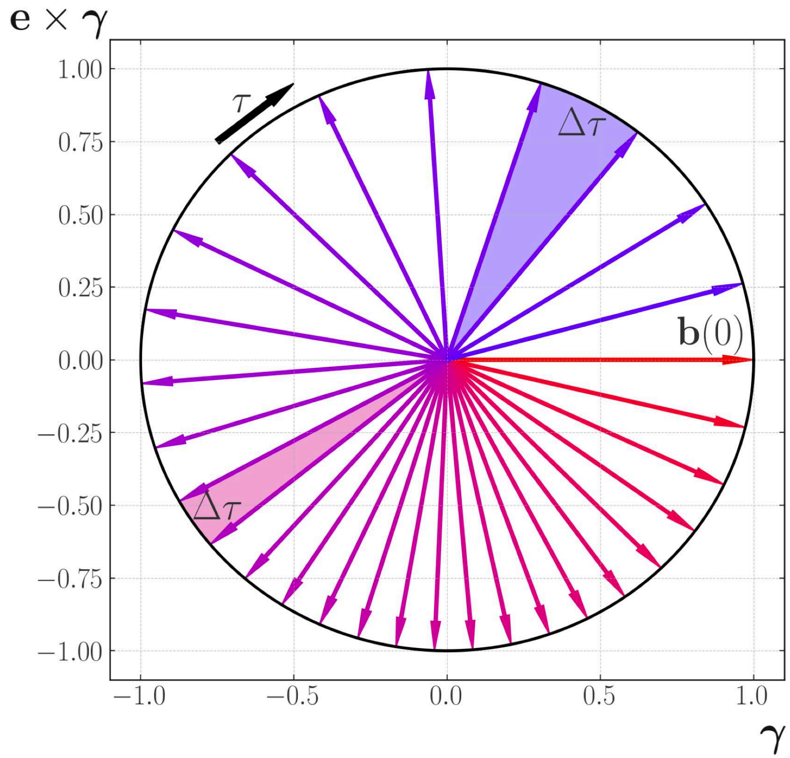

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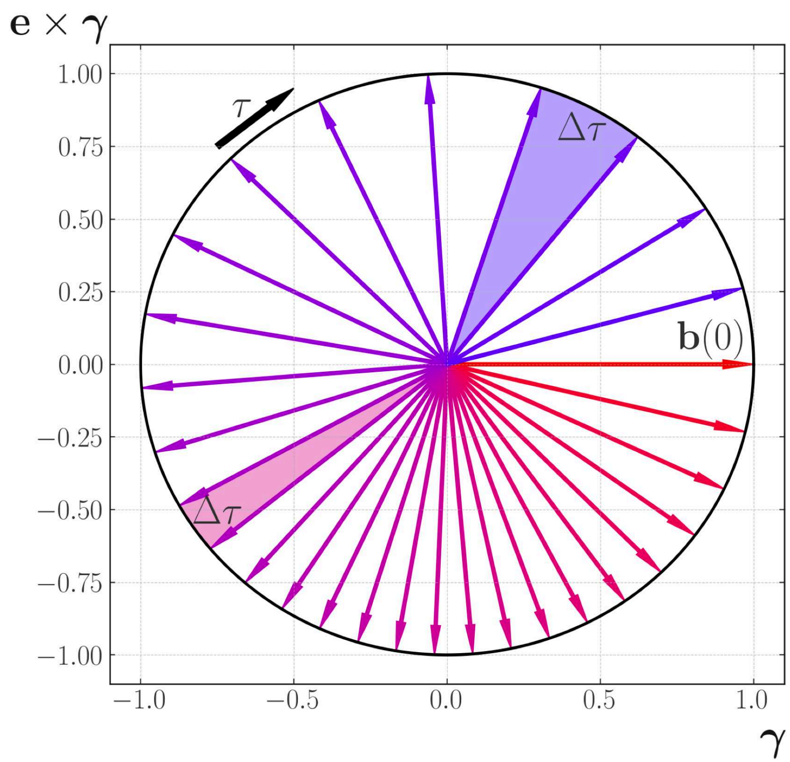

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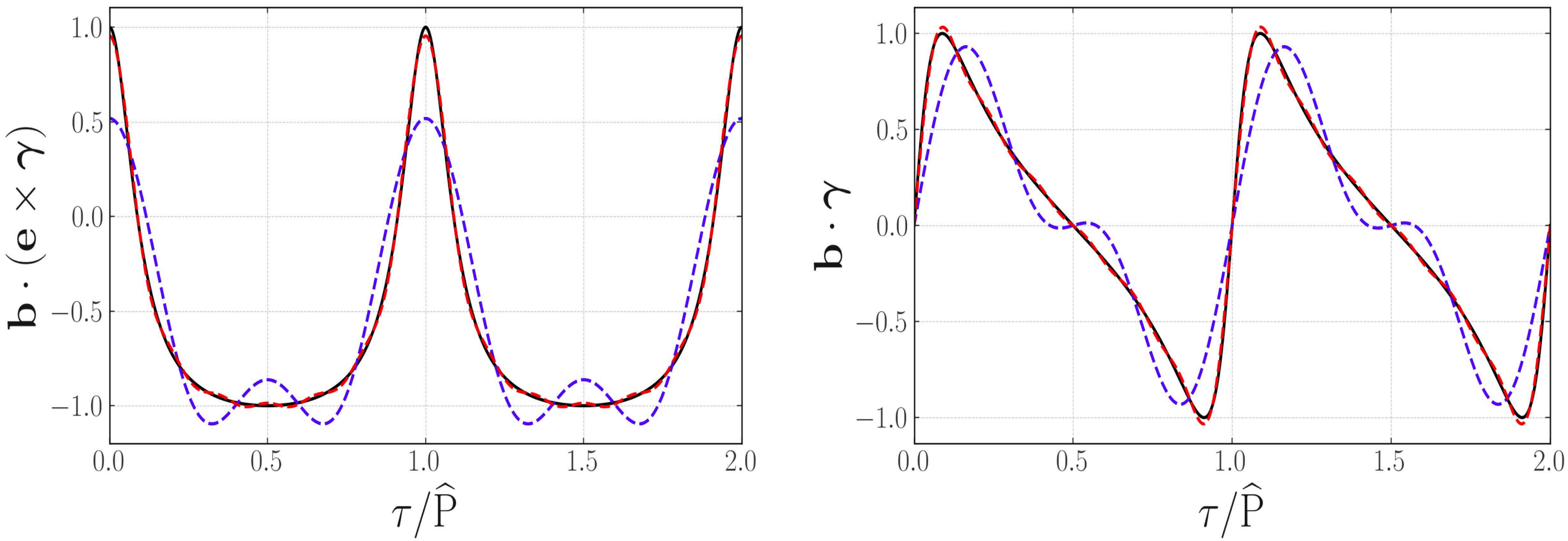
– **Extremal CUQ**: CUQ with $r \to 1 \;\Rightarrow\; \mathrm{P} \to \infty$ [AP '97]

– CUQ in a Pure State: $\mathbf{b}(0) = \mathbf{e} \times \boldsymbol{\gamma}$ and $r = 0.85$.



$\mathbf{b}(t)$-projections along different axes have different profiles, but $|\mathbf{b}(t)| = 1$

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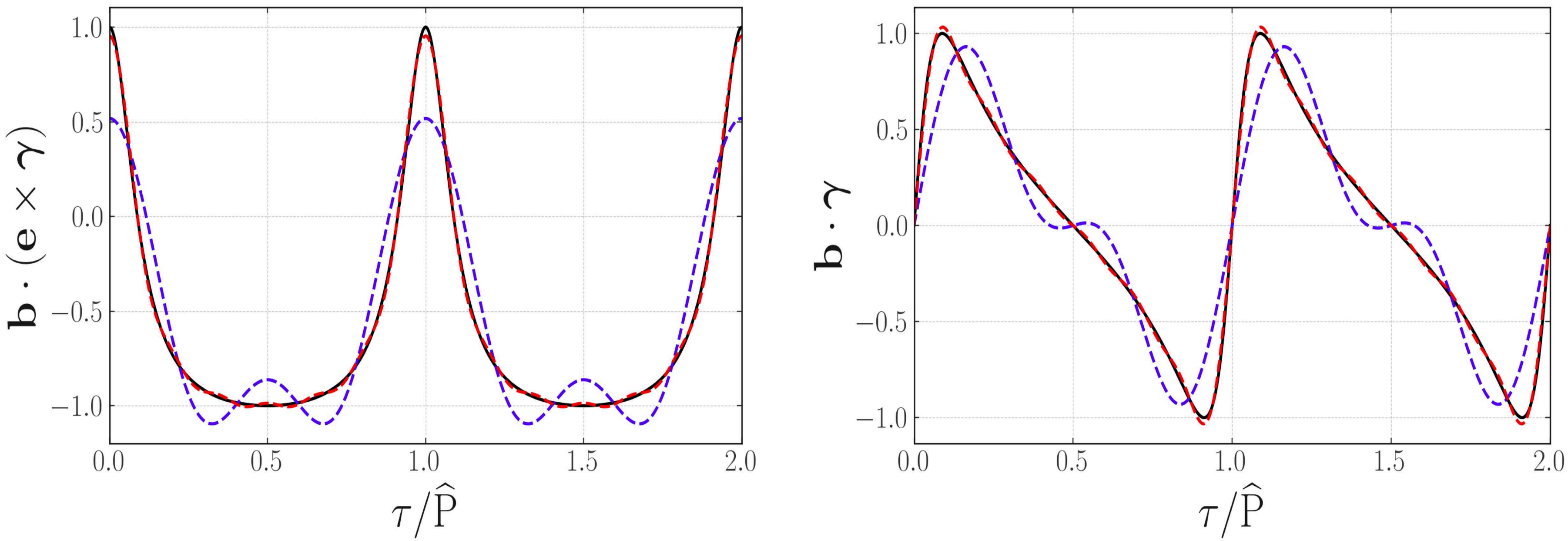

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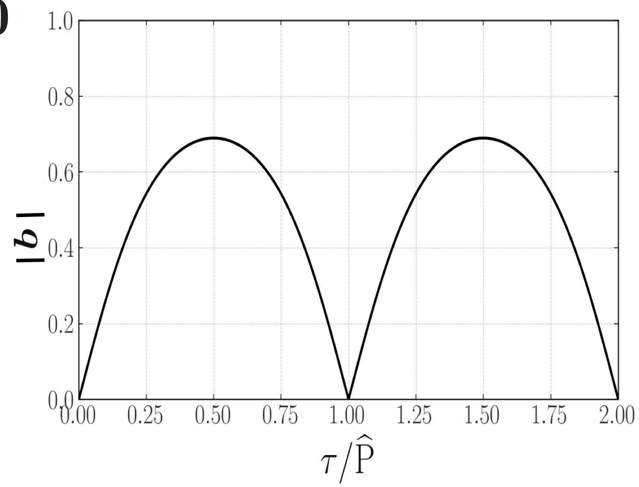
– CUQ in an initially Mixed State: $\mathbf{b}(0) = \mathbf{0}$

$$|\mathbf{b}(t)|^2 \;=\; 1 \;-\; \frac{(1-r^2)^2}{\left[1 - r^2\cos(2\pi t/\mathrm{P})\right]^2}\,.$$

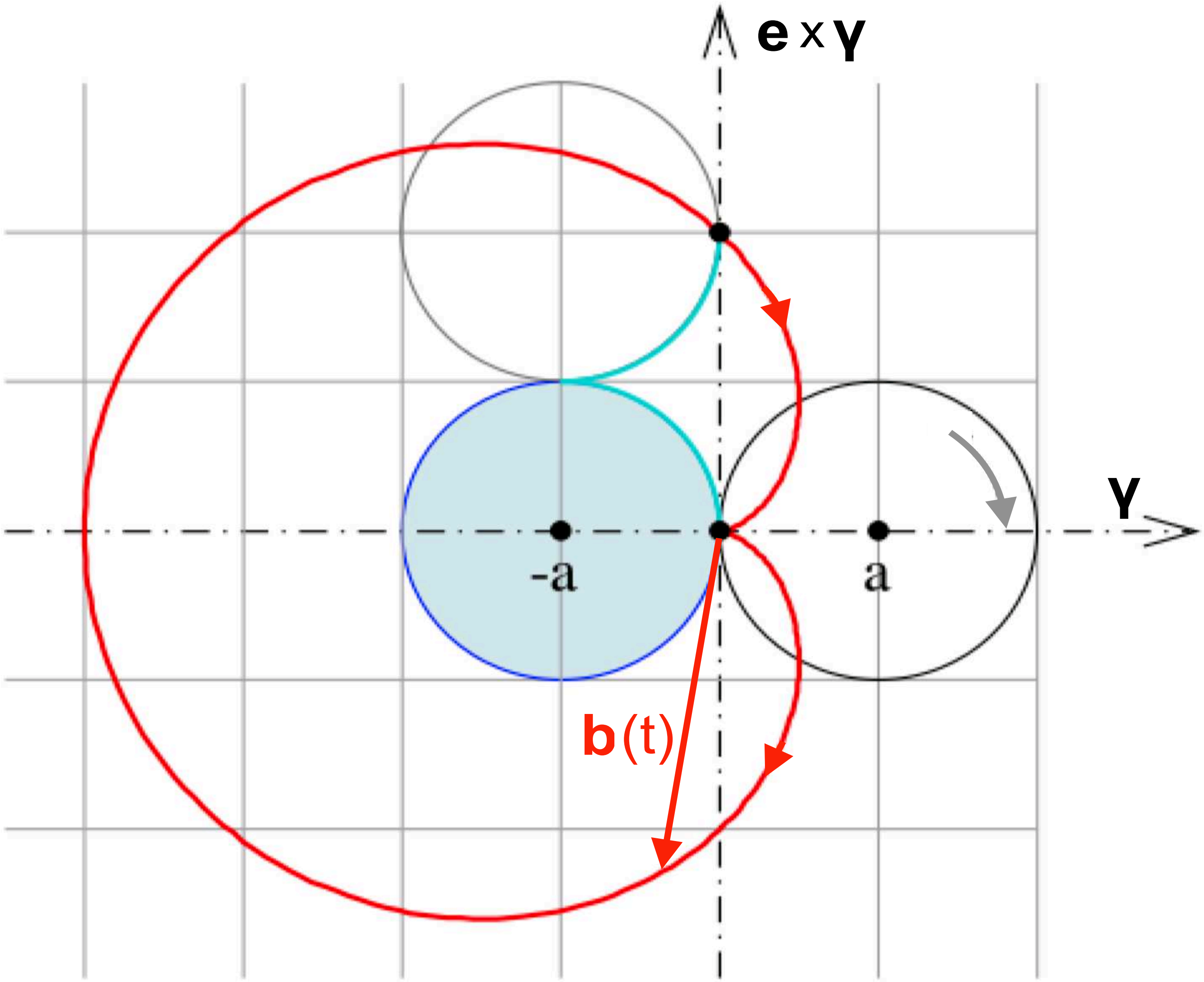
$\Rightarrow$ Coherence-Decoherence oscillations

with the same period P.



– **Fully mixed CUQ with $\mathrm{b}(0) = 0$: Coherence-decoherence oscillations**

Motion of $\mathbf{b}(t) \perp \mathrm{e}$ resembles a Cardioid:



– Entanglement entropy of unstable qubits in the codecaying frame

[Karamitros, McKelvey, AP '23]

$$S[\widehat{\rho}] \;\equiv\; -\,\mathrm{Tr}\big(\widehat{\rho}\,\ln\widehat{\rho}\big) \;=\; \ln 2 - \frac{1}{2}(1+|\mathbf{b}|)\,\ln(1+|\mathbf{b}|) - \frac{1}{2}(1-|\mathbf{b}|)\,\ln(1-|\mathbf{b}|)$$

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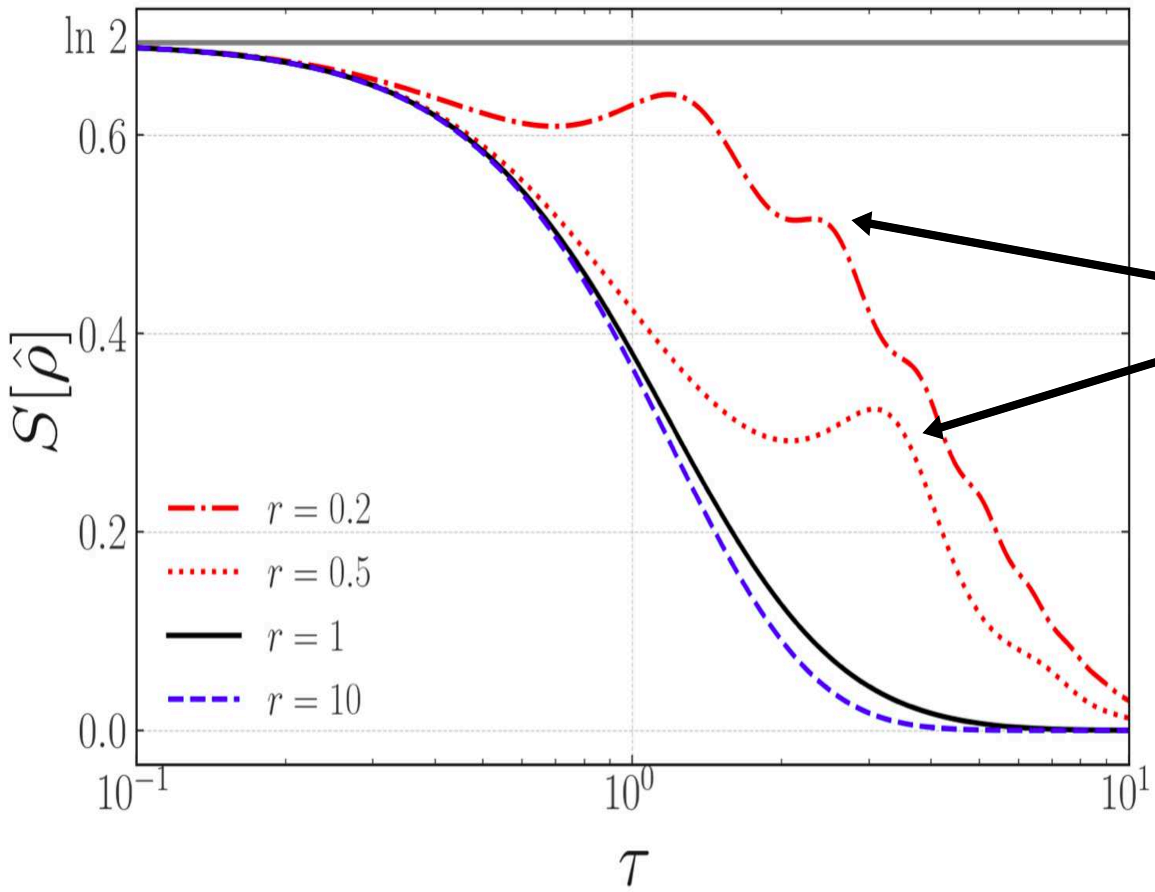
$$\theta_{\mathbf{e}\boldsymbol{\gamma}} \equiv \angle(\mathbf{e}\,,\boldsymbol{\gamma}) = 75^\circ:$$



Entropy oscillations enter through coherence-decoherence oscillations

• $B^0\bar{B}^0$-Meson System as a CUQ

[Karamitros, McKelvey, AP '25]

– Bloch-sphere description of meson–antimeson systems:

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\left|\frac{q}{p}\right|^2 &= \left|\frac{(\mathrm{H}_{\mathrm{eff}})_{21}}{(\mathrm{H}_{\mathrm{eff}})_{12}}\right| = \sqrt{\frac{1+r^2-2r\sin\theta_{\mathbf{e}\boldsymbol{\gamma}}}{1+r^2+2r\sin\theta_{\mathbf{e}\boldsymbol{\gamma}}}}\,.
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– Bloch-sphere description of meson–antimeson systems:

$$\Delta E \;=\; 2|\mathbf{E}|\,\mathrm{Re}\Big(\sqrt{1 - r^2 - 2ir\cos\theta_{\mathbf{e}\boldsymbol{\gamma}}}\Big)\,,$$
$$\Delta\Gamma \;=\; -\,4|\mathbf{E}|\,\mathrm{Im}\Big(\sqrt{1 - r^2 - 2ir\cos\theta_{\mathbf{e}\boldsymbol{\gamma}}}\Big)\,,$$
$$\left|\frac{q}{p}\right|^2 \;=\; \left|\frac{(\mathrm{H_{eff}})_{21}}{(\mathrm{H_{eff}})_{12}}\right| \;=\; \sqrt{\frac{1 + r^2 - 2r\sin\theta_{\mathbf{e}\boldsymbol{\gamma}}}{1 + r^2 + 2r\sin\theta_{\mathbf{e}\boldsymbol{\gamma}}}}\,.$$

Introduce the two dimensionless quantities,

$$\delta_M \;=\; \frac{|q/p|^4 - 1}{|q/p|^4 + 1} \quad \& \quad \eta \;=\; \frac{(\Delta E)\left(\frac{1}{2}\Delta\Gamma\right)}{(\Delta E)^2 - \left(\frac{1}{2}\Delta\Gamma\right)^2}\,,$$

to reexpress the Bloch-sphere parameters as follows:

$$r \;\equiv\; \frac{|\boldsymbol{\Gamma}|}{2|\mathbf{E}|} \;=\; \frac{\sqrt{1+4\eta^2} \mp \sqrt{1-\delta_M^2}}{\sqrt{4\eta^2 + \delta_M^2}}\,, \quad \text{for } r \lessgtr 1\,,$$

$$\sin\theta_{\mathbf{e}\boldsymbol{\gamma}} \;=\; -\,\frac{1+r^2}{2r}\delta_M\,, \quad |\mathbf{E}|^2 \;=\; \frac{(\Delta E)^2 - \left(\frac{1}{2}\Delta\Gamma\right)^2}{4\,(1 - r^2)}\,.$$

– Experimental meson–antimeson mixing data: [PDG '24]

Meson	$\Delta E = \Delta m$ [ps^{-1}]	$\Delta\Gamma$ [ps^{-1}]	$\lvert q/p\rvert - 1$
K^0	$0.005293 \pm 9 \times 10^{-6}$	$0.01 \pm 5 \times 10^{-6}$	-0.003239 ± 10^{-6}
D^0	0.01 ± 0.001	0.03 ± 0.003	$(-5.00 \pm 0.04) \times 10^{-3}$
$B^0_{\rm d}$	0.5069 ± 0.0019	$(0.7 \pm 7) \times 10^{-3}$	$(1.0 \pm 0.8) \times 10^{-3}$
$B^0_{\rm s}$	17.765 ± 0.006	0.084 ± 0.005	$(0.1 \pm 1.4) \times 10^{-3}$

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$\Downarrow$

[Karamitros, McKelvey, AP '25]

Meson	$r \equiv \lvert\mathbf{\Gamma}\rvert/(2\lvert\mathbf{E}\rvert)$	$\theta_{\mathbf{e}\boldsymbol{\gamma}}[^\circ]$	$\lvert\mathbf{E}\rvert$[ps^{-1}]
K^0	$0.945 \pm 2 \times 10^{-3}$	$179.6322 \pm 1\times 10^{-4}$	$2.64652 \times 10^{-3} \pm 7\times 10^{-8}$
D^0	1.5 ± 0.2	179 ± 2	$(5.00 \pm 0.04)\times 10^{-3}$
$B^0_{\rm d}$	$(1 \pm 4) \times 10^{-3}$	270 ± 90	0.253 ± 0.001
$B^0_{\rm s}$	$(2.4 \pm 0.2) \times 10^{-3}$	182.7 ± 33.8	8.9 ± 0.1

$\boldsymbol{r \geq 1}$: Unstable qubit is overdamped exhibiting no oscillations

– $B^0_d \bar{B}^0_d$-meson mixing data:

[U. Nierste '04, A. Lenz et al '24]

	$\Delta E\,[\mathrm{ps}^{-1}]$	$\Delta\Gamma\,[\mathrm{ps}^{-1}]$	$\lvert q/p\rvert - 1$
Experiment	0.5069 ± 0.019	$(0.7 \pm 7)\times 10^{-3}$	$(-1.00 \pm 0.8)\times 10^{-3}$
Theory	0.535 ± 0.021	$\left(2.7 \pm 0.4\right) \times 10^{-3}$	$(2.6 \pm 0.3) \times 10^{-4}$
	$r \equiv \lvert\mathbf{\Gamma}\rvert/(2\lvert\mathbf{E}\rvert)$	$\theta_{\mathbf{e}\boldsymbol{\gamma}}[^\circ]$	$\lvert\mathbf{E}\rvert[\mathrm{ps}^{-1}]$
Experiment	$(1 \pm 4) \times 10^{-3}$	-90 ± 90	0.253 ± 0.001
Theory	$(2.5 \pm 0.4) \times 10^{-3}$	-5 ± 3	0.28 ± 0.01

$\Rightarrow$ New physics may turn $B^0_d \bar{B}^0_d$ into a CUQ, with $\Delta\Gamma = 0 \longrightarrow \theta_{\mathbf{e}\boldsymbol{\gamma}} = -90^\circ$.

– $B_{\rm d}^0\bar{B}_{\rm d}^0$-meson mixing data: [U. Nierste '04, A. Lenz et al '24]

	ΔE [ps^{-1}]	$\Delta\Gamma$ [ps^{-1}]	$\lvert q/p\rvert - 1$
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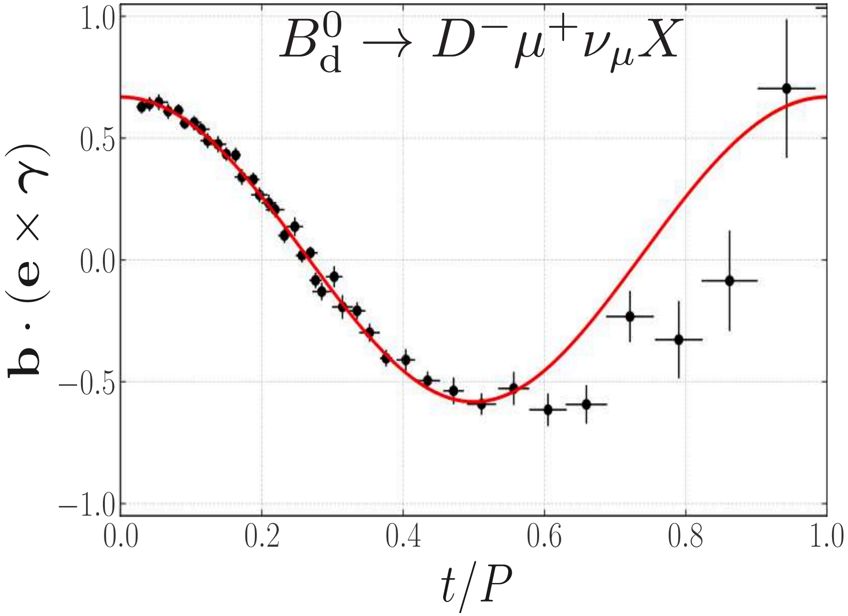
– How can we observe any CUQ dynamics? [Karamitros, McKelvey, AP '25]

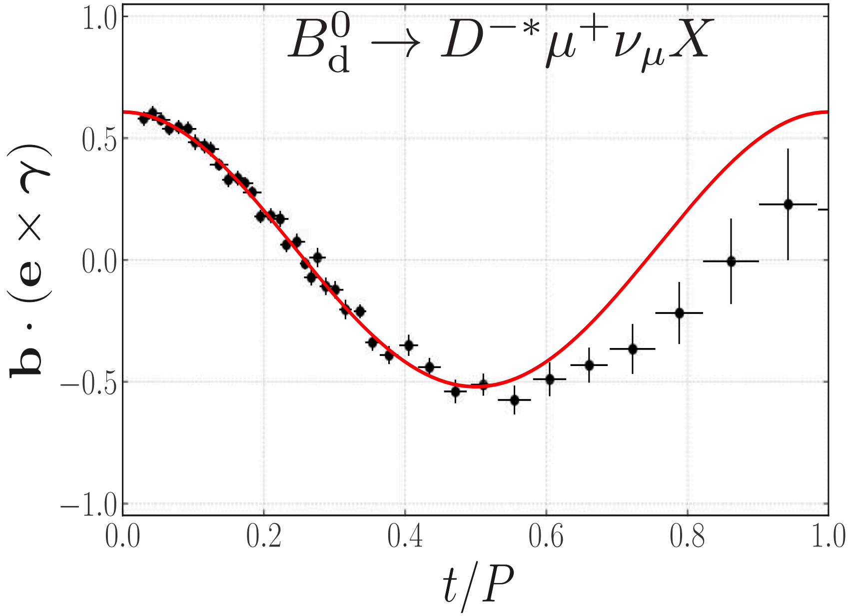
$\Rightarrow$ Look for *non*-sinusoidal oscillations of flavour asymmetries, beyond the principal Fourier harmonic $n = 1$:

$$\delta(t) \;=\; \mathbf{b}(t)\cdot(\mathbf{e}\times\boldsymbol{\gamma}) \;=\; d_0 + \sum_{n=1}^{\infty} d_n \cos(n\omega t)\,,$$

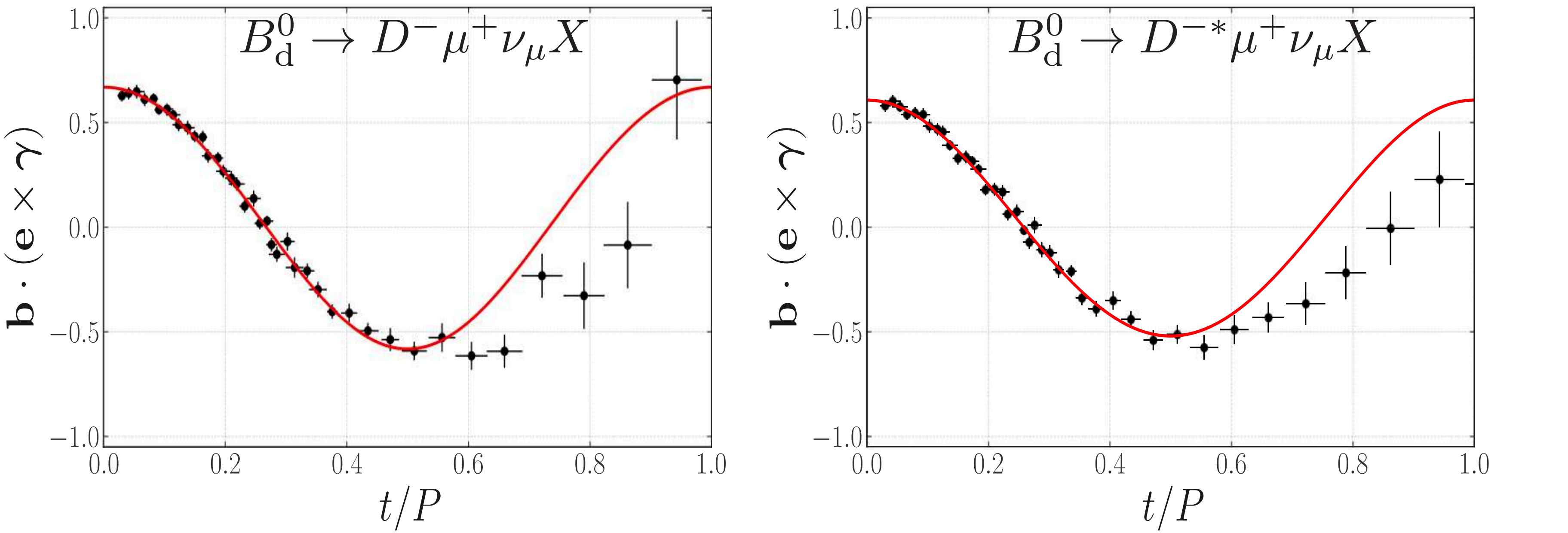
with $\omega \equiv 2\pi/\mathrm{P} = 2|\mathbf{E}|\sqrt{1-r^2}$.

– **Experimental data fits with three Fourier modes (d_0, d_1, d_2):** [LHCb '12]





– Experimental data fits with three Fourier modes (d_0, d_1, d_2): [LHCb '12]



– Analysis of Anharmonicity: [Karamitros, McKelvey, AP '25]

-	$B_d^0 \to D^- \mu^+ \nu_\mu X$ (2012)		$B_d^0 \to D^{-*} \mu^+ \nu_\mu X$ (2012)	
	$d_n \pm \delta d_n$	p-value	$d_n \pm \delta d_n$	p-value
d_0	0.06 ± 0.07	45%	0.04 ± 0.06	58%
d_1	0.625 ± 0.004	$< 0.01\%$	0.564 ± 0.003	$< 0.01\%$
d_2	-0.013 ± 0.009	18%	0.007 ± 0.008	37%

Null hypothesis: $d_0 = d_1 = d_2 = 0$.

• CUQs Beyond Particle Physics

Potential applications of CUQs to systems Beyond the Standard Model:

- Higgs boson as a CUQ via a CP-odd mixing with another scalar. [AP '97]

- Heavy Majorana neutrinos as (extremal) CUQs $\rightarrow$ Resonant Leptogenesis

- CUQs $\rightarrow$ Critical Unstable Qu<u>Trits</u> $\rightarrow$ <u>Tri</u>-Resonant Leptogenesis

- Axion(s)–photon mixing as a CUQ system, e.g. in stars. [G. Raffelt, L. Stodolsky '88]

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Beyond Particle Physics: [R.P. Feynman '86]

- Probing the new oscillation phenomena in CUQs with quantum gates.

- Quantum simulation of non-Hermitian effective Hamiltonians like $\mathrm{H}_{\mathrm{eff}}^{\mathrm{CUQ}}$.

• SUMMARY

- CUQs is a novel class of unstable qubits characterised by:

  (i) $\mathrm{e} \perp \gamma$ and (ii) $0 < r \equiv |\boldsymbol{\Gamma}|/(2|\mathbf{E}|) \leq 1$

  $\Rightarrow$ Two energy eigenstates of $\mathsf{H}_{\mathrm{eff}}^{\mathrm{CUQ}}$ have equal lifetimes

- CUQs exhibit two atypical behaviours in their *co-decaying* frame:

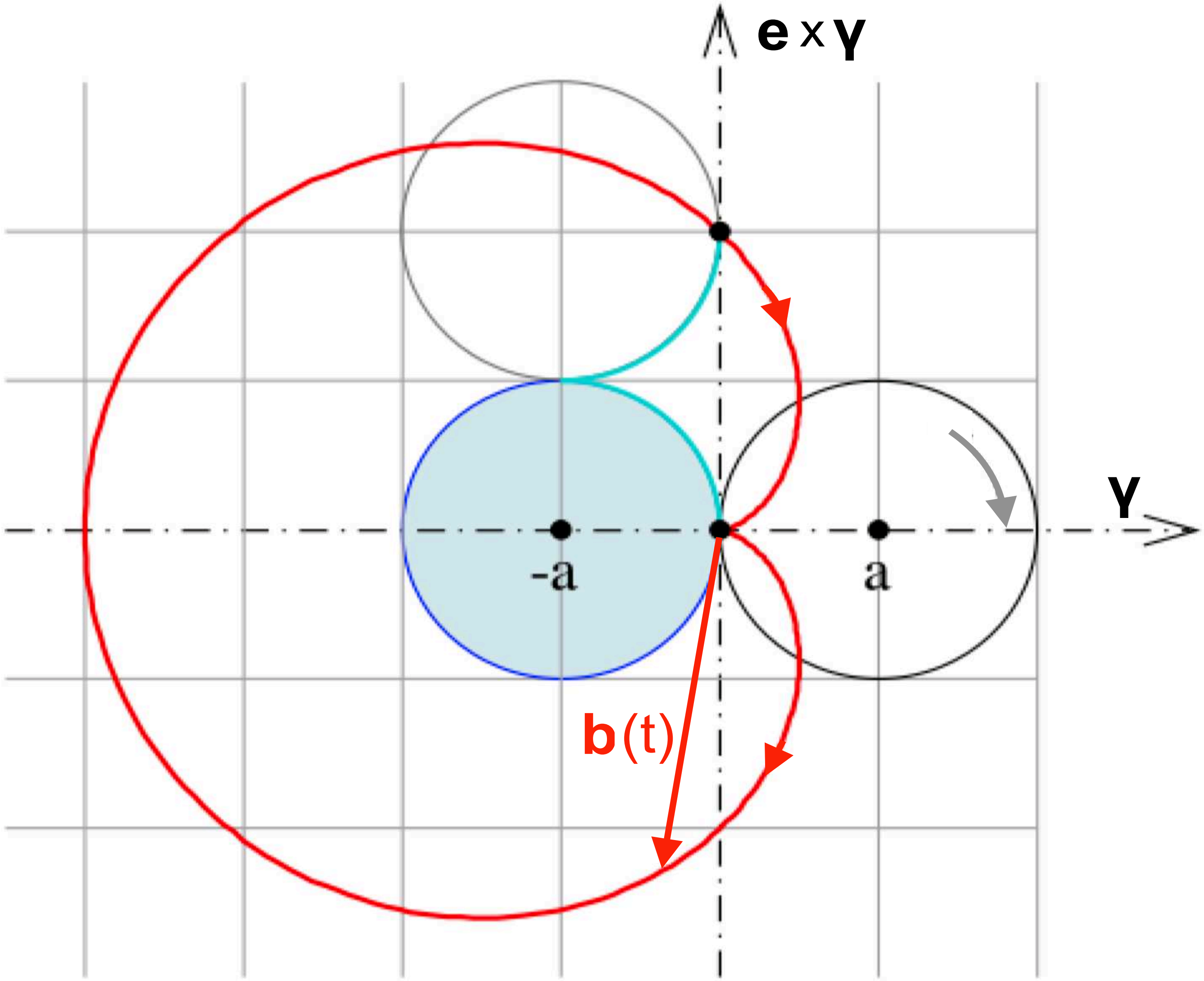
  (i) Coherence-Decoherence oscillations

  (ii) Bloch vector b sweeps out *unequal* areas in equal intervals of time

- Quasi-CUQs with $\big||\theta_{\mathrm{e}\gamma}| - 90^\circ\big| \lesssim 10^\circ$ retain most features of CUQs.

- Anharmonicity observables can probe deeper the structure of $\mathsf{H}_{\mathrm{eff}}$, due to non-sinusoidal oscillations (as in the $B^0\bar{B}^0$-meson system).

- Beyond Particle Physics (to be explored): quantum simulation of $\mathsf{H}_{\mathrm{eff}}^{\mathrm{CUQ}}$, quantum entanglement/dissipation, quantum computing . . .



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Message from the Editor-in-Chief

The multidisciplinary journal *Universe* is aiming to follow and, hopefully, to lead to the largest extent as possible the ever-self renovating threads which weave mathematical theories with our understanding of the magnificent natural world. On behalf of all the distinguished members of the Advisory and Editorial Boards, I extend my welcome to this journal and look forward to hearing from the interested contributors and learning about their valuable research.

Aims

Universe (ISSN 2218-1997) is an international peer-reviewed open access journal focused on theoretical, experimental, and observational progress in fundamental and applied physics from circumterrestrial space to cosmological scenarios. The journal focuses on publishing research articles, reviews, and communications, but other article types will also be considered. Our aim is to encourage scientists to publish their research results in as much detail as possible. There is no restriction on the maximum length of the papers.

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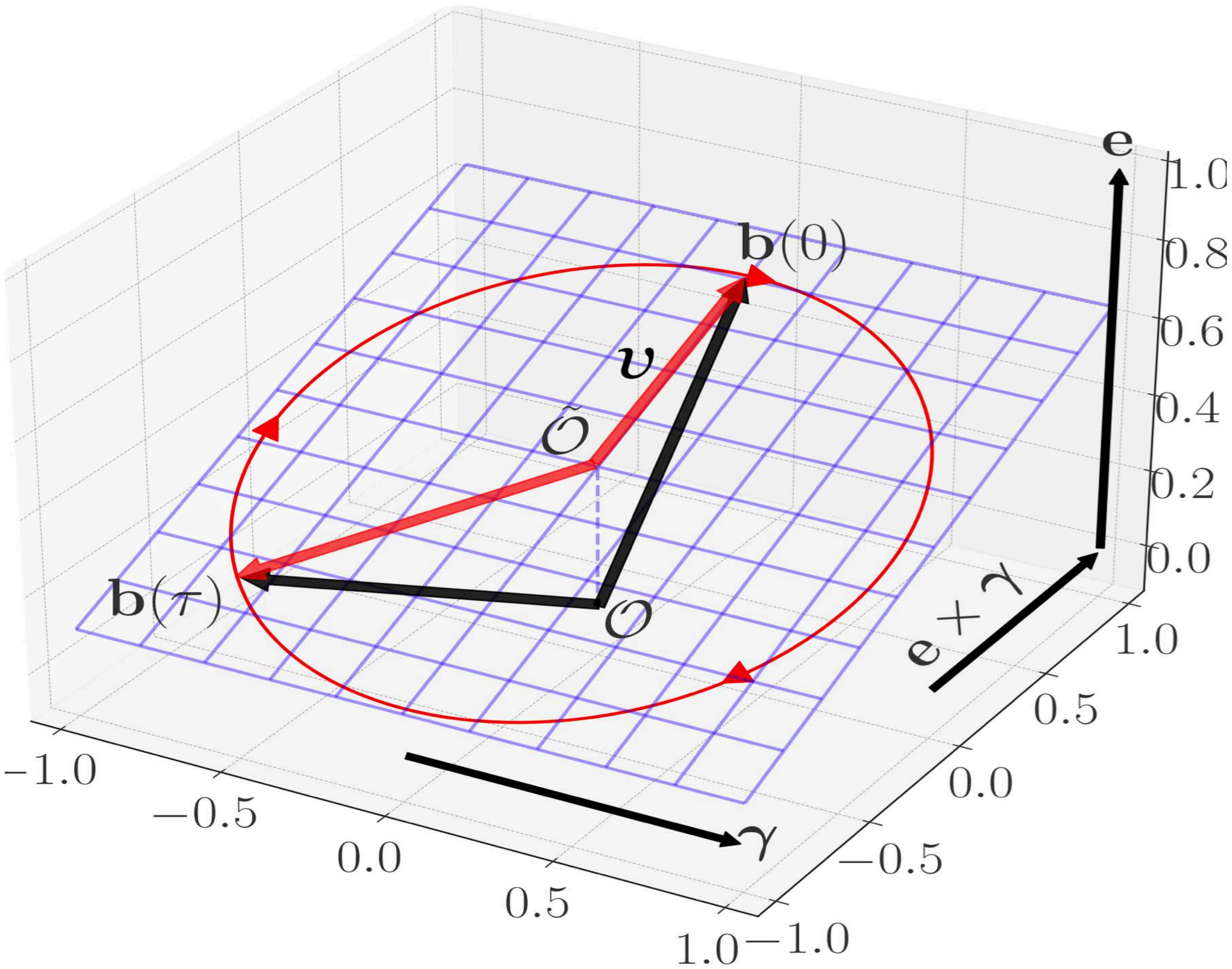
Back-Up Slides

- **Plane of oscillation for a CUQ with an arbitrary initial condition:**

  e.g., $\mathbf{b}(0) = 0.6\,\mathbf{e} + 0.8\,\mathbf{e}\times\boldsymbol{\gamma} \not\perp \mathbf{e}$ [Karamitros, McKelvey, AP '25]



Torsion of $\mathbf{b}(\tau)$:

$$\mathcal{T} \propto \dddot{\mathbf{b}}\cdot\big(\ddot{\mathbf{b}}\times\dot{\mathbf{b}}\big) = 0\,,$$

with $\dot{\mathbf{b}} \equiv \mathrm{d}\mathbf{b}/\mathrm{d}\tau$.

$$\Downarrow$$

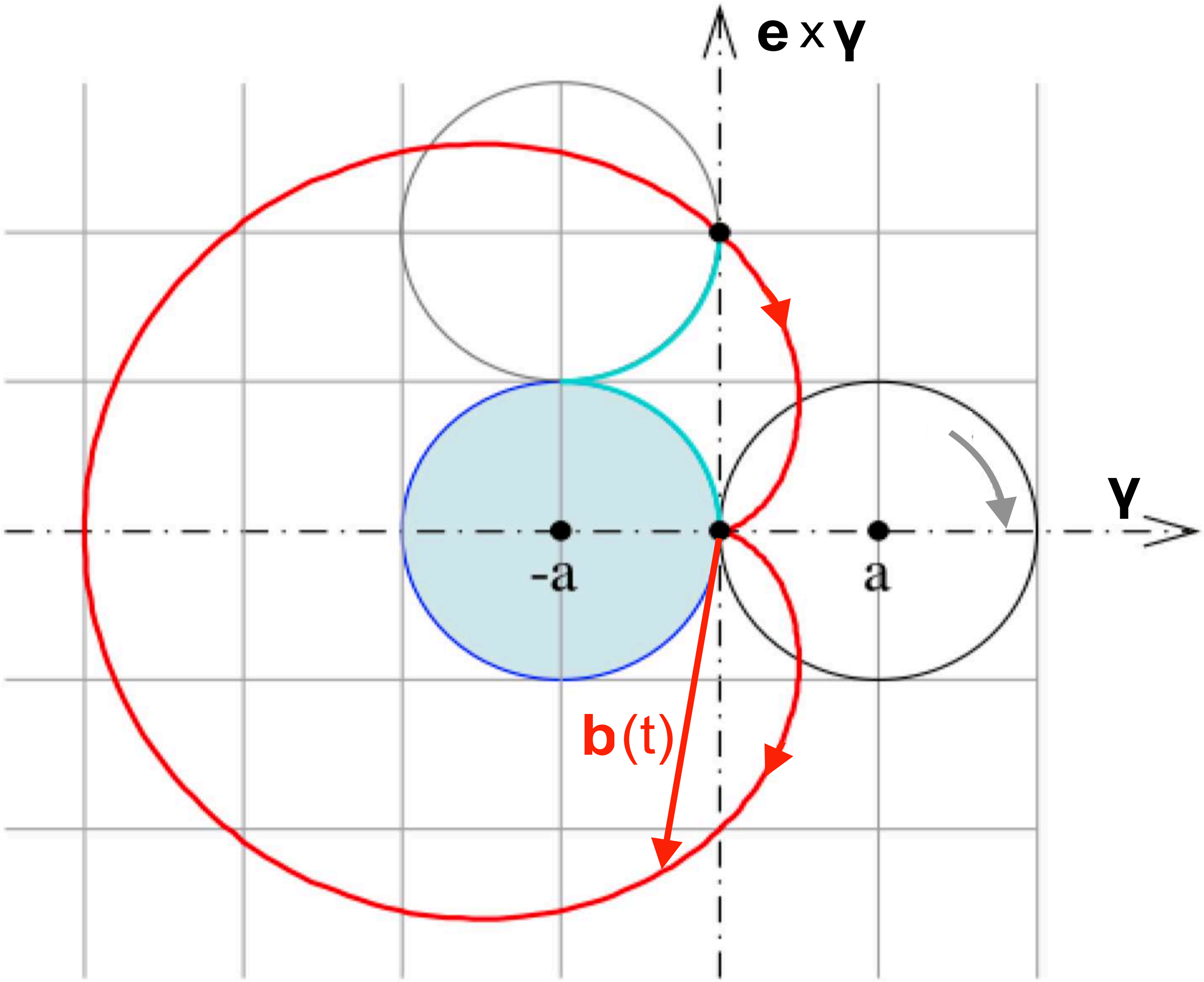
Motion of $\mathbf{b}(\tau)$ restricted on a plane

Oscillation plane spanned by: $\boldsymbol{\gamma}$ & $\widehat{\boldsymbol{v}} = \frac{1}{N}\Big[\mathbf{e}\times\boldsymbol{\gamma} + r\Big(\boldsymbol{\gamma}\times\mathbf{b}(0)\Big)\times\boldsymbol{\gamma}\Big]$

For $\mathbf{b}(0) = \mathbf{0} \longrightarrow \widehat{\boldsymbol{v}} = \mathbf{e}\times\boldsymbol{\gamma} \perp \mathbf{e}$

• **Fully mixed CUQ with $\mathbf{b}(0) = 0$: Coherence-decoherence oscillations**

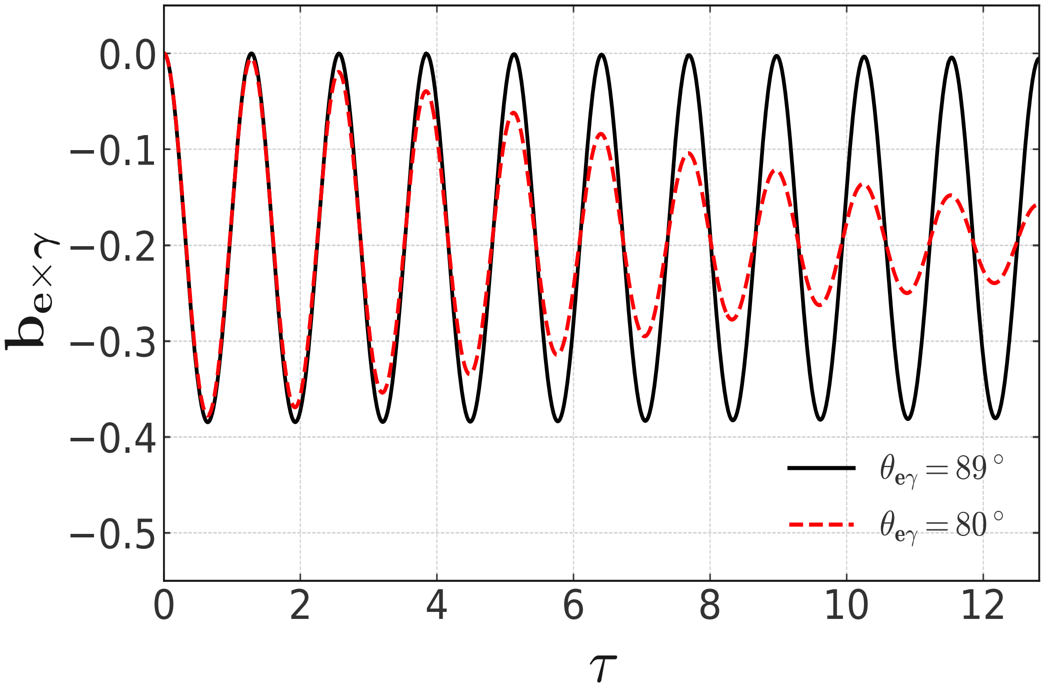
Motion of $\mathbf{b}(t) \perp \mathbf{e}$ resembles a Cardioid $\longrightarrow$ **Quantonomy**

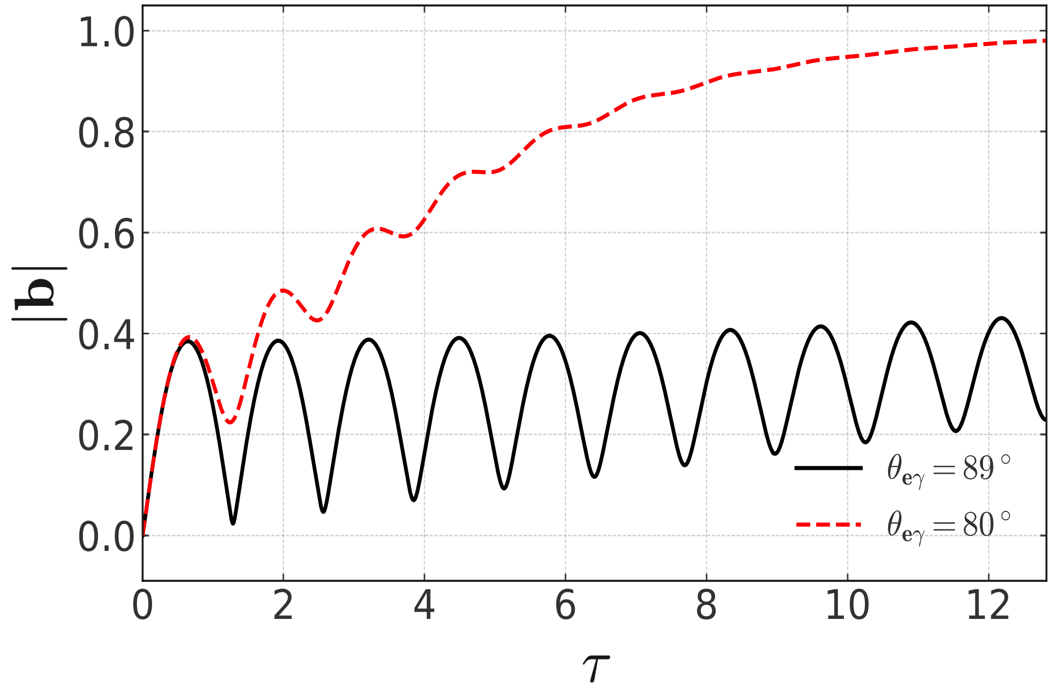


• Quasi-CUQ Scenarios: Coherence-Decoherence Oscillations

[Karamitros, McKelvey, AP '23]

Parameter $r = 0.2$ & Initial condition: $\mathbf{b}(0) = \mathbf{0}$





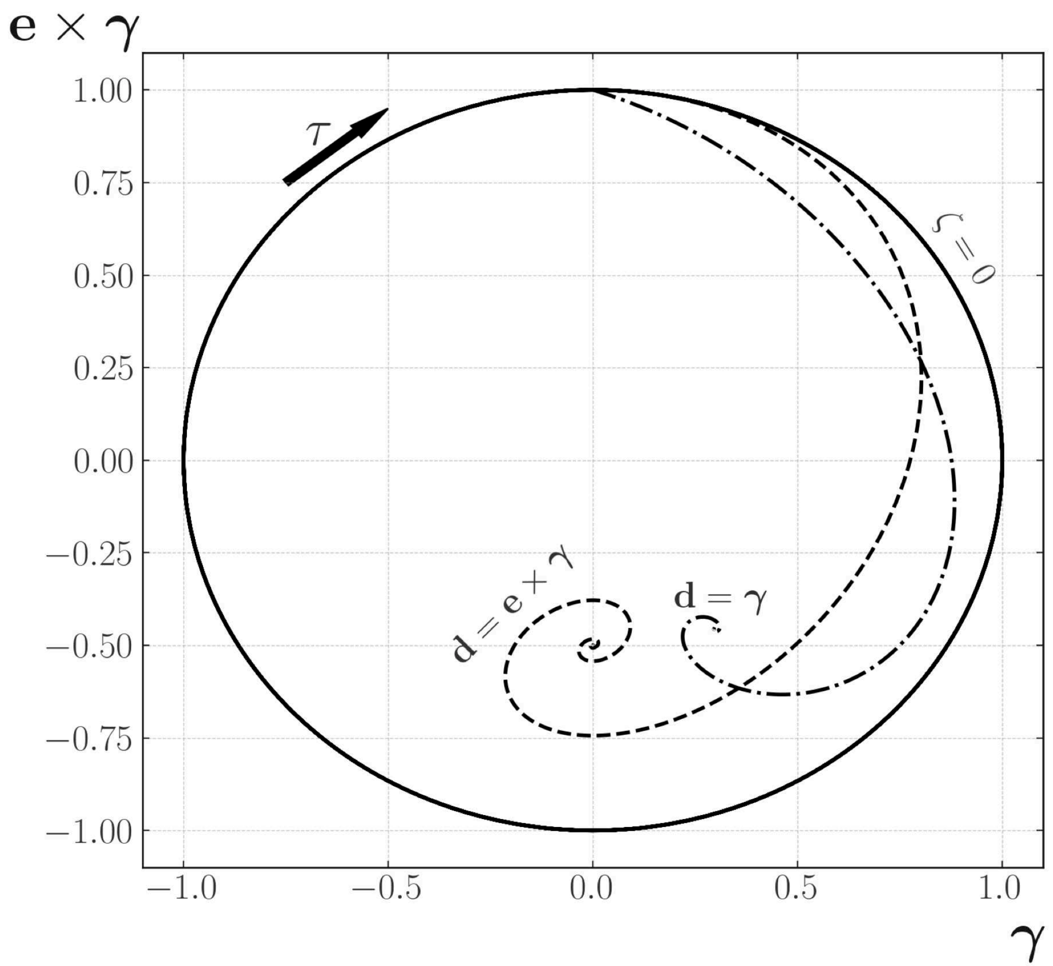
$\longrightarrow$ Motion of $\mathbf{b}(t)$ is no longer contained on a plane

• Decoherence due to CUQs interaction with the environment

[Karamitros, McKelvey, AP '23]

Parameter $r = 0.5$ & Initial condition: $\mathbf{b}(0) = \mathbf{e} \times \boldsymbol{\gamma}$

[G. Lindblad '76, . . . P. Huet, M. Peskin '95]



Decoherent dissipation:

$$\frac{\mathrm{d}}{\mathrm{d}t}\rho = -i\,[\mathsf{E}\,,\rho] - \frac{1}{2}\{\Gamma\,,\rho\} - [D\,,[D\,,\rho]]\,,$$

with $\mathsf{D} = D^0\mathbf{1}_2 - \mathbf{D}\cdot\boldsymbol{\sigma}$

$$\Downarrow$$

$$\frac{\mathrm{d}\mathbf{b}}{\mathrm{d}\tau} = -\frac{1}{r}\mathbf{e}\times\mathbf{b} + \boldsymbol{\gamma} - (\boldsymbol{\gamma}\cdot\mathbf{b})\,\mathbf{b} + \zeta(\mathbf{d}\cdot\mathbf{b})\,\mathbf{d} - \zeta\mathbf{b}\,,$$

with $\zeta \equiv 4\,|\mathbf{D}|^2/|\boldsymbol{\Gamma}| \;\to\; 1$

and $\mathbf{d} \equiv \mathbf{D}/|\mathbf{D}|$