Critical Unstable Qubits in Particle Physics and Beyond

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Based on:

- D. Karamitros, T. McKelvey, AP, PRD111 (2025) 096020 [MITP-25-020]
- D. Karamitros, T. McKelvey, AP, PRD108 (2023) 016006
- AP, NPB504 (1997) 61

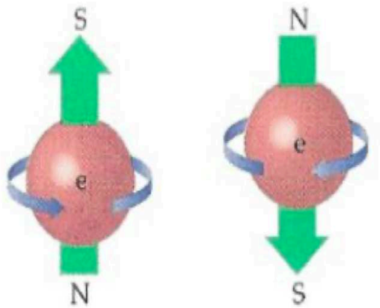
Outline:

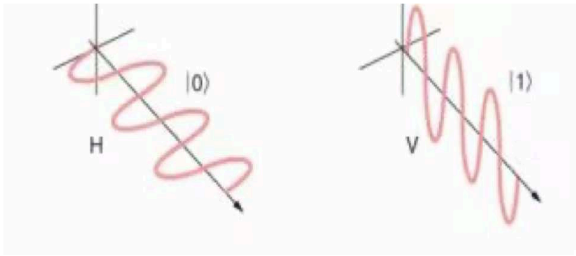
- UN celebrates 100 years of Quantum Science since '1925'
- Qubits in the Bloch Sphere Formalism
- Unstable Qubits in the Co-Decaying Frame
- Critical Unstable Qubits (CUQs)
- $B^0\bar{B}^0$-Meson System as a CUQ
- CUQs Beyond Particle Physics
- Summary

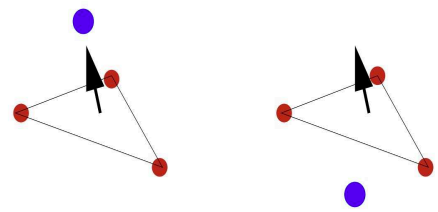
• Qubits in the Bloch Sphere Formalism

– **Qubit**: two-level quantum system that can be in a superposition of two *distinct* quantum states *or* bits, e.g. $|0\rangle$ and $|1\rangle$.

Examples: spin-states of e^- linearly polarized light Ammonia molecule







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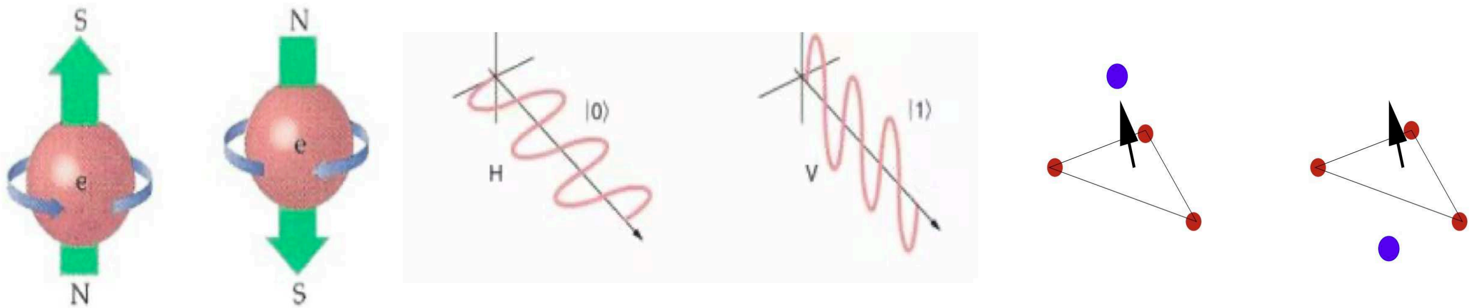

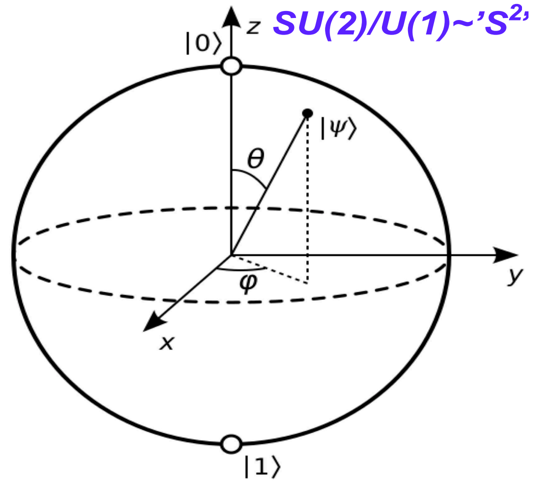

– **Bloch sphere rep:** $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$, [F. Bloch '46]

with $\alpha = \cos\left(\frac{\theta}{2}\right)$ & $\beta = e^{i\varphi}\sin\left(\frac{\theta}{2}\right)$

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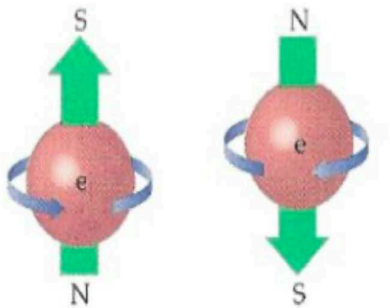

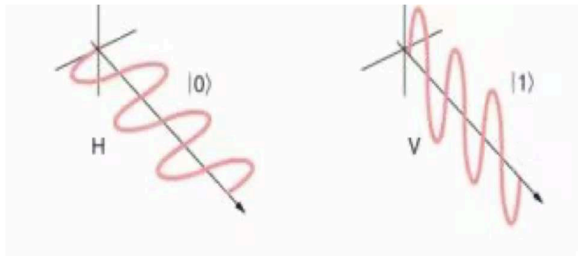

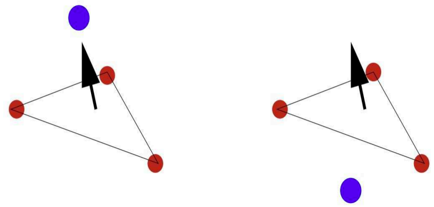

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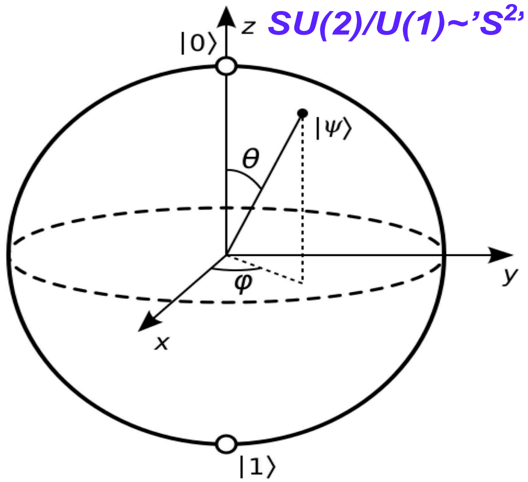

– **Time evolution of qubit:** [E. Schrödinger '1925]

$$i\,\frac{\mathrm{d}}{\mathrm{d}t}\,|\psi\rangle \;=\; \mathsf{H}\,|\psi\rangle\,,$$

with H is a 2D Hamiltonian written in the qubit basis: $\{|0\rangle\,,|1\rangle\}$.

– **Density Matrix:** an operator ρ that encodes all the information of a quantum system, for both *pure* and *mixed* states,

$$\rho \; = \; \sum_{k=1}^{N} w_k \, |\psi^k\rangle \, \langle\psi^k| \,, \quad \text{with } \rho^\dagger = \rho \;\; \& \;\; \mathrm{Tr}\, \rho = 1$$

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– **Time evolution of qubit reformulated:** [J. von Neumann '27, I.I. Rabi '37]

$$\frac{\mathrm{d}}{\mathrm{d}t}\rho \;=\; -i\,[\mathsf{H}\,,\rho] \quad \Rightarrow \quad \frac{\mathrm{d}}{\mathrm{d}t}\mathbf{b} \;=\; -2\,\mathbf{E}\times\mathbf{b}\,,$$

where $\mathsf{H} \;\equiv\; E_\mu\sigma^\mu = E^0\mathbf{1}_2 - \mathbf{E}\cdot\boldsymbol{\sigma}$ and $\mathsf{H} = \mathsf{H}^\dagger$

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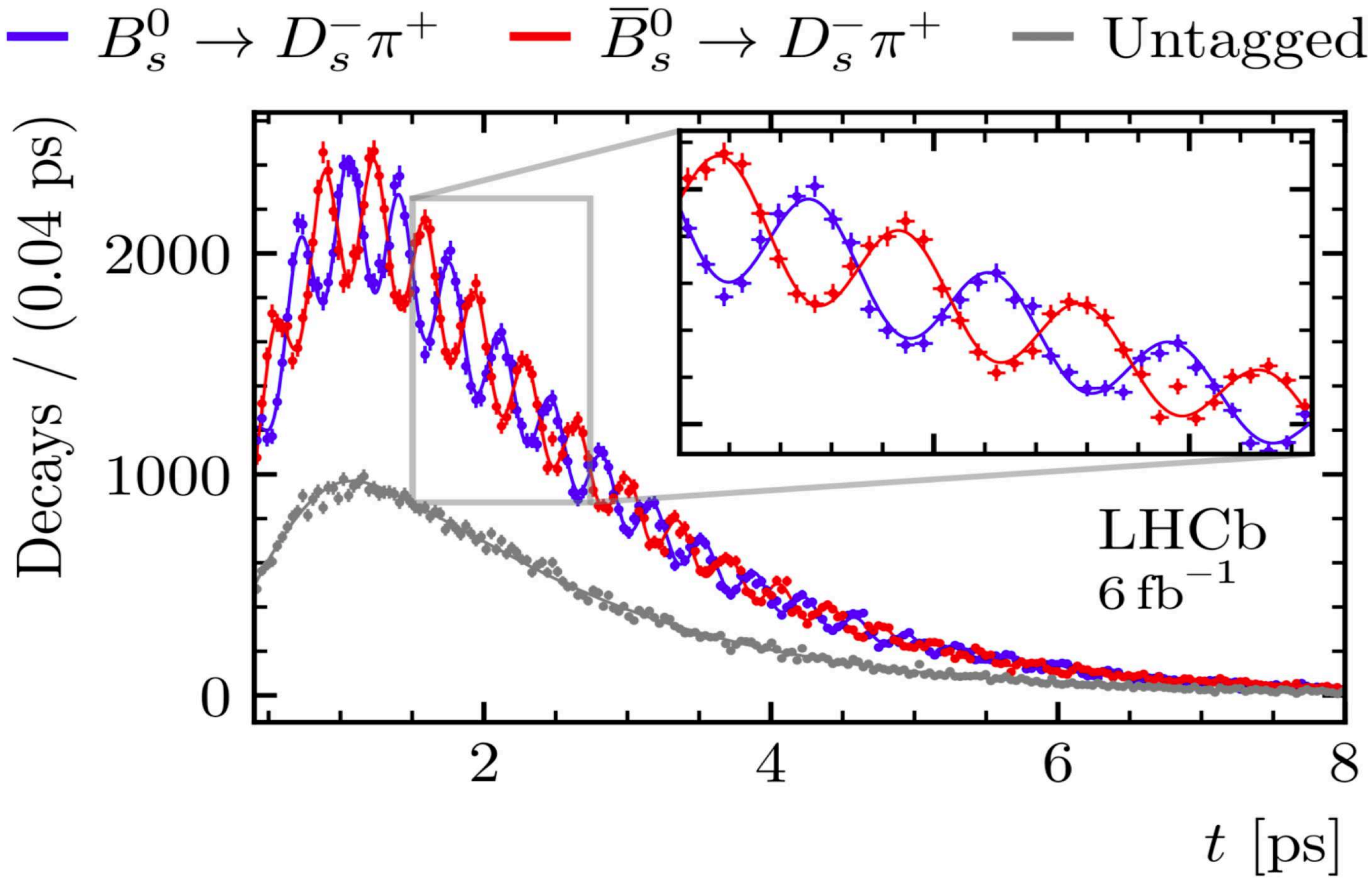

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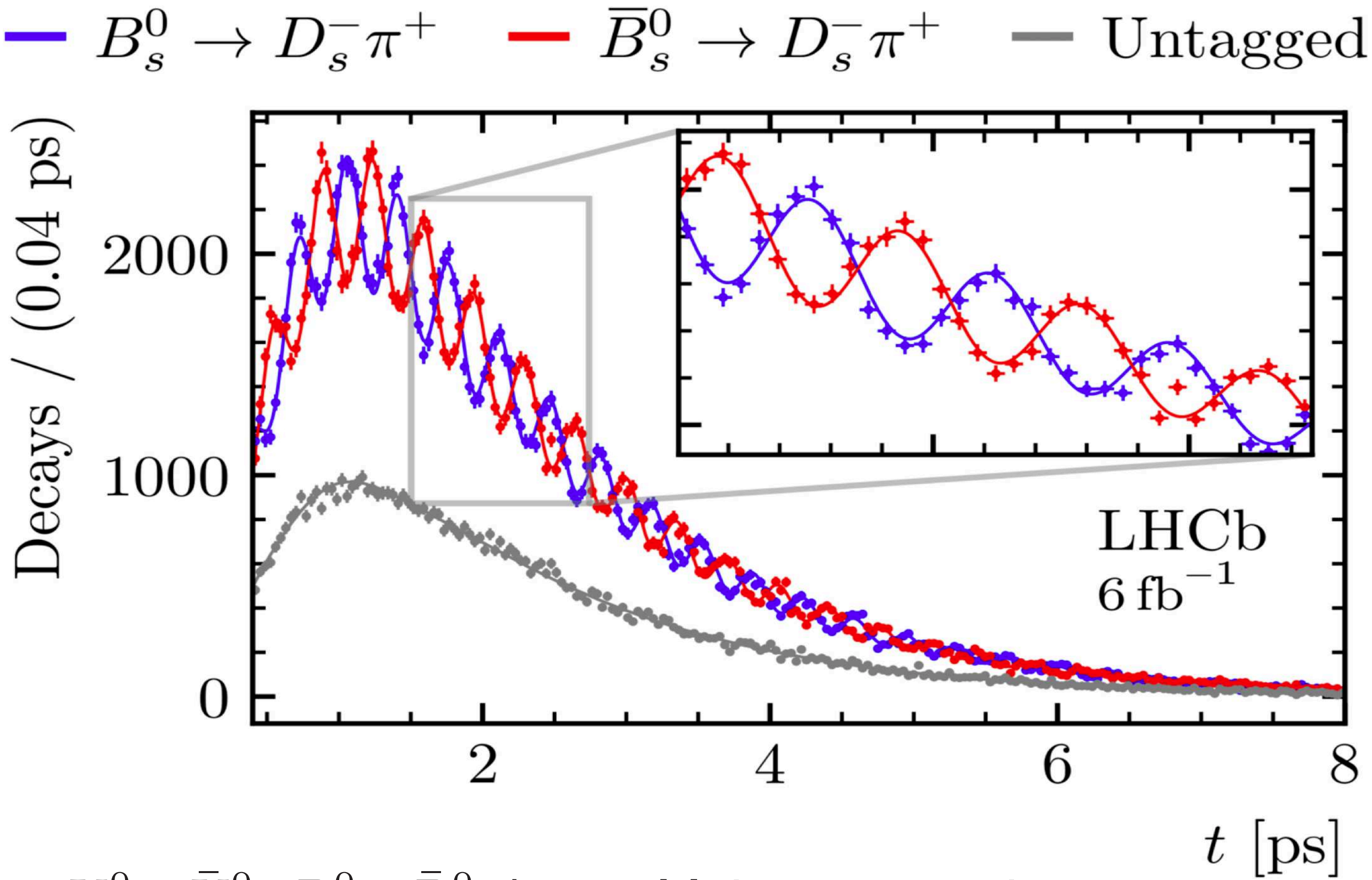

Other systems: $K^0 - \bar{K}^0$, $D^0 - \bar{D}^0$, heavy Majorana neutrinos . . .

At large t, they approach a pure state || the long-lived state of the system.

– Time evolution of unstable qubit:

[V. Weisskopf, E. Wigner '30;
T. D. Lee, R. Oehme, C.-N. Yang '57
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$$i\,\frac{\mathrm{d}}{\mathrm{d}t}\,|\Psi\rangle \;=\; \mathsf{H}_{\mathsf{eff}}\,|\Psi\rangle \quad\Rightarrow\quad \frac{\mathrm{d}}{\mathrm{d}t}\,\rho \;=\; -i\,[\mathsf{E}\,,\rho] \;-\; \frac{1}{2}\{\Gamma\,,\rho\}\,,$$

where $\mathsf{H}_{\mathsf{eff}} \;\equiv\; \mathsf{E} - \frac{i}{2}\Gamma \;\neq\; \mathsf{H}^{\dagger}_{\mathsf{eff}}$, with $\mathsf{E} \equiv E_\mu \sigma^\mu = E^0 \mathbf{1}_2 - \mathbf{E}\cdot\boldsymbol{\sigma}$
and $\Gamma \equiv \Gamma_\mu \sigma^\mu = \Gamma^0 \mathbf{1}_2 - \boldsymbol{\Gamma}\cdot\boldsymbol{\sigma}$.

Probability leakage from the qubit subspace:

$$\frac{\mathrm{d}}{\mathrm{d}t}\,\mathsf{Tr}\,\rho \;=\; -\mathsf{Tr}\,(\Gamma\rho) \;\neq\; 0 \quad\Rightarrow\quad \mathsf{Tr}\,\rho(t) \;\neq\; 1\,, \quad \text{for } t > 0.$$

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$\widehat{\rho}$: co-decaying density matrix $\;\longrightarrow\;$ $\mathbf{b}$: co-decaying Bloch vector.

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– **Time evolution of unstable qubit in the co-decaying frame:**

$$\frac{\mathrm{d}}{\mathrm{d}t}\,\widehat{\rho} \;=\; -i\,[\mathsf{E}\,,\widehat{\rho}] \;-\; \frac{1}{2}\{\Gamma\,,\widehat{\rho}\} \;+\; \mathrm{Tr}\,(\Gamma\widehat{\rho})\,\widehat{\rho}\,,$$

[Karamitros, McKelvey, AP '23, '25]

$$\Rightarrow \quad \boxed{\frac{\mathrm{d}\mathbf{b}}{\mathrm{d}t} \;=\; -2\,\mathbf{E}\times\mathbf{b} \;+\; \boldsymbol{\Gamma} \;-\; (\boldsymbol{\Gamma}\cdot\mathbf{b})\,\mathbf{b}}$$

Note: differential equation non-linear in $\mathbf{b}$, but first-order in t.

– **Master evolution equation of unstable qubit (co-decaying frame):**

$$\frac{\mathrm{d}\mathbf{b}}{\mathrm{d}\tau} = -\frac{1}{r}\mathbf{e}\times\mathbf{b} + \boldsymbol{\gamma} - (\boldsymbol{\gamma}\cdot\mathbf{b})\,\mathbf{b}$$

with $\tau \equiv |\mathbf{\Gamma}|\,t$, $r \equiv |\mathbf{\Gamma}|/(2|\mathbf{E}|)$, and unit vectors: $\mathbf{e} \equiv \mathbf{E}/|\mathbf{E}|$, $\boldsymbol{\gamma} \equiv \mathbf{\Gamma}/|\mathbf{\Gamma}|$.

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– **Energy- and decay-width differences of the eigenstates of $\mathsf{H}_{\mathsf{eff}}$:**

$$\Delta E = 2|\mathbf{E}|\,\mathrm{Re}\left(\sqrt{1-r^2-2ir\cos\theta_{\mathbf{e}\boldsymbol{\gamma}}}\right),$$

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with $\theta_{\mathbf{e}\boldsymbol{\gamma}} = \angle(\mathbf{e}\,,\boldsymbol{\gamma})$.

– **Large-time behaviour of b when $\mathrm{e} \perp \gamma$:**

$$\mathbf{b}(\tau\to\infty) \;=\; \frac{\sqrt{r^2-1}}{r}\,\boldsymbol{\gamma} \;-\; \frac{1}{r}\,\mathbf{e}\times\boldsymbol{\gamma}$$

approaches the long-lived state of $\mathsf{H}_{\mathsf{eff}}$, but only possible for $r > 1$.

• Critical Unstable Qubits (CUQs)

[Karamitros, McKelvey, AP '23, '25]

– **CUQ:** unstable qubit having (i) $\mathbf{E} \perp \mathbf{\Gamma}$ and (ii) $r \equiv |\mathbf{\Gamma}|/(2|\mathbf{E}|) < 1$.

$\Rightarrow$ the two eigenstates of H_{eff} have equal lifetimes: $\Delta\Gamma = 0$.

$\Rightarrow$ Oscillations in the *co-decaying* frame continue indefinitely.

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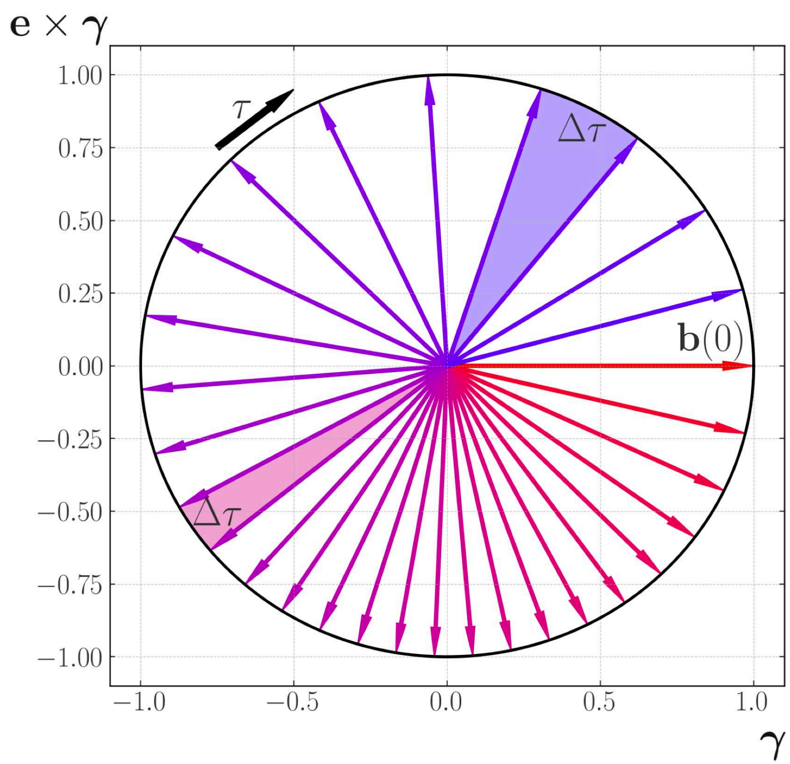
– **The angular speed of b:**

$$\dot{\varphi} \;=\; -\frac{1}{r} \,-\, \sin\varphi\,.$$

Inhomogeneous oscillation frequency rotating *slower* in the lower half plane:

$$\tan\frac{\theta}{2} \;=\; -\sqrt{\frac{1+r}{1-r}}\,\tan\frac{\omega t}{2}\,,$$

with $\theta = \varphi + \frac{\pi}{2}$ & $\omega = 2|\mathbf{E}|\sqrt{1-r^2}$.



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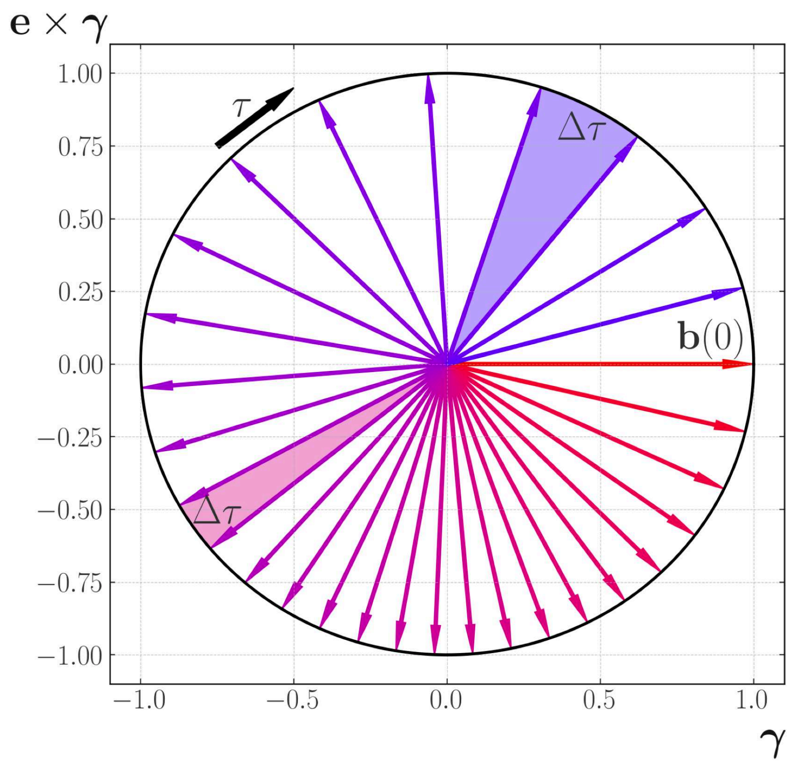

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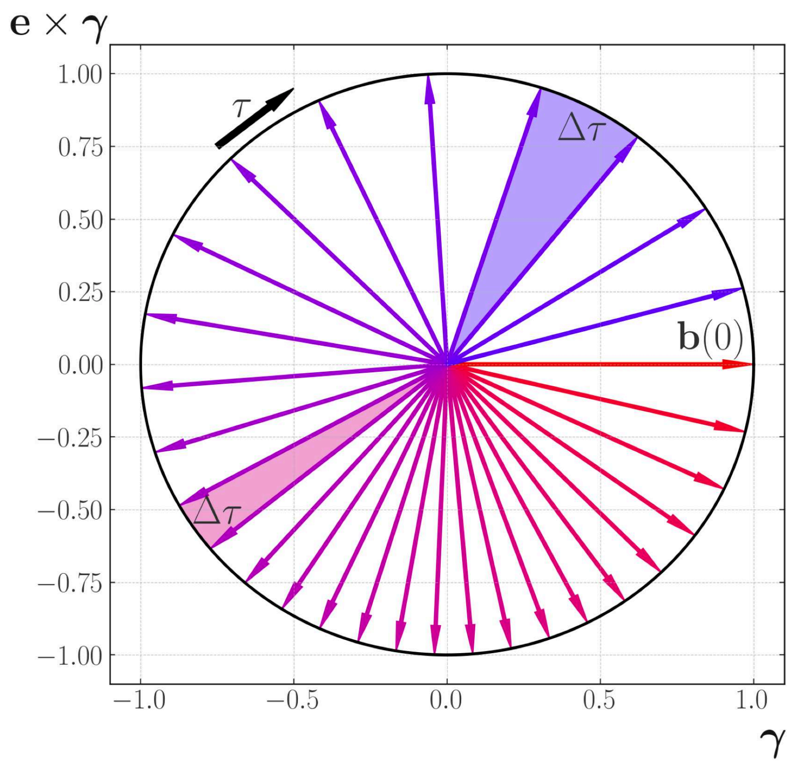

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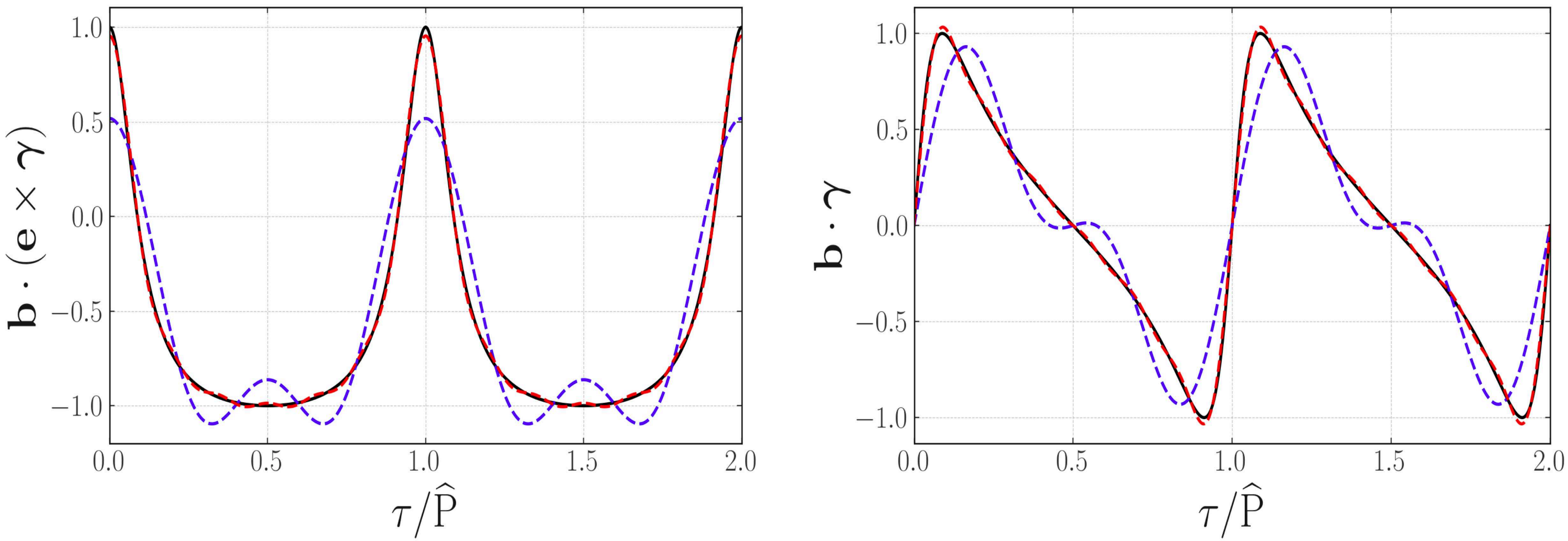
– **Extremal CUQ**: CUQ with $r \to 1 \;\Rightarrow\; \mathsf{P} \to \infty$ [AP '97]

– CUQ in a Pure State: $\mathbf{b}(0) = \mathbf{e} \times \boldsymbol{\gamma}$ and $r = 0.85$.



$\mathbf{b}(t)$-projections along different axes have different profiles, but $|\mathbf{b}(t)| = 1$

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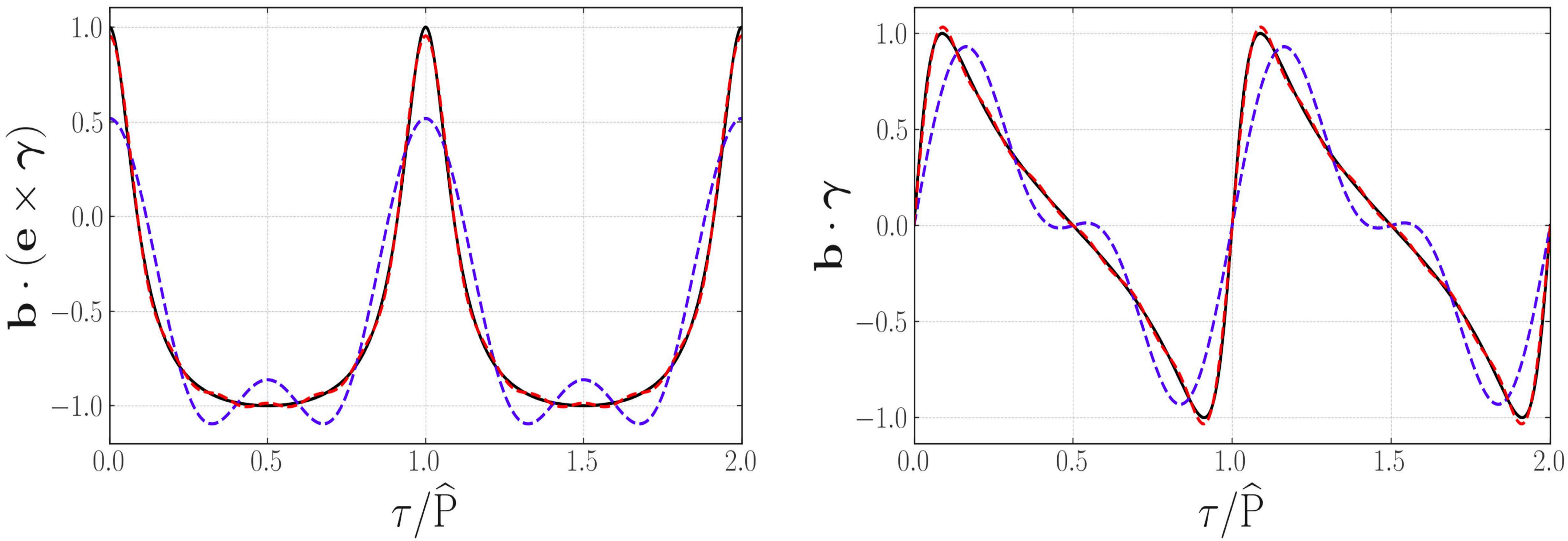

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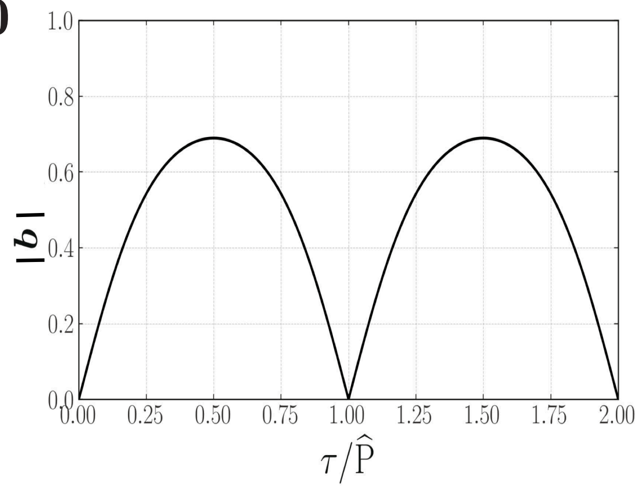
– CUQ in an initially Mixed State: $\mathbf{b}(0) = \mathbf{0}$

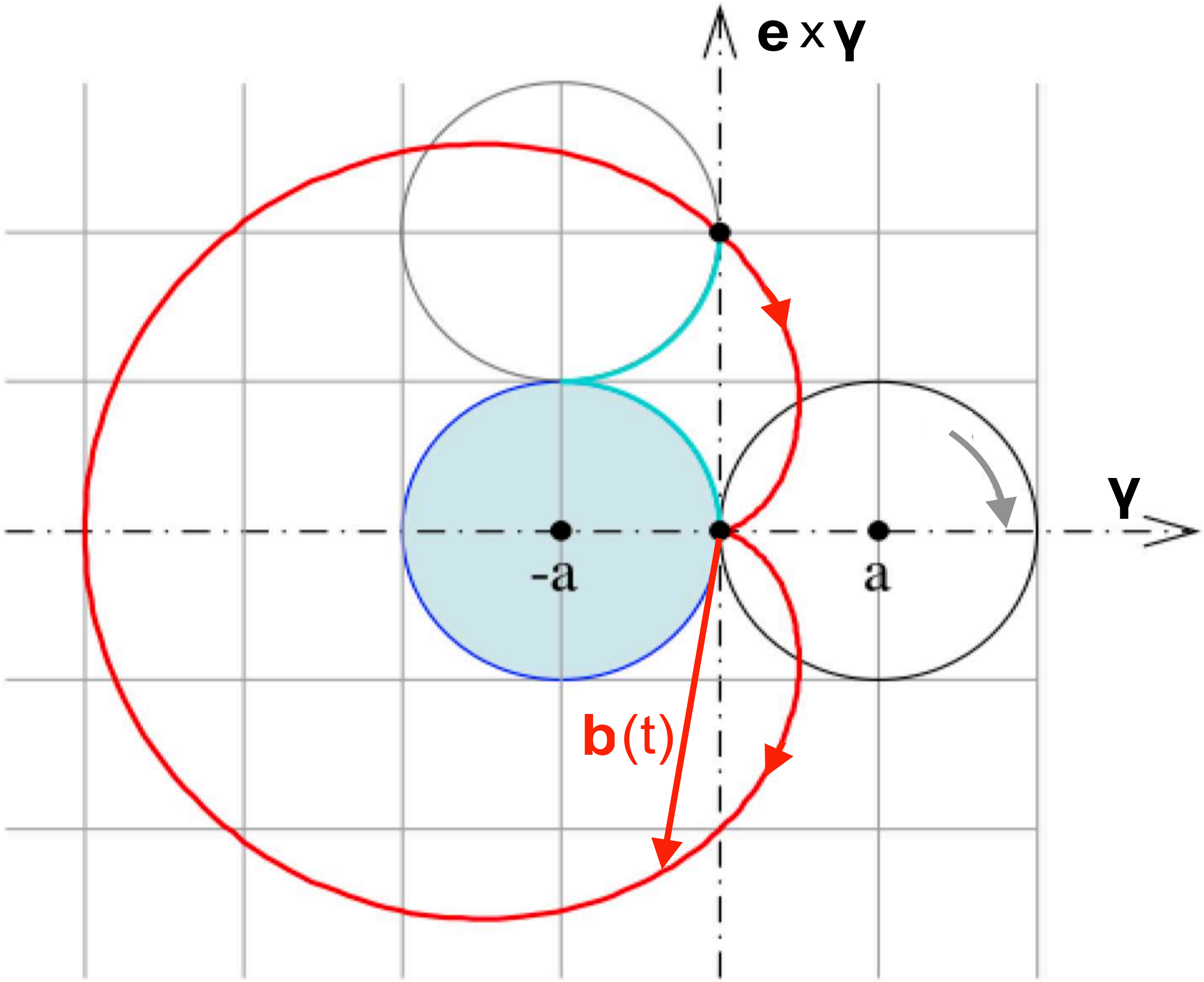
$$|\mathbf{b}(t)|^2 \;=\; 1 \;-\; \frac{(1-r^2)^2}{\left[1 - r^2\cos(2\pi t/\mathrm{P})\right]^2}\,.$$



⇒ Coherence-Decoherence oscillations
with the same period P.

– **Fully mixed CUQ with $\mathrm{b}(0) = 0$: Coherence-decoherence oscillations**

Motion of $\mathbf{b}(t) \perp \mathbf{e}$ resembles a Cardioid:



– Entanglement entropy of unstable qubits in the codecaying frame

[Karamitros, McKelvey, AP '23]

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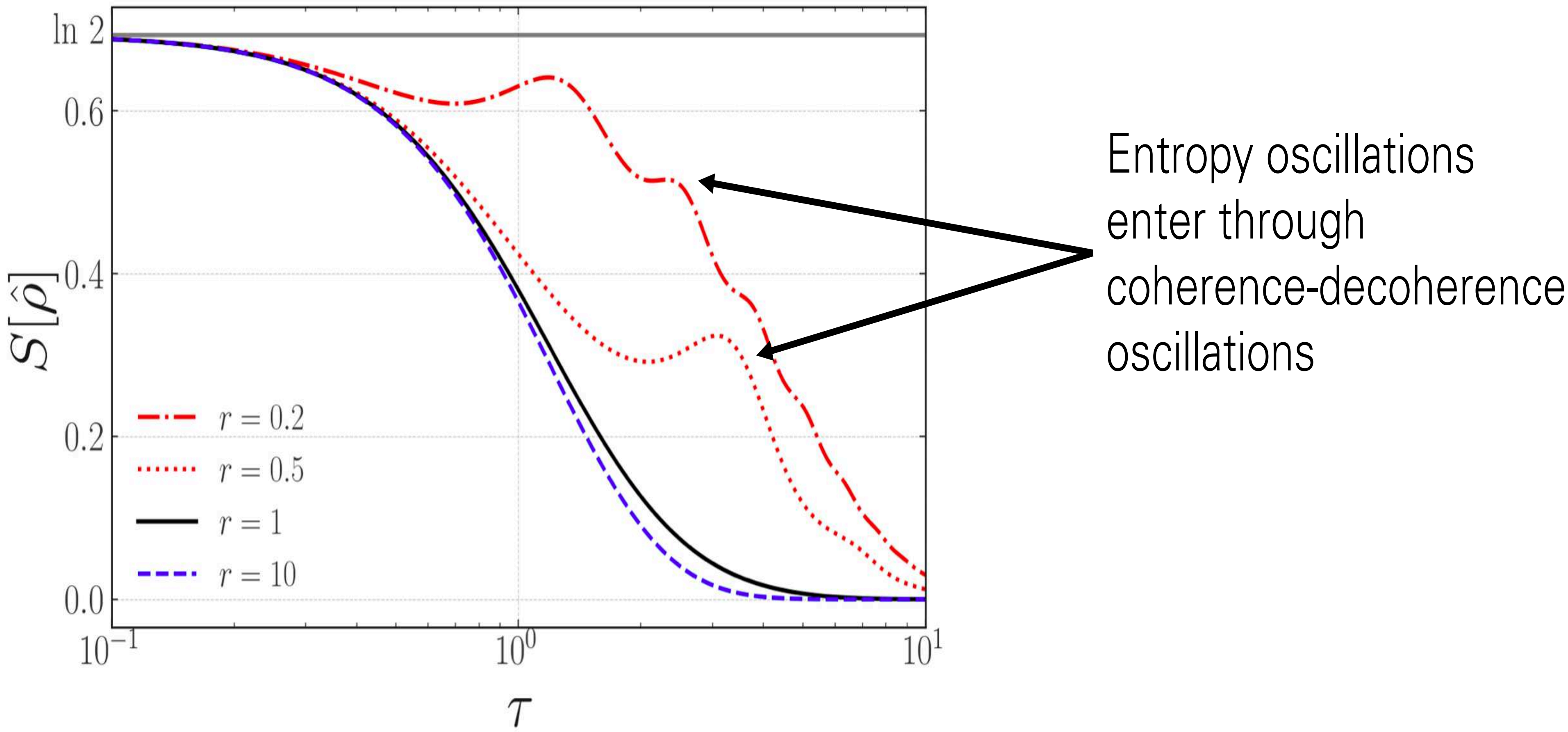
$$\theta_{\mathbf{e}\boldsymbol{\gamma}} \;\equiv\; \angle(\mathbf{e}\,,\boldsymbol{\gamma}) \;=\; 75^\circ:$$



• $B^0\bar{B}^0$-Meson System as a CUQ

[Karamitros, McKelvey, AP '25]

– Bloch-sphere description of meson–antimeson systems:

$$\Delta E = 2|\mathbf{E}|\,\mathrm{Re}\left(\sqrt{1 - r^2 - 2ir\cos\theta_{\mathbf{e}\gamma}}\right),$$

$$\Delta\Gamma = -4|\mathbf{E}|\,\mathrm{Im}\left(\sqrt{1 - r^2 - 2ir\cos\theta_{\mathbf{e}\gamma}}\right),$$

$$\left|\frac{q}{p}\right|^2 = \left|\frac{(\mathrm{H_{eff}})_{21}}{(\mathrm{H_{eff}})_{12}}\right| = \sqrt{\frac{1 + r^2 - 2r\sin\theta_{\mathbf{e}\gamma}}{1 + r^2 + 2r\sin\theta_{\mathbf{e}\gamma}}}\,.$$

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Introduce the two dimensionless quantities,

$$\delta_M \;=\; \frac{|q/p|^4-1}{|q/p|^4+1} \quad \& \quad \eta \;=\; \frac{(\Delta E)\left(\frac{1}{2}\Delta\Gamma\right)}{(\Delta E)^2-\left(\frac{1}{2}\Delta\Gamma\right)^2}\,,$$

to reexpress the Bloch-sphere parameters as follows:

$$r \;\equiv\; \frac{|\boldsymbol{\Gamma}|}{2|\mathbf{E}|} \;=\; \frac{\sqrt{1+4\eta^2}\mp\sqrt{1-\delta_M^2}}{\sqrt{4\eta^2+\delta_M^2}}\,, \qquad \text{for } r \lessgtr 1\,,$$

$$\sin\theta_{\mathbf{e}\boldsymbol{\gamma}} \;=\; -\,\frac{1+r^2}{2r}\delta_M\,, \qquad |\mathbf{E}|^2 \;=\; \frac{(\Delta E)^2-\left(\frac{1}{2}\Delta\Gamma\right)^2}{4\,(1-r^2)}\,.$$

– Experimental meson–antimeson mixing data: [PDG '24]

Meson	$\Delta E = \Delta m$ [ps^{-1}]	$\Delta\Gamma$ [ps^{-1}]	$\lvert q/p\rvert - 1$
K^0	$0.005293 \pm 9 \times 10^{-6}$	$0.01 \pm 5\times 10^{-6}$	-0.003239 ± 10^{-6}
D^0	0.01 ± 0.001	0.03 ± 0.003	$(-5.00 \pm 0.04)\times 10^{-3}$
$B_{\rm d}^0$	0.5069 ± 0.0019	$(0.7 \pm 7)\times 10^{-3}$	$(1.0 \pm 0.8)\times 10^{-3}$
$B_{\rm s}^0$	17.765 ± 0.006	0.084 ± 0.005	$(0.1 \pm 1.4)\times 10^{-3}$

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$\Downarrow$ [Karamitros, McKelvey, AP '25]

Meson	$r \equiv \lvert\mathbf{\Gamma}\rvert/(2\lvert\mathbf{E}\rvert)$	$\theta_{\mathbf{e}\boldsymbol{\gamma}}[^\circ]$	$\lvert\mathbf{E}\rvert$[ps^{-1}]
K^0	$0.945 \pm 2 \times 10^{-3}$	$179.6322 \pm 1\times 10^{-4}$	$2.64652 \times 10^{-3} \pm 7\times 10^{-8}$
D^0	1.5 ± 0.2	179 ± 2	$(5.00 \pm 0.04)\times 10^{-3}$
$B^0_{\rm d}$	$(1 \pm 4) \times 10^{-3}$	270 ± 90	0.253 ± 0.001
$B^0_{\rm s}$	$(2.4 \pm 0.2) \times 10^{-3}$	182.7 ± 33.8	8.9 ± 0.1

$\boldsymbol{r \geq 1}$: Unstable qubit is overdamped exhibiting no oscillations

– $B^0_{\mathrm{d}}\bar{B}^0_{\mathrm{d}}$**-meson mixing data:** [U. Nierste '04, A. Lenz et al '24]

	$\Delta E\,[\text{ps}^{-1}]$	$\Delta\Gamma\,[\text{ps}^{-1}]$	$\lvert q/p\rvert - 1$
Experiment	0.5069 ± 0.019	$(0.7 \pm 7)\times 10^{-3}$	$(-1.00 \pm 0.8)\times 10^{-3}$
Theory	0.535 ± 0.021	$\left(2.7 \pm 0.4\right) \times 10^{-3}$	$(2.6 \pm 0.3) \times 10^{-4}$
	$r \equiv \lvert\mathbf{\Gamma}\rvert/(2\lvert\mathbf{E}\rvert)$	$\theta_{\mathbf{e}\boldsymbol{\gamma}}[^\circ]$	$\lvert\mathbf{E}\rvert[\text{ps}^{-1}]$
Experiment	$(1 \pm 4) \times 10^{-3}$	-90 ± 90	0.253 ± 0.001
Theory	$(2.5 \pm 0.4) \times 10^{-3}$	-5 ± 3	0.28 ± 0.01

$\Rightarrow$ New physics may turn $B^0_{\mathrm{d}}\bar{B}^0_{\mathrm{d}}$ into a CUQ, with $\Delta\Gamma = 0 \longrightarrow \theta_{\mathbf{e}\boldsymbol{\gamma}} = -90^\circ$.

– $B^0_{\rm d}\bar{B}^0_{\rm d}$-meson mixing data:

[U. Nierste '04, A. Lenz et al '24]

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⇒ New physics may turn $B^0_{\rm d}\bar{B}^0_{\rm d}$ into a CUQ, with $\Delta\Gamma = 0 \longrightarrow \theta_{\mathbf{e}\boldsymbol{\gamma}} = -90°$.

– How can we observe any CUQ dynamics?

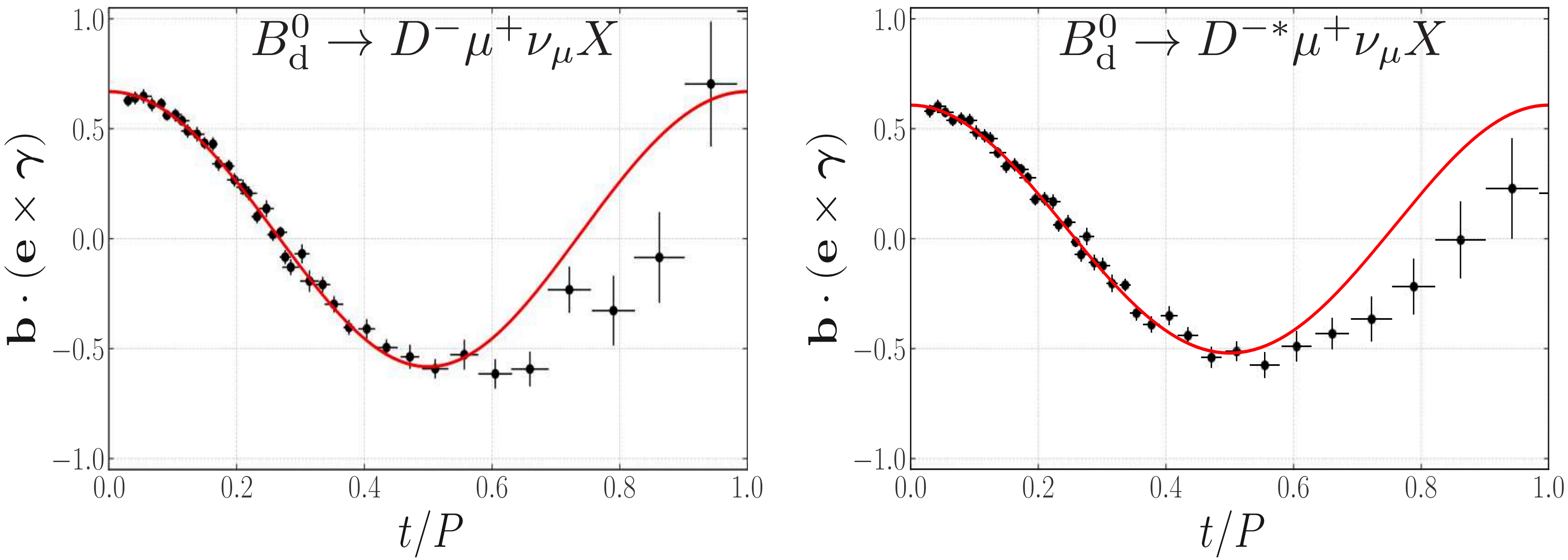
[Karamitros, McKelvey, AP '25]

⇒ Look for *non*-sinusoidal oscillations of flavour asymmetries, beyond the principal Fourier harmonic $n = 1$:

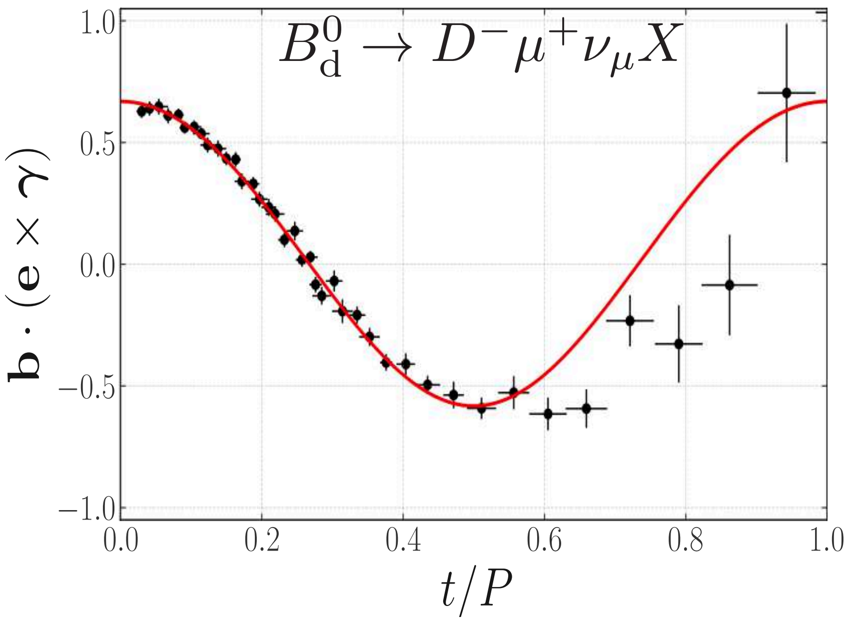
$$\delta(t) \;=\; \mathbf{b}(t)\cdot(\mathbf{e}\times\boldsymbol{\gamma}) \;=\; d_0 + \sum_{n=1}^{\infty} d_n \cos(n\omega t)\,,$$

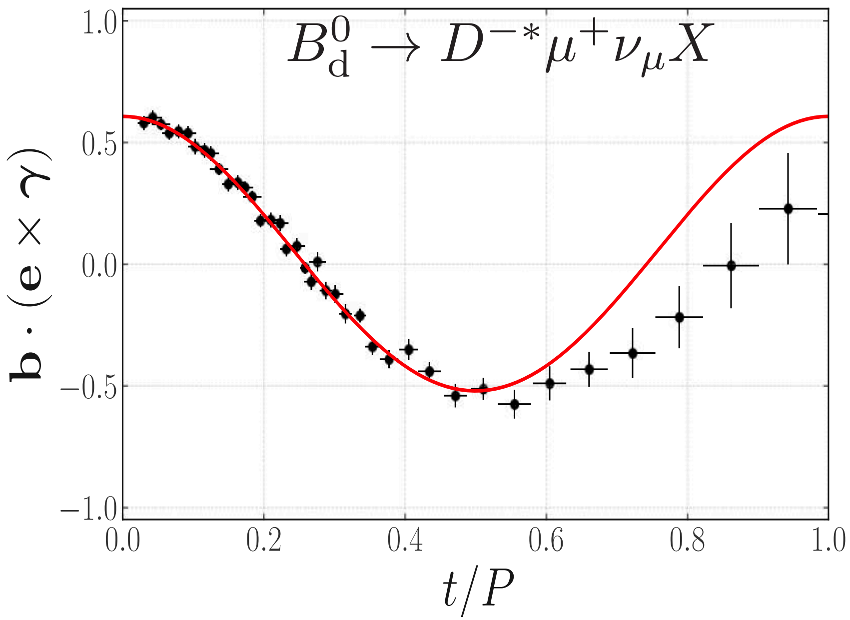
with $\omega \equiv 2\pi/\mathrm{P} = 2|\mathbf{E}|\sqrt{1-r^2}$.

– Experimental data fits with three Fourier modes (d_0, d_1, d_2): [LHCb '12]



– Experimental data fits with three Fourier modes (d_0, d_1, d_2): [LHCb '12]





– Analysis of Anharmonicity: [Karamitros, McKelvey, AP '25]

-	$B_d^0 \to D^- \mu^+ \nu_\mu X$ (2012)		$B_d^0 \to D^{-*} \mu^+ \nu_\mu X$ (2012)	
	$d_n \pm \delta d_n$	p-value	$d_n \pm \delta d_n$	p-value
d_0	0.06 ± 0.07	45%	0.04 ± 0.06	58%
d_1	0.625 ± 0.004	$< 0.01\%$	0.564 ± 0.003	$< 0.01\%$
d_2	-0.013 ± 0.009	18%	0.007 ± 0.008	37%

Null hypothesis: $d_0 = d_1 = d_2 = 0$.

• CUQs Beyond Particle Physics

Potential applications of CUQs to systems Beyond the Standard Model:

- Higgs boson as a CUQ via a CP-odd mixing with another scalar. [AP '97]
- Heavy Majorana neutrinos as (extremal) CUQs → Resonant Leptogenesis
- CUQs → Critical Unstable QuTrits → Tri-Resonant Leptogenesis
- Axion(s)–photon mixing as a CUQ system, e.g. in stars. [G. Raffelt, L. Stodolsky '88]

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Beyond Particle Physics: [R.P. Feynman '86]

- Probing the new oscillation phenomena in CUQs with quantum gates.
- Quantum simulation of non-Hermitian effective Hamiltonians like $H_{\rm eff}^{\rm CUQ}$.

• SUMMARY

- **CUQs is a novel class of unstable qubits characterised by:**

  (i) $\mathbf{e} \perp \boldsymbol{\gamma}$ **and (ii)** $0 < r \equiv |\boldsymbol{\Gamma}|/(2|\mathbf{E}|) \leq 1$

  $\Rightarrow$ **Two energy eigenstates of $H_{\rm eff}^{\rm CUQ}$ have equal lifetimes**

- **CUQs exhibit two atypical behaviours in their *co-decaying* frame:**

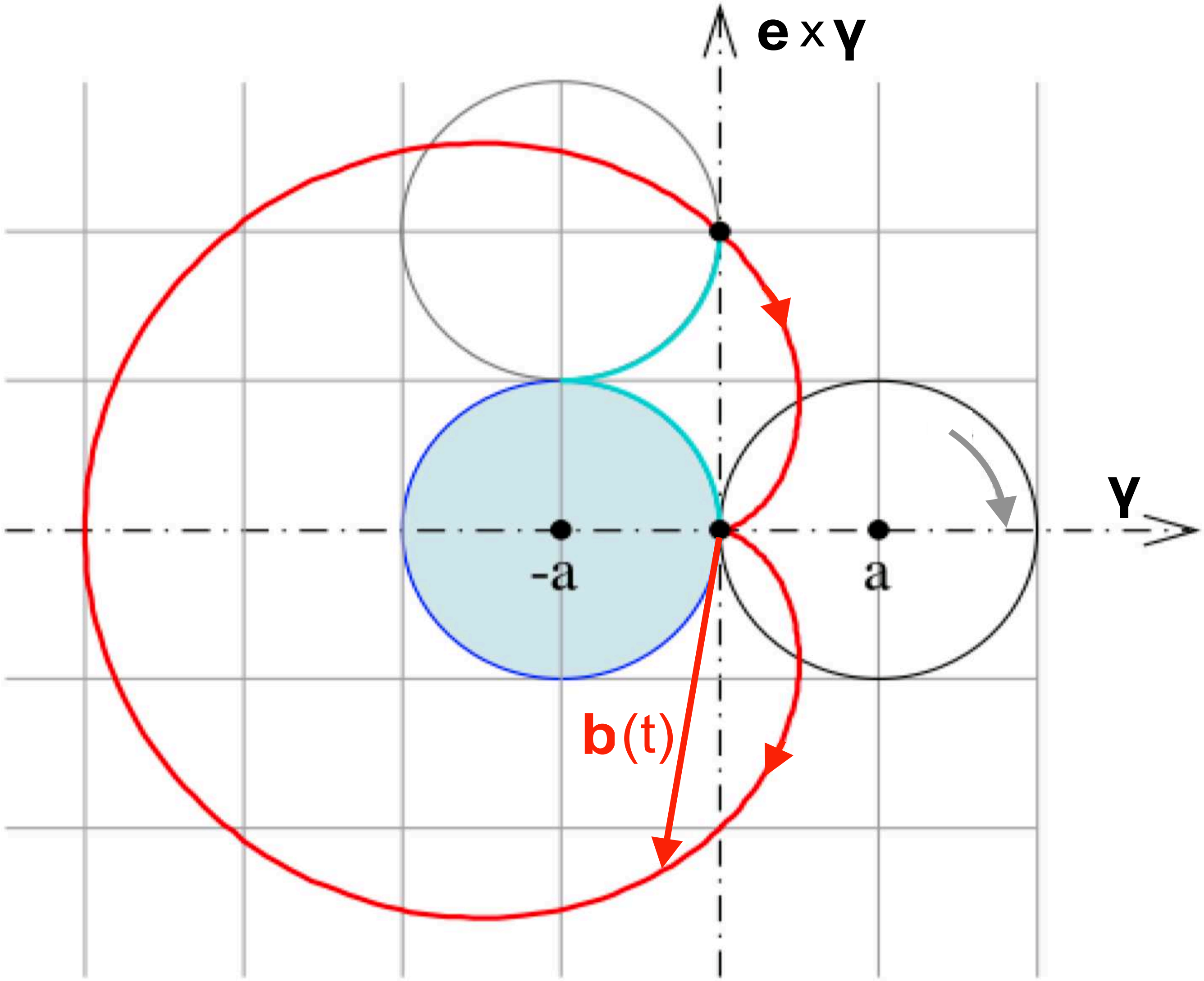
  (i) Coherence-Decoherence oscillations

  (ii) Bloch vector $\mathbf{b}$ sweeps out *unequal* areas in equal intervals of time

- **Quasi-CUQs with $\big||\theta_{e\gamma}| - 90^\circ\big| \lesssim 10^\circ$ retain most features of CUQs.**

- **Anharmonicity observables can probe deeper the structure of $H_{\rm eff}$, due to non-sinusoidal oscillations (as in the $B^0\bar{B}^0$-meson system).**

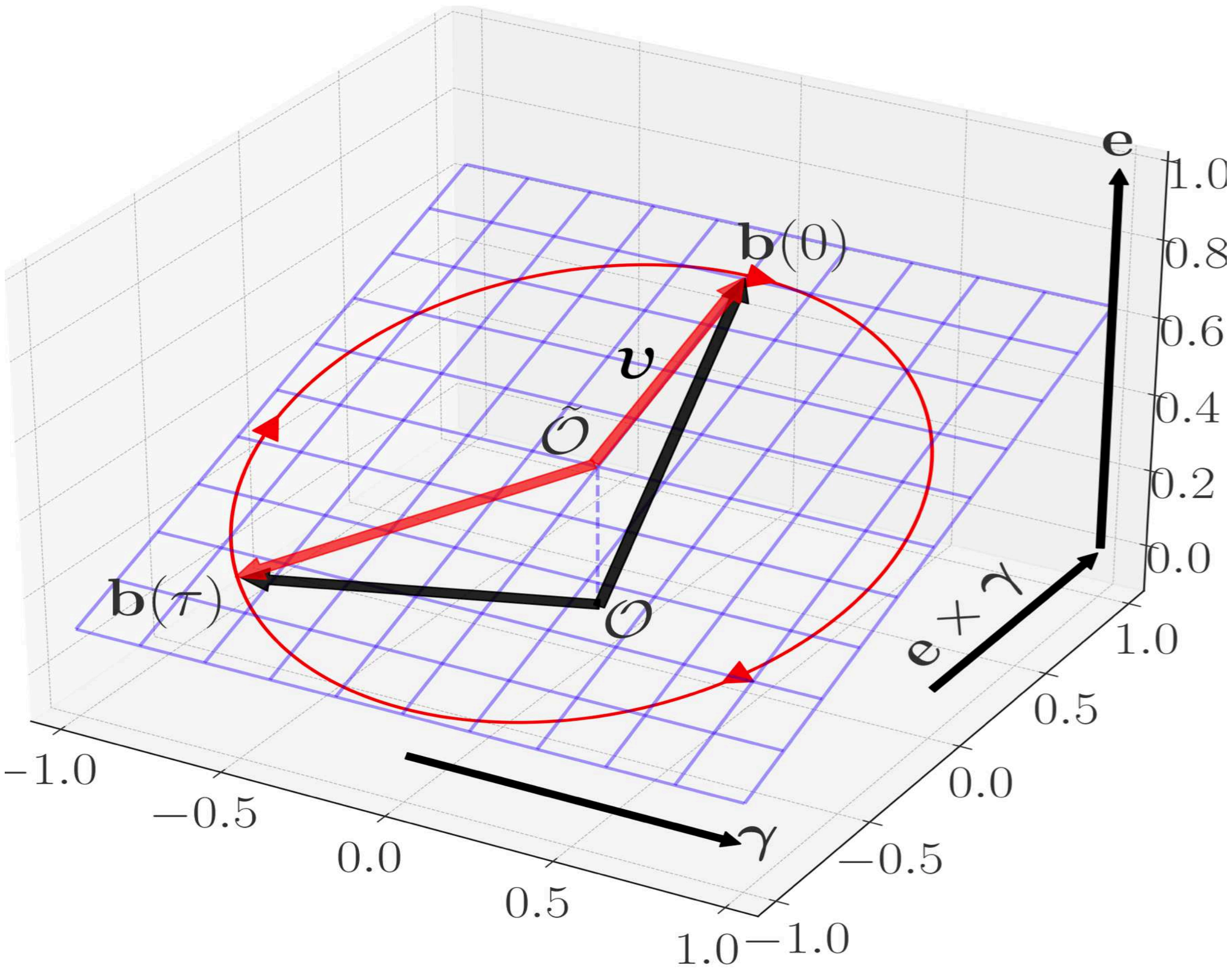
- Beyond Particle Physics (to be explored): **quantum simulation of $H_{\rm eff}^{\rm CUQ}$, quantum entanglement/dissipation, quantum computing . . .**


e x γ
γ
-a
a
b(t)

Back-Up Slides

- **Plane of oscillation for a CUQ with an arbitrary initial condition:**

  e.g., $\mathbf{b}(0) = 0.6\,\mathbf{e} + 0.8\,\mathbf{e}\times\boldsymbol{\gamma} \not\perp \mathbf{e}$ [Karamitros, McKelvey, AP '25]



Torsion of $\mathbf{b}(\tau)$:

$$\mathcal{T} \propto \dddot{\mathbf{b}}\cdot\big(\ddot{\mathbf{b}}\times\dot{\mathbf{b}}\big) = 0\,,$$

with $\dot{\mathbf{b}} \equiv d\mathbf{b}/d\tau$.

$$\Downarrow$$

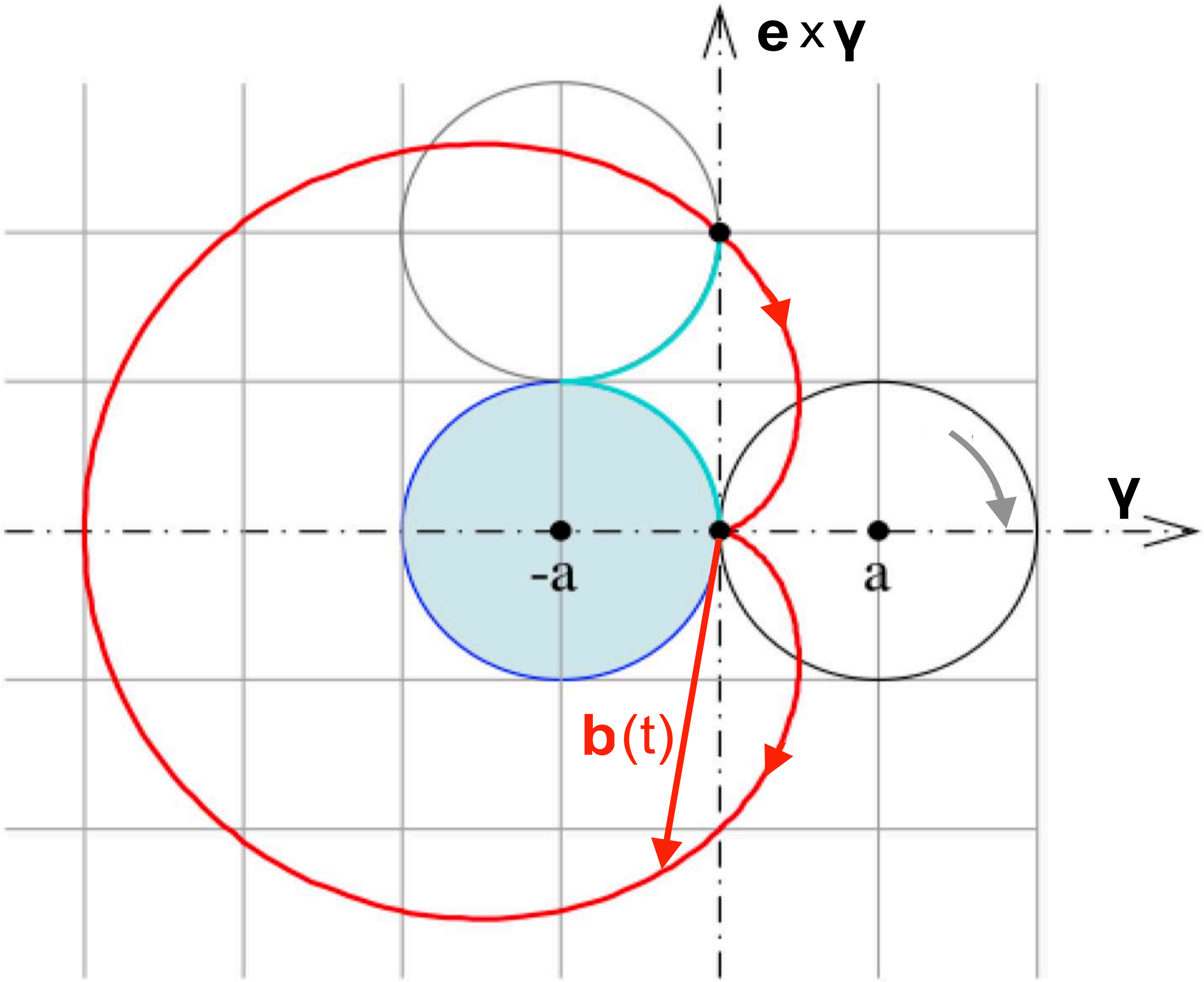
Motion of $\mathbf{b}(\tau)$ restricted on a plane

Oscillation plane spanned by: $\boldsymbol{\gamma}$ & $\widehat{\boldsymbol{v}} = \frac{1}{N}\Big[\mathbf{e}\times\boldsymbol{\gamma} + r\Big(\boldsymbol{\gamma}\times\mathbf{b}(0)\Big)\times\boldsymbol{\gamma}\Big]$

For $\mathbf{b}(0) = \mathbf{0} \longrightarrow \widehat{\boldsymbol{v}} = \mathbf{e}\times\boldsymbol{\gamma} \perp \mathbf{e}$

- **Fully mixed CUQ with $\mathbf{b}(0) = 0$: Coherence-decoherence oscillations**

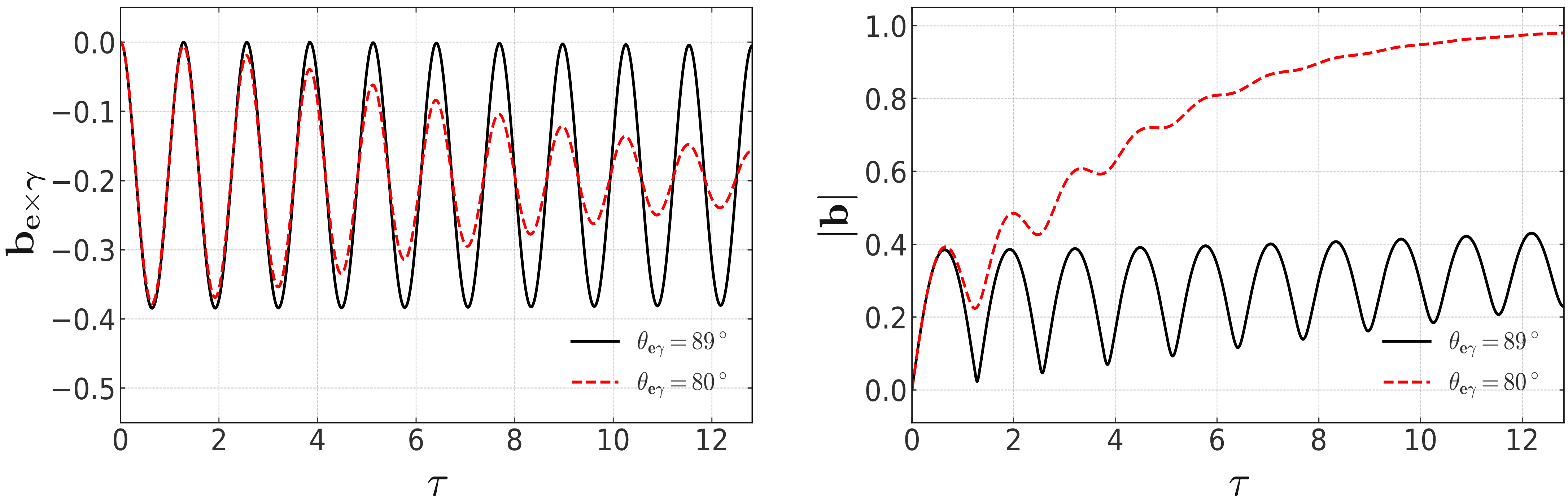
Motion of $\mathbf{b}(t) \perp \mathbf{e}$ resembles a Cardioid $\longrightarrow$ **Quantonomy**



• Quasi-CUQ Scenarios: Coherence-Decoherence Oscillations

[Karamitros, McKelvey, AP '23]

Parameter $r = 0.2$ & Initial condition: $\mathbf{b}(0) = \mathbf{0}$

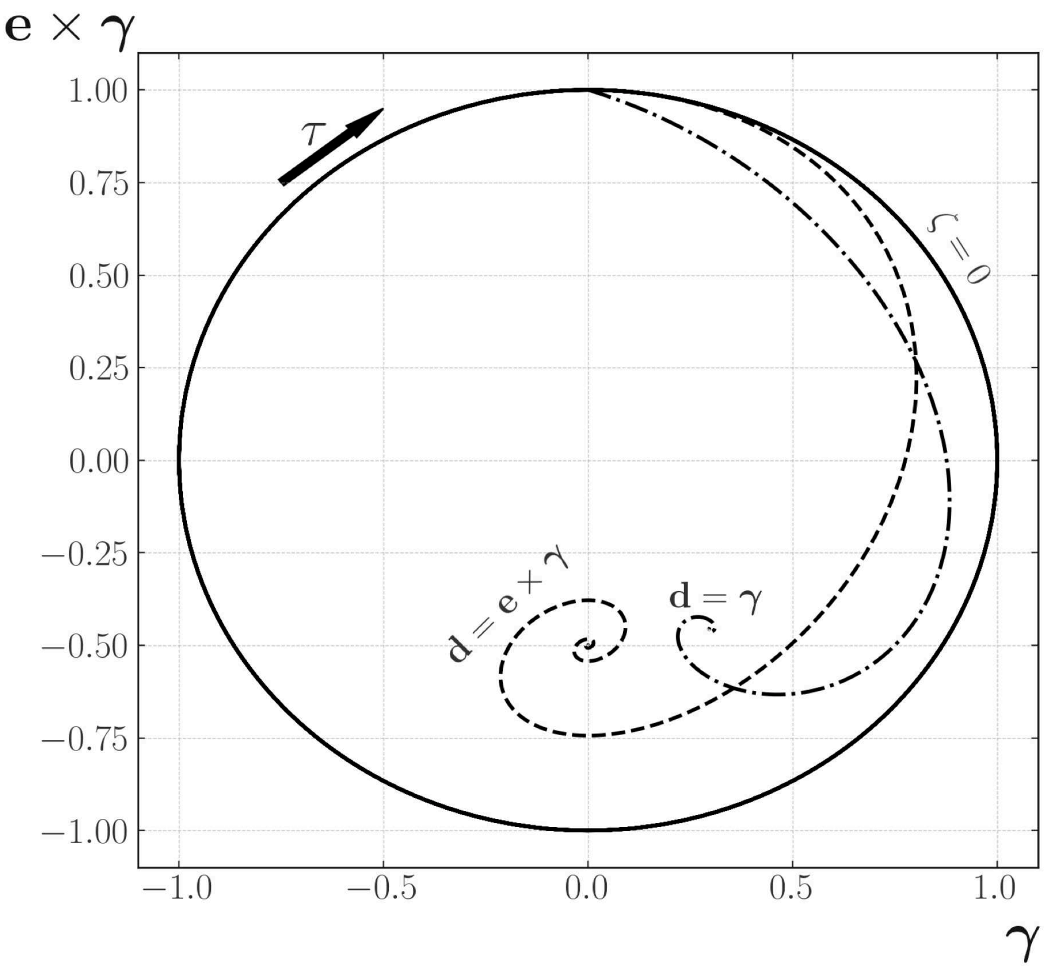


→ Motion of $\mathbf{b}(t)$ is no longer contained on a plane

- **Decoherence due to CUQs interaction with the environment**

[Karamitros, McKelvey, AP '23]

Parameter $r = 0.5$ & Initial condition: $\mathbf{b}(0) = \mathbf{e} \times \boldsymbol{\gamma}$ [G. Lindblad '76, ... P. Huet, M. Peskin '95]



Decoherent dissipation:

$$\frac{\mathrm{d}}{\mathrm{d}t}\rho = -i\,[\mathsf{E}\,,\rho] - \frac{1}{2}\{\Gamma\,,\rho\} - [D\,,[D\,,\rho]]\,,$$

with $\mathsf{D} = D^0\mathbf{1}_2 - \mathbf{D}\cdot\boldsymbol{\sigma}$

$$\Downarrow$$

$$\frac{\mathrm{d}\mathbf{b}}{\mathrm{d}\tau} = -\frac{1}{r}\mathbf{e}\times\mathbf{b} + \boldsymbol{\gamma} - (\boldsymbol{\gamma}\cdot\mathbf{b})\,\mathbf{b} + \zeta(\mathbf{d}\cdot\mathbf{b})\,\mathbf{d} - \zeta\mathbf{b}\,,$$

with $\zeta \equiv 4\,|\mathbf{D}|^2/|\mathbf{\Gamma}| \to 1$
and $\mathbf{d} \equiv \mathbf{D}/|\mathbf{D}|$