Unifying the Higgs Mass Hierarchy, the Cosmological Constant, and the Axion Quality Puzzles

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Scales in Nature

- Scale of quantum gravity, related to Newtons constant, $G_N = 6.7 \times 10^{-39}\ \mathrm{GeV}^{-2}$, $M_P = 2.435 \times 10^{18}$ GeV

- Fermi scale, associated with electroweak interactions, $G_F = 1.17 \times 10^{-5}\ \mathrm{GeV}^{-2}$, $M_W = 80.38$ GeV

- QCD scale of strong interactions, $\Lambda_{\mathrm{QCD}} \sim 100$ MeV

- Cosmological constant, or vacuum energy, or Dark Energy, $\epsilon_{vac} = \left(2.24 \times 10^{-3}\mathrm{eV}\right)^4$

Are all these scales fundamental and independent?

What the multitude of these scales tells us?

Several hierarchy problems

- Higgs mass hierarchy: $m_H/M_P \simeq 10^{-16}$ (no solution yet)

- Cosmological Constant Problem: $(\epsilon_{\rm vac}/M_P)^{\frac{1}{4}} \simeq 10^{-31}$ (no solution yet)

- QCD scale, $\Lambda/M_P \simeq 10^{-19}$ (perhaps, because the strong coupling runs logarithmically, and $e^{-\frac{\alpha_s(M_P)}{\beta(\alpha_s(M_P))}} \ll 1$)

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Problems are coming from different sectors of the theory

Higgs-gravity action:

$$S = \int d^4x\sqrt{-g}\left[\frac{M_P^2}{2}R + m_H^2 H^\dagger H + \epsilon_{vac} + \dots\right]$$

four independent parameters

KSVZ axion (as an example):

Ψ - superheavy quark
Φ - PQ scalar field

$$L = \dots + f\bar{\Psi}\Psi\Phi + m_{\not{PQ}}\bar{\Psi}\Psi$$

Axion quality problem: the PQ symmetry breaking mass $m_{\not{PQ}} \neq 0$ does not make the theory inconsistent, and PQ symmetry is broken by the anomaly, so there are no convincing arguments for its absence.

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Yes, in Weyl-invariant Einstein-Cartan gravity

Karananas, MS, Zell, arXiv:2406.11956, arXiv:2506.11836

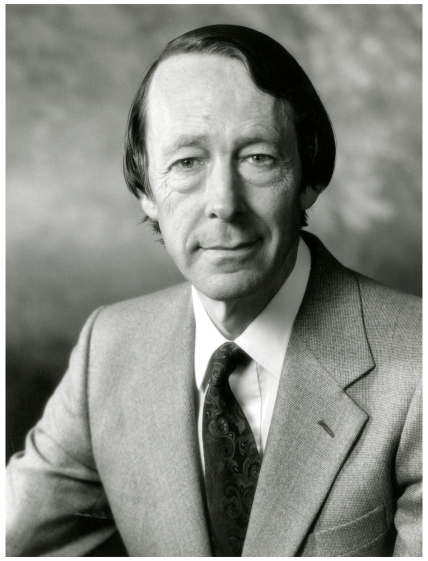
Gravity as a gauge theory

Existence of electromagnetic field - U(1) global invariance of fermion Lagrangian promoted to be local

Gluons, W^+, W^- Z and γ of the Standard Model - SU(3)xSU(2)xU(1) global invariance of SM fermion Lagrangian promoted to be local

Existence of gravitational field - Poincare invariance of SM fermion Lagrangian promoted to be local?

Gauging of the Poincare group: Utiyama (1956), Kibble (1961) , Sciama (1962,1964).



Gravity as a gauge theory: localising the Lorentz group

Basic gauge fields

- e_μ^A - tetrad one-form (frame field, translations), A=0,1,2,3. 4x4=16 fields
- ω_μ^{AB} - spin connection one form (gauge field of the local Lorentz group, 4x3/2x4=24 fields).

Field strength, curvature:

$$F_{\mu\nu}^{AB} = \partial_\mu \omega_\nu^{AB} - \partial_\nu \omega_\mu^{AB} + \omega_{\mu C}^{A} \omega_\nu^{CB} - \omega_{\nu C}^{A} \omega_\mu^{CB}$$

Field strength, torsion:

$$T_{\mu\nu}^{A} = \partial_\mu e_\nu^{A} - \partial_\nu e_\mu^{A} + \omega_{\mu B}^{A} e_\nu^{B} - \omega_{\nu B}^{A} e_\mu^{B}$$

Gravity as a gauge theory

Lowest order invariants:

$$F \equiv \frac{1}{8\sqrt{g}} \epsilon_{ABCD} \epsilon^{\mu\nu\rho\sigma} F^{AB}_{\mu\nu} e^{C}_{\rho} e^{D}_{\sigma}$$

$$\tilde{F} \equiv \frac{1}{\sqrt{g}} \epsilon^{\mu\nu\rho\sigma} e_{\rho C} e_{\sigma D} F^{CD}_{\mu\nu}$$

Irreducible components of the torsion: $T_{\mu\nu\rho} = e_{\mu A} T^{A}_{\nu\rho}\,,$

$$v_\mu = T^{\nu}{}_{\mu\nu}\;, \quad a_\mu = \epsilon_{\mu\nu\rho\sigma} T^{\nu\rho\sigma}\;, \quad \tau_{\mu\nu\rho} = \frac{2}{3}\left(T_{\mu\nu\rho} - v_{[\nu} g_{\rho]\mu} - T_{[\nu\rho]\mu}\right)$$

EC gravity and the SM

- Pure gravity gauge action of lowest dimensionality:

$$S_{\text{gravity}} = M_P^2 \left[aF + b\tilde{F} + c v_\mu v^\mu + d a_\mu a^\mu + e a_\mu v^\mu + f \tau^{\mu\nu\rho} \tau_{\mu\nu\rho} \right]$$

M_P is the Planck mass.

- Standard Model action S_{SM} :
make it invariant with respect to the local Lorentz group, using covariant derivatives and tetrads replacing the flat metric, e.g. for fermions: $D\Psi = d\Psi + \frac{1}{8}\omega_{IJ}[\gamma^I, \gamma^J]\Psi$

- Mixing terms S_{mixing}:
non-minimal couplings $h^2 F$, $h^2 \tilde{F}$, $h^2 a_\mu^2$, etc

EC gravity and the SM

- Particle content of the theory - just SM (or ν MSM) fields plus graviton (only 2 out of 40 degrees of freedom hidden in spin-connection and tetrad are physical and propagating. Torsion is non-dynamical.)

- Non-polynomial kinetic term for the Higgs field, new 4-fermion interactions, and interactions between fermions and the Higgs field. Applications: Higgs inflation, Dark Matter production

Weyl-invariant EC gravity

Weyl symmetry changes locally the scale of rulers,
theory does not have any intrinsic mass parameters

Why having Weyl symmetry is interesting?

- Flat space limit - conformal symmetry, leading to conformal field theory (CFT)

- Symmetry of free Maxwell equations

- The Lagrangian of the Standard Model is invariant under the full conformal group if the mass of the Higgs boson is put to zero. Perhaps, the observed smallness of the Fermi scale in comparison with the Planck scale is a consequence of this, 125 GeV $\simeq M_H \ll M_P \sim 10^{19}$ GeV?

- The energy of the ground state in CFTs is equal to zero. Perhaps, this is relevant for the explanation of the amazing smallness of the cosmological constant?

Weyl symmetry

What is Weyl symmetry in EC gravity? Local change of the rulers,

$$e_\mu^A \to \Omega(x) e_\mu^A, \quad \omega_\mu^{AB} \to \omega_\mu^{AB},$$

$$a_\mu \to a_\mu, \;\; v_\mu \to v_\mu + 3\Omega\partial_\mu\Omega^{-1}, \;\; \tau_{\mu\nu\rho} \to \Omega^2 \tau_{\mu\nu\rho}$$

Torsion vector is an Abelian gauge field of the Weyl symmetry. It can be used for construction of Weyl-invariant actions. No quantum Weyl anomaly (similar statement for Weyl conformal geometry, Ghilencea 2309.11372).

Weyl-invariant EC gravity and the Standard Model

Dimensionful couplings are not admitted.

The only Weyl invariant structures which are ghost free for any parameter choice:

$$\frac{1}{f^2}F^2 + \frac{1}{\tilde{f}^2}\tilde{F}^2 + L_{\mathrm{SM}}$$

gauge couplings of Lorentz group

plus non-minimal Higgs-gravity couplings like $h^2F,\ h^2\tilde{F},\ a_\mu^2 h^2, \cdots$, plus cross term $F\tilde{F}$

Spectrum of the theory

Remove non-dynamical degrees of freedom.

Result: gravity in metric formulation + SM with non-polynomial kinetic term for the Higgs field, new 4-fermion interactions, and interactions between fermions and the Higgs field, containing one new scalar particle ϕ.

Demonstration: field strengths in terms of torsion and torsion-free curvature and covariant derivatives:

$$F = \frac{R}{2} + v^{\mu}_{;\mu} - \frac{1}{3} v_{\mu} v^{\mu} + \frac{1}{48} a_{\mu} a^{\mu} + \frac{1}{4} \tau_{\mu\nu\rho} \tau^{\mu\nu\rho}$$

$$\tilde{F} = -a^{\mu}_{;\mu} + \frac{2}{3} a_{\mu} v^{\mu} - \frac{1}{2} \epsilon^{\mu\nu\rho\sigma} \tau_{\lambda\mu\nu} \tau^{\lambda}_{\rho\sigma}$$

Spectrum of the theory

1. Introduce auxiliary fields χ (dilaton) and ϕ (ALP- axion like particle),

$$S_{\rm gr} = \int d^4x\sqrt{g}\left[\chi^2 F + M_P^2\phi\tilde{F} - \frac{f^2\chi^4}{4} - \frac{\tilde{f}^2 M_P^4\phi^2}{4}\right]$$

2. Use the Weyl invariance and replace $\chi = \dfrac{M_P}{\sqrt{2}}$

(spontaneous breaking of the Weyl symmetry, generating all the mass scales)

$$S_{\rm gr} = M_P^2\int d^4x\sqrt{g}\left[\frac{F}{2} + \phi\tilde{F} - \frac{\tilde{f}^2 M_P^2\phi^2}{4} - \frac{f^2 M_P^2}{16}\right]$$

3. Solve for torsion components and remove them with the help of equations of motion. The auxiliary field ϕ becomes dynamical, the substitution $\phi \propto a^2$ gives a field with canonical mass dimension.

Spectrum of the theory

Roughly speaking, this new scalar particle is associated with torsion, and leads to axion-like field that can be used for solution of the strong CP-problem (Lattanzi and Mercuri ‘2010, Mavromatos and Pilaftsis, ‘2012, Karananas ‘2018,…)

Features:

- The gauge coupling f determines the vacuum energy and the Higgs boson mass,

$$\epsilon_{vac} = f^2 M_P^4/16, \qquad m_H^2 = -f^2 \xi_h M_P^2/4$$

- The gauge coupling $\tilde{f}$ determines the scalar mass,

$$m_a^2 \propto \tilde{f}^2 (1 + 6\xi_h) M_P^2/48$$

=> Very small Lorentz gauge couplings lead to the smallness of the vacuum energy, Higgs boson mass and new scalar a. Unification of 3 puzzles!

Radiative corrections and non-perturbative effects

- Weyl-invariant perturbation theory: the zero values of f and $\tilde{f}$ are the fixed points of RG evolution.

- Tree masses and tree value of the vacuum energy do not match the observations. Non-perturbative effects: gravity-Higgs instantons may lead to $m_H \propto \exp(-S) M_P$, where S is the instanton action. It is possible that $S \gg 1$ (*Shkerin, MS'18, Shkerin, MS, Zell'21*)

- Black hole contributions to m_H?

- If the non-perturbative contribution to the vacuum energy $\propto m_H^4$, f should be tuned ☹️ to make the cosmological constant small, $f \sim 10^{-32}$. If no contribution, $f \sim 10^{-62}$.

Strong CP-problem

Interactions of a:

- $\partial_\mu a A^\mu$, $\partial_\mu a V^\mu$ - induced by EC-nature of gravity,

  where A and V are axial and vector fermonic currents

- $\propto \frac{\zeta}{M_P}[aG\tilde{G} \; + \; a\bar{q}\gamma_5 q]$ - induced by the requirement of

  quantum Weyl symmetry

Strong CP problem is solved if the QCD induced mass is smaller than the mass induced by gravity:

$$\frac{\tilde{f}}{\zeta} \lesssim 10^{-5} \frac{m_\pi f_\pi}{M_P^2} \frac{\sqrt{m_u m_d}}{m_u + m_d} \sim \mathcal{O}\left(10^{-43}\right) ,$$

where ζ is related to the strength of the a-coupling to quarks and gluons.

To compare with the known axion solutions

QCD without axion:

- One "unnatural" number, $\theta \lesssim 10^{-10}$

QCD with axion:

- 6 new degrees of freedom (KSVZ - one complex scalar field and a new massive quark, DFSZ - two complex scalar fields, one is the doublet with respect to the SU(2) weak isospin and another is a singlet).

- Two "unnatural" numbers:

- Ratio of EW scale and PQ scale: $v_{EW}/F_{PQ} \lesssim 10^{-7}$

- Quality of PQ symmetry: $m_{\not{PQ}}/F_{PQ} \lesssim 10^{-25}$

- What is the need for the PQ symmetry, besides the solution of the strong CP problem?

Conclusions

The Weyl-invariant Einstein-Cartan gravity
is a theory with the following properties:

- In comparison with the SM, it has one extra scalar degree of freedom - the ALP, allowing to solve the strong CP problem.

- The cosmological constant, the tree value of the Higgs boson mass and the gravitationally induced ALP mass are all proportional to the gauge couplings of the Lorentz group. They are required to be very small (gravity is the weakest gauge force in Nature).

Conclusions

Open problems, currently under investigation:

- Non-perturbative generation of the Higgs mass?
- Cosmological applications:
    - Higgs-axion inflation, isocurvature perturbations?
    - Axion behaviour in the early Universe, axion as dark matter?
- Theory is not renormalisable by standard methods