Gravitational Waves from Gravitational Particle Production

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dgapa

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Scalar spectator

Consider a scalar χ (spin 0) which only interacts with gravity and/or the inflaton ϕ

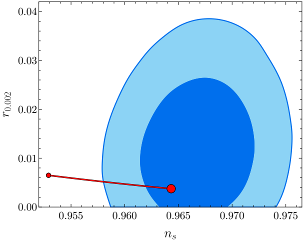
$$\mathcal{S} = \int d^4x \sqrt{-g} \left[-\frac{1}{2} M_P^2 R + \frac{1}{2} (\partial_\mu \phi)^2 + \frac{1}{2} (\partial_\mu \chi)^2 - \frac{1}{2} m_\chi^2 \chi^2 \right.$$

$$\left. - \underbrace{6 \lambda M_P^4 \tanh^2 \left(\frac{\phi}{\sqrt{6} M_P} \right)}_{\text{T-model inflation}} - \frac{1}{2} \underbrace{\sigma \phi^2 \chi^2}_{\phi\text{-coupling}} - \underbrace{y \phi \bar{\psi} \psi}_{\text{reheating}} + \mathcal{L}_{\text{SM}} \right]$$

Inflationary couplings normalized by

$$\lambda \simeq \frac{3\pi^2 A_{S*}}{N_*^2} , \qquad T_{\text{reh}} \simeq \left(\frac{9\lambda}{20\pi^4 g_{\text{reh}}} \right)^{1/4} y M_P$$

R. Kallosh, A. Linde, JCAP 10 (2013) 033; Planck + BICEP/Keck PRL 127 (2021) 151301



Gravitational particle production

Introducing *conformal time*, $dt = a\, d\tau$, and the re-scaled field $X = a\chi$,

$$\left(\partial_\tau^2 - \nabla^2 + a^2 m_{\rm eff}^2\right) X \;=\; 0\,, \qquad m_{\rm eff}^2 \;=\; m_\chi^2 + \sigma\phi^2 + \frac{1}{6} R$$

Quantize as a superposition of oscillators

$$\hat{X}(\tau, \mathbf{x}) \;=\; \int \frac{d^3\boldsymbol{k}}{(2\pi)^{3/2}}\, e^{-i\boldsymbol{k}\cdot\mathbf{x}} \left[X_k(\tau)\hat{a}_{\boldsymbol{k}} + X_k^*(\tau)\hat{a}^\dagger_{-\boldsymbol{k}}\right]\,, \qquad [\hat{a}_{\boldsymbol{k}}, \hat{a}^\dagger_{\boldsymbol{k}'}] = \delta(\boldsymbol{k} - \boldsymbol{k}')\,, \quad \hat{a}_{\boldsymbol{k}}|0\rangle \;=\; 0$$

obtaining

$$X_k'' + \omega_k^2 X_k \;=\; 0\,, \quad \text{with} \quad \omega_k^2 \;=\; k^2 + a^2 m_{\rm eff}^2$$

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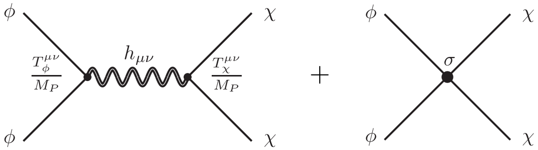
For a mode inside the horizon,

$$\omega_k^2 \;=\; \underbrace{k^2}_{\text{free particle}} + \mathcal{O}\left(\underbrace{\frac{a^2H^2}{k^2}}_{\text{interactions}}\right) \;>\; 0$$

Gravitational particle production

For a mode inside the horizon, the interaction picture works during reheating,



$$\mathcal{L}_I = -\frac{1}{M_P^2} h_{\mu\nu}(T^{\mu\nu}_\phi + T^{\mu\nu}_\chi) - \frac{1}{2}\sigma\phi^2\chi^2$$

$$|\mathcal{M}|^2 = \frac{1}{8}\frac{\rho_\phi^2}{m_\phi^4}(\sigma-\lambda)^2$$

$$\frac{\partial f_\chi}{\partial t} - H|\boldsymbol{P}|\frac{\partial f_\chi}{\partial|\boldsymbol{P}|} = \frac{1}{P^0}\int\frac{d^3\boldsymbol{K}}{(2\pi)^3 n_\phi}\frac{d^3\boldsymbol{P}'}{(2\pi)^3 2P'^0}(2\pi)^4\delta^{(4)}(K-P-P')|\overline{\mathcal{M}}|^2$$

$$\times\left[\underbrace{f_\phi(K)}_{=(2\pi)^3 n_\phi\delta^{(3)}(\boldsymbol{K})}(1+f_\chi(P))(1+f_\chi(P')) - f_\chi(P)f_\chi(P')(1+f_\phi(K))\right]$$

Gravitational particle production

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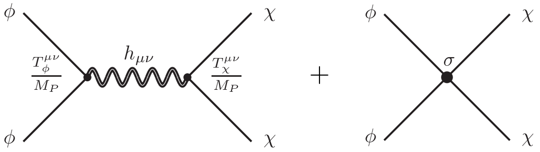


$$\mathcal{L}_I = -\frac{1}{M_P^2} h_{\mu\nu}\left(T_\phi^{\mu\nu} + T_\chi^{\mu\nu}\right) - \frac{1}{2}\sigma\phi^2\chi^2$$

$$|\mathcal{M}|^2 = \frac{1}{8}\frac{\rho_\phi^2}{m_\phi^4}(\sigma - \lambda)^2$$

$$\frac{\partial f_\chi}{\partial t} - H|\boldsymbol{P}|\frac{\partial f_\chi}{\partial |\boldsymbol{P}|} = \frac{\pi|\overline{\mathcal{M}}|^2}{2m_\phi^2}\delta\left(|\boldsymbol{P}| - m_\phi\right)\left(1 + 2f_\chi(|\boldsymbol{P}|)\right)$$

$$\Rightarrow \quad f_\chi(|\boldsymbol{P}|, t) \equiv \frac{1}{2}\left[\exp\left(2f_\chi^c(|\boldsymbol{P}|, t)\right) - 1\right], \qquad f_\chi^c(q, t) = \frac{\sqrt{3}\pi\hat{\sigma}^2\rho_{\rm end}^{3/2}M_P}{16m_\phi^7} q^{-9/2}\theta(q-1)\theta\left(\frac{a(t)}{a_{\rm end}} - q\right)$$

where $q = \frac{|\boldsymbol{P}|}{m_\phi}\left(\frac{a}{a_{\rm end}}\right)$

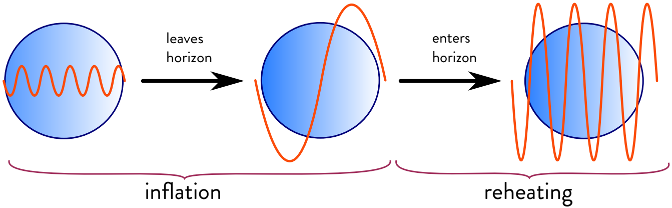
Gravitational particle production

Light scalar fields are unstable during inflation

$$X_k'' + \omega_k^2 X_k \;=\; 0\,, \quad \text{with} \quad \omega_k^2 \;=\; k^2 + 2(aH)^2 \left[\frac{m_\chi^2}{2H^2} + \frac{\sigma\phi^2}{2H^2} - 1\right]$$

For a mode that is outside the horizon ($k/aH \ll 1$),

$$\omega_k^2 \;<\; 0 \qquad \text{if} \qquad m_\chi^2 < 2H^2\,,\; \sigma/\lambda \ll 1 \qquad \text{(tachyonic instability)}$$



No free particle state during inflation $\Rightarrow$ no perturbative picture

Gravitational particle production

Light scalar fields are unstable during inflation

$$X_k'' + \omega_k^2 X_k = 0\,, \quad \text{with} \quad \omega_k^2 = k^2 + 2(aH)^2\left[\frac{m_\chi^2}{2H^2} + \frac{\sigma\phi^2}{2H^2} - 1\right]$$

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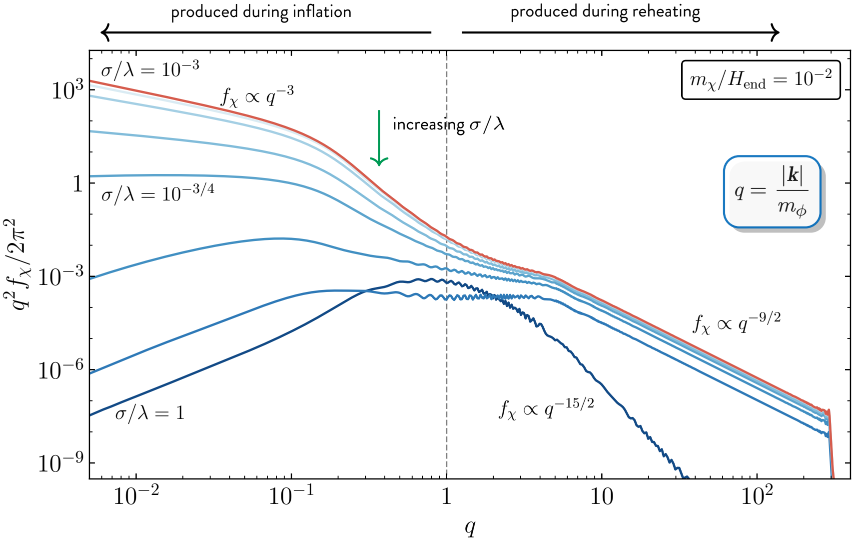
$$X_k(\tau) \simeq \frac{1}{2}\sqrt{-\pi\tau}\, e^{i\frac{\pi}{2}(\nu+\frac{1}{2})} H_\nu^{(1)}(-k\tau) \qquad \Rightarrow \qquad f_\chi = \frac{1}{2\omega_k}|\omega_k X_k - iX_k'|^2 \propto k^{-3\sqrt{1-2\Lambda/3}}$$

with $\Lambda = \frac{2m_{\chi,\text{eff}}^2}{3H_\text{I}^2}$ and $\nu = \sqrt{\frac{9}{4} - \frac{3}{2}\Lambda}$

L. Parker, PRL 21 (1968), 562
L. Ford, PRD 35 (1987), 2955 ...

N. Herring, D. Boyanovsky and A. Zentner, PRD 101 (2020) , 083516
K. Kaneta, S. Lee, and K. Oda, JCAP 09 (2022), 018
S. Ling and A. Long, PRD 103 (2021), 103532
E. Kolb, A. Long, Rev. Mod. Phys. 96 (2024), 045005
A. Chakraborty, S. Cléry et al., PRD 112 (2025), 043511

Weak inflaton coupling



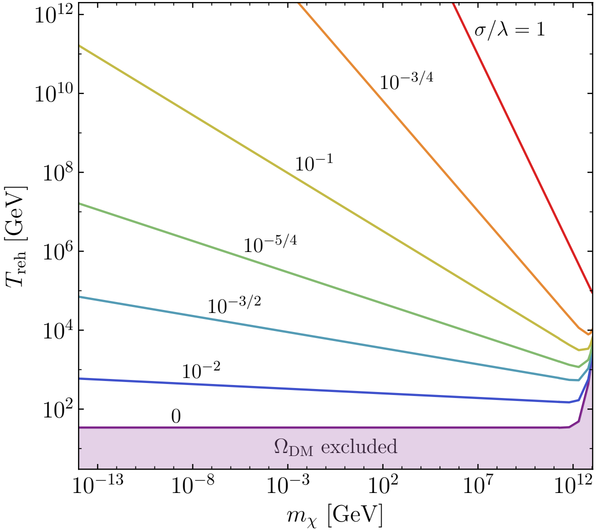
Spectator as Dark Matter

Relic abundance

$$\Omega_{\rm DM} \simeq \frac{\rho_\chi}{\rho_c} \propto \frac{m_\chi T_{\rm reh}}{M_P^2} \underbrace{\int dq\, q^2 f_\chi(q)}_{}$$

$$\sigma/\lambda \lesssim 10^{-3} : \qquad \propto\ m_\chi^{-1}$$

$$\sigma/\lambda \gtrsim 10^{-3} : \qquad \propto\ F(\sigma/\lambda)$$

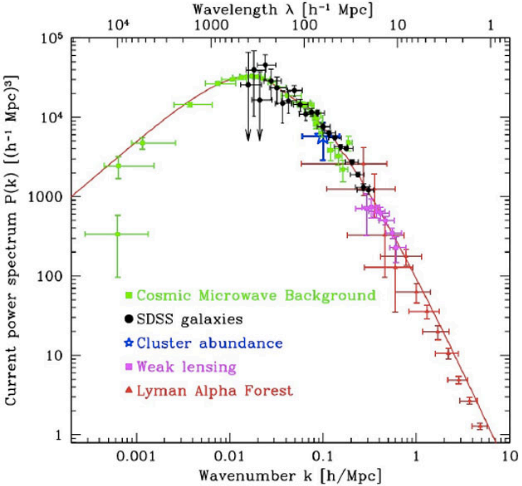


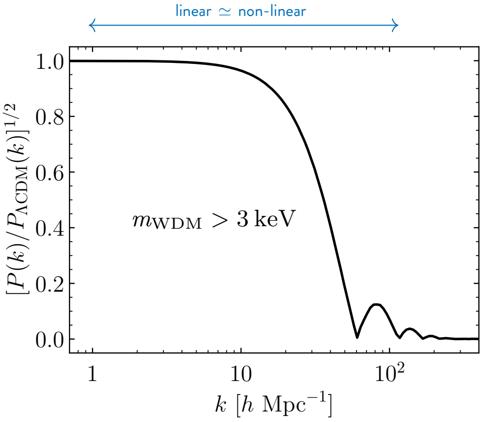
For even larger masses, see

E. Kolb, D. Chung and A. Riotto, AIP Conf. Proc. 484 (1999) 91

D. Racco, S. Verner and W. Xue, JHEP 09 (2024), 129

Limits from structure formation





$$\boxed{m_{\mathrm{DM}} = m_{\mathrm{WDM}} \left(\frac{T_\star}{T_{\mathrm{WDM}}}\right) \sqrt{\frac{\langle q^2 \rangle}{\langle q^2 \rangle_{\mathrm{WDM}}}}}$$

G. Ballesteros, MG and M. Pierre, JCAP 03 (2021), 101

Spectator as Dark Matter

Relic abundance

$$\Omega_{\rm DM} \simeq \frac{\rho_\chi}{\rho_c} \propto \frac{m_\chi T_{\rm reh}}{M_P^2} \int dq\, q^2 f_\chi(q)$$

Structure formation constraint

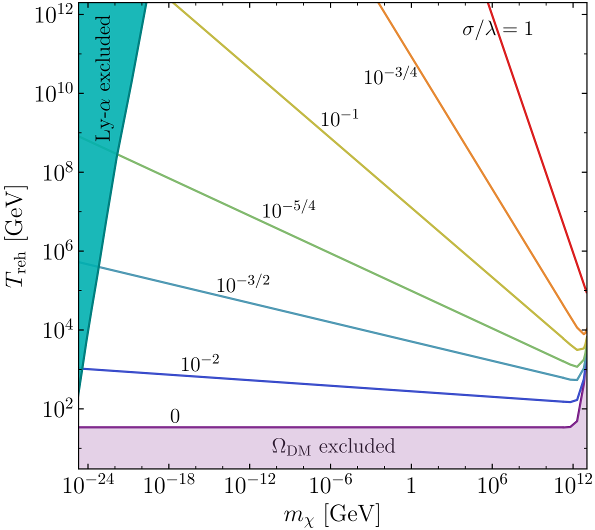
$$m_{\rm WDM} > (1.9 - 5.3)\ {\rm keV} \quad ({\rm Ly}\text{-}\alpha)$$

$$m_{\rm DM} = m_{\rm WDM} \left(\frac{T_\star}{T_{\rm WDM}} \right) \sqrt{\frac{\langle q^2 \rangle}{\langle q^2 \rangle_{\rm WDM}}}$$

For strong couplings, see

MG, M. Pierre and S. Verner, PRD 107 (2023), 043530

MG and A. Pereyra-Flores, JCAP 08 (2024), 043



Isocurvature in the CMB

CDI: cold dark matter density isocurvature

NDI: neutrino density isocurvature

NVI: neutrino velocity isocurvature

However, they have not been detected,

$$\beta_{\rm iso} = \frac{\Delta^2_{\mathcal{S}}}{\Delta^2_\zeta + \Delta^2_{\mathcal{S}}} < \begin{cases} 2.5\% \text{ (CDI)} \\ 7.4\% \text{ (NDI)} \\ 6.8\% \text{ (NVI)} \end{cases}$$

This constraint applies only at

large scales ($k_* = 0.002\,\mathrm{Mpc}^{-1}$)

At smaller scales, ¯_(ツ)_/¯

Y. Akrami et al. [Planck], Astron. Astrophys. 641, A10 (2020)

Isocurvature spectrum of spectator field

The **full** isocurvature spectrum is given by

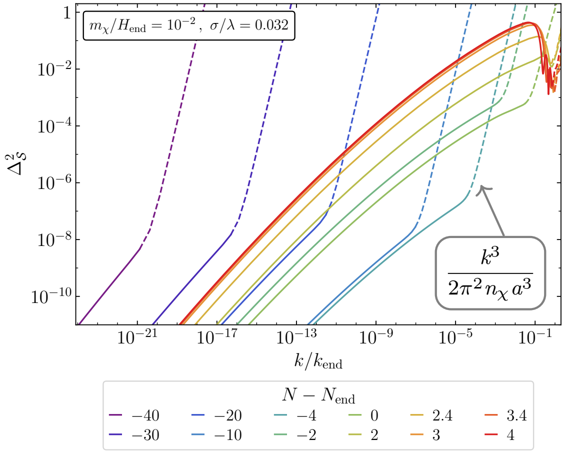
D. Chung, E. Kolb, A. Riotto, L. Senatore, PRD 72, 023511 (2005)

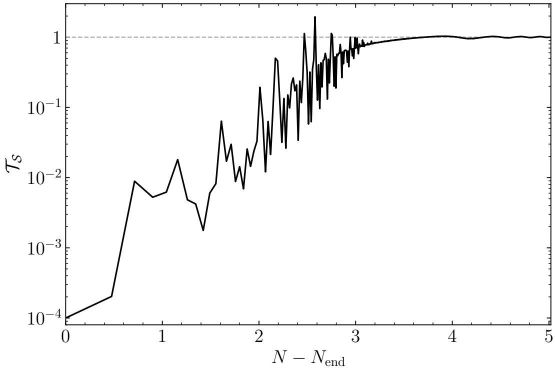
$$\Delta^2_{\mathcal{S}}(k) = \frac{k^3}{2\pi^2\rho_\chi^2}\int d^3\boldsymbol{x}\,\langle :\delta\rho_\chi(\boldsymbol{x})::\delta\rho_\chi(0):\rangle\, e^{-i\boldsymbol{k}\cdot\boldsymbol{x}} = \frac{k^3}{(2\pi)^5\rho_\chi^2}\int d^3\boldsymbol{p}\,P_\chi(p,|\boldsymbol{p}-\boldsymbol{k}|)$$

where

$$\begin{aligned} P_\chi(p,q) &= |\chi'_p|^2|\chi'_q|^2 - H\Big[\chi_p\chi_p'^*|\chi'_q|^2 + \chi_q\chi_q'^*|\chi'_p|^2 - H(\chi_p\chi_p'^*)(\chi'_q\chi_q^*) + \text{h.c.}\Big] \\ &\quad + \left(\frac{p^2+q^2-k^2}{2a^2} + m_{\text{eff}}^2 + H^2\right)\Big[(\chi_p\chi_p'^*)(\chi_q\chi_q'^*) - H\left(\chi_p\chi_p'^*|\chi_q|^2 + \chi_q\chi_q'^*|\chi_p|^2\right) + \text{h.c.}\Big] \\ &\quad + H^2\left(|\chi'_p|^2|\chi_q|^2 + |\chi_p|^2|\chi'_q|^2\right) + \left(\frac{p^2+q^2-k^2}{2a^2} + m_{\text{eff}}^2 + H^2\right)^2|\chi_p|^2|\chi_q|^2 \end{aligned}$$

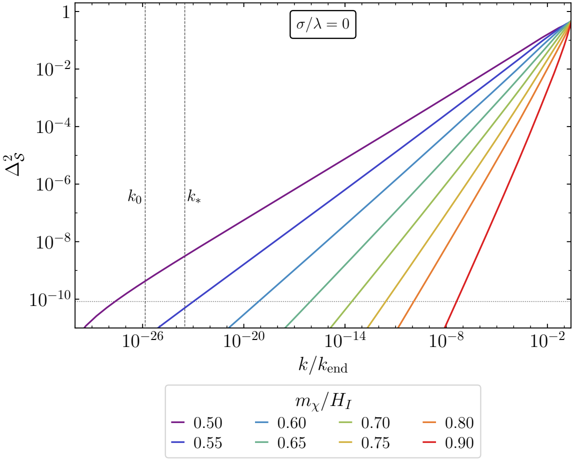
Isocurvature spectrum of spectator field

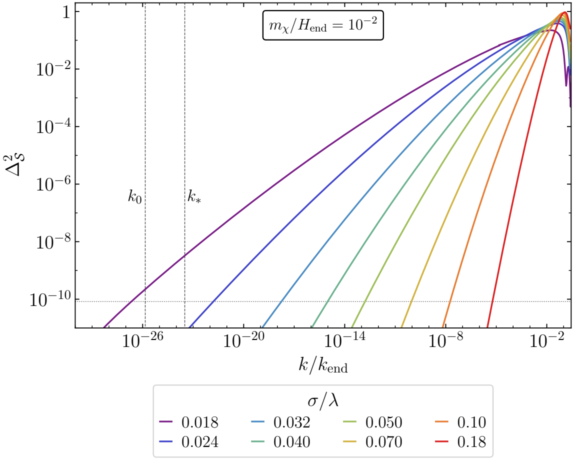




$$\Delta^2_{\mathcal{S}}(\tau, k) \;=\; \mathcal{T}_{\mathcal{S}}(\tau)\Delta^2_{\mathcal{S}}(k)\,, \qquad (k < aH)$$

Isocurvature in gravitational production





$$\Delta^2_{\mathcal{S}}(k) \simeq \Lambda^2 \left(\frac{k}{k_{\rm end}}\right)^{2\Lambda} \ln\left(\frac{k}{k_{\rm end}}\frac{k_{\rm end}}{H_I}\right), \quad \Lambda = \frac{2m^2_{\chi,\rm eff}}{3H^2_{\rm I}}$$

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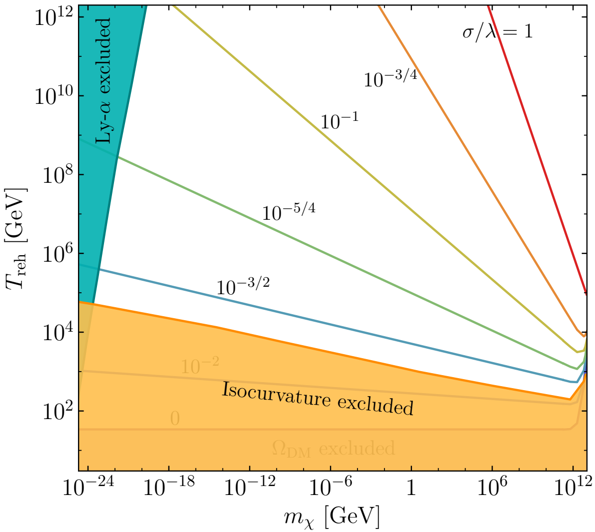
Isocurvature

$$\Delta^2_{\mathcal{S}}(k) = \frac{k^3}{2\pi^2\rho_\chi^2}\int d^3\mathbf{x}\,\langle\delta\rho_\chi(\mathbf{x})\delta\rho_\chi(0)\rangle\, e^{-i\mathbf{k}\cdot\mathbf{x}}$$



Induced gravitational waves

Gravitational waves sourced by the particle production

$$h''_{ij} + 2\mathcal{H}h'_{ij} - \nabla^2 h_{ij} \;=\; P^{ab}_{ij}\{4\partial_a\Phi\partial_b\Phi + 2\partial_a\chi\partial_b\chi\}$$

In the uniform density gauge

$$\nabla^2\Phi \;=\; \frac{a^2}{2M_P^2}\left(\delta\rho_\chi - \frac{3\mathcal{H}}{a^2}\chi'\chi\right)$$

with

$$h_{ij}(\eta,\mathbf{x}) = \sum_\lambda \int \frac{d^3k}{(2\pi)^{3/2}}\, e^{i\mathbf{k}\cdot\mathbf{x}}\epsilon^\lambda_{ij}(\mathbf{k})h_{\mathbf{k},\lambda}(\eta)$$

one gets

$$h_{\boldsymbol{k},\lambda}(N) \;=\; -\frac{1}{M_P^4}\int^N dN'\mathcal{G}_{\boldsymbol{k}}(N,N')\left(\frac{a(N')}{H(N')}\right)^2\int\frac{d^3\boldsymbol{p}}{(2\pi)^{3/2}}\frac{\epsilon^\lambda_{ij}\boldsymbol{p}^i(\boldsymbol{k}-\boldsymbol{p})^j}{|\boldsymbol{p}|^2|\boldsymbol{k}-\boldsymbol{p}|^2}\,\delta\rho_{\chi,\boldsymbol{p}}(N')\delta\rho_{\chi,\boldsymbol{k}-\boldsymbol{p}}(N') + \cdots$$

Define

S. Garcia-Saenz et al. JCAP 03 (2023) 057

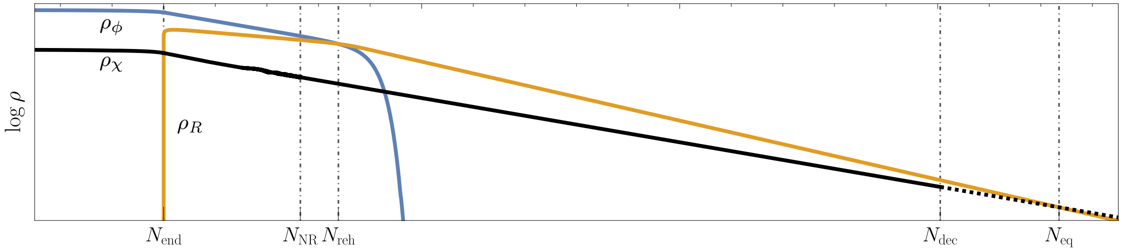
$$\langle h_{\boldsymbol{k}}(N)h_{\boldsymbol{k}'}(N)\rangle \;=\; \frac{2\pi^2}{k^3}\Delta_h^2(k)\,\delta^{(3)}(\boldsymbol{k}-\boldsymbol{k}')$$

Induced gravitational waves

$$\Omega_{\mathrm{GW},\mathcal{S}} = \frac{1}{12}\left(\frac{k}{aH}\right)^2 \Delta_h^2(k)$$

$$\Delta_h^2(k) = 2\left[\frac{1}{8}\int^N dN'\,\mathcal{G}_{\boldsymbol{k}}(N,N')\,\mathcal{T}_\mathcal{S}(N')\left(\frac{a(N')H(N')}{k}\right)^2\left(\frac{\rho_\chi(N')}{H^2(N')M_P^2}\right)^2\right]^2 g(k) + \cdots$$

$$g(k) = k^2\int_0^\infty dp\,p\int_{|k-p|}^{k+p} dq\,q\,\frac{(k^4-2k^2(p^2+q^2)+(p^2-q^2)^2)^2}{p^7\,q^7}\,\Delta_\mathcal{S}^2(p)\Delta_\mathcal{S}^2(q)$$



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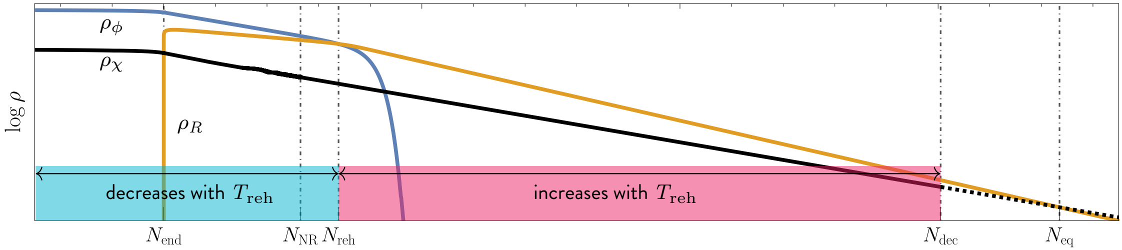
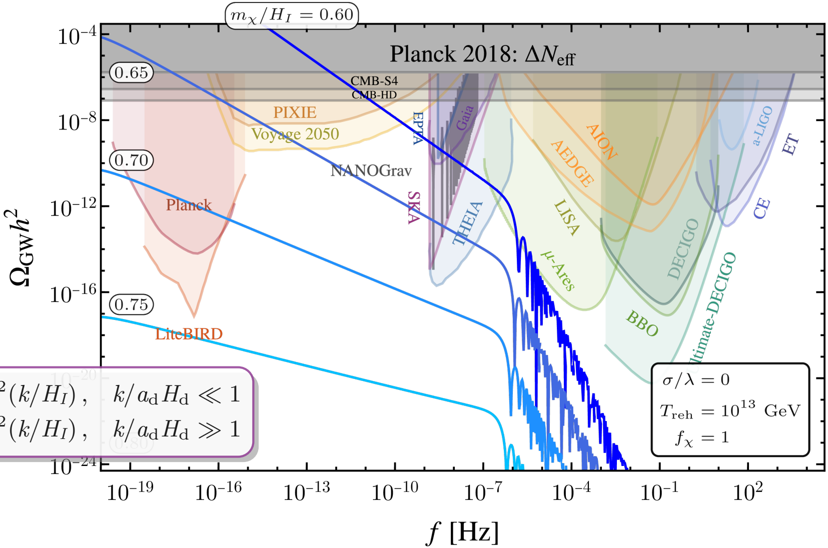
$$g(k) = k^2 \int_0^\infty dp\, p \int_{|k-p|}^{k+p} dq\, q\, \frac{(k^4 - 2k^2(p^2+q^2) + (p^2-q^2)^2)^2}{p^7\, q^7}\, \Delta_{\mathcal{S}}^2(p)\Delta_{\mathcal{S}}^2(q)$$



Massive unstable spectator (curvaton-like)



$$\Omega_{\mathrm{GW},s} \propto \begin{cases} k^{4\Lambda-2}\ln^2(k/H_I)\,, & k/a_{\mathrm{d}} H_{\mathrm{d}} \ll 1 \\ k^{4\Lambda-4}\ln^2(k/H_I)\,, & k/a_{\mathrm{d}} H_{\mathrm{d}} \gg 1 \end{cases}$$

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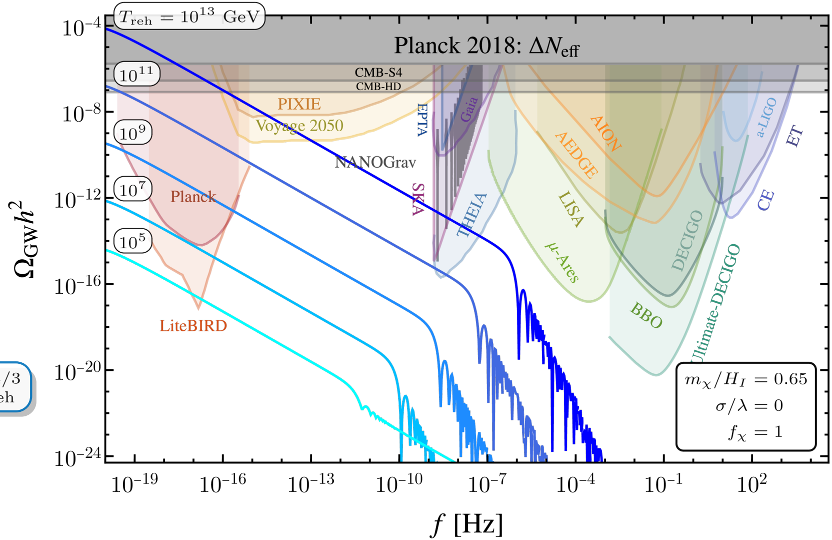
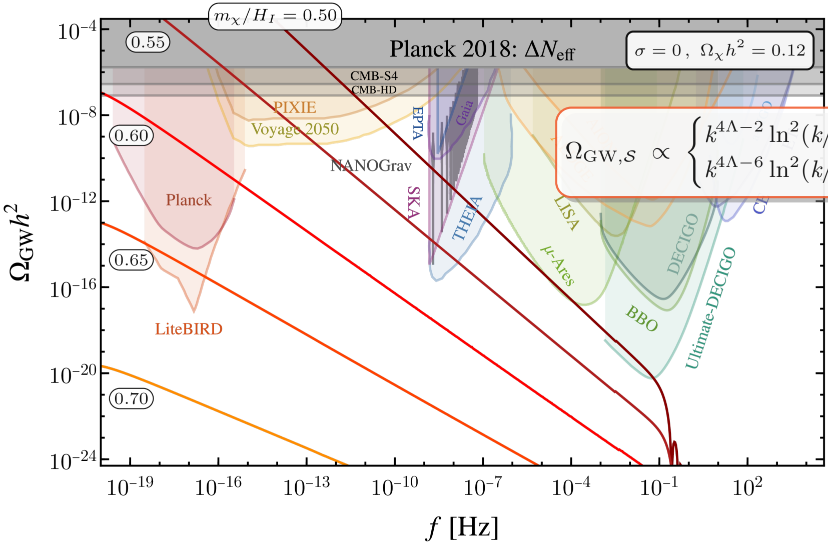


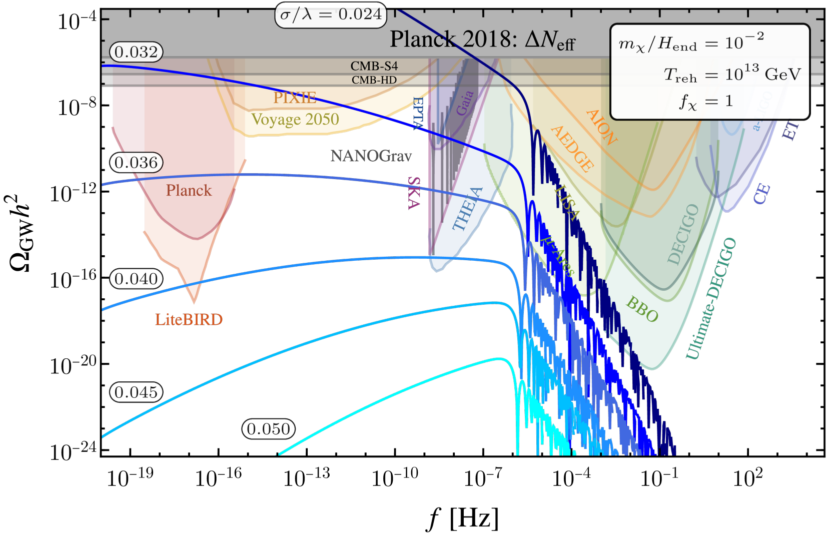
$$\Omega_{\rm GW,\mathcal{S}} \propto T_{\rm reh}^{4/3}$$

Massive stable spectator (dark matter)

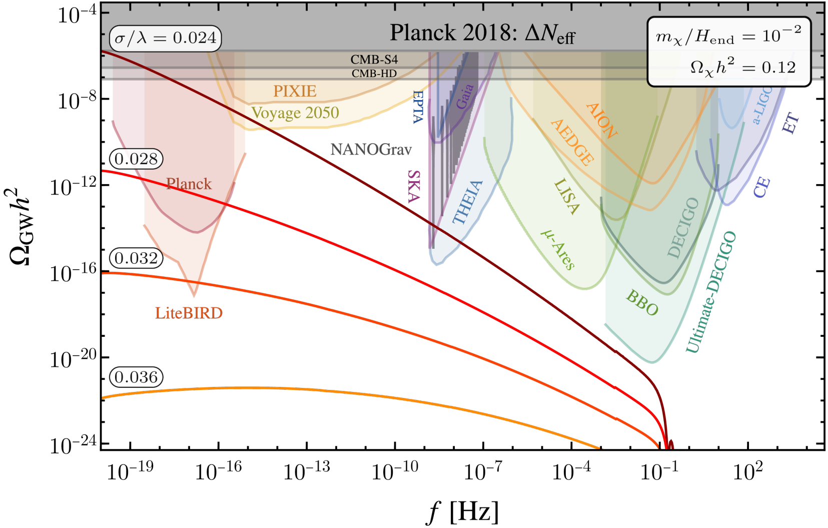


$$\Omega_{{\rm GW},\mathcal{S}} \;\propto\; \begin{cases} k^{4\Lambda-2}\,\ln^2(k/H_I)\,, & k/a_{\rm reh}H_{\rm reh} \ll 1 \\ k^{4\Lambda-6}\,\ln^2(k/H_I)\,, & k/a_{\rm reh}H_{\rm reh} \gg 1 \end{cases}$$

Light unstable spectator (curvaton-like)

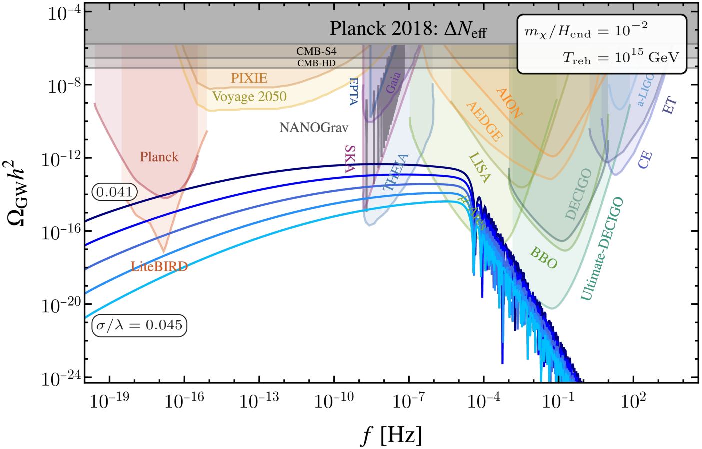


Light stable spectator (dark matter)



Conclusion

- Observable signals across multiple detector bands (CMB, PTAs, space-based interferometers, ground-based detectors), with potential sweet-spot signals for curvaton-like scenarios



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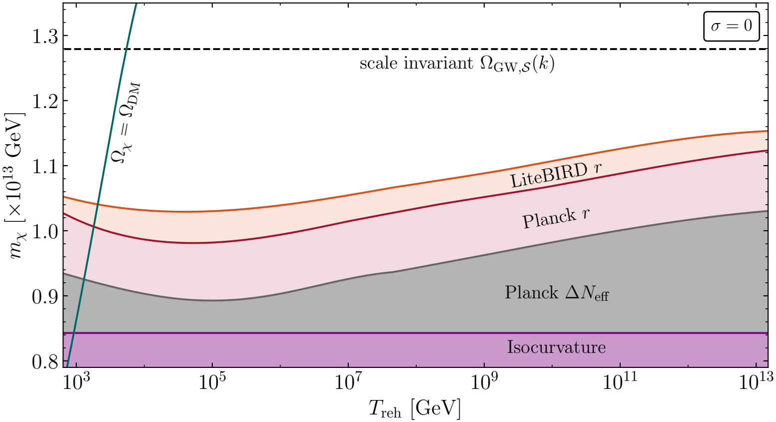
- Tighter constraints than isocurvature at CMB scales, for curvaton- and DM-like scenarios



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- Tighter constraints than isocurvature at CMB scales, for curvaton- and DM-like scenarios
- Plenty more to do:
  - Connected piece and non-Gaussianity

$$\langle\delta\rho_\chi(\mathbf{k}_1)\delta\rho_\chi(\mathbf{k}_2)\delta\rho_\chi(\mathbf{k}_3)\delta\rho_\chi(\mathbf{k}_4)\rangle = \underbrace{\langle\delta\rho_\chi(\mathbf{k}_1)\delta\rho_\chi(\mathbf{k}_2)\delta\rho_\chi(\mathbf{k}_3)\delta\rho_\chi(\mathbf{k}_4)\rangle_d}_{\sim\Delta^2_{\mathcal{S}}(\mathbf{k}_1)\Delta^2_{\mathcal{S}}(|\mathbf{k}_1-\mathbf{q}_1|)} + \underbrace{\langle\delta\rho_\chi(\mathbf{k}_1)\delta\rho_\chi(\mathbf{k}_2)\delta\rho_\chi(\mathbf{k}_3)\delta\rho_\chi(\mathbf{k}_4)\rangle_c}_{\sim\mathcal{T}(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3,\mathbf{k}_4)}$$

  - High frequency signal from field power spectrum

$$h''_{ij}+2\mathcal{H}h'_{ij}-\nabla^2 h_{ij} = P^{ab}_{ij}\{4\partial_a\Phi\partial_b\Phi + 2\partial_a\chi\partial_b\chi\} \quad\Rightarrow\quad \Omega_{\mathrm{GW},\mathcal{S}} = \int_0^\infty dp\, p\int_{|k-p|}^{k+p} dq\,\Delta^2_\chi(p)\Delta^2_\chi(q)\cdots$$

  - PBHs, using a configuration space approach

Thank you!