(In)consistency of the Real 2HDM

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Introduction

Real Two-Higgs-Doublet Model: Usual 2HDM with softly-broken Z_2 symmetry for Natural Flavour Conservation, with CP invariance imposed on scalar potential.

$$\begin{aligned} V &= m_{11}^2 \Phi_1^\dagger \Phi_1 + m_{22}^2 \Phi_2^\dagger \Phi_2 - \left[m_{12}^2 \Phi_1^\dagger \Phi_2 + \text{h.c.} \right] \\ &+ \frac{1}{2}\lambda_1 \left(\Phi_1^\dagger \Phi_1\right)^2 + \frac{1}{2}\lambda_2 \left(\Phi_2^\dagger \Phi_2\right)^2 + \lambda_3 \left(\Phi_1^\dagger \Phi_1\right)\left(\Phi_2^\dagger \Phi_2\right) + \lambda_4 \left|\Phi_1^\dagger \Phi_2\right|^2 \\ &+ \left\{ \frac{1}{2}\lambda_5 \left(\Phi_1^\dagger \Phi_2\right)^2 + \text{h.c.} \right\}. \end{aligned}$$

The issue: CP is broken (hard) by phase in CKM matrix. Should propagate into renormalization of CPV phase[$(m_{12}^{2\,*})^2 \lambda_5$].

Highlighted by D. Fontes, M. Löschner, J.C. Romão, & J.P. Silva, 2103.05002: performed a state-of-the-art 3-loop calculation of A^0 tadpole; found that leading imaginary divergent contribution canceled!

We demonstrate that imaginary divergent contributions should indeed show up, but that they will first appear at 7 loops.

Outline

Framework for analyzing divergences

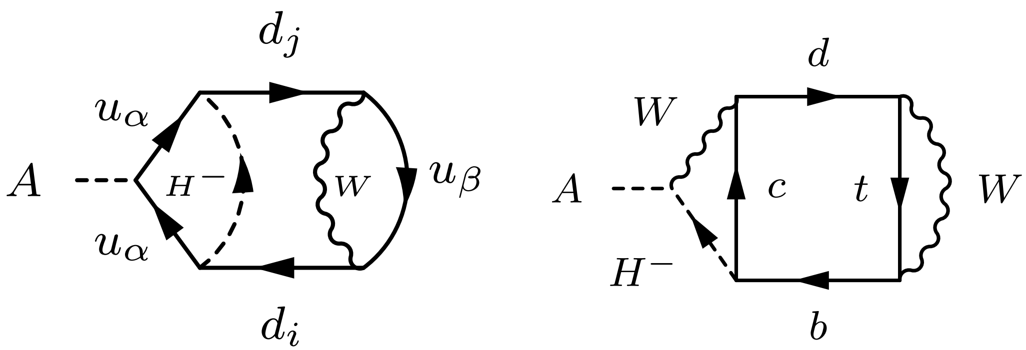
Necessary ingredients for CP-violating divergences
- Type II
- Type I

Practical consequences?

Conclusions

Framework for analyzing divergences

D. Fontes et al., arXiv:2103.05002: (sample diagrams)



- 4 insertions of CKM matrix: lowest order that can give rise to Jarlskog invariant $J = \left|\mathrm{Im}(V_{\alpha i} V_{\beta j} V^*_{\alpha j} V^*_{\beta i})\right|$
- 208 nonvanishing fully-massive 3-loop vacuum* diagrams for each combination of fermion flavours *no external momentum injections
- Two independent software chains; valuable stress-test of tools
- Computed the leading $1/\varepsilon^3$ divergence
- CPV divergence found! But it canceled upon summation over all possible quark combinations!!

Need different strategy to understand cancellation and to go beyond 3 loops.

Framework for analyzing divergences

All we care about (in this talk) is whether we'll need a counter-term for $\mathrm{Im}(\lambda_5)$. $\Rightarrow$ Equivalent to asking whether $\mathrm{Im}(\lambda_5)$ runs under renormalization-group (RG) flow.

Exploit known properties of RG equations:
(1) Spontaneous symmetry breaking does not affect RGEs of quartic couplings – can work in unbroken phase. Simplifies sum over all quarks to just a trace of products of Yukawa matrices.
(2) Mass terms don't affect RGEs of quartic couplings – can treat all particles as massless. (IR regulator does not affect renormalizability.)

... and of Feynman diagrams:
(3) Coefficient of *local* divergence is polynomial in momenta; nonlocal divergences (involving logs) guaranteed to be canceled by lower-order counterterms – can use dimensional analysis.

(4) Consider properties of *pairs* of diagrams that are related by a well-defined transformation, and look for cancellations of imaginary divergence.

Ingredients for CP Violation: Jarlskog invariant

Reparametrization-invariant measure of the CP violation in the CKM matrix Jarlskog, ZPhysC, PRL 1985

$$J = \left|\mathrm{Im}(V_{\alpha i} V_{\beta j} V^*_{\alpha j} V^*_{\beta i})\right|, \qquad (\alpha \neq \beta, i \neq j)$$

More useful here to express it in terms of the quark Yukawa couplings. Define combinations of quark Yukawa matrices:

$$\widehat{H}_u = Y_u Y_u^\dagger \qquad \widehat{H}_d = Y_d Y_d^\dagger$$

Minimal combination that yields an imaginary part involves 12 powers of Y's: Botella & Silva, PRD 1995

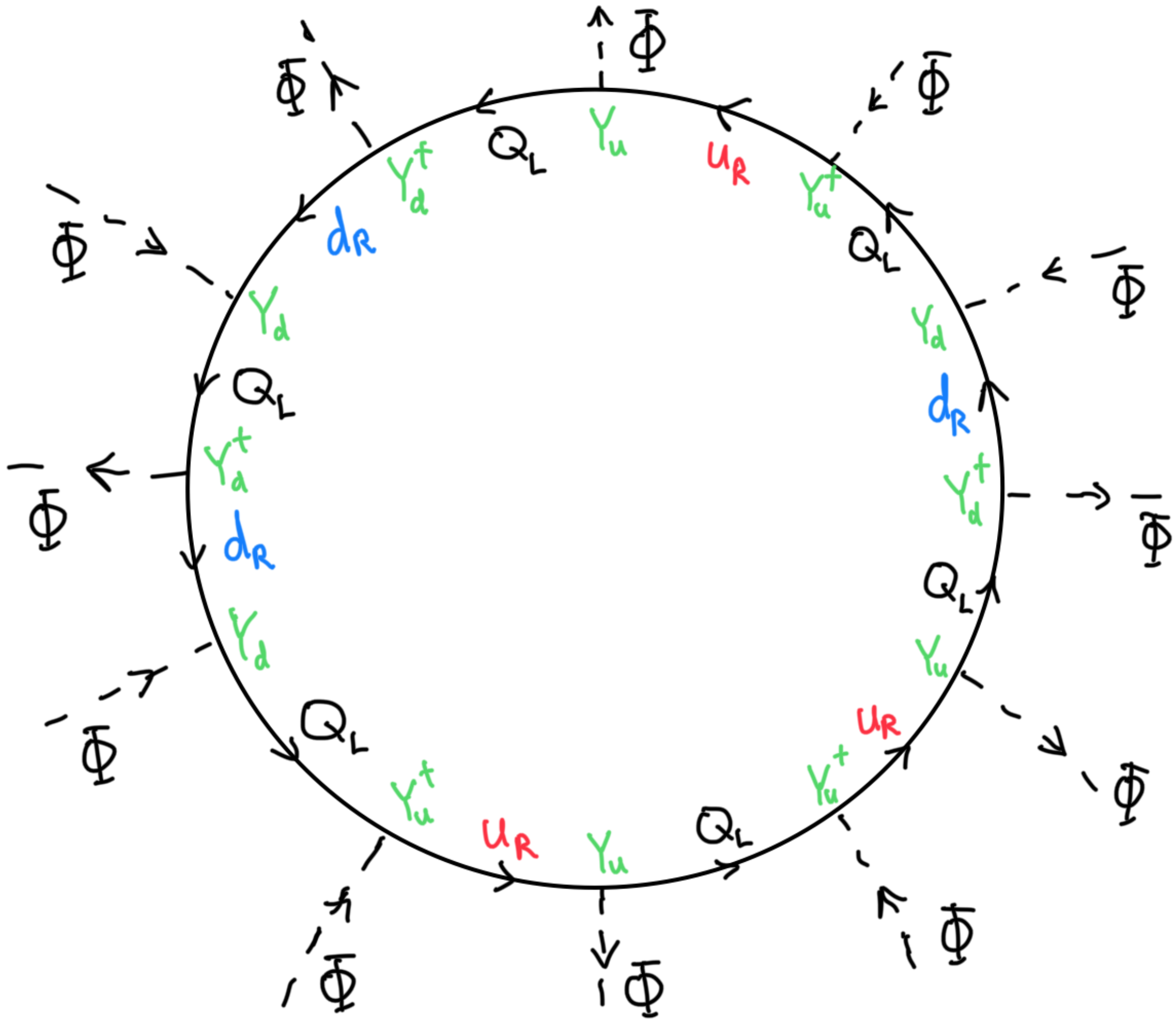
$$\mathcal{J} = \mathrm{Tr}\left(\widehat{H}_u \widehat{H}_d \widehat{H}_u^2 \widehat{H}_d^2\right) \qquad \text{where } \mathrm{Im}(\mathcal{J}) \propto J$$

(Any fewer powers of $\widehat{H}_i$ gives a Hermitian matrix inside the trace.)

In the unbroken phase, this trace arises from the sum over quark generations in diagrams involving one big quark circle with 12 scalars attached.

$\Rightarrow$ Draw the simplest loop diagrams that involve $\mathcal{J}$ or $\mathcal{J}^*$, then try to build the non-Hermitian operators $\left(\Phi_1^\dagger \Phi_2\right)^2$ or $\Phi_1^\dagger \Phi_2$.

Ingredients for CP Violation: Jarlskog invariant



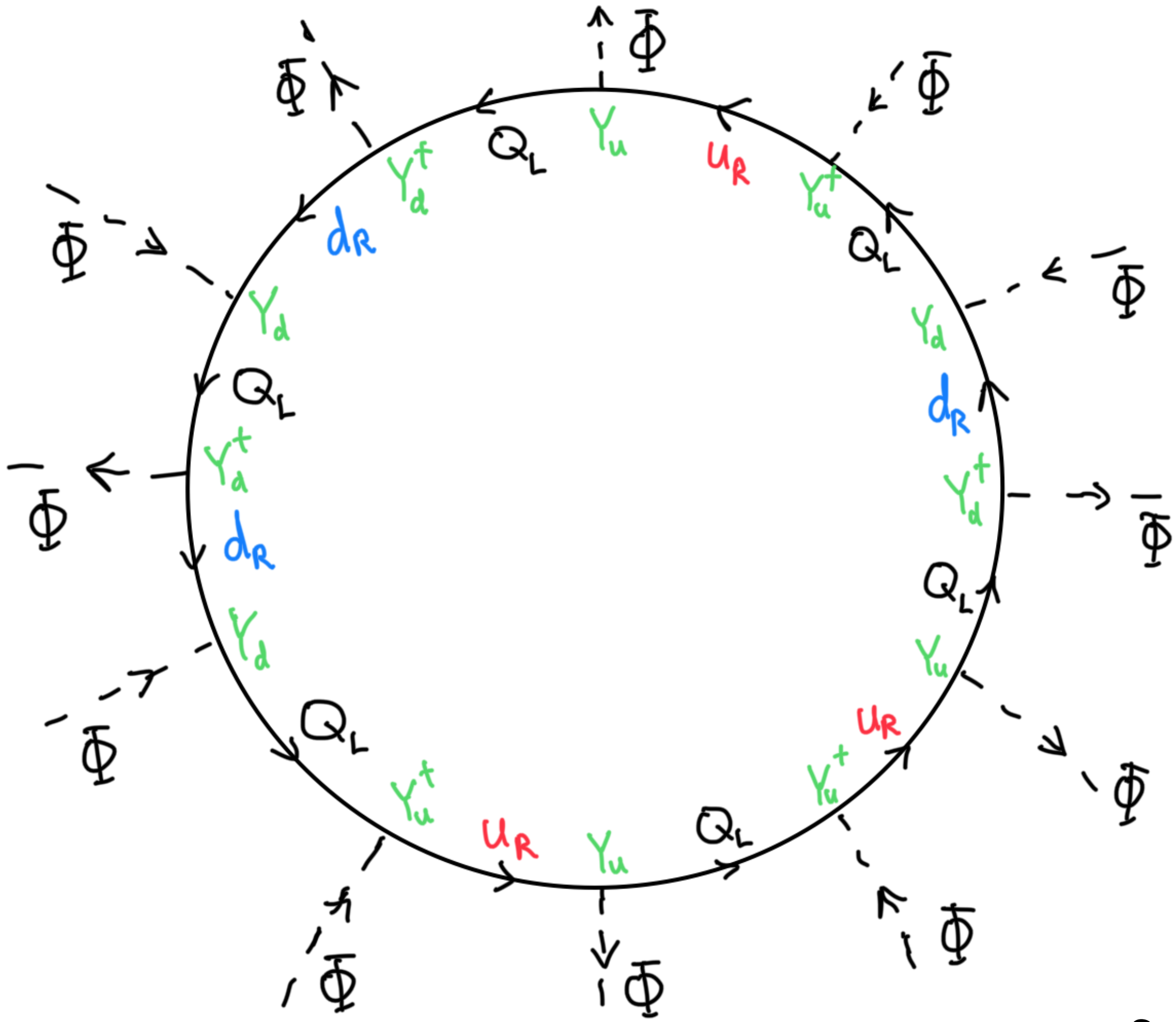
4-point operator:
connect 8 legs
$\Rightarrow$ min. 5 loops.

2-point operator:
connect 10 legs
$\Rightarrow$ min. 6 loops.

Immediately explains cancellation of divergent CPV at 3 loops when summed over quark generations!

(3 loops, broken phase: enough CKM's, but too few quark mass insertions.)

Ingredients for CP Violation: Jarlskog invariant



Type I:

6 incoming Φ_2's

6 outgoing Φ_2's

Type II:

3 incoming Φ_1's

3 outgoing Φ_1's

3 incoming Φ_2's

3 outgoing Φ_2's

Cannot actually construct desired $\left(\Phi_1^\dagger\Phi_2\right)^2$ or $\Phi_1^\dagger\Phi_2$ operators, without adding another ingredient!
Need to, e.g., convert two outgoing Φ_2's into Φ_1's.

Ingredients for CP violation: breaking the would-be $U(1)$

Consider the quark Yukawa couplings after imposing Natural Flavour Conservation:

$$\mathcal{L}_{Yuk} = -Y^d_{ij}\bar{Q}_{Li}\Phi_1 d_{Rj} - Y^u_{ij}\bar{Q}_{Li}\tilde{\Phi}_2 u_{Rj} + \text{h.c.}$$

(for Type II; replace Φ_1 with Φ_2 for Type I.)

We normally enforce this by imposing a Z_2 symmetry.

But we could equally well have achieved this form for the Yukawa couplings by imposing a global $U(1)$ symmetry, e.g.:

$$\Phi_1 \to e^{-i\theta}\Phi_1, \qquad \Phi_2 \to e^{i\theta}\Phi_2$$

with Q_L invariant and

Ferreira & Silva 2011

$$u_R \to e^{i\theta}u_R, \qquad d_R \to e^{-i\theta}d_R \qquad \text{(Type I)}$$
$$u_R \to e^{i\theta}u_R, \qquad d_R \to e^{i\theta}d_R \qquad \text{(Type II)}$$

(For Type II, this is equivalent to the Peccei-Quinn $U(1)$.)

Ingredients for CP violation: breaking the would-be $U(1)$

Global $U(1)$ symmetry $\Phi_1 \to e^{-i\theta}\Phi_1, \quad \Phi_2 \to e^{i\theta}\Phi_2$ is broken only by $\lambda_5 \neq 0$ (and softly broken by $m_{12}^2 \neq 0$).

$$\begin{aligned} V &= m_{11}^2 \Phi_1^\dagger \Phi_1 + m_{22}^2 \Phi_2^\dagger \Phi_2 - \left[m_{12}^2 \Phi_1^\dagger \Phi_2 + \text{h.c.} \right] \\ &+ \frac{1}{2}\lambda_1 \left(\Phi_1^\dagger \Phi_1\right)^2 + \frac{1}{2}\lambda_2 \left(\Phi_2^\dagger \Phi_2\right)^2 + \lambda_3 \left(\Phi_1^\dagger \Phi_1\right)\left(\Phi_2^\dagger \Phi_2\right) + \lambda_4 \left|\Phi_1^\dagger \Phi_2\right|^2 \\ &+ \left\{ \frac{1}{2}\lambda_5 \left(\Phi_1^\dagger \Phi_2\right)^2 + \text{h.c.} \right\}. \end{aligned}$$

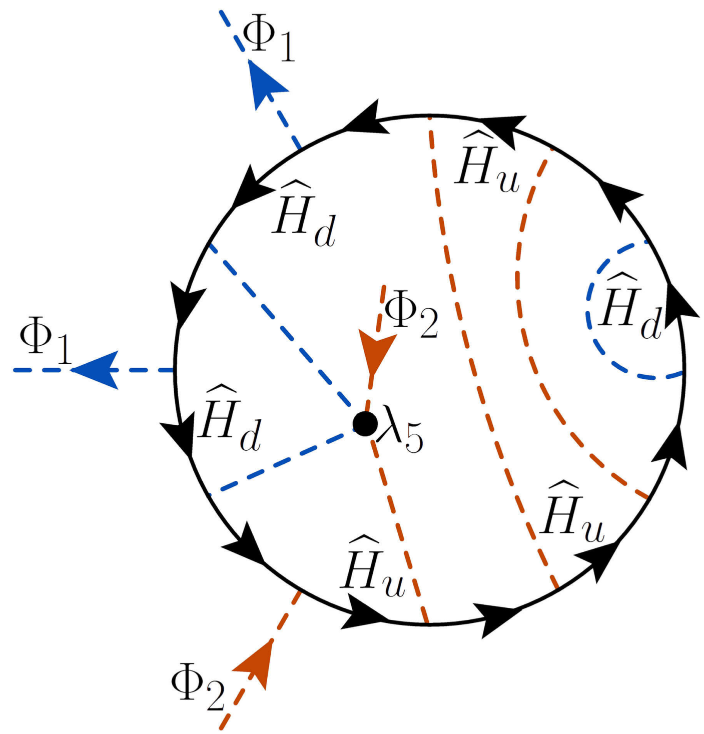
In the (softly-broken) $U(1)$ model, $\lambda_5 = 0$ and complex phase of m_{12}^2 can be trivially rotated away. $\Rightarrow$ Protected to all orders from divergent CPV by global $U(1)$! Pilaftsis 1998

Diagrams involving only Yukawa vertices preserve the $U(1)$: Cannot generate $\left(\Phi_1^\dagger \Phi_2\right)^2$ or $\Phi_1^\dagger \Phi_2$ without a $U(1)$-breaking coupling insertion (converts e.g. two outgoing Φ_2's into Φ_1's).
$\Rightarrow$ Minimum 6 loops for $\left(\Phi_1^\dagger \Phi_2\right)^2$.

Ingredients for CP violation: breaking the would-be $U(1)$

12 Yukawas plus a λ_5 vertex successfully generates diagrams for $\left(\Phi_1^\dagger \Phi_2\right)^2$ proportional to $\mathcal{J}$, in both Type I and Type II 2HDMs!

(Sample diagram: Type II)



Superficial degree of divergence is zero; no reason for divergent piece to vanish.

Are we done? (No.)

Have to consider possible cancellations among diagrams.

From this point, have to study Type I and Type II separately. Start with Type II because it's simpler.

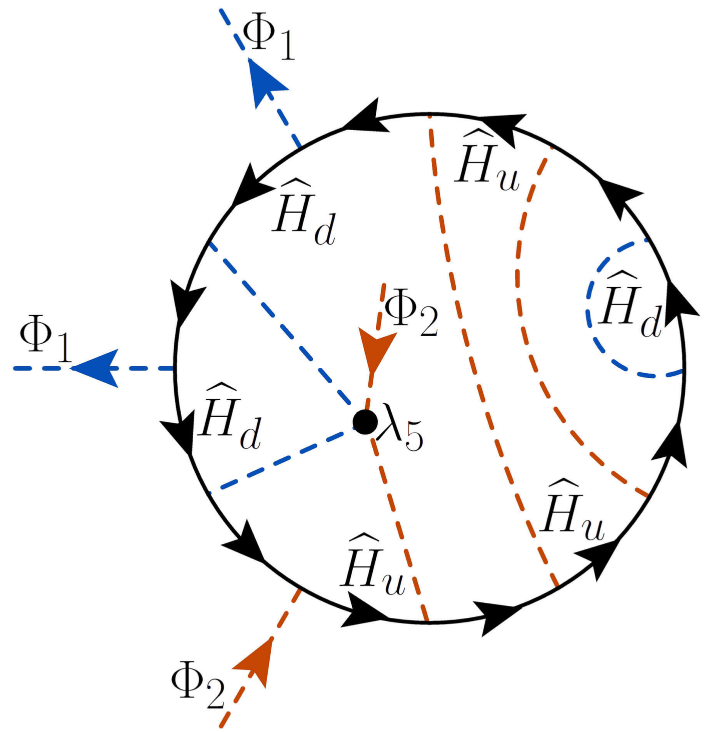
Ingredients for CP violation (Type II): breaking a generalized CP transformation

Consider a 6-loop diagram proportional to $\lambda_5 \mathcal{J}$ that generates $\left(\Phi_1^\dagger \Phi_2\right)^2$.

For each such diagram, we can construct another diagram that generates $\left(\Phi_1^\dagger \Phi_2\right)^2$ by applying a generalized CP transformation: Branco et al. 2012

- exchanging $u_R \leftrightarrow d_R$, $(\mathcal{J} \to \mathcal{J}^*)$
- exchanging $\Phi_1 \leftrightarrow \tilde{\Phi}_2$. $(\lambda_5 \to \lambda_5)$

New diagram is proportional to $\lambda_5 \mathcal{J}^*$!
(recall $\mathcal{J} = \mathrm{Tr}\left(\widehat{H}_u \widehat{H}_d \widehat{H}_u^2 \widehat{H}_d^2\right)$)

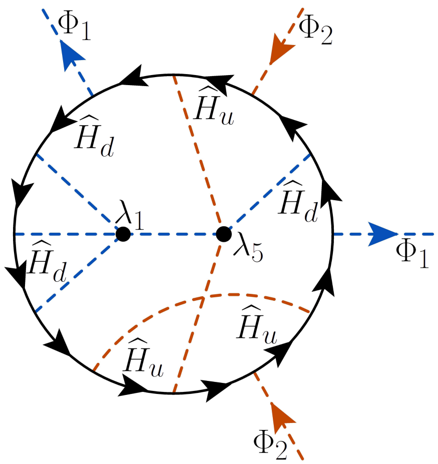


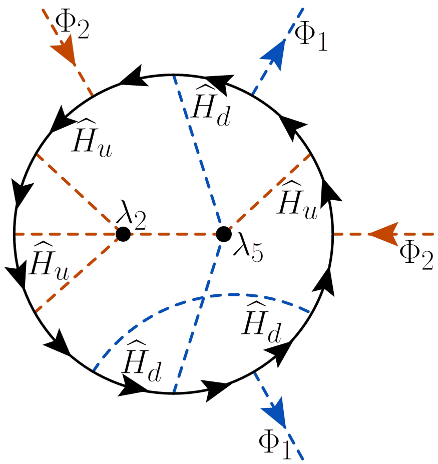
Because the coefficient of the divergent piece(s) cannot depend on masses, and the structure of the diagram is otherwise identical (at zero external momenta), the imaginary part of the divergence must cancel between this pair of diagrams!

Ingredients for CP violation (Type II): breaking a generalized CP transformation

To break this cancellation, we have to go to 7 loops by adding an interaction that is not invariant under the GCP transformation:

- a hypercharge gauge boson (distinguishes u_R from d_R);
- a λ_1 or λ_2 vertex (distinguishes Φ_1 from Φ_2 if $\lambda_1 \neq \lambda_2$);
- another pair of Yukawas in the quark loop (either $Y_u Y_u^\dagger$ or $Y_d Y_d^\dagger$).



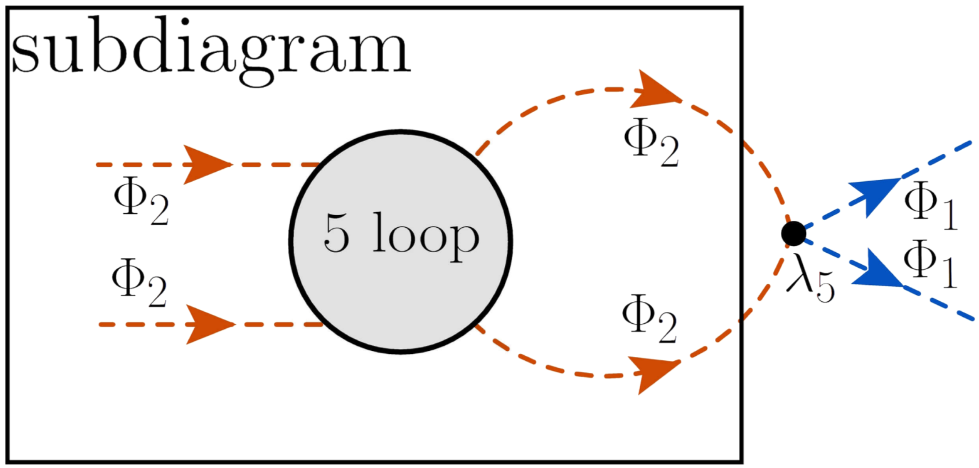


Predict the parameter dependence of the RGE for Im(λ_5)!

$$\frac{d\,\mathrm{Im}(\lambda_5)}{d\ln\mu} = \frac{\lambda_5 \mathrm{Im}(\mathcal{J})}{(16\pi^2)^7}\left[a_1(\lambda_1 - \lambda_2) + a_2 g'^2 + a_3(y_t^2 - y_b^2 + \ldots)\right]$$

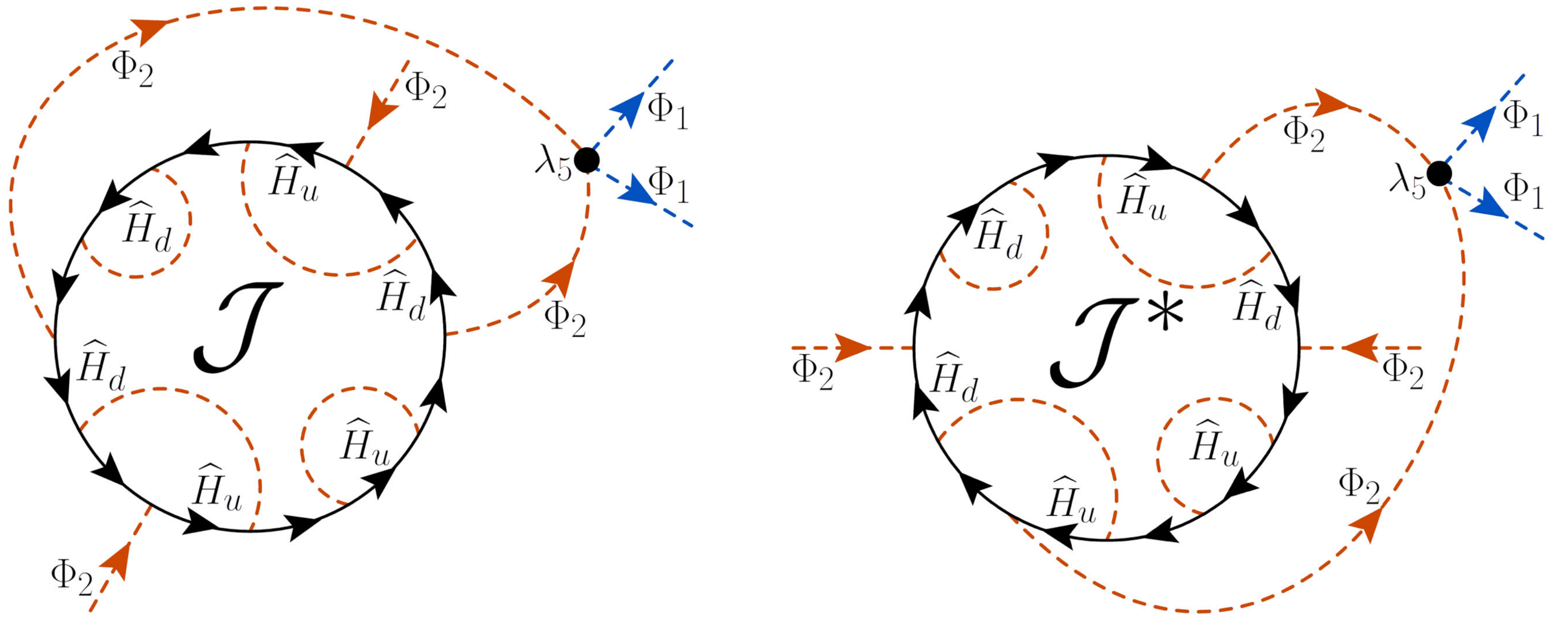
Ingredients for CP violation (Type I): breaking the subdiagram structure

Type I 2HDM: structure of 6-loop diagrams more constrained: only Φ_2 couples to quarks, so the two external Φ_1's must be attached to the λ_5 vertex.



Ingredients for CP violation (Type I): breaking the subdiagram structure

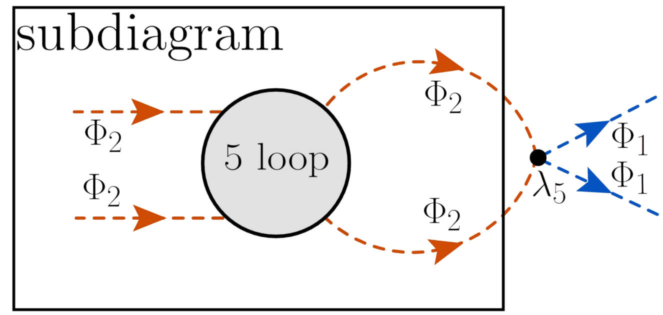
For each such diagram $\propto \mathcal{J}$, can construct another diagram $\propto \mathcal{J}^*$ by complex-conjugating the subdiagram and moving the λ_5 insertion to the other pair of Φ_2 legs.



The only structural difference between these diagrams is that the injection of momentum flowing in the 6th loop happens in a different place. Does the imaginary part cancel?

Ingredients for CP violation (Type I): breaking the subdiagram structure

- Treat the sum of all 5-loop subdiagrams proportional to $\mathcal{J}$ as a formfactor, which depends on the momenta of its four legs.



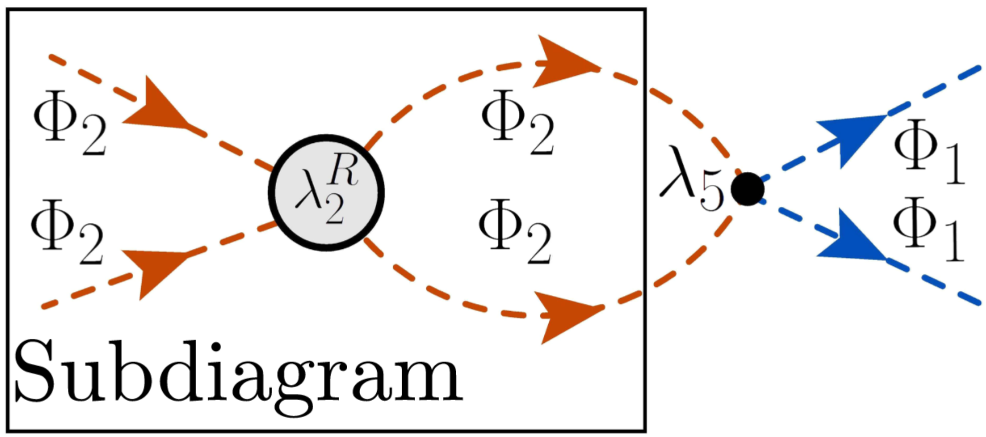
- Any parts of this formfactor that will later contribute to divergences of the 6-loop diagram must be an analytic dimensionless function of Lorentz-invariant combinations of the momenta of its four legs.

- If this formfactor has a piece which is antisymmetric when the relevant momenta are swapped, then the imaginary divergence will not cancel. Ex: $(k_a^2 - k_b^2)/f(k_i^2)$

- But an antisymmetric piece of a dimensionless formfactor lacks a well-defined zero-momentum limit: unphysical, must = 0!

Ingredients for CP violation (Type I): breaking the subdiagram structure

The formfactor for the 5-loop subdiagram is "actually" just the renormalization of the operator $(\Phi_2^\dagger \Phi_2)^2$, which is hermitian and thus cannot acquire an imaginary part (divergent or otherwise) in the zero-momentum limit.



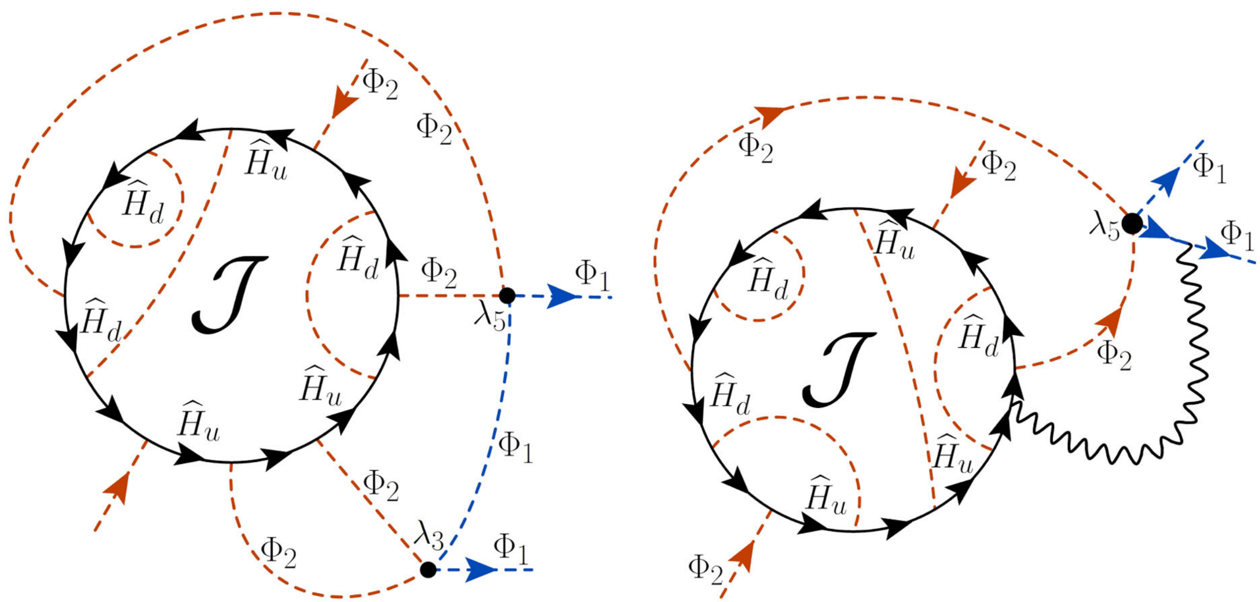
The coefficient of this operator being dimensionless guarantees that momentum dependence cannot circumvent this conclusion.

To break the cancellation, must break the subdiagram structure.

Ingredients for CP violation (Type I): breaking the subdiagram structure

To break the cancellation, again have to go to 7 loops: attach something to *both* the 5-loop formfactor *and* an external Φ_1 leg:

- a $\lambda_3 \left(\Phi_1^\dagger \Phi_1\right)\left(\Phi_2^\dagger \Phi_2\right)$ or $\lambda_4 \left|\Phi_1^\dagger \Phi_2\right|^2$ vertex;
- an $SU(2)_L$ or hypercharge gauge boson.



Predict the parameter dependence of the RGE for $\mathrm{Im}(\lambda_5)$!

$$\frac{d\,\mathrm{Im}(\lambda_5)}{d \ln \mu} = \frac{\lambda_5 \mathrm{Im}(\mathcal{J})}{(16\pi^2)^7}\left[b_1\lambda_3 + b_2\lambda_4 + b_3 g'^2 + b_4 g^2\right]$$

Practical consequences?

The imaginary part of λ_5 runs starting at 7 loops: Real 2HDM is formally nonrenormalizable starting at this order.

Estimate size of Im(λ_5) assuming Im(λ_5) = 0 at Planck scale (for some unknown reason): take a_i, b_i $\sim$ number of diagrams, get Im(λ_5) $\sim 10^{-22}$ (versus Re(λ_5) $\sim$ 1).

So tiny that it has no conceivable observable effect!

- Contribution to eEDM orders of magnitude smaller than SM
- Mixing angle between A^0 and $h^0, H^0 \lll 1$

Can continue to use the Real 2HDM... but have to accept that the question of why Im(λ_5) = 0 at a high scale is not answered.

Probably more sensible to accept that the 2HDM is likely to be complex (and somewhat tuned) rather than strictly real.

$$\left(\frac{1\ \text{TeV}}{M}\right)^2 \text{Im}(\lambda_5) \times f(\sin^2\beta, \cos^2\beta) \lesssim 0.5 - 1\%$$

from $|d_e| < 4.1 \times 10^{-30} e\,\text{cm}$ (JILA 2022) Altmannshofer, Gori, Hamer, & Patel, 2020

Conclusions

Principle of QFT: Can't impose a symmetry on one part of a theory if it is violated in another (interacting) part of the theory.

Properties of divergent piece of Feynman diagrams (or sums thereof) are remarkably constrained by known features of renormalization: allows exploitation of overlapping approximate symmetries order-by-order to build up required "ingredients" for a CPV divergence in the scalar potential.

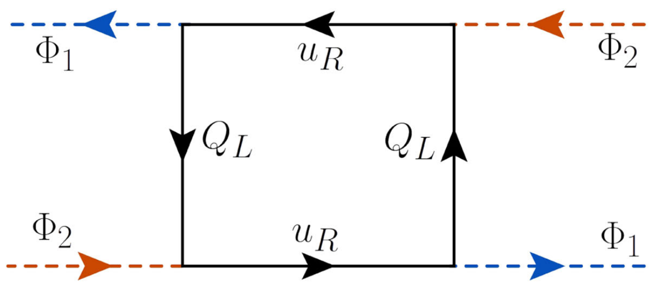
Real 2HDM is an artificial construct... but the CKM structure together with various approximate symmetries is very effective at sequestering the CP violation $\Rightarrow$ RGE starts at 7 loops!

BACKUP SLIDES

2HDM without Z_2 symmetry:

Compare "Type III" 2HDM without Z_2: divergent CPV already at 1-loop from BSM CPV in flavour-violating Yukawa matrices.

New sources of CPV in flavour-violating Yukawa matrices.



$$(\delta\lambda_5)_{\text{div}} \propto \text{Tr}\left(Y_u^{(1)} Y_u^{(2)\dagger} Y_u^{(1)} Y_u^{(2)\dagger}\right) + \cdots$$

Has a nonzero imaginary part in general, already at 1-loop.

Symmetries of the 2HDM and the role of λ_5

Corollary: If one wants a real 2HDM that is guaranteed in an obvious way to be safe from CPV "leaks" (and hence theoretically consistent), use the softly-broken-$U(1)_{PQ}$ 2HDM (with $\lambda_5 = 0$).

- Scalar mass spectrum is more constrained than in softly-broken-Z_2 model due to perturbativity and bounded-from-below constraints, but still fully viable phenomenologically.

- One coupling degree of freedom is removed from triple- and quartic-scalar couplings: $U(1)_{PQ}$ model is more predictive (less general) than Z_2 version.

λ_5 dependence shows up in triple-Higgs couplings including $h^0 H^+ H^-$ coupling: $U(1)_{PQ}$ restricts the charged Higgs contribution to $h^0 \to \gamma\gamma$.

$\mathcal{J}$ and the Jarlskog invariant

We define $\mathcal{J}$ in the unbroken phase as

$$\mathcal{J} = \mathrm{Tr}\left(\widehat{H}_u \widehat{H}_d \widehat{H}_u^2 \widehat{H}_d^2\right) \qquad \text{where } \widehat{H}_u = Y_u Y_u^\dagger,\ \widehat{H}_d = Y_d Y_d^\dagger$$

(following Botella & Silva, 1995)

This is related to the original Jarlskog invariant J according to (in Type II):

$$\begin{aligned} \mathrm{Im}(\mathcal{J}) &= \frac{2^6}{v_1^6 v_2^6} \mathrm{Im}\left\{\mathrm{Tr}\left(V^\dagger M_U^2 V M_D^2 V^\dagger M_U^4 V M_D^4\right)\right\} \\ &= \frac{2^6}{v_1^6 v_2^6} T(M_U^2)\, B(M_D^2)\, J, \end{aligned}$$

where

$$\begin{aligned} T(M_U^2) &= (m_t^2 - m_c^2)(m_t^2 - m_u^2)(m_c^2 - m_u^2), \\ B(M_D^2) &= (m_b^2 - m_s^2)(m_b^2 - m_d^2)(m_s^2 - m_d^2). \end{aligned}$$

In terms of measured CKM elements and quark masses, this gives

$$\mathrm{Im}(\mathcal{J}) \simeq 2 \times 10^{-24} / \sin^6\beta \cos^6\beta$$

(For Type I, replace $v_1 \to v_2$ and $\cos\beta \to \sin\beta$.)