

Hierarchies and conformal UV Completions

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Scalars 2025: Higgs bosons and cosmology

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The Success of the Standard Model

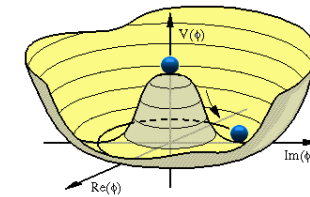
The SM is the endpoint of a very successful development: d=4 renormalizable gauge theory

$$\begin{array}{lll} \text{QED} \Rightarrow & \text{QCD} \Rightarrow & \text{SM} \\ U(1)_{em} \Rightarrow & SU(3)_c \Rightarrow & SU(3)_c \times SU(2)_L \times U(1)_Y \end{array}$$

new ingredient:

Higgs = fundamental scalar

→ spontaneous symmetry breaking



based on QFT in 4d, symmetries & quantum effects

→ excellent agreement of theory and experiment

→ known deficiencies ... → BSM!

Exper. facts, hints, problems:

- Electro-weak scale \ll Planck scale
- Gauge couplings almost unify
- Neutrino masses & large mixings
- Flavour: Patterns of masses & mixings
- Baryon asymmetry of the Universe
- Dark Matter
- Dark Energy

Theoretical problems:

SM does not exist without cutoff

(triviality, vacuum stability)

Gauge hierarchy problem – became worse

Gauge unification & charge quantization

Strong CP problem

Unification with gravity

3 generations, reps., d=4, many parameters

Standard Model Hierarchy Problems

The (old) hierarchy problem: quantum stability of scales

- SM with a cutoff $\Lambda \gg M_H$

$$\delta M_H^2 = \frac{\Lambda^2}{32\pi^2 V^2} (6M_W^2 + 3M_Z^2 + 3M_H^2 - 12M_t^2) \sim \Lambda^2 \gg M_H^2$$

- big quantum corrections **pull M_H to $\Lambda \rightarrow$ problem!**
- SM is renomalizable, no cutoff \rightarrow no problem!
 $\Lambda \leftrightarrow$ new physics (= embedding, typically more scales)

The (new) little hierarchy problem: (so far) nothing showed up

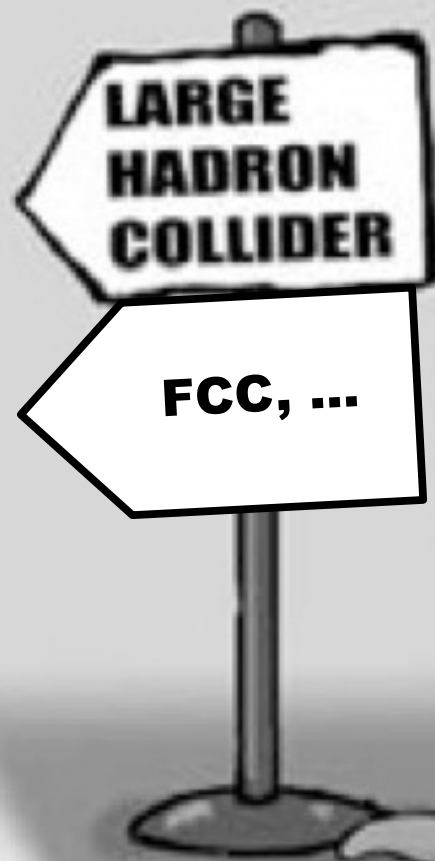
- Λ = scale of some (composite) dynamics: condensates generate GBs, PGBs:

$$\mathcal{L}_{\text{kin}} = f^2 \partial_\mu \Sigma^\dagger \partial^\mu \Sigma \rightarrow \text{radiative: } M_W, \text{ potential:}$$

$$\mu^2 = c \frac{g^2}{16\pi^2} \Lambda^2 \sim c g^2 f^2, \quad \lambda = c' \frac{g^2}{f^2} \frac{1}{16\pi^2} \Lambda^2 \sim c' g^2$$

- $f = 200\text{-}300 \text{ GeV} \leftrightarrow$ correct EW scale (M_W) $\rightarrow \Lambda$ at most 3 TeV \leftrightarrow LHC

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- when will BSM physics finally show up
- just a bit later than expected – why?
 - or is it very far away?

The Problem: Two or more Scalars (Scales)

- SM has just **one scale** $\langle\Phi\rangle = v \rightarrow$ all masses $\sim v$, no problem
- **simplest case:** scalars φ, Φ with masses m, M and **$m \ll M$**
- $\varphi^+\varphi$ and $\Phi^+\Phi$ are singlets \rightarrow **portal term** $\lambda_{\text{mix}}(\varphi^+\varphi)(\Phi^+\Phi)$
- quantum corrections $\sim M^2$ drive m to the (heavy) scale M
 \rightarrow vastly different explicit scalar scales are generically unstable

SM embeddings need SSB \leftrightarrow more scalars

- gauge extensions: LR, PS, GUTs \rightarrow must be broken...
- even for SUSY GUTS \rightarrow doublet-triplet splitting...
- also for fashionable Higgs-portal scenarios...

\rightarrow generic conflict between BSM indications and more scales

Mitigating Hierarchy Problems

central issue: scalar portals: $\lambda_p H^+ H \Phi^+ \Phi$ with $\lambda_p = \mathcal{O}(1)$

→ quantum corrections: $\delta(m_H)^2 \sim (M_\Phi)^2 \rightarrow m_H \ll M_\Phi$ is a fine-tuning

in general a sum of contributions: $\delta(m_H)^2 = \text{sum of diagrams} \sim \Lambda^2$

postulate: $\delta M_H^2 = \frac{\Lambda^2}{32\pi^2 V^2} (6M_W^2 + 3M_Z^2 + 3M_H^2 - 12M_t^2) = 0$ Veltman condition

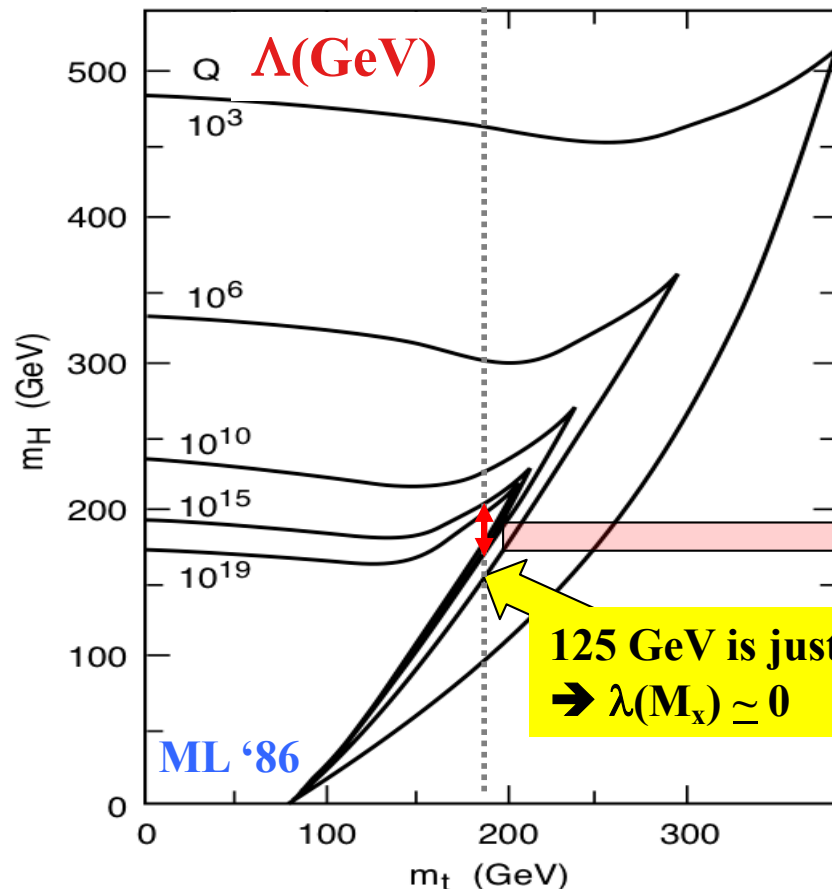
...but there is no reason (symmetry, mechanism) for this relation...

→ potential directions:

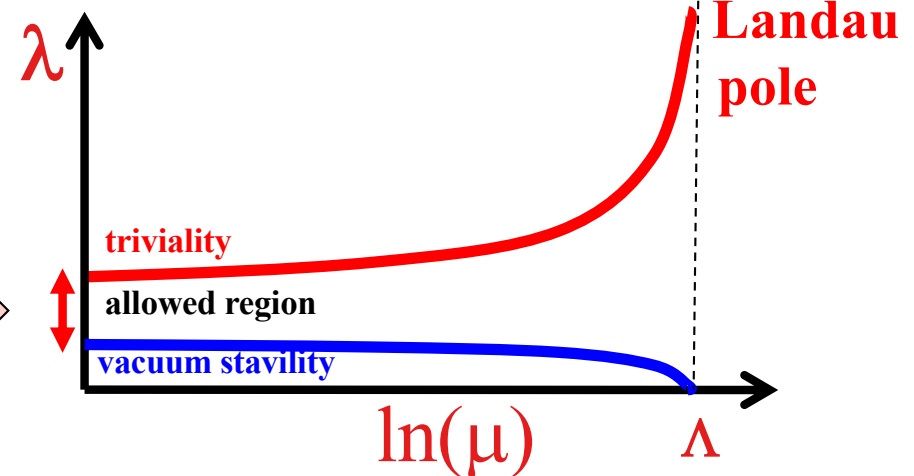
- a) supersymmetry (superpartners)
- b) loop suppression $\leftrightarrow 1/16\pi^2$, accidental symmetries, composite Φ
- c) unsuppressed portals to dark sectors (invisible BSM)
- d) a natural explanation for a very tiny $\lambda_p \ll 1$ (at quantum level)
conformal: $G = G_1 * G_2 + \text{orthogonal representations} + \dots$

Experimental Observations

- SM is a renormalizable QFT like QED w/o hierarchy problem
- Cutoff “ Λ ” has no meaning → triviality, vacuum stability



$$126 \text{ GeV} < m_H < 174 \text{ GeV}$$



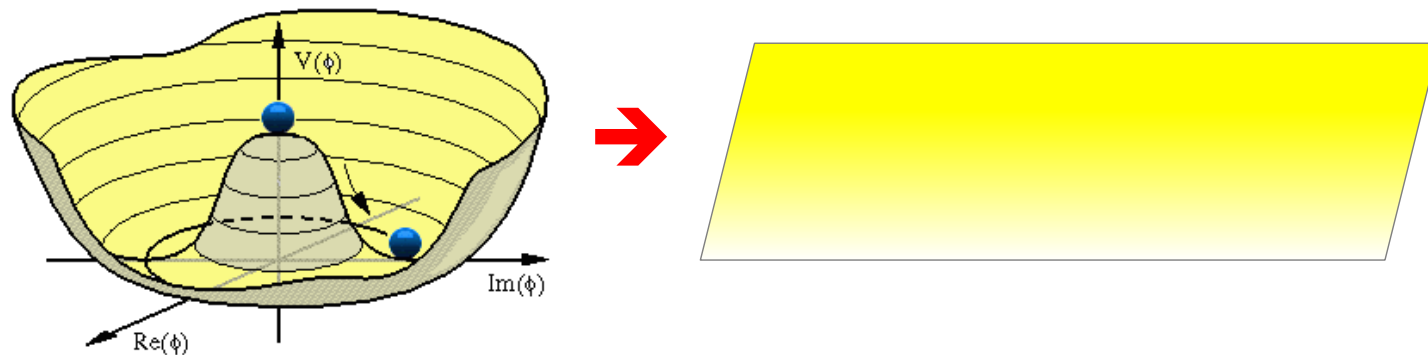
→ SM quantum corrections OK over large scales distances

Important observation:

- a remarkable relation between the weak scale, m_t , m_H , gauge couplings and Λ
- connected to log divergences – not to quadartic divergences \leftrightarrow HP?

Is there a Message?

- $\lambda(M_X) \simeq 0$? \rightarrow remarkable log cancellations of unrelated parameters
- **remember: μ is the only single scale of the SM \rightarrow special role**
- if in addition $\mu^2 = 0 \rightarrow V(M_X) \simeq 0 \rightarrow$ **C-SM with no scale at all**
 \rightarrow Mexican hat becomes flat due to conspiring quantum effects

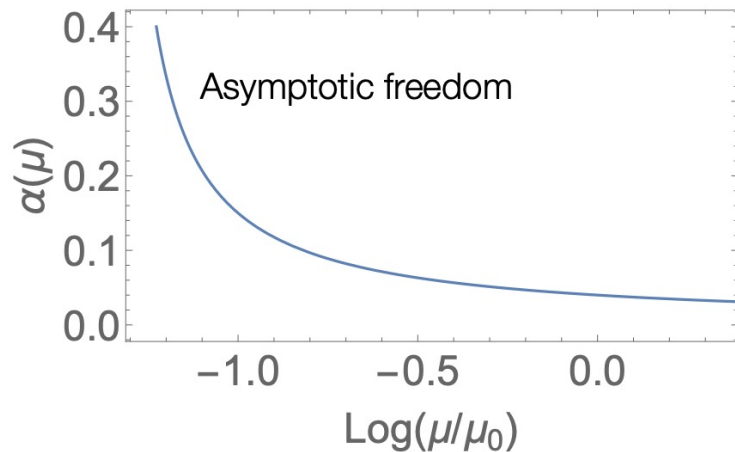


- alternatively: All scalar and Yukawa couplings dissolve
i.e. composite scalars \rightarrow potential dissolves (no metastability issues)
- **In both cases tempting:**
LE broken conformal (or shift) symmetry \leftrightarrow HP?

UV-Completion & Conformal Symmetry

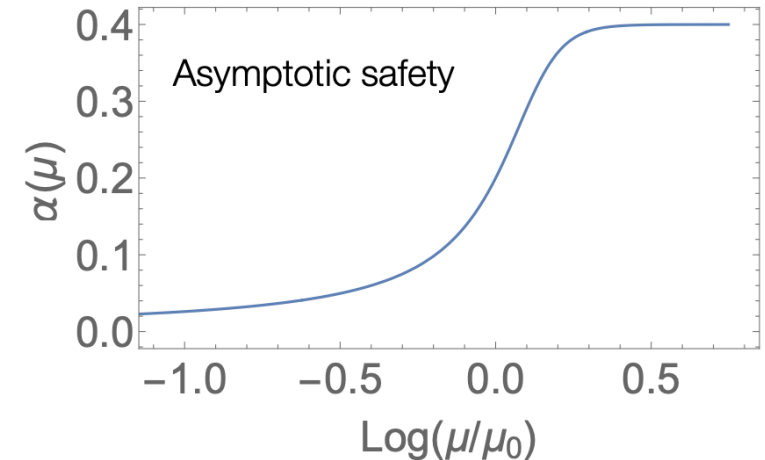
Successful theories should have a meaningful UV-completion

→ vanishing β -functions (UV fixedpoints) \leftrightarrow restored scale symmetry



**Interacting
UV-fixedpoint →**

← trivial fixedpoint



Interacting UV-fixedpoints:

- scalar and Yukawa couplings tend to have Landau poles, instability...
- all couplings... requires carefully selected particle content → explanation?

Trivial fixedpoints:

- no fundamental scalars
- no Yukawa couplings
- asymptotically free non-abelian gauge theories w/o scalars → easy

Higgs Portals to Hidden Sectors

- SM scalar Φ plus some new scalar φ (or more scalars)
- CS \rightarrow no scalar mass terms
- the scalar portal $\lambda_{\text{mix}}(\varphi^+\varphi)(\Phi^+\Phi)$ must exist

\rightarrow a condensate of $\langle\varphi^+\varphi\rangle$ produces $\lambda_{\text{mix}}\langle\varphi^+\varphi\rangle(\Phi^+\Phi) = \mu^2(\Phi^+\Phi)$
 \rightarrow effective mass term for Φ

- no CA... \rightarrow breaking only $\ln(\Lambda)$
 \rightarrow implies a TeV-ish condensate for φ to obtain $\langle\Phi\rangle = 246$ GeV
- Many model building possibilities / phenomenological aspects:
 - φ could be an effective field of some hidden sector DSB
 - further particles could exist in hidden sector; e.g. confining...
 - extra hidden U(1) potentially problematic \leftrightarrow U(1) mixing
 - avoid Yukawas which couple visible and hidden sector \rightarrow phenomenology safe due to Higgs portal \rightarrow suppressed TeV-ish BSM physics!

SM \otimes hidden SU(3)_H Gauge Sector

Holthausen, Kubo, Lim, ML

- hidden SU(3)_H:

$$\mathcal{L}_H = -\frac{1}{2}\text{Tr } F^2 + \text{Tr } \bar{\psi}(i\gamma^\mu D_\mu - yS)\psi$$

gauge fields ; $\psi = 3_H$ with SU(3)_F ; **S = real singlet scalar**

- SM coupled by S via a Higgs portal:

$$V_{\text{SM}+S} = \lambda_H (H^\dagger H)^2 + \frac{1}{4}\lambda_S S^4 - \frac{1}{2}\lambda_{HS} S^2 (H^\dagger H)$$

- no scalar mass terms
- use similarity to QCD, use NJL approximation, ...
- χ -ral symmetry breaking in hidden sector: SU(3)_L × SU(3)_R → SU(3)_V → generation of TeV scale
- transferred into the SM sector through the singlet S
- **dark pions are PGBs: naturally stable → DM**

Many more Models along this Direction

SM + extra singlet or doublet: Φ, φ

Nicolai, Meissner Farzinnia, He, Ren, Foot, Kobakhidze, Volkas, Hill, ...

Minimal B-L extension: $SU(3)_c \times SU(2)_L \times U(1)_Y \times U(1)_{B-L}$ Iso, Okada, Orikasa

SM + high rep. QCD scalar: J. Kubo, K.S. Lim, ML

Minimal LR-model: $SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ Holthausen, ML, Schmidt

SM \otimes $SU(N)_H$ with new N-plet in a hidden sector

Ko, Carone, Ramos, Holthausen, Kubo, Lim, ML, Hambye, Strumia , ...

SM + QCD colored scalar which condenses at TeV scale Kubo, Lim, ML

SM \otimes [$SU(2)_X \otimes U(1)_X$]

Altmannshofer, Bardeen, Bauer, Carena, Lykken

... more ...

Since SM- does not work \Rightarrow more \Rightarrow observable effects:

- Higgs & other scalars (singlet, hidden sector, ...) \rightarrow little hierarchy is natural
- dark matter candidates \leftrightarrow hidden sectors & Higgs portals
- consequences for neutrino masses, ...

Conformal Little Higgs

conformal little Higgs: Ahmed, ML, Saake, 2309.07845, PRD 109.075041

1) All scalars (including Higgs) are GBs or PGBs

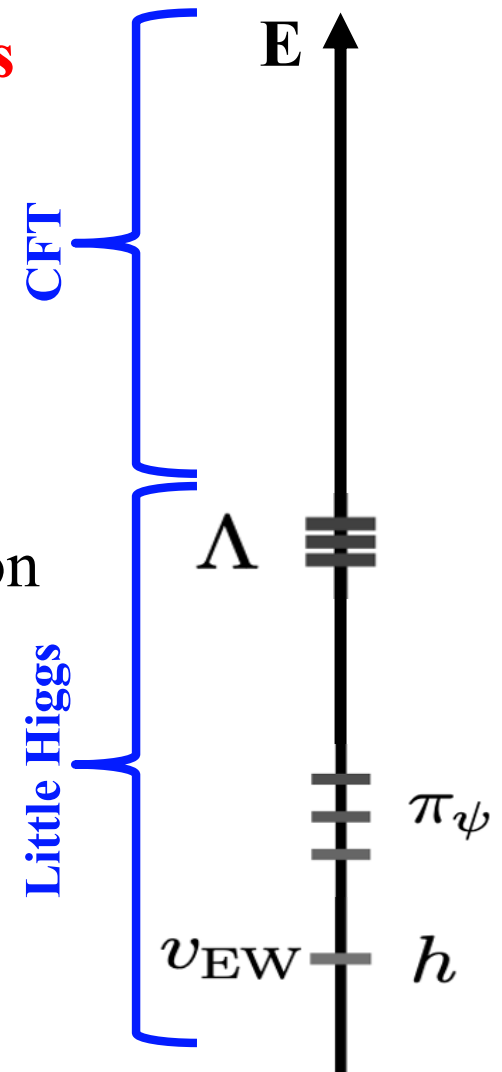
- scale $\Lambda \simeq 10$ TeV little Higgs model
- symmetry explanation of the LHP
- all λ 's and Yukawa couplings dissolve at Λ

2) conformal non-abelian UV completion

- Λ becomes scale of a dimensional transmutation
- no new scalars or scales \leftrightarrow HP

Remarks:

- realized for SM, but works for extended Higgs sectors
- can be combined with neutrino masses, DM, BAU, ...
- gravity – comments if time allows



A little Higgs reminder

Λ = scale of some (composite) dynamics

- condensates generate GBs, PGBs

$\mathcal{L}_{\text{kin}} = f^2 \partial_\mu \Sigma^\dagger \partial^\mu \Sigma \rightarrow$ radiative: M_W , potential:

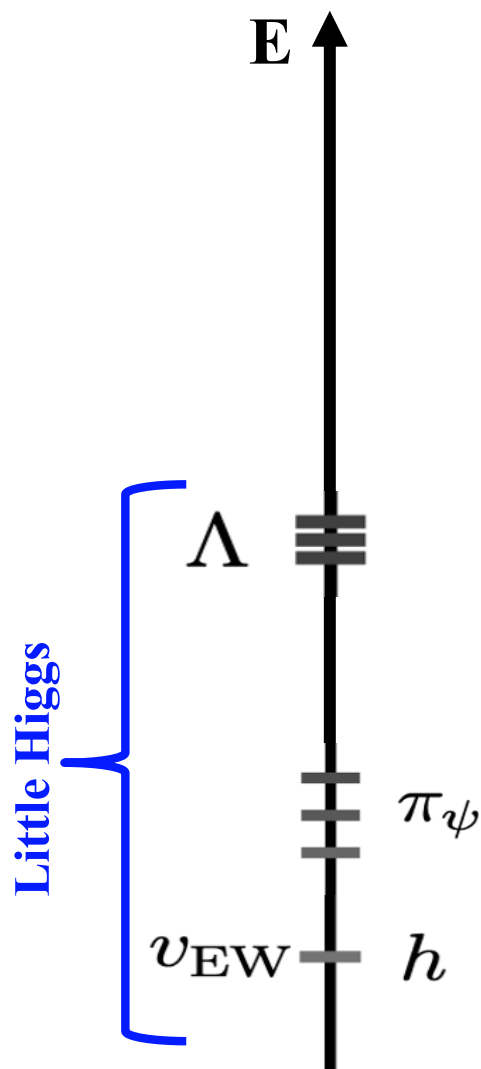
$$\mu^2 = c \frac{g^2}{16\pi^2} \Lambda^2 \sim c g^2 f^2, \quad \lambda = c' \frac{g^2}{f^2} \frac{1}{16\pi^2} \Lambda^2 \sim c' g^2$$

- $f = 200\text{-}300 \text{ GeV} \leftrightarrow$ correct EW scale (M_W)
 $\rightarrow \Lambda$ at most 2-3 TeV: exp. excluded operators
 \rightarrow spectrum may contain lower lying states?
c.f. techni- ρ in technicolor \rightarrow S parameter...

- **little Higgs:** f can be $O(\text{TeV}) \rightarrow \Lambda = 5\text{-}10 \text{ TeV}$

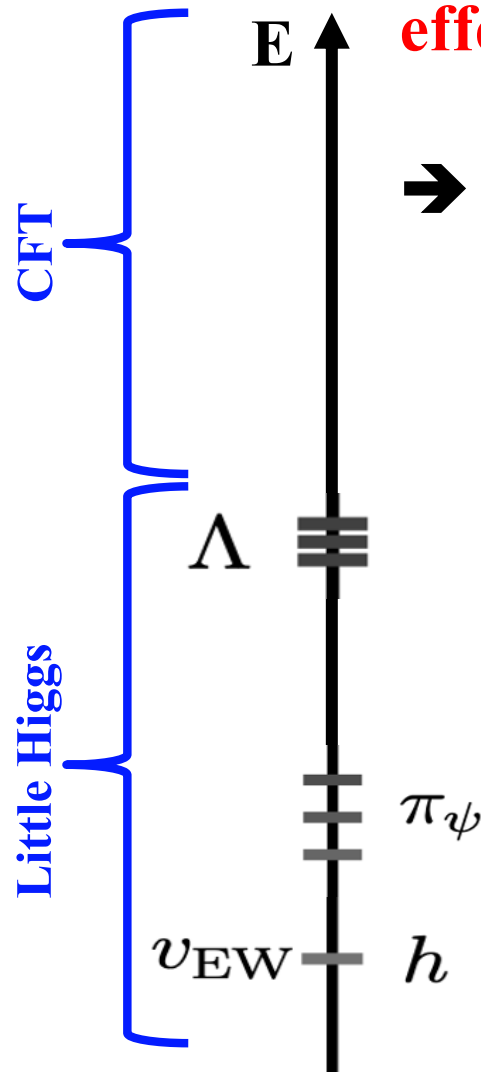
$$\mu^2 \sim \frac{g^2}{16\pi^2} f^2 \log \frac{\Lambda^2}{f^2} \sim \frac{g^2}{8\pi^2} f^2 \log(4\pi)$$

- important: **all** scalar dof are GBs or PGBs
- lower lying bound states more remote



Conformal UV Completion

Idea: Make the little Higgs model to be the effective theory of a conformal completion



→ **Suitable conformal theory:**

- no fundamental scalars, no scales, χ -ral fermions
- non-abelian gauge group → asymptotically free
 - trivial UV fixepoint
 - $\beta=0 \leftrightarrow$ no conformal anomaly
 - IR dimensional transmutation like χ -ral QCD
- condensation → **effective little Higgs model**
- dynamical transmutation **no y 's or λ 's beyond Λ**
 - no Λ^2 corrections

Conformal Little Higgs Models

Ahmed, ML, Saake, arXiv: 2309.07845, PRD 109.075041

Exemplification for ``bested little Higgs'' model:

➔ UV completion without introducing any elementary/fundamental scalars

- confining non-abelian gauge symmetry $SU(N_c)$ - we take $N_c = 2$

- new fermions:

➔ ``technifermions''

four light flavors

	$SU(N_c)$	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$
$\tilde{\psi} \equiv \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}$	\square	1	\square	0
$\psi' \equiv \begin{pmatrix} \psi_3 \\ \psi_4 \end{pmatrix}$	\square	1	1	$-\frac{1}{2}$
$\chi \times N_m$	\square	1	1	$+\frac{1}{2}$

- $SU(2)_L \subset SU(4)_L$ and the custodial group $SU(2)_L' \subset SU(4)_L$, respectively

- conjugate fields transform under the subgroups of $SU(4)_R$

- global symmetry breaking coset $SU(4)_L \times SU(4)_R / SU(4)_V$

- condensation ➔ flavor symmetry breaking

The Higgs Sector

- condensation → 15 Goldstone bosons
- transform under the custodial symmetry $SO(4) \simeq SU(2)_L \times SU(2)_R \subset SU(4)_V$
as $15_{SU(4)V} = (2,2) + (2,2) + (3,1) + (1,3) + (1,1)$
- Goldstone matrix: $U = \exp\left[i\Pi/\sqrt{2}f\right]$
- where
$$\Pi = \begin{pmatrix} \sigma^a \Delta_1^a + \eta/\sqrt{2} & -i\Phi_H \\ i\Phi_H^\dagger & \sigma^a \Delta_2^a - \eta/\sqrt{2} \end{pmatrix}$$
- with bi-doublet
$$\Phi_H \equiv \left(\tilde{H}_1 + i\tilde{H}_2, \quad H_1 + iH_2 \right); \quad \tilde{H}_i \equiv i\sigma_2 H_i^*$$

where H_i are Higgs doublets under $SU(2)_L$
- and the triplets
$$\sigma^a \Delta^a = \begin{pmatrix} \Delta^0 & \sqrt{2}\Delta^+ \\ \sqrt{2}\Delta^- & -\Delta^0 \end{pmatrix}$$

Including the Planck Scale

The Planck Scale from CS Breaking

Conformal Gravity (C-GR):

- more symmetry \rightarrow power counting renormalizable
- C-GR may have a ghost \rightarrow ...
- spontaneous generation of M_{Pl} \rightarrow Einstein-Hilbert gravity
- most economic and simple way:

$$\frac{\xi_S}{2} S^2 R \rightarrow \frac{\xi_S}{2} \langle S \rangle^2 R \rightarrow \frac{M_{\text{Pl}}^2}{2} R$$

$$M_{\text{Pl}} = \sqrt{\xi_S \langle S \rangle}$$

Brans+Dicke,'61; Fujii,'74; Englert+Truffin+Grastmans,'76; Minkowsky,'77;.....

Idea: Generate M_{Planck} from conformal gravity \otimes SU(N)

\rightarrow gauge assisted condensate via SU(N) field $\rightarrow M_{\text{Planck}}$ = effective scale

Kubo, ML, Schmitz, Yamada similar ideas: Donoghue, Menezes, ...

$$S_C = \int d^4x \sqrt{-g} \left[-\hat{\beta} S^\dagger S R + \hat{\gamma} R^2 - \frac{1}{2} \text{Tr} F^2 + \right. \\ \left. + g^{\mu\nu} (D_\mu S)^\dagger D_\nu S - \hat{\lambda} (S^\dagger S)^2 + a R_{\mu\nu} R^{\mu\nu} + b R_{\mu\nu\alpha\beta} R^{\mu\nu\alpha\beta} \right]$$

R = Ricci curvature scalar, $R_{\mu\nu}$ = Ricci tensor, $R_{\mu\nu\alpha\beta}$ = Riemann tensor

F = field-strength tensor of the $SU(N_c)$ gauge theory, S = complex scalar in fund. rep. $\rightarrow N_c$

\rightarrow most general diffeomorphism invariance, gauge invariance, and global scale invariance

Condensation in $SU(N_c)$ gauge sector

\rightarrow dimensional transmutation: $\langle S^\dagger S \rangle \rightarrow$ effective Planck mass

$$M_{\text{planck}} = 2 \beta f_0 = \frac{N_c \beta}{16\pi^2} (2 \lambda f_0) \left(1 + 2 \ln \frac{2 \lambda f_0}{\Lambda^2} \right) \quad \text{with} \quad f_0 = \langle S^\dagger S \rangle$$

\rightarrow Effectively normal gravity with a dynamically generated M_{Planck}

What about the portal of S with the SM scalar Φ ? \rightarrow effective S + gravity suppresses portal
de Boer, Kubo, ML, Reinig to appear soon...

Dilaton-Scalaron Inflation

Effective Jordan-frame Lagrangian:

$$\frac{\mathcal{L}_{\text{eff}}^J}{\sqrt{-g_J}} = -\frac{1}{2} B(\chi) M_{\text{Pl}}^2 R_J + G(\chi) R_J^2 + \frac{1}{2} g_J^{\mu\nu} \partial_\mu \chi \partial_\nu \chi - U(\chi) \rightarrow \text{auxiliary field } \Psi \rightarrow$$

$$\frac{\mathcal{L}_{\text{eff}}^J}{\sqrt{-g_J}} = -\left[\frac{1}{2} B(\chi) M_{\text{Pl}}^2 - 2G(\chi) \psi\right] R_J + \frac{1}{2} g_J^{\mu\nu} \partial_\mu \chi \partial_\nu \chi - U(\chi) - G(\chi) \psi^2$$

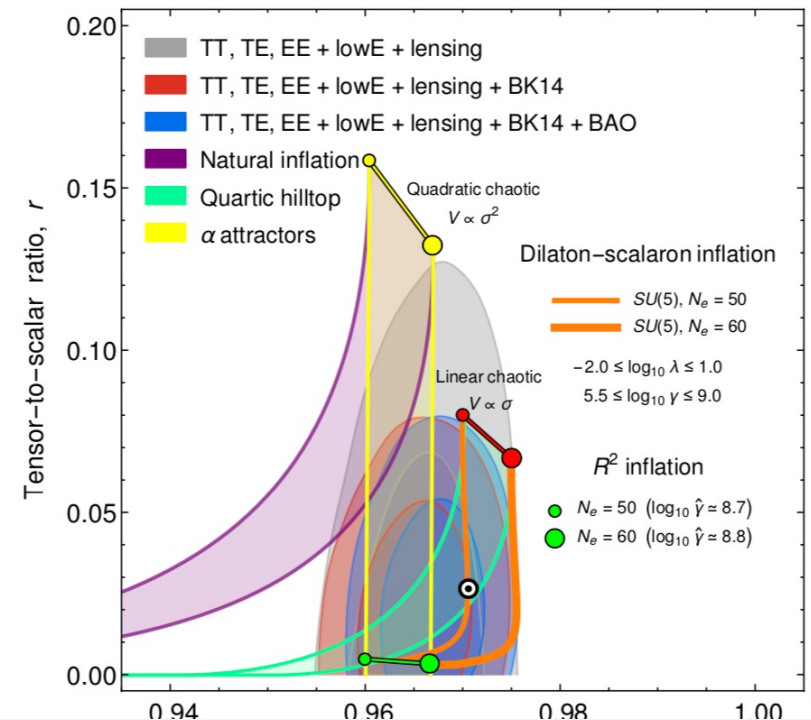
Weyl rescaling: $g_{\mu\nu} = \Omega^2 g_{\mu\nu}^J$ $\Omega^2 = e^{\Phi(\phi)}$, $\Phi(\phi) = \frac{\sqrt{2} \phi}{\sqrt{3} M_{\text{Pl}}}$

Einstein-frame scalar potential:

$$V(\chi, \phi) = e^{-2\Phi(\phi)} \left[U(\chi) + \frac{M_{\text{Pl}}^4}{16 G(\chi)} \left(B(\chi) - e^{\Phi(\phi)} \right)^2 \right]$$

→ Slow role inflation

→ fits data very well!



Conclusions

➤ The Standard Model

- works perfectly – no problems besides triviality, metastability
- list of unanswered questions / problems \leftrightarrow BSM
- lots of progress: DM, ν 's, GR waves, ... + many new ideas
- hierarchy problem worsened due to the little hierarchy problem
- remarkable coincidence of parameters: flat Higgs potential @HE

➤ Conformal portals to dark sectors

- dimensional transmutation in dark sector + portal $O(1) \rightarrow$ SM

➤ Conformal little Higgs

- a natural explanation of LHP: all scalar dof are GBs or PGBs
- conformal UV completion: avoid to reintroduce problems (fund. scalars)
- non-abelian gauge theory with fermions, gauge bosons and no scale
 - ➔ dimensional transmutation at multi TeV-ish Λ

➤ Including the Planck scale

- conformal gravity with dimensional transmutation