Goofy Symmetries

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based on:

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Outline

- The original GOOFy transformation
- Definition of Goofy transformations / *regular* vs. *goofy*
- RG stability of regular / goofy transformations in general
- Additional goofy transformations in 2HDM
- Applications of goofy transformations
- Conclusions

Disclaimer

This is a very young topic, all input is welcome and important.

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e.g. 2012: YOLO, . . . , 2020: lost, 2021: cringe, . . . , 2023: goofy

The first Goofy transformation

2HDM: Scalar doublets $\Phi_{a=1,2}(x)$ in $(\mathbf{2},-1/2)$ of $\mathrm{SU}(2)_\mathrm{L}\times\mathrm{U}(1)_\mathrm{Y}$.

The invariant scalar potential conventionally written as

$m_{11,22},\ \lambda_{1,2,3,4}\in\mathbb{R}$
$m_{12},\ \lambda_{5,6,7}\in\mathbb{C}$

$$
\begin{aligned}
V(\Phi^*,\Phi) \;=\;& m_{11}^2\Phi_1^\dagger\Phi_1+m_{22}^2\Phi_2^\dagger\Phi_2+\lambda_1(\Phi_1^\dagger\Phi_1)^2+\lambda_2(\Phi_2^\dagger\Phi_2)^2+\lambda_3(\Phi_1^\dagger\Phi_1)(\Phi_2^\dagger\Phi_2)+\lambda_4(\Phi_1^\dagger\Phi_2)(\Phi_2^\dagger\Phi_1)\\
&-\left\{m_{12}^2\Phi_1^\dagger\Phi_2\right\}+\left\{\lambda_5(\Phi_1^\dagger\Phi_2)^2+[\lambda_6(\Phi_1^\dagger\Phi_1)+\lambda_7(\Phi_2^\dagger\Phi_2)]\Phi_1^\dagger\Phi_2\right\}+\text{h.c.}.
\end{aligned}
$$

Community agrees, **all possible** (exact global) symmetries of 2HDM are known:
[Ivanov '06, '07], [Ferreira, Haber, Maniatis, Nachtmann, Silva '11], [Deshpande, Ma '78], [Ginzburg, Krawczyk '05], [Nishi '11], [Pilaftsis '12], ...

$$\mathrm{CP1}\,,\quad \mathbb{Z}_2\,,\quad \mathrm{U}(1)\,,\quad \mathrm{CP2}\,,\quad \mathrm{CP3}\,,\quad \mathrm{SU}(2)\,.$$

However, **FGOO**≡[Ferreira, Grzadkowski, Ogreid, Osland '23] found the relations

$$m_{11}^2=-m_{22}^2\,,\qquad \lambda_1=\lambda_2\,,\qquad \lambda_6=-\lambda_7\,.$$

- These are renormalization group (RG) stable *to all orders*.
- Do **not** correspond to any of the known *regular* 2HDM symmetries.

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Consider **canonical** (gauge-)kinetic terms

$$\mathcal{K} \;=\; (D_\mu\Phi_1)^\dagger(D^\mu\Phi_1) + (D_\mu\Phi_2)^\dagger(D^\mu\Phi_2)\;\;.$$

Applied to the **canonical** (gauge-)kinetic terms the transformation maps

$$\mathcal{K} \;\mapsto\; -\mathcal{K}\;,$$

If imposed as an exact symmetry this would enforce $\mathcal{K} = 0$.

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If imposed as an exact symmetry this would enforce $\mathcal{K} = 0$.

$\Rightarrow$ This transformation cannot be an *exact* symmetry in a *dynamical* theory ($\mathcal{K} \neq 0$).
But how can relations (1) then be RG stable?

My definition of Goofy transformations

Any theory:

$$\mathcal{L}[\phi] \;=\; \mathcal{K}_{(kin.)}[\phi] \;-\; \mathcal{V}_{(pot.)}[\phi]\;.$$

- How come, when talking about symmetry transformations, we usually only consider constraining the potential $\mathcal{V}[\phi]$?
- Naive perception: $\mathcal{K}[\phi]$ is invariant under *all possible* sym. transformations...

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Defining criterion:

Regular transformations act **trivially** on kinetic term. $\longleftrightarrow$ ***Goofy*** transformations act **non-trivially** on kinetic term.

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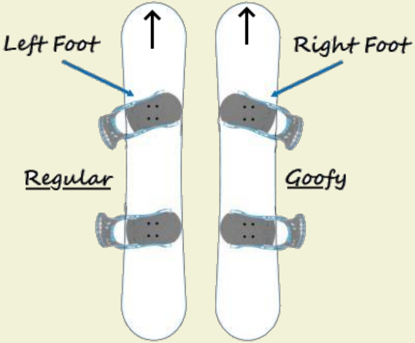
Crucial: **Transformation** $\neq$ **Symmetry.**

Transformations (regular/goofy) can be physically important, even if explicitly broken.

Regular vs. Goofy

in Surfing, Skateboarding, Snowboarding, . . .



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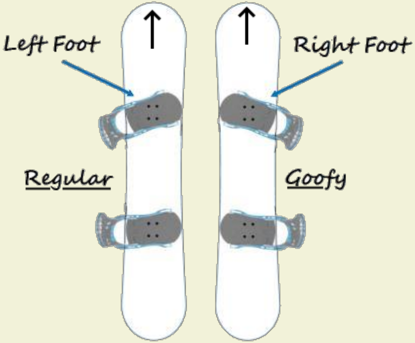

RG stability of Goofy parameter relations

There are parameter relations derived from a transformation acting on $V(\Phi^*, \Phi)$.

Central question:

How can these parameter relations be RG stable, even if the corresponding transformation is explicitly broken by canonical (gauge-)kinetic terms $\mathcal{K}$?

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This is a folk wisdom, based on 't Hooft's technical naturalness argument: ['t Hooft '79]

$$\beta_g \equiv \frac{\mathrm{d}\, g}{\mathrm{d}\mu} \propto g \qquad \textbf{iff} \qquad g \to 0 \text{ enhances the symmetry.}$$

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We will present a general (non-perturbative) formal argument in [De Boer, AT to appear].

It is based on the fact that *prospective* symmetries can be viewed as (linear) maps in the parameter space of a theory because they act as *outer automorphisms*. [Fallbacher, AT '15]

In short: Couplings transform covariantly $\Rightarrow$ β-functions trafo covariantly.
Symmetries of β-functions $\geq$ symmetries of theory.
[De Boer, AT to appear]

Computing RG fixed points

[AT '25], [De Boer, AT to appear]

Consider theory with fields $\phi_a,\ \phi^*_{a=1,\dots,N} \in \mathbb{C}$ and couplings $\lambda_{i=1,\dots,K}$.
If there is a transformation that acts on the fields as (A, B unitary)

$$T: \qquad \vec{\phi} \mapsto A\vec{\phi}\,, \qquad \vec{\phi}^* \mapsto B^*\vec{\phi}^*\,,$$

($A = B$: regular, $A \neq B$: goofy)

that can *equivalently* be represented as a mapping in the space of couplings⋆

$$T: \qquad \vec{\lambda} \mapsto \mathcal{O}\vec{\lambda}\,,$$

($\mathcal{O}$ can be derived from A and B)

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$$T: \qquad \vec{\lambda} \mapsto \mathcal{O}\vec{\lambda}\,, \qquad (\mathcal{O} \text{ can be derived from } A \text{ and } B)$$

then the full coupled system of (generally non-linear) beta functions

$$\beta_{\vec{\lambda}} \equiv \mu \frac{\mathrm{d}\,\vec{\lambda}}{\mathrm{d}\mu} = \vec{f}(\lambda_1, \lambda_2, \dots)\,,$$

transforms covariantly, and **in the same irreps as the couplings themselves.**
This poses non-perturbative, all-order constraints on $\beta_{\vec{\lambda}}$ of the form $\mathcal{O}\beta_{\vec{\lambda}} = \vec{f}\Big|_{\vec{\lambda}\to\mathcal{O}\vec{\lambda}}$.

Computing RG fixed points

- The existence of such transformations* imposes strong all order exact constraints on system of β functions.

  Namely, the covariant β functions are spanned by covariant combination of couplings.
- This argument does not require such transformations to be conserved, the mere existence is enough.
- This argument holds at the non-perturbative level.
- The more of such possible transformations exist, the more restricted is the overall system of β functions.

- **If** the transformations are *imposed* to be conserved as symmetries
  $\Longrightarrow$ Non-trivially transforming covariant combination of λ_i's must vanish,
  $\Longrightarrow$ Beta functions of nontrivially transforming λ_i's are forced to $\beta_{\lambda_i} = 0$.

This is the completed version of 't Hooft's argument.

[AT '25], [De Boer, AT to appear]

*All *outer automorphism* transformations are of this kind.

Example for a regular transformation in 2HDM

- *In absence of other symmetries*, most general $\mathrm{SU}(2)$ Higgs-basis change is outer automorphism $\Phi' = U\Phi$, $\Phi'^* = U^*\Phi^*$. Couplings transform covariantly.

see e.g. [Ferreira, Haber, et al. '10], [AT '18], [Bednyakov '18],...

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  E.g. the following combination transforms as $\mathrm{SU}(2)$ triplet (vector)

$$\vec{\Lambda} \;:=\; const. \times \left(\mathrm{Re}(\lambda_6+\lambda_7), -\mathrm{Im}(\lambda_6+\lambda_7), \frac{1}{2}(\lambda_1-\lambda_2)\right)^{\mathrm{T}} .$$

  Our argument then implies that

  explicitly known to six loops [Bednyakov '18, '24]

$$\beta_{\vec{\Lambda}} \;\propto\; \vec{\Lambda}\,.$$

- This is because contributions from other vectors do not contribute to $\beta_{\vec{\Lambda}}$:
  - $\vec{M}(m_{11}^2, m_{22}^2, m_{12}^2)$ wrong mass dimension. (our argument for scaling outer automorphism)
  - $\mathbf{5}$-plet $\tilde{\Lambda}(\lambda_3, \lambda_4, \dots)$, $\mathbf{3} \subset (\mathbf{5}\otimes\mathbf{5}\otimes\dots)$, but these contractions vanish for single $\mathbf{5}$.

$\Rightarrow$ This shows $\vec{\Lambda} = 0$ is RG fixed point to all orders in scalar+gauge corrections.

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- Fixed point $\vec{\Lambda} = 0$, implied by any trafo that requires $\lambda_1 = \lambda_2$ and $\lambda_6 = -\lambda_7$.
- E.g. $\mathrm{CP2}$ implies $\vec{\Lambda} = 0$. And this survives **soft** breaking ($m_{11}^2 \neq \pm m_{22}^2$).

What is different for Goofy transformations?

- Most general basis for (gauge-)kinetic terms

$$\mathcal{K} = (D_\mu \Phi_i)^\dagger K_{ij} (D^\mu \Phi_j) \qquad \text{with} \qquad K_{ij} = K_{ji}^* \,.$$

- Wave function renormalization coeffs. K_{ij} trafo *covariantly* under goofy trafos.
- In view of RGEs, K_{ij} should be treated exactly like couplings of the potential.
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- Starting from canonical kinetic terms $K_{ij} = \delta_{ij}$, goofy transformations

$$\Phi \mapsto A\Phi\ , \qquad \Phi^\dagger \mapsto \Phi^\dagger B^\dagger\ . \qquad \text{Here,}\quad (B^\dagger A)\quad \text{is unitary + must be hermitean.}$$

$\Rightarrow$ There exists a basis where effect of the most general transformation on the **canonical** (gauge-)kinetic terms is given by

$$\mathcal{K} \;\mapsto\; \kappa_1\,(D_\mu\Phi_1)^\dagger(D^\mu\Phi_1) + \kappa_2\,(D_\mu\Phi_2)^\dagger(D^\mu\Phi_2) \qquad \text{with} \qquad \kappa_1 = \pm 1,\ \kappa_2 = \pm 1\ .$$

$\curvearrowright$ Can use std. RGEs + (propagator and gauge vertex) counting to **restore** covariants κ_i.

Back to original question: RG stability of goofy parameter relations

FGOO 2HDM parameter relations: $m_{11}^2 = -m_{22}^2, \quad \lambda_1 = \lambda_2, \quad \lambda_6 = -\lambda_7$.

- Consider e.g. $\beta_{\lambda_1 - \lambda_2}$. Under FGOO trafo, must trafo like $(\lambda_1 - \lambda_2)$, i.e. as a $\mathbf{1}'$.
- Other $\mathbf{1}'$ covariants are $(\lambda_6 + \lambda_7)$ and κ_1, κ_2. ($m_{11}^2 + m_{22}^2$ has wrong mass dimension)

$\Rightarrow$ Beta function $\beta_{\lambda_1-\lambda_2}$ to all orders(!) schematically *can only* be given by

$$\beta_{\lambda_1-\lambda_2} \;=\; (\lambda_1 - \lambda_2)\, f_+(\lambda_i) \;+\; (\lambda_6 + \lambda_7)\, g_+(\lambda_i) \;+\; \underbrace{\kappa_1^n \kappa_2^m}_{n+m\,=\,\mathrm{odd}}\, h_+(\lambda_i)\;,$$

where f_+, g_+, h_+ are functions which are invariant under the trafo.

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- Consider e.g. $\beta_{\lambda_1-\lambda_2}$. Under FGOO trafo, must trafo like $(\lambda_1 - \lambda_2)$, i.e. as a $\mathbf{1}'$.
- Other $\mathbf{1}'$ covariants are $(\lambda_6 + \lambda_7)$ and κ_1, κ_2. ($m_{11}^2 + m_{22}^2$ has wrong mass dimension)

$\Rightarrow$ Beta function $\beta_{\lambda_1-\lambda_2}$ to all orders(!) schematically *can only* be given by

$n + m = \text{odd}$

$$\beta_{\lambda_1-\lambda_2} \;=\; (\lambda_1 - \lambda_2)\, f_+(\lambda_i) \;+\; (\lambda_6 + \lambda_7)\, g_+(\lambda_i) \;+\; \kappa_1^n \kappa_2^m\, h_+(\lambda_i)\;,$$

where f_+, g_+, h_+ are functions which are invariant under the trafo.

- Turns out for 2HDM:
  The global sign of the kinetic term does not enter the β *functions* $\Leftrightarrow h_+ = 0$.
- Analogous arguments hold for $\beta_{m_{11}^2+m_{22}^2}$ and $\beta_{\lambda_6+\lambda_7}$.
- This shows that **if** FGOO relation is imposed, it is **not** violated in the RG flow.

$\curvearrowright$ Goofy symmetries are *explicitly* broken by $\mathcal{K} \neq 0$, but this breaking is **soft**!

More new goofy transformations in 2HDM

[AT '25]

Explicit action of flavor-/CP-type global-sign-flipping **Goofy** transformations

$$\vec{\Phi} \equiv \left(\Phi_1, \Phi_2, \Phi_1^*, \Phi_2^*\right)^{\mathrm{T}} , \qquad \vec{\Phi} \mapsto \begin{pmatrix} S & \mathbf{0} \\ \mathbf{0} & -S^* \end{pmatrix} \vec{\Phi} , \qquad \text{or} \qquad \vec{\Phi} \mapsto \begin{pmatrix} \mathbf{0} & X \\ -X^* & \mathbf{0} \end{pmatrix} \vec{\Phi} .$$

Goofy trafo.					parameter relations						accidental regular sym.
	m_{11}^2	m_{22}^2	m_{12}^2	λ_1	λ_2	λ_3	λ_4	λ_5	λ_6	λ_7	
$\mathcal{P}_G$ $(\equiv \mathbb{1} \oplus -\mathbb{1})$	0	0	0								$-$
$\mathbb{Z}_{2,G}$ $(\equiv \sigma_3 \oplus -\sigma_3)$	0	0							0	0	$-$
$\mathrm{CP1}_G$	0	0	${-m_{12}^2}^*$					λ_5^*	λ_6^*	λ_7^*	$-$
$\mathrm{CP2}_G$ (FGOO)		$-m_{11}^2$			λ_1					$-\lambda_6$	$-$
$\mathrm{U}(1)_G$	0	0	0					0	0	0	$\mathrm{U}(1)$
$\mathrm{CP3}_G$	0	0	0		λ_1			$\lambda_1 - \lambda_3 - \lambda_4$	0	0	CP2
$\mathrm{SU}(2)_G$	0	0	0		λ_1		$\lambda_1 - \lambda_3$	0	0	0	$\mathrm{SU}(2)$
$\mathrm{CP2}_G^{\mathrm{soft}}$					λ_1					$-\lambda_6$	$-$

Parameter relations for goofy transformations, RG stable to all orders in scalar and gauge quantum corrections.

These are *genuinely new* goofy transformations. FGOO discussed: $\mathrm{CP2}_G$ + all regular.

Applications of Goofy transformations

- An entirely new class of possible transformations and associated RG stable fixed points have been missed so far in *all* QFTs, all models, ...
- Bare scalar mass terms $\phi^\dagger\phi$ explicitly break some goofy transformations (non-zero mass gaps are possible).
  $\Rightarrow$ Goofy symmetries can be instrumental in solving EW hierarchy problem.
  [AT '25](x2), [de Boer, Goertz, Incrocci '25]
- Relative-kinetic-term-sign-flipping goofy trafos are **not** RG stable but expose RG sensitivity to *generation dependent* sign flips $\longrightarrow$ connection to flavor.
- Goofy transformations can constrain non-canonical kinetic terms.

  In particular, the non-trivial Kähler potential of SUSY theories. $\Rightarrow$ Possibility to remove a major roadblock for predictivity of many classes of flavor models (discrete symmetries, modular symmetries, ...)
  [Chen, Fallbacher, (Omura), Ratz, Staudt '12,'13], [Chen, Ramos-Sánchez, Ratz '19]

“Dynamical classicalization” and a request

- If goofy relations are imposed only on V, the “couplings” of the kinetic terms (wave function renromalization coefficients) K_{ij} still run under RG.
- In this case, vanishing kinetic term(s) are RG fixed points (symmetry is enhanced there).
- Approaching these points in RG flow, the theory approaches a regime where one or more of the fields become non-propagating
  “quasi-classical background” or “auxiliary” fields.

$\Rightarrow$ RG evolution *dynamically* approaches a quasi-classical regime!

- To fully explore this regime, and systematically explore the all-order constraints on WFR (anomalous dimensions), RGEs should be formulated starting from the most general possible basis.

$\curvearrowright$ Could track effect of goofy trafos on K_{ij} and get all-order constraints on WFR (anomalous dimensions), just as for the other couplings.

Working exclusively in the canonical basis one is blind to both of these effects!

Remark on non-canonical kinetic terms

[AT 2508.02646, footnote [18]]

One can find an **invariant** but **non-canonical** kinetic term (here for original FGOO trafo)

$$\widetilde{\mathcal{K}} \;:=\; (D_\mu\Phi_1)^\dagger(D^\mu\Phi_1) - (D_\mu\Phi_2)^\dagger(D^\mu\Phi_2)\ .$$

$\widetilde{\mathcal{K}}$ ***preserves*** the original FGOO transformation! $\Rightarrow$ The FGOO trafo is **not** goofy w.r.t. $\widetilde{\mathcal{K}}$.

In a theory with $\widetilde{\mathcal{K}}$, FGOO is a *regular* symmetry trafo!

One can restore canonical kinetic terms via a "goofy" basis change:

$$\begin{pmatrix}\Phi_1'\\ \Phi_2'\\ \Phi_1'^{\,*}\\ \Phi_2'^{\,*}\end{pmatrix} \equiv \begin{pmatrix}1&&&\\&1&&\\&&1&\\&&&-1\end{pmatrix}\begin{pmatrix}\Phi_1\\ \Phi_2\\ \Phi_1^*\\ \Phi_2^*\end{pmatrix}. \quad \text{In this basis:} \quad \begin{pmatrix}\Phi_1'\\ \Phi_2'\\ \Phi_1'^{\,*}\\ \Phi_2'^{\,*}\end{pmatrix} \xmapsto{\text{FGOO}} \begin{pmatrix}&&&1\\&&1&\\&1&&\\1&&&\end{pmatrix}\begin{pmatrix}\Phi_1'\\ \Phi_2'\\ \Phi_1'^{\,*}\\ \Phi_2'^{\,*}\end{pmatrix}.$$

In this basis, $\widetilde{\mathcal{K}}'$ is canonical and the original FGOO trafo clearly is a regular trafo!
The FGOO trafo here corresponds to $\mathrm{CP1}\times\Pi_2\ (\simeq \mathrm{CP1})$, enforcing the parameter relations

$$m_{11}'^{\,2} = +\,m_{22}'^{\,2}\,, \qquad \lambda_1' = \lambda_2'\,, \qquad \lambda_6' = +\lambda_7'\,.$$

This certainly is RG stable to all orders. But it does *not* explain the RG stability of the FGOO relations in presence of a *canonical* kinetic term (which breaks the FGOO trafo).

Conclusions

- *Goofy* transformations by definition do not leave invariant the kinetic terms.
- Explicit breaking of goofy trafos in kinetic terms can be soft, and parameter relations enforced by goofy trafos can be stable to all orders in RG evolution.
- Symmetry of β functions $\geq$ symmetry of action. [AT '25], [De Boer, AT to appear]
- It is *mandatory* to include goofy transformations to understand all (partial) RG fixed points [RG fixed hyperplanes] of *any* QFT. [AT '25], [De Boer, AT to appear]
- Many of the most important puzzles in our theoretical understanding of Nature may be related to goofy transformations (e.g. EW hierarchy, flavor, . . .)
- Parameter regions of exact goofy symmetry are points where a QFT dynamically approaches a quasi-classical regime for some of the fields.
- There are many goofy avenues worth exploring...

. . . I cannot think about QFT *without* goofy transformations anymore.

Thank You!

Image credits: PNGaaa.com, W a l t D i s n e y. ”Hawaiian Holiday” 1937

Backup slides

2HDM – Regular and global-sign-flipping goofy

Explicit action of global **regular** flavor- and CP-type transformations: [AT '25]

$$\vec{\Phi} \equiv \left(\Phi_1, \Phi_2, \Phi_1^*, \Phi_2^*\right)^{\mathrm{T}} \;, \qquad \vec{\Phi} \mapsto \begin{pmatrix} S & \mathbf{0} \\ \mathbf{0} & S^* \end{pmatrix} \vec{\Phi} \,, \qquad \text{or} \qquad \vec{\Phi} \mapsto \begin{pmatrix} \mathbf{0} & X \\ X^* & \mathbf{0} \end{pmatrix} \vec{\Phi} \,.$$

Flavor- and CP-type **Goofy versions** of these transformations:

$$\vec{\Phi} \mapsto \begin{pmatrix} S & \mathbf{0} \\ \mathbf{0} & -S^* \end{pmatrix} \vec{\Phi} \,, \qquad \text{or} \qquad \vec{\Phi} \mapsto \begin{pmatrix} \mathbf{0} & X \\ -X^* & \mathbf{0} \end{pmatrix} \vec{\Phi} \,.$$

Note: all of these are "global-sign-flipping goofy transformations" $\mathcal{K} \mapsto -\mathcal{K}$.

Explicit choices for matrix generators of 2HDM transformations (ξ, ψ, $\theta \in \mathbb{R}$)

$$\mathbb{Z}_{2,(G)}: \quad S = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \equiv \sigma_3 \;, \qquad \mathrm{CP1}_{(G)}: \quad X = \mathbb{1}_2 \;,$$

$$\mathrm{U(1)}_{(G)}: \quad S = \begin{pmatrix} e^{-i\xi} & 0 \\ 0 & e^{i\xi} \end{pmatrix} \;, \qquad \mathrm{CP2}_{(G)}: \quad X = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \equiv \varepsilon \;,$$

$$\mathrm{SU(2)}_{(G)}: \quad S = \begin{pmatrix} e^{-i\xi}\cos\theta & -e^{-i\psi}\sin\theta \\ e^{i\psi}\sin\theta & e^{i\xi}\cos\theta \end{pmatrix} \;, \qquad \mathrm{CP3}_{(G)}: \quad X = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \;.$$

2HDM – Relative-sign-flipping goofy transformations

We can also construct transformations that act with $\kappa_1 = -\kappa_2 = \pm 1$.
Action of goofy transformations with relative sign flip of the 2HDM kinetic terms in flavor- or CP-type subvariants:

$$\vec{\Phi} \mapsto \begin{pmatrix} A & \mathbf{0} \\ \mathbf{0} & B^* \end{pmatrix} \vec{\Phi}\,, \qquad \text{or} \qquad \vec{\Phi} \mapsto \begin{pmatrix} \mathbf{0} & C \\ D^* & \mathbf{0} \end{pmatrix} \vec{\Phi}\,.$$

With examples of explicit matrix generators given by

$$\begin{aligned}
&\mathbb{Z}_{2,G}^{-}: && A = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix},\ B^* = \mathbb{1}, && C = D^* = 0\,, \\
&\mathrm{CP4}_{G}^{-}: && A = B^* = 0, && C = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix},\ D^* = \mathbb{1}\,. \\
&\mathbb{Z}_{4,G}^{-}: && A = B^* = \begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix}, && C = D^* = 0\,.
\end{aligned}$$

It turns out that none of these transformations is radiatively stable.
Reason: *The <u>relative</u> sign of the kinetic terms <u>does</u> enter the β functions of the 2HDM.*

Goofy transformations 101 – Goofy basis change

Consider multiplet of complex scalars $\phi_{a=1,\ldots,N} \in \mathbb{C}$ and hermitean conjugate fields $\phi_a^\dagger$.

Standard "canonical" kinetic term

$$\mathcal{K}[\phi, \phi^\dagger] \;=\; \partial_\mu \phi^\dagger \, \partial^\mu \phi \;=\; \partial_\mu \phi_a^\dagger \, \delta^{ab} \, \partial^\mu \phi_b \;.$$

Field space metric $\kappa^{ab}\phi_a^\dagger\phi_b$ is only unity, $\kappa^{ab} = \delta^{ab}$, in the **canonical basis.**
In any other basis $\kappa^{ab} \neq \delta^{ab}$. Recall that ϕ and $\phi^\dagger$ are independent degrees of freedom. In particular, we can always do a general *passive* **field redefinition**

$$\phi' \equiv V\phi \;, \qquad \phi'^\dagger \equiv \phi^\dagger U^\dagger \;.$$

Regular field redefinition: $U = V$, Goofy redefinition: $U \neq V$.
The kinetic term then reads

$$\mathcal{K}[\phi, \phi^\dagger] = \mathcal{K}[V^\dagger \phi', \phi'^\dagger U] = \partial_\mu \phi'^\dagger \left(U V^\dagger\right) \partial^\mu \phi' \;, \quad \Rightarrow \; \kappa = U V^\dagger \;.$$

This is nothing else than rotating the kinetic term **to** / **from** a non-canonical basis.

Note: this discussion holds for any term $(\phi^\dagger\phi)_{\mathbf{1}_0}$, (gauge-covariant) derivatives play no role here.

Goofy transformations 101 – Goofy basis change

We actually do this all the time in going to the canonical basis in the first place!
In the most general basis, the kinetic terms are

$$\mathcal{K} = \kappa^{ab}\,(\partial_\mu\phi_a)^\dagger(\partial^\mu\phi_b) \qquad \text{with} \qquad \kappa^{ab} = \kappa^{ba\,*}\ .$$

Wave function renormalization coefficients $\kappa^{ab} \equiv$ “couplings” of kinetic term.
Any hermitean matrix κ can be written as

$$\kappa \;=\; U^\dagger \begin{pmatrix} k_1 & 0 & \\ 0 & k_2 & \\ & & \ddots \end{pmatrix} U\,, \qquad (U \text{ unitary, eigenvalues } k_a \in \mathbb{R})$$

U’s can be absorbed in $\phi^\dagger$, ϕ by a *regular* field redefinition.
Canonical diagonal entries are obtained by subsequent rescaling (no sum)

$$\phi_a = \sqrt{k_a}\,\phi'_a\,, \qquad \text{and} \qquad \phi_a^\dagger = \sqrt{k_a}\,\phi_a'^\dagger\ .$$

Note: if any one of the $k_a < 0$, this “rescaling” <u>is</u> a *goofy* transformation.

Goofy transformations $\mathbf{101}$ – Active goofy transformations

Goofy transformations as *active* transformations (we assume unitary A, and B)

$$\phi \mapsto A\phi\,, \qquad \phi^\dagger \mapsto \phi^\dagger B^\dagger\,.$$

For canonical kinetic term this corresponds to a mapping

$$\phi_a^\dagger\,\delta^{ab}\,\phi_b \mapsto \phi_a^\dagger (B^\dagger A)^{ab}\phi_b\,.$$

For the action to stay real valued, $(B^\dagger A)$ is required to be hermitean and unitary. Therefore, $(B^\dagger A)^2 = \mathbb{1}$ is of order 2. This means there is a basis in which

$$\sum_a \partial_\mu \phi_a^\dagger\,\partial^\mu \phi_a \quad \longmapsto \quad \sum_a \kappa_a\,\partial_\mu \phi_a^\dagger\,\partial^\mu\phi_a \qquad \text{with} \qquad \kappa_a = \pm 1 \quad \text{(uncorrelated signs)}\,.$$

In this basis goofy transformation corresponds to pure sign flips of kinetic terms.