General Form of Effective Operators from Hidden Sectors

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Introduction

The Standard Model describes nature well up to energies of order the weak scale but leaves several deep questions unanswered. These include the resolution of the (little) hierarchy problem, the nature of dark matter, the origin of neutrino masses and the source of the baryon asymmetry.

New physics at or below the weak scale is, in general, highly constrained by experiment. The least constrained models involve a hidden sector composed of particles with no charge under the SM gauge groups.

Hidden sectors at or below the weak scale can provide solutions to several of the puzzles of the SM, including all of the four mentioned above.

Hidden sectors communicate with the SM through interactions of the form

$$\mathcal{L} \supset \mathcal{O}_{\rm SM} \mathcal{O}_{\rm HS}$$

The SM operators O_{SM} act as "portals" to the hidden sector.

The lowest dimensional portal operators are the most likely to give rise to observable effects.

Focus on the three lowest dimension portal operators:

Higgs portal $\mathcal{O}_{\rm SM} \equiv H^\dagger H$

Neutrino portal $\mathcal{O}_{\rm SM} \equiv \ell H$

Hypercharge portal $\mathcal{O}_{\rm SM} \equiv B_{\mu\nu}$

There have been searches for hidden sectors at colliders and beam dumps. Their reach is limited if the hidden sector is heavy.

In this case, we can still obtain sensitivity from electroweak precision observables.

The resulting constraints have been worked out for portals to some specific weakly coupled hidden sectors with a small number of degrees of freedom.

However, the results cannot be directly applied to more general hidden sectors, especially if the hidden sector is strongly coupled or has many degrees of freedom.

Consider a general hidden sector that couples through the Higgs, neutrino or hypercharge portals.

- For each portal, the forms of the leading dimension-six terms in the low-energy effective theory are fixed and independent of the hidden sector dynamics. In some cases, their signs are fixed by unitarity and causality.
- The coefficients of these dimension-six terms can be thought of as the analogue for hidden sectors of the oblique parameters in universal theories.
- For each portal, we perform a global fit of these dimension-six terms to electroweak precision observables, Higgs measurements and diboson data and determine the current bounds on their coefficients.

We assume that portal coupling is small enough to be treated perturbatively and also that all the hidden sector states have masses above the weak scale.

Preview of the Results

For the Higgs portal, two independent operators are generated.

$$\mathcal{O}_H = (H^\dagger H)^3 \quad \text{and} \quad \mathcal{O}_{H\Box} = (H^\dagger H)\Box(H^\dagger H)$$

For some range of scaling dimensions of the operator O_{HS}, sign of $O_{H\Box}$ is fixed.

For the neutrino portal, a unique operator is generated.

$$\mathcal{O}_{\ell H} = (\ell H)^\dagger i\bar{\sigma}^\mu \partial_\mu (\ell H)$$

For the hypercharge portal, a unique operator is again generated.

$$\mathcal{O}_{2B} = -\frac{1}{2}(\partial_\rho B_{\mu\nu})(\partial^\rho B^{\mu\nu})$$

For some range of scaling dimensions of the operator O_{HS}, the signs of $O_{\ell H}$ and O_{2B} are fixed.

Higher Dimensional Operators from Hidden Sectors

The Higgs Portal

Consider a Higgs portal interaction,

$$\mathcal{L} \supset -\lambda H^\dagger H \mathcal{O}_S$$

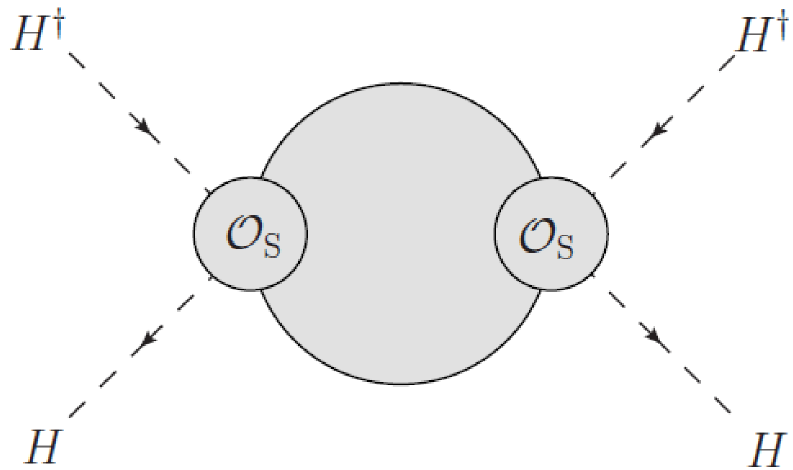
Here O_S is an arbitrary gauge invariant operator in the hidden sector.

It could, for example, be an elementary or composite operator of a weakly coupled sector or a primary or secondary operator of a strongly coupled theory that is conformal in the ultraviolet.

Assume the coupling λ is small enough to be treated perturbatively. We will proceed systematically in powers of λ.

Integrating out the hidden sector at order λ, the leading effect is a correction to the Higgs mass term. No observable consequences.

At order λ^2, we consider the process $H(p_1)\ H(p_2) \rightarrow H(p_3)\ H(p_4)$ and match to the low energy effective field theory.



The matrix element factorizes into hidden sector and SM matrix elements.

$$\frac{1}{2}\langle p_3, p_4| T\{(-i\lambda)^2 \int d^4x \int d^4y [H^\dagger H \mathcal{O}_S](x)[H^\dagger H \mathcal{O}_S](y)\} |p_1, p_2\rangle$$
$$= -\frac{\lambda^2}{2} \int d^4x \int d^4y \, \langle\Omega| T\{\mathcal{O}_S(x)\mathcal{O}_S(y)\} |\Omega\rangle \, \langle p_3, p_4| T\{H^\dagger H(x) H^\dagger H(y)\} |p_1, p_2\rangle$$

The hidden sector matrix element governs the hidden sector dynamics.

Expressed in terms of the Kallen-Lehmann spectral representation,

$$\langle\Omega|\, T\{\mathcal{O}_{\mathcal{S}}(x)\mathcal{O}_{\mathcal{S}}(y)\}\, |\Omega\rangle \;=\int_0^\infty dM^2 \rho(M^2) D_F(x-y, M^2)\,.$$

Here $D_F(x\text{-}y,\, M^2)$ is the Feynman propagator in position space. The spectral weight $\rho(M^2)$ is positive.

This is to be combined with the SM matrix element. Upon doing so, the total matrix element reduces to a sum of two terms of the form

$$-\lambda^2 \int_0^\infty dM^2 \rho(M^2) \frac{i}{p^2 - M^2 + i\epsilon}\,.$$

Here $p = (p_1 - p_3)$ for the first term and $(p_1 - p_4)$ for the second.

Since all the hidden sector states are massive, $\varrho(M^2) = 0$ for M^2 below some specific value, ${M_{IR}}^2$, corresponding to the mass of the lightest state. Since we are considering $p^2 < {M_{IR}}^2$, we can expand in powers of p^2/M^2 . We obtain

$$+i\lambda^2 \int_0^\infty dM^2 \frac{\rho(M^2)}{M^2} \left[\left(1 + \frac{(p_1 - p_3)^2}{M^2} + \ldots \right) + \left(1 + \frac{(p_1 - p_4)^2}{M^2} + \ldots \right) \right].$$

The constant terms can be absorbed into the Higgs quartic and have no observable effects.

However, the p^2/M^2 terms correspond to a genuine dimension-six operator in the low-energy effective field theory

$$-\frac{\alpha}{M_{\rm IR}^2}(H^\dagger H)\Box(H^\dagger H) \equiv -\frac{\alpha}{M_{\rm IR}^2}\mathcal{O}_{H\Box} \qquad \text{with} \qquad \alpha > 0.$$

Here $O_{H\Box}$ is a standard operator in the Warsaw basis.

The coefficient α of this term is positive!

Where does the positivity of α come from?

- The positivity of the spectral density function, which follows from unitarity.
- The form of the Kallen-Lehmann spectral representation, which is dictated by causality. (The spectral representation is a special case of a dispersion relation).

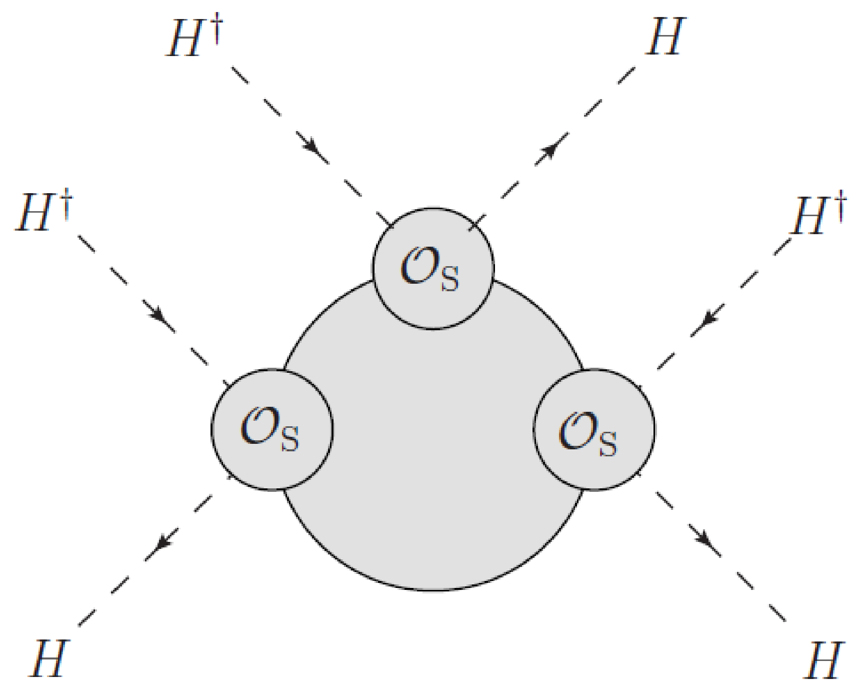
Loop diagrams involving the SM fields will give rise to other dimension-six terms at order λ^2 at the matching scale. However, these effects are loop-suppressed and therefore subleading.

We have implicitly assumed that the integral over M^2 does not diverge in the ultraviolet. In general, this assumption may not be valid.

Consider the case the operator O_S has a definite scaling dimension Δ_S in the far ultraviolet. Then the coefficient α of the dimension-six operator is UV-sensitive at order λ^2 for values of $\Delta_S > 3$. For $\Delta_S \leq 3$ our conclusion that $\alpha > 0$ is expected to be valid.

If O_S does not have a definite scaling dimension, our conclusion that $\alpha > 0$ is expected to be valid provided $\rho(M^2)$ does not grow any faster than M^2 in the far ultraviolet.

At order λ^3, the only operator we generate at leading order is $(H^\dagger H)^3 \equiv \mathcal{O}_H$



There is no restriction on the sign of its coefficient.

We see that for the Higgs portal, two independent operators are generated.

$$\mathcal{O}_H = (H^\dagger H)^3 \quad \text{and} \quad \mathcal{O}_{H\Box} = (H^\dagger H)\Box(H^\dagger H)$$

What is the effect of $O_{H\Box}$ on low energy observables?

If we replace two of the Higgs fields in $O_{H\Box}$ by their VEVs, we obtain a correction to the kinetic term of the physical Higgs boson. All the couplings of the Higgs to the SM fermions and gauge bosons are rescaled by a common factor. The Higgs production rate is affected, but not the branching ratios.

What about O_H?

This corrects the Higgs trilinear and quartic couplings. Since these have not been measured, this operator is poorly constrained.

The Neutrino Portal

Consider a neutrino portal interaction,

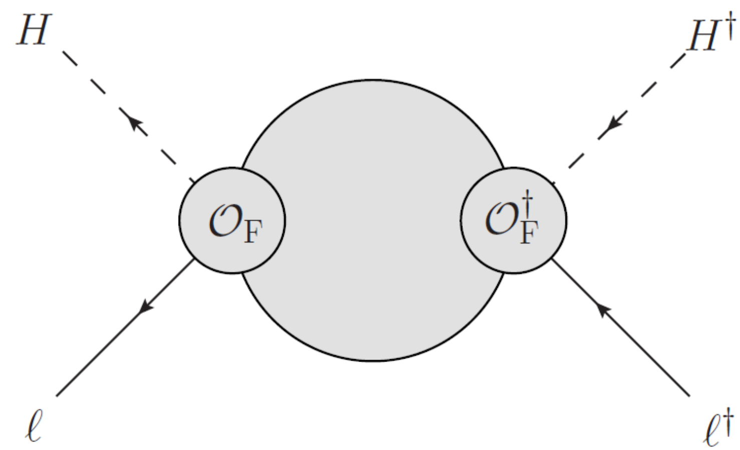
$$\mathcal{L} \supset -y\mathcal{O}_F \ell H + \text{h.c.}$$

Here O_F is a hidden sector operator that transforms as a left-chiral fermion under the Lorentz group.

Assume that the hidden sector does not violate lepton number. For now, restrict to a single SM generation.

As before, assume that the coupling y is small enough to be treated perturbatively. We will proceed systematically in powers of y.

Leading effect arises at order y^2.



We consider the process $l(p_1)\, H(p_2) \rightarrow l(p_3)\, H(p_4)$ and match to the low energy effective field theory.

As before, the matrix element factorizes into hidden sector and SM pieces.

$$\langle p_3, x(p_3), p_4| T \left\{ (-iy)^2 \int d^4x \int d^4y [(\ell H)^\dagger {\mathcal{O}_F}^\dagger](x)[\mathcal{O}_F \ell H](y) \right\} |p_1, x(p_1), p_2\rangle =$$

$$(-iy)^2 \int d^4x \int d^4y \, \langle p_3, x(p_3), p_4| T \left\{ (\ell H)^\dagger(x) \ell H(y) \right\} |p_1, x(p_1), p_2\rangle \, \langle \Omega | T\{ {\mathcal{O}_F}^\dagger(x) \mathcal{O}_F(y) \} |\Omega\rangle$$

The hidden sector matrix element is $\langle\Omega| T\{\mathcal{O}_F{}^\dagger(x)\mathcal{O}_F(y)\} |\Omega\rangle$.

Employing the Kallen-Lehmann representation and combining this with the SM contribution we obtain the matrix element

$$(-iy)^2 x^\dagger(p_3)\left[\int_0^\infty dM^2 \frac{\rho(M^2)(ip_\mu\bar{\sigma}^\mu)}{p^2-M^2+i\epsilon}\right] x(p_1)$$

where the spectral density function $\rho(M^2)$ is again positive.

Expanding out the matrix element in powers of p^2/M^2, we obtain

$$iy^2\, x^\dagger(p_3)(p_\mu\bar{\sigma}^\mu)x(p_1)\int_0^\infty \frac{dM^2}{M^2}\rho(M^2)\ .$$

This corresponds to a dimension-six term in the low energy effective theory,

$$\frac{\alpha}{M_{\mathrm{IR}}^2}(\ell H)^\dagger i\bar{\sigma}^\mu\partial_\mu(\ell H) \equiv \frac{\alpha}{M_{\mathrm{IR}}^2}\mathcal{O}_{\ell H}\ .$$

Here $\alpha > 0$ due to the positivity of $\rho(M^2)$.

Loop diagrams involving the SM fields will give rise to other dimension-six terms at order y^2 at the matching scale. However, these effects are loop-suppressed and therefore subleading.

We have implicitly assumed that the integral over M^2 does not diverge in the ultraviolet. In general, this assumption may not be valid.

Consider the case the operator O_F has a definite scaling dimension Δ_F in the far ultraviolet. Then the coefficient α of the dimension-six operator is UV-sensitive at order λ^2 for $\Delta_F > 5/2$. For $\Delta_F \leq 5/2$ our conclusion that $\alpha > 0$ is expected to be valid.

If O_F does not have a definite scaling dimension, our conclusion that $\alpha > 0$ is expected to be valid provided $\rho(M^2)$ does not grow with M^2, but instead falls or remains constant.

We see that for the neutrino portal, for a single generation of SM fermions, only one dimension-six operator is generated at leading order.

$$\mathcal{O}_{\ell H} = (\ell H)^\dagger i\bar{\sigma}^\mu \partial_\mu (\ell H)$$

Generalizing to three generations of SM fermions, this becomes

$$\frac{\alpha^{ij}}{M_{\rm IR}^2}(\ell_i H)^\dagger i\bar{\sigma}^\mu \partial_\mu (\ell_j H)$$

What is the effect of O_{lH} on low energy observables?

If we replace the two Higgs fields in O_{lH} by their VEVs, we obtain a correction to the kinetic term of the neutrino without a corresponding correction to the kinetic term of the corresponding charged lepton. This affects the coupling of the neutrino to the W and Z gauge bosons. Very strongly constrained by precise measurements of muon lifetime and Z boson line shape.

We can express the neutrino portal operator $O_{\ell H}$ in the Warsaw basis.

$$\mathcal{O}_{\ell H} = \frac{1}{4}\left[\mathcal{O}_{H\ell}^{(1)} - \mathcal{O}_{H\ell}^{(3)}\right]$$

Here the Warsaw basis operators are defined as

$$\mathcal{O}_{H\ell}^{(1)} \equiv (H^\dagger i \overleftrightarrow{D}_\mu H)(\bar{\ell}\gamma^\mu \ell),$$
$$\mathcal{O}_{H\ell}^{(3)} \equiv (H^\dagger i \overleftrightarrow{D}_\mu^I H)(\bar{\ell}\tau^I \gamma^\mu \ell).$$

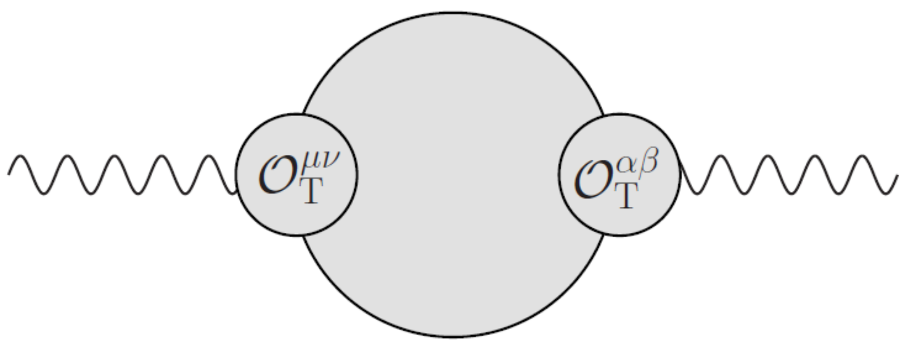
The Hypercharge Portal

Consider a hypercharge portal interaction,

$$\mathcal{L} \supset -\epsilon B_{\mu\nu} \mathcal{O}_T^{\mu\nu}$$

Here O_T is a hidden sector operator that transforms as a two-index antisymmetric tensor under the Lorentz group.

Assume again that the coupling ϵ is small enough to be treated perturbatively. We will proceed systematically in powers of ϵ.



The leading effect is a correction to the hypercharge gauge boson propagator.

The matrix element again factorizes into hidden sector and SM pieces.

$$\frac{1}{2}\langle 0|\, T\Big\{B_\rho(x')B_\sigma(y')(-i\epsilon)^2\int d^4x\int d^4y[B_{\mu\nu}\mathcal{O}_T^{\mu\nu}](x)[B_{\alpha\beta}\mathcal{O}_T^{\alpha\beta}](y)\Big\}\ |0\rangle =$$
$$-\frac{\epsilon^2}{2}\int d^4x\int d^4y\,\langle\Omega|\,T\left\{\mathcal{O}_T^{\mu\nu}(x)\mathcal{O}_T^{\alpha\beta}(y)\right\}|\Omega\rangle\,\langle 0|\,T\left\{B_\rho(x')B_\sigma(y')B_{\mu\nu}(x)B_{\alpha\beta}(y)\right\}|0\rangle$$

Expressed in terms of the Kallen-Lehmann spectral representation,

$$\langle\Omega|\,T\left\{\mathcal{O}_T^{\mu\nu}(x)\mathcal{O}_T^{\alpha\beta}(y)\right\}|\Omega\rangle = \int dM^2 D_F(x-y,M^2)\int\frac{d^4q}{(2\pi)^4}\delta(q^2-M^2)\pi^{\mu\nu\alpha\beta}(q)\ .$$

Lorentz symmetry constrains the form of $\pi^{\mu\nu\alpha\beta}(q)$.

$$\pi^{\mu\nu\alpha\beta}(q) = \rho_\epsilon(q^2)\epsilon^{\mu\nu\alpha\beta} + \rho_0(q^2)\Pi_0^{\mu\nu\alpha\beta} + \rho_1(q^2)\Pi_1^{\mu\nu\alpha\beta}(q)$$

Here

$$\Pi_0^{\mu\nu\alpha\beta} \equiv g^{\mu\alpha}g^{\nu\beta} - g^{\mu\beta}g^{\nu\alpha},$$
$$\Pi_1^{\mu\nu\alpha\beta}(q) \equiv -g^{\mu\alpha}q^\nu q^\beta + g^{\mu\beta}q^\nu q^\alpha - g^{\nu\beta}q^\mu q^\alpha + g^{\nu\alpha}q^\mu q^\beta\ .$$

Combining this with the SM matrix element we obtain for the correction to the propagator

$$4\epsilon^2 \left(g_{\rho\sigma} - \frac{p_\rho p_\sigma}{p^2}\right) \frac{i}{p^2 + i\epsilon} \int_0^\infty dM^2 \, \frac{-\rho_0(M^2) + p^2 \rho_1(M^2)}{p^2 - M^2 + i\epsilon} \, .$$

Expanding out the matrix element in powers of p^2/M^2, we obtain

$$4\epsilon^2 \left(g_{\rho\sigma} - \frac{p_\rho p_\sigma}{p^2}\right) \frac{i}{p^2 + i\epsilon} \left\{ \int_0^\infty \frac{dM^2}{M^2} \rho_0(M^2) - p^2 \int_0^\infty \frac{dM^2}{M^4} \left(-\rho_0(M^2) + M^2 \rho_1(M^2)\right) \right\} .$$

The first term simply corrects the kinetic term of the hypercharge gauge boson and has no observable effect.

The second term corresponds to a dimension-six operator of the form

$$\frac{\alpha}{M_{\text{IR}}^2} \mathcal{O}_{2B} \qquad \text{where} \qquad \alpha > 0 \qquad \text{and} \qquad \mathcal{O}_{2B} \equiv -\frac{1}{2} (\partial_\lambda B_{\mu\nu})(\partial^\lambda B^{\mu\nu}).$$

Here $\alpha > 0$ due to the positivity of $M^2 \rho_1(M^2) - \rho_0(M^2)$!

Loop diagrams involving the SM fields will give rise to other dimension-six terms at order ϵ^2 at the matching scale. However, these effects are loop-suppressed and therefore subleading.

We have implicitly assumed that the integral over M^2 does not diverge in the ultraviolet. In general, this assumption may not be valid.

Consider the case the operator O_T has a definite scaling dimension Δ_T in the far ultraviolet. Then the coefficient α of the dimension-six operator is UV-sensitive at order ϵ^2 for $\Delta_T > 3$. For $\Delta_T \leq 3$ our conclusion that $\alpha > 0$ is expected to be valid.

If O_T does not have a definite scaling dimension, our conclusion that $\alpha > 0$ is expected to be valid provided that the combination $M^2\rho_1(M^2) - \rho_0(M^2)$ does not grow any faster than M^2 in the far ultraviolet.

We can express the hypercharge portal operator O_{2B} in the Warsaw basis.

$$\frac{\alpha}{M_{\rm IR}^2}\mathcal{O}_{2B} = -\frac{\alpha}{M_{\rm IR}^2}g'^2\Bigg[\mathcal{O}_{HD}+\frac{1}{4}\mathcal{O}_{H\Box}
+\left(Y_q\left[\mathcal{O}_{Hq}^{(1)}\right]_{ii}+Y_\ell\left[\mathcal{O}_{H\ell}^{(1)}\right]_{ii}+Y_u\left[\mathcal{O}_{Hu}\right]_{ii}+Y_d\left[\mathcal{O}_{Hd}\right]_{ii}+Y_e\left[\mathcal{O}_{He}\right]_{ii}\right)
+\left(Y_q^2\left[\mathcal{O}_{qq}^{(1)}\right]_{iijj}+Y_\ell^2\left[\mathcal{O}_{\ell\ell}\right]_{iijj}+Y_u^2\left[\mathcal{O}_{uu}\right]_{iijj}+Y_d^2\left[\mathcal{O}_{dd}\right]_{iijj}+Y_e^2\left[\mathcal{O}_{ee}\right]_{iijj}
+2Y_qY_\ell\left[\mathcal{O}_{\ell q}^{(1)}\right]_{iijj}+2Y_qY_u\left[\mathcal{O}_{qu}^{(1)}\right]_{iijj}+2Y_qY_d\left[\mathcal{O}_{qd}^{(1)}\right]_{iijj}+2Y_qY_e\left[\mathcal{O}_{qe}\right]_{iijj}
+2Y_\ell Y_u\left[\mathcal{O}_{\ell u}\right]_{iijj}+2Y_\ell Y_d\left[\mathcal{O}_{\ell d}\right]_{iijj}+2Y_\ell Y_e\left[\mathcal{O}_{\ell e}\right]_{iijj}
+2Y_uY_d\left[\mathcal{O}_{ud}^{(1)}\right]_{iijj}+2Y_uY_e\left[\mathcal{O}_{eu}\right]_{iijj}+2Y_dY_e\left[\mathcal{O}_{ed}\right]_{iijj}\right)\Bigg]$$

Here Y_f is the hypercharge of the SM fermion f.

$$\left\{Y_q, Y_\ell, Y_u, Y_d, Y_e\right\} = \left\{\frac{1}{6}, -\frac{1}{2}, \frac{2}{3}, -\frac{1}{3}, -1\right\}$$

Summary of SMEFT Operators from Higgs, Neutrino and Hypercharge Portals

Higgs portal			
$\mathcal{O}_H$	$(H^\dagger H)^3$	$\mathcal{O}_{H\Box}$	$(H^\dagger H)\Box(H^\dagger H)$

Neutrino portal			
$\mathcal{O}_{H\ell}^{(1)}$	$(H^\dagger i \overleftrightarrow{D}_\mu H)(\bar{\ell}_p \gamma^\mu \ell_r)$	$\mathcal{O}_{H\ell}^{(3)}$	$(H^\dagger i \overleftrightarrow{D}^I_\mu H)(\bar{\ell}_p \tau^I \gamma^\mu \ell_r)$

Hypercharge portal			
$\mathcal{O}_{HD}$	$(H^\dagger D_\mu H)^\star (H^\dagger D^\mu H)$	$\mathcal{O}_{H\Box}$	$(H^\dagger H)\Box(H^\dagger H)$
$\mathcal{O}_{Hq}^{(1)}$	$(H^\dagger i \overleftrightarrow{D}_\mu H)(\bar{q}_p \gamma^\mu q_r)$	$\mathcal{O}_{H\ell}^{(1)}$	$(H^\dagger i \overleftrightarrow{D}_\mu H)(\bar{\ell}_p \gamma^\mu \ell_r)$
$\mathcal{O}_{Hu}$	$(H^\dagger i \overleftrightarrow{D}_\mu H)(\bar{u}_p \gamma^\mu u_r)$	$\mathcal{O}_{Hd}$	$(H^\dagger i \overleftrightarrow{D}_\mu H)(\bar{d}_p \gamma^\mu d_r)$
$\mathcal{O}_{He}$	$(H^\dagger i \overleftrightarrow{D}_\mu H)(\bar{e}_p \gamma^\mu e_r)$	plus four-fermion operators	

Constraints on Portal-Generated Operators

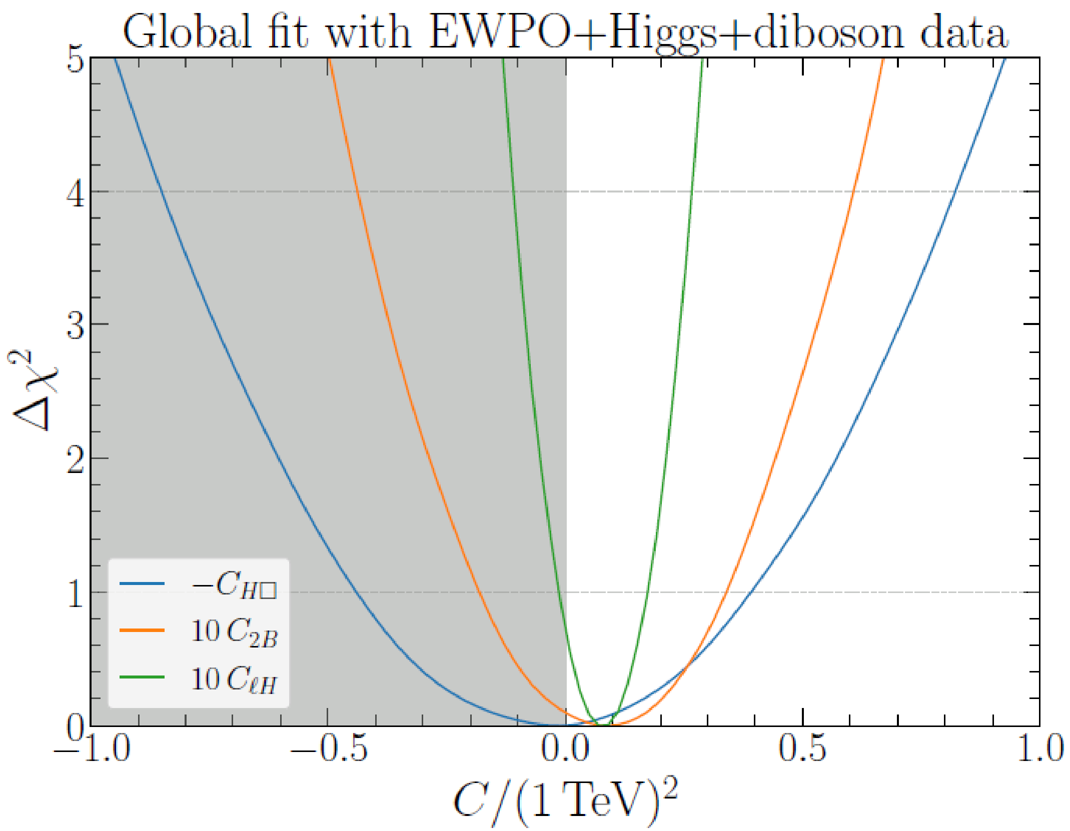


Figure 7: We plot $\Delta\chi^2$ as a function of Wilson coefficients of the Higgs, neutrino, and hypercharge portal operators for a matching scale $\Lambda = 1$ TeV. The plot incorporates data from EWPO, Higgs, and diboson observables and takes into account renormalization group evolution. The gray-shaded region is forbidden for some range of scaling dimensions of the hidden sector operator $\mathcal{O}_{\rm HS}$, as discussed above.

No significant preference for any of the portals over the SM.

Conclusions

We have considered hidden sectors that couple through the Higgs, neutrino and hypercharge portals and shown that, when the hidden sector is integrated out, the forms of the leading dimension-six operators in the low energy effective theory are fixed and independent of the dynamics in the hidden sector.

For the Higgs portal, two independent operators are generated, one of which has a sign that, under certain conditions, is fixed by unitarity and causality.

For the neutrino portal, for a single SM generation and assuming lepton number conservation, a unique operator is generated. This operator has a sign that, under certain conditions, is fixed by unitarity and causality.

For the hypercharge portal, a unique operator is generated, which has a sign that, under certain conditions, is fixed by unitarity and causality.

We fit the coefficients of these terms to electroweak precision observables, Higgs and diboson data. No significant preference for any of the portals over the SM.