Is our vacuum global in an economical 331 model?

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What are 331 models?

331-models replace the SM gauge group $SU(3)_c \times SU(2)_L \times U(1)_Y$ with $SU(3)_c \times SU(3)_L \times U(1)_X$

The $SU(3)_L$-gauge group has one additional diagonal generator compared to the $SU(2)_L$:
$\Rightarrow$ Freedom in electric charge definition: $Q = T_3 + \beta T_8 + X$ (compare to SM: $Q = T_3 + Y/2$).

Two types of models:

- $\beta = \pm\sqrt{3}$ (**too complicated scalar sector**):
  F. Pisano and V. Pleitez, Phys. Rev. D **46**, 410 (1992).
  P. H. Frampton, Phys. Rev. Lett. **69**, 2889 (1992).
- $\beta = \pm\frac{1}{\sqrt{3}}$ (**we choose this**):
  M. Singer, J. W. F. Valle and J. Schechter, Phys. Rev. D **22**, 738 (1980).

Economical Scalar Sector (with $\beta = -\frac{1}{\sqrt{3}}$)

$$\rho = \begin{pmatrix} \rho_1^+ \\ \rho_2^0 \\ \rho_3^+ \end{pmatrix} \sim (1, 3, \frac{2}{3}), \quad \eta = \begin{pmatrix} \eta_1^0 \\ \eta_2^- \\ \eta_3^0 \end{pmatrix}, \; \chi = \begin{pmatrix} \chi_1^0 \\ \chi_2^- \\ \chi_3^0 \end{pmatrix} \sim (1, 3, -\frac{1}{3})$$

Typical electroweak vacuum is:

$$\langle\rho\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_\rho \\ 0 \end{pmatrix}, \quad \langle\eta\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} v_\eta \\ 0 \\ 0 \end{pmatrix}, \quad \langle\chi\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ 0 \\ v_\chi \end{pmatrix}$$

The symmetry breaking pattern is:

$$SU(3)_L \times U(1)_X \xrightarrow{v_\chi} SU(2)_L \times U(1)_Y \xrightarrow{v_\rho,\, v_\eta} \times U(1)_{\rm em},$$

where $v_\chi, u \gtrsim 1\mathrm{TeV}$ and $v_\rho, v_\eta = \mathcal{O}(100\mathrm{GeV})$

Scalar potential

Scalar potential with softly broken $\mathbb{Z}_2$ symmetry $\chi \to -\chi$:

$$
\begin{aligned}
V = & \mu_\eta^2 \eta^\dagger \eta + \mu_\rho^2 \rho^\dagger \rho + \mu_\chi^2 \chi^\dagger \chi + \lambda_\eta (\eta^\dagger \eta)^2 + \lambda_\rho (\rho^\dagger \rho)^2 + \lambda_\chi (\chi^\dagger \chi)^2 \\
& + \lambda_{\eta\rho} (\eta^\dagger \eta)(\rho^\dagger \rho) + \lambda_{\eta\chi} (\eta^\dagger \eta)(\chi^\dagger \chi) + \lambda_{\rho\chi} (\rho^\dagger \rho)(\chi^\dagger \chi) \\
& + \lambda'_{\eta\rho} (\eta^\dagger \rho)(\rho^\dagger \eta) + \lambda'_{\eta\chi} (\eta^\dagger \chi)(\chi^\dagger \eta) + \lambda'_{\rho\chi} (\rho^\dagger \chi)(\chi^\dagger \rho) \\
& + \frac{1}{2} \lambda''_{\eta\chi} (\chi^\dagger \eta)^2 - \frac{f}{\sqrt{2}} \epsilon^{ijk} \eta_i \rho_j \chi_k + \text{h.c.},
\end{aligned}
$$

where $f > 0$ and $\lambda''_{\eta\chi} < 0$.

What do we want? Find all the possible minima of the potential.
And when is our minimum the global one?
If our minimum is not global, when is it at least metastable?

The potential seems quite complicated. How to get the full vacuum structure? $\Rightarrow$ **Orbit space method!**

Orbit space method

- Write the potential in terms of gauge invariant quantities. They can be divided into two types: field norms and dimensionless angular variables.
- **Orbit space**: the space of dimensionless angular orbit parameters
- The minimum of the potential is located on the edge of the orbit space
- The problem is reduced into finding the shape of the orbit space edge.
- The ordit space method has the advantage: one doesn't have to track each multiplet component individually. The number of parameters is therefore greatly reduced. The downside is that for more complicated potentials the shape of the orbit space can be nontrivial.
- There is luckily a systematic method to find the orbit space boundary: the P-matrix method.

Orbit parameters

Basis invariants:

$$p_1 = \eta^\dagger \eta, \quad p_2 = \rho^\dagger \rho, \quad p_3 = \chi^\dagger \chi,$$
$$p_4 = \mathrm{Re}\chi^\dagger \eta, \quad p_5 = \mathrm{Re}\chi^\dagger \rho, \quad p_6 = \mathrm{Re}\eta^\dagger \rho,$$
$$p_7 = \mathrm{Re}\,\det(\eta\rho\chi), \quad p_8 = \mathrm{Im}\chi^\dagger \eta, \quad p_9 = \mathrm{Im}\chi^\dagger \rho,$$
$$p_{10} = \mathrm{Im}\eta^\dagger \rho, \quad p_{11} = \mathrm{Im}\,\det(\eta\rho\chi).$$

Orbit parameters:

$$\vartheta_1^2 = \frac{p_4^2 + p_8^2}{p_1 p_3}, \quad \vartheta_2^2 = \frac{p_5^2 + p_9^2}{p_2 p_3}, \quad \vartheta_3^2 = \frac{p_6^2 + p_{10}^2}{p_1 p_2}, \quad \vartheta_4 = \frac{p_7}{\sqrt{p_1 p_2 p_3}}.$$

The potential in terms of field norms and orbit parameters ϑ:

$$\begin{aligned} V = {} & \mu_\eta^2 |\eta|^2 + \mu_\rho^2 |\rho|^2 + \mu_\chi^2 |\chi|^2 + \lambda_\eta |\eta|^4 + \lambda_\rho |\rho|^4 + \lambda_\chi |\chi|^4 \\ & + [\lambda_{\eta\chi} + (\lambda'_{\eta\chi} - |\lambda''_{\eta\chi}|)\vartheta_1^2] |\eta|^2 |\chi|^2 + (\lambda_{\rho\chi} + \lambda'_{\rho\chi}\vartheta_2^2) |\rho|^2 |\chi|^2 \\ & + (\lambda_{\eta\rho} + \lambda'_{\eta\rho}\vartheta_3^2) |\eta|^2 |\rho|^2 - \sqrt{2} |f| \, \vartheta_4 \, |\eta| |\rho| |\chi|. \end{aligned}$$

The edge of orbit space

The orbit space lies within a hypercube;

$$-1 \leq \vartheta_i, \vartheta_4 \leq 1$$

But what is its exact shape? Use **P-matrix method!**
The P-matrix is defined by

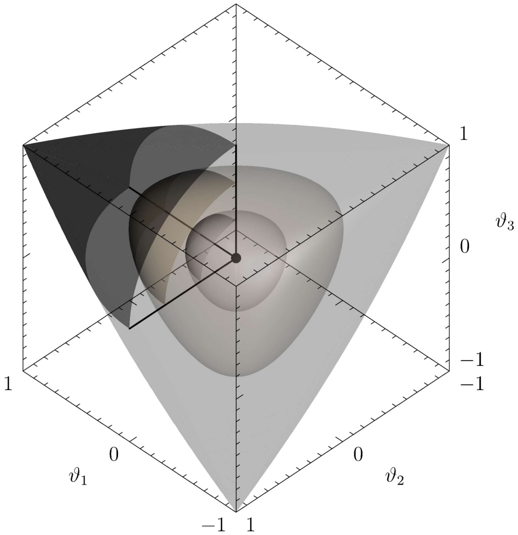
$$P_{ij} = \frac{\partial p_i}{\partial \Phi_a^\dagger} \frac{\partial p_j}{\partial \Phi^a}$$

The equations for different features of orbit space boundary are obtained by computing minors of the P-matrix: $\det P = 0, \qquad \text{rank}\, P = k.$

Features are: vertices, edges, faces, cells and so on.

Each feature corresponds to a minimum of certain type. Like different number VEVs, charge breaking and so on.

Orbit Space

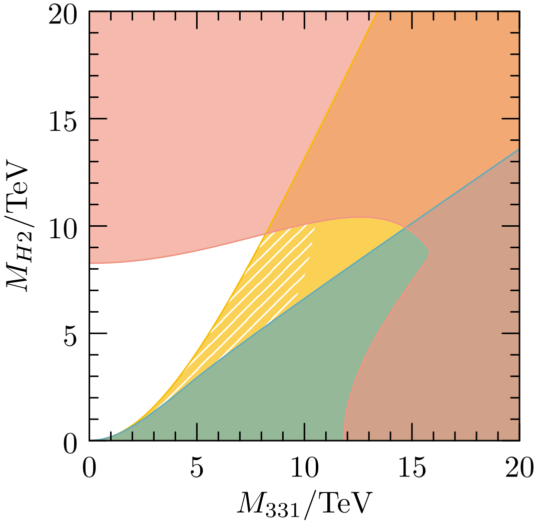


All possible vacuum configurations

Name	Orbit configuration	χ^T	ρ^T	η^T
V_O	$\lvert\chi\rvert = \lvert\eta\rvert = \lvert\rho\rvert = 0$	$(0,0,0)$	$(0,0,0)$	$(0,0,0)$
V_χ	$\lvert\eta\rvert = \lvert\rho\rvert = 0$	$\lvert\chi\rvert\,(0,0,1)$	$(0,0,0)$	$(0,0,0)$
V_ρ	$\lvert\eta\rvert = \lvert\chi\rvert = 0$	$(0,0,0)$	$\lvert\rho\rvert\,(0,1,0)$	$(0,0,0)$
V_η	$\lvert\rho\rvert = \lvert\chi\rvert = 0$	$(0,0,0)$	$(0,0,0)$	$\lvert\eta\rvert\,(0,0,1)$
$V_{\rho\chi}^{\perp}$	$\lvert\eta\rvert = 0,\ \vartheta_2 = 0$	$\lvert\chi\rvert\,(0,0,1)$	$\lvert\rho\rvert\,(0,1,0)$	$(0,0,0)$
$V_{\rho\chi}^{\parallel}$	$\lvert\eta\rvert = 0,\ \vartheta_2 = 1$	$\lvert\chi\rvert\,(0,0,1)$	$\lvert\rho\rvert\,(0,0,1)$	$(0,0,0)$
$V_{\eta\chi}^{\perp}$	$\lvert\rho\rvert = 0,\ \vartheta_1 = 0$	$\lvert\chi\rvert\,(0,0,1)$	$(0,0,0)$	$\lvert\eta\rvert\,(1,0,0)$
$V_{\eta\chi}^{\parallel}$	$\lvert\rho\rvert = 0,\ \vartheta_1 = 1$	$\lvert\chi\rvert\,(0,0,1)$	$(0,0,0)$	$\lvert\eta\rvert\,(0,0,1)$
$V_{\eta\rho}^{\perp}$	$\lvert\chi\rvert = 0,\ \vartheta_3 = 0$	$(0,0,0)$	$\lvert\rho\rvert\,(0,1,0)$	$\lvert\eta\rvert\,(1,0,0)$
$V_{\eta\rho}^{\parallel}$	$\lvert\chi\rvert = 0,\ \vartheta_3 = 1$	$(0,0,0)$	$\lvert\rho\rvert\,(1,0,0)$	$\lvert\eta\rvert\,(1,0,0)$
V_{tip}	$\vartheta_i^2 = 1,\ \vartheta_4 = 0$	$\lvert\chi\rvert\,(0,0,1)$	$\lvert\rho\rvert\,(0,0,1)$	$\lvert\eta\rvert\,(0,0,1)$
V_{EW}^{+}	$\vartheta_i = 0,\ \vartheta_4 = 1$	$\frac{1}{\sqrt{2}}v_\chi(0,0,1)$	$\frac{1}{\sqrt{2}}v_\rho(0,1,0)$	$\frac{1}{\sqrt{2}}v_\eta(1,0,0)$
V_{EW}^{-}	$\vartheta_i = 0,\ \vartheta_4 = -1$	$\lvert\chi\rvert\,(1,0,0)$	$\lvert\rho\rvert\,(0,1,0)$	$\lvert\eta\rvert\,(0,0,1)$

and so on...

Metastability



Summary

- We obtain the full vacuum structure of the usual pheno-potential for the first time
- This was done with **Orbit Space Method** and **P-matrix method**
- There can be dangerous deeper minima in 331 potential, not taken into account previously

$\beta = (\pm)\frac{1}{\sqrt{3}}$

M. Singer, J. W. F. Valle and J. Schechter, Phys. Rev. D **22**, 738 (1980).

Fermion representations:

$$\text{Triplets:} \quad L_{L,i} = \begin{pmatrix} \nu_i \\ e_i \\ N_i \end{pmatrix}_L \sim (1, 3, -\frac{1}{3}), \quad Q_{L,1} = \begin{pmatrix} u \\ d \\ U \end{pmatrix}_L \sim (3, 3, \frac{1}{3})$$

$$Antitriplets : Q_{L,2} = \begin{pmatrix} s \\ c \\ D_1 \end{pmatrix}_L \sim (3, 3^*, 0), Q_{L,3} = \begin{pmatrix} b \\ t \\ D_2 \end{pmatrix}_L \sim (3, 3^*, 0)$$

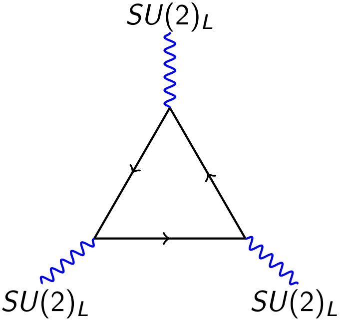
$N_{L,i}$ = new neutrino-like fields, U = new up-type quark, D_1, D_2 = new down-type quarks.

Minimal Scalar Sector:

$$\eta = \begin{pmatrix} \eta^+ \\ \eta^0 \\ \eta'^+ \end{pmatrix} \sim (1, 3, \frac{2}{3}), \quad \rho = \begin{pmatrix} \rho^0 \\ \rho^- \\ \rho'^0 \end{pmatrix}, \ \chi = \begin{pmatrix} \chi^0 \\ \chi^- \\ \chi'^0 \end{pmatrix} \sim (1, 3, -\frac{1}{3})$$

Anomaly cancellation in 331-model

$$\text{Standard model} = SU(3)_C \times SU(2)_L \times U(1)_Y$$
$$331 - \text{model} = SU(3)_C \times SU(3)_L \times U(1)_X$$



Pure $SU(3)_L$ anomaly cancels only if number of fermion triplets equals the number of antitriplets.