NATIONAL SCIENCE CENTRE
POLAND

Bubble Wall Velocity from Hydrodynamics

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Based on:
JHEP 2025.6: 118, [2411.16580]
with T. Krajewski, M. Lewicki and M. Zych

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Why study first-order transitions?

Bariogenesis

Necessary conditions:	SM
1) Baryon number violation	✓
2) C and CP violation	X
3) Out-of-equilibrium dynamics	X

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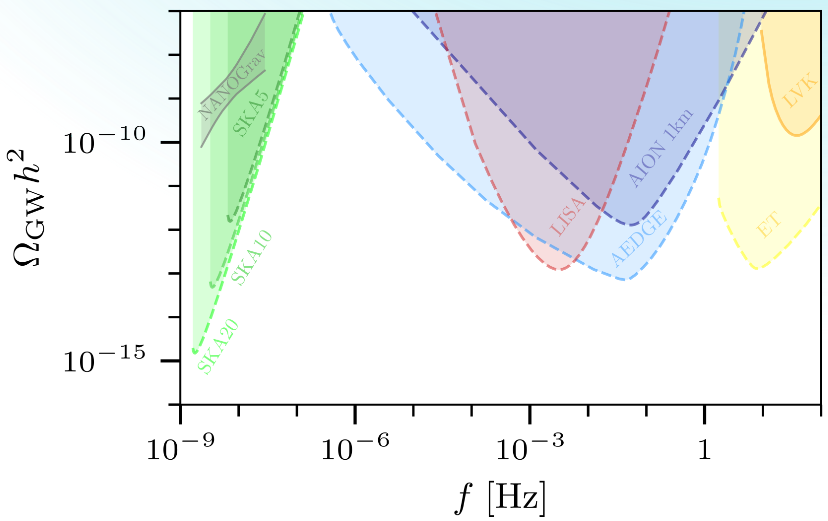
Gravitational waves



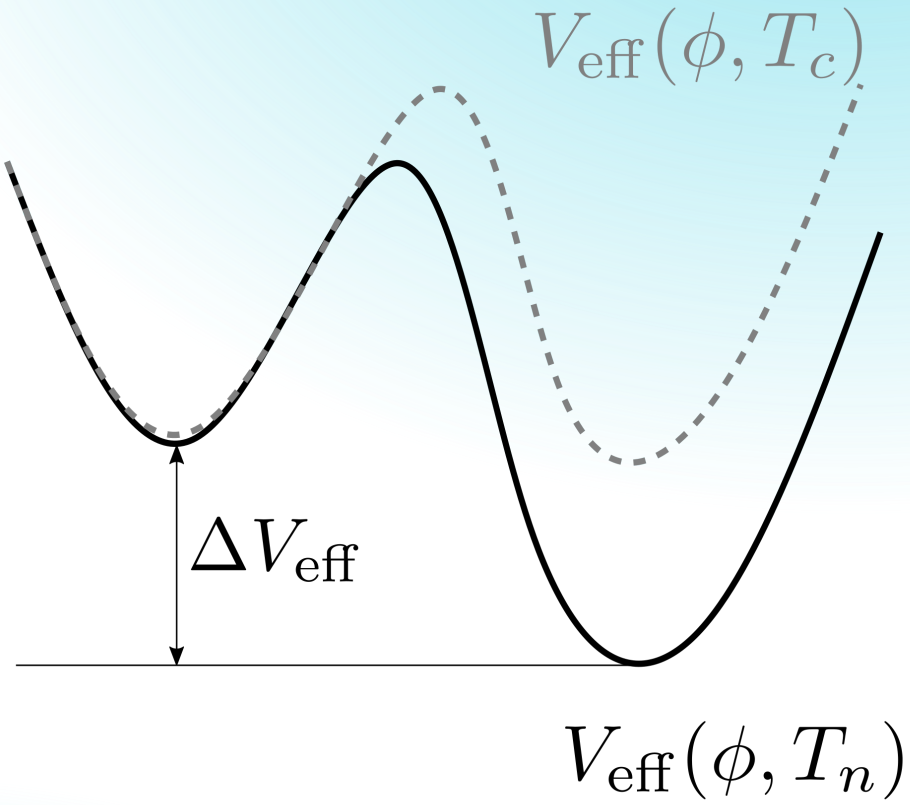
Transition Parameters

- Critical and nucleation temperatures: T_c, T_n
- Transition strength:

$$\alpha_\theta = \frac{\bar{\theta}_b - \bar{\theta}_s}{3\omega_s}, \quad \bar{\theta} = e - \frac{p}{c_b^2}$$

where p is pressure, e and ω are energy and enthalpy densities while c_b is sound speed in a broken phase.

- Terminal bubble wall velocity ξ_w



Primordial plasma

In equilibrium plasma energy-momentum tensor reads

$$T^{\mu\nu}_{\text{fl}} = \omega u^{\mu} u^{\nu} - g^{\mu\nu} p,$$

where u^{μ} is the plasma four velocity.

Hydrodynamics EOMs in the shockwave are

$$2\frac{v}{\xi} = \gamma^2 (1 - v\xi)\left[\frac{\mu^2(\xi, v)}{c^2} - 1\right]\partial_\xi v, \quad \partial_\xi w = w\left(1 + \frac{1}{c^2}\right)\gamma^2 \mu \partial_\xi v.$$

with $\mu(\xi, v) = \frac{\xi - v}{1 - \xi v}$ and $\xi = r/t$

Matching conditions:

$$w_+ \gamma_+^2 v_+ = w_- \gamma_-^2 v_-$$

$$w_+ \gamma_+^2 v_+^2 + p_+ = w_- \gamma_-^2 v_-^2 + p_- \qquad + \quad ??$$

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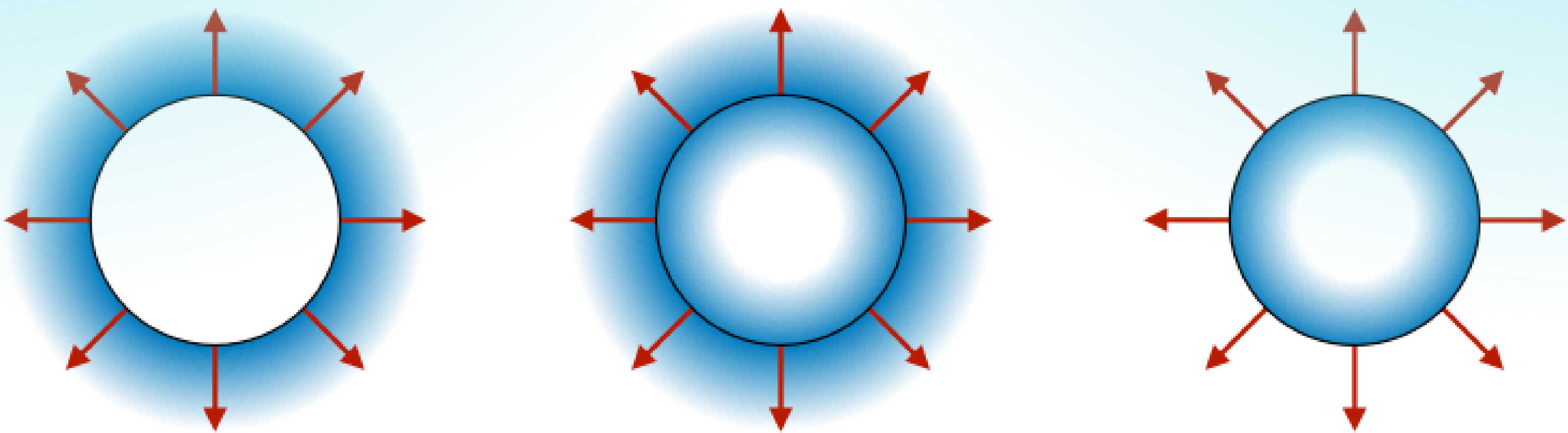
Solution types

Deflagration	Hybrid	Detonation



$\xi_w \leq c_s$	$c_s < \xi_w < v_J$	$v_J \leq \xi_w$

Local thermal equilibrium

Entropy conserved $\Longrightarrow$

New matching condition[1]

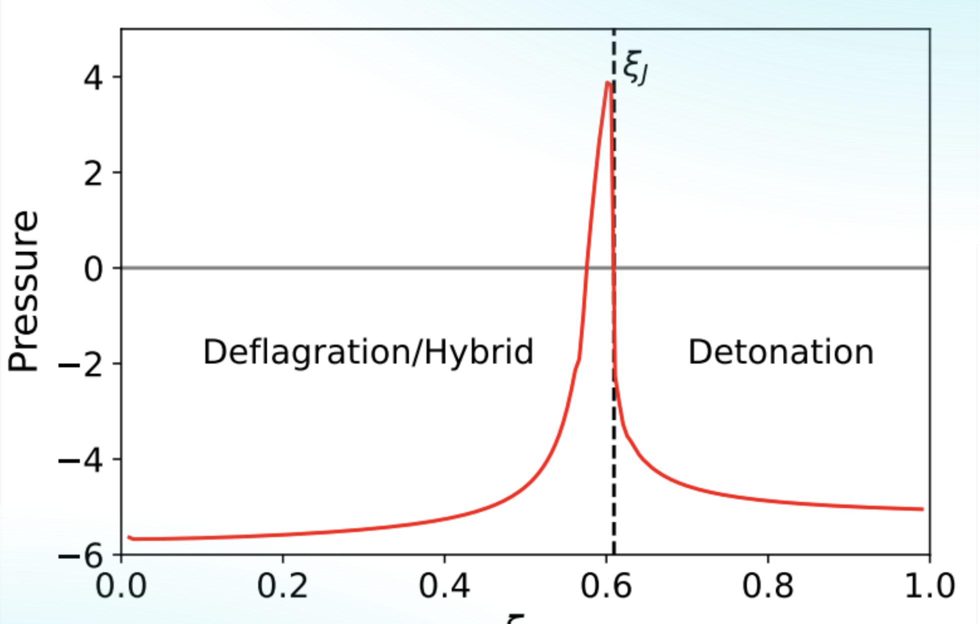
$$s_+ v_+ \gamma_+ = s_- v_- \gamma_-$$

In LTE ξ_w is fixed by

- tansition strength α_θ
- enthalpy ratio $\Psi_n = \frac{\omega_b(T_n)}{\omega_s(T_n)}$
- sound speed $c_s, \ c_b$

No stable detonations exist in LTE, only run-away solutions with $\xi_w \to 1$.

Net pressure on the wall

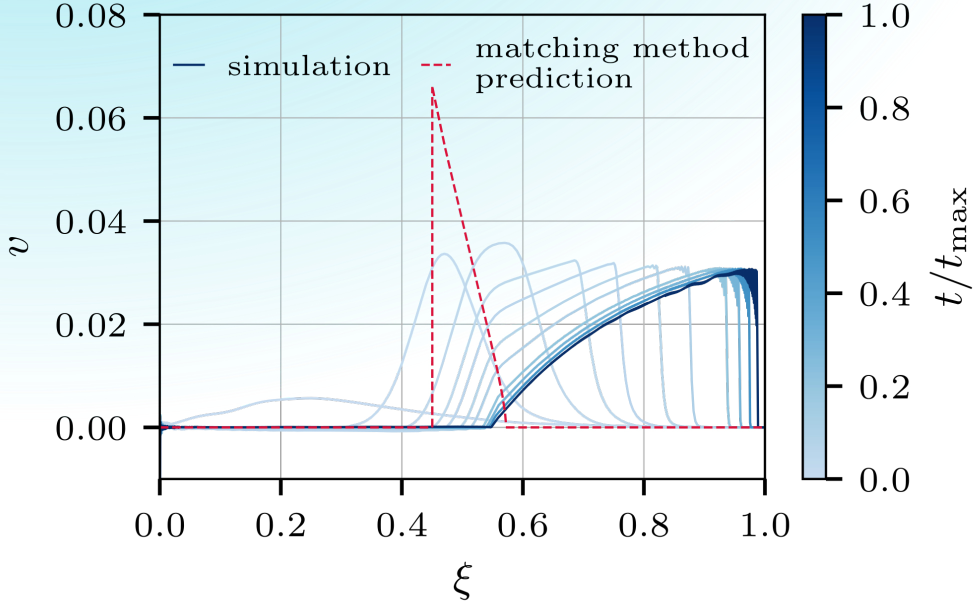


Credits: M. Zych

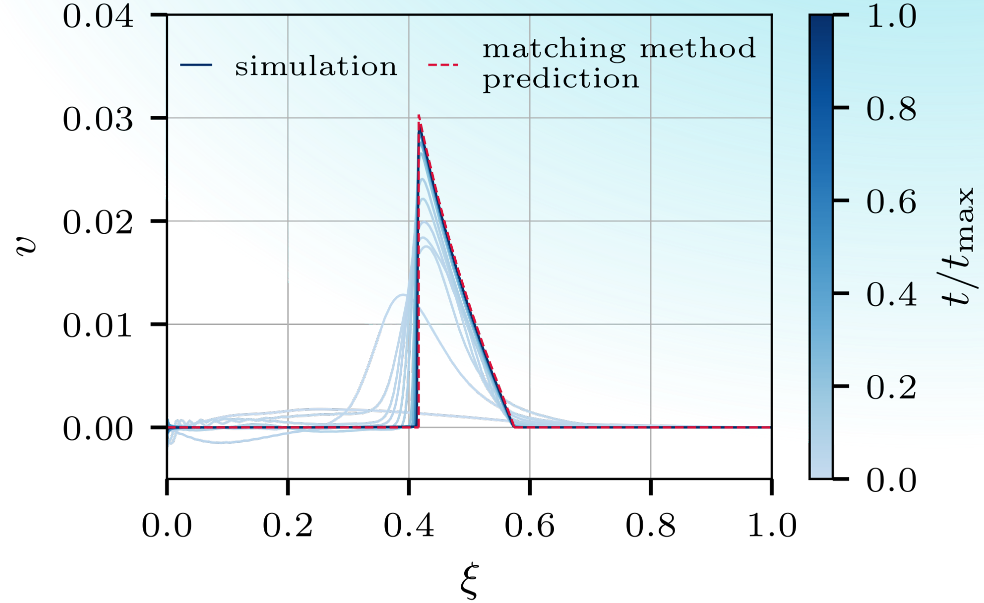
[1] W.-Y. Ai et al. *Model-independent bubble wall velocities in local thermal equilibrium*, JCAP **07** (2023), arXive: 2303.10171.

Stable LTE profile or run-away?

Real-time EOM solutions fall into two categories:[2]



Ultrarelativistic detonation (run-away in LTE)



Stable deflagration/hybrid

[2] T. Krajewski at al., *Hydrodynamical constraints on the bubble wall velocity,* Phys. Rev. D. , (2023), arXive:2303.18216. Plots curtesy of M. Zych.

New matching condition beyond LTE

Dissipation model:

$$\partial_\mu (s u^\mu) = \frac{\eta}{T} (u^\mu \partial_\mu \phi)^2.$$

Entropy produced in the wall

$$\Delta S = \int_{\text{wall}} dz \frac{\eta}{T} (u^\mu \partial_\mu \phi)^2.$$

Using the ansatz for the background field profile

$$\phi(z) = \frac{v_0}{2} \left(1 - \tanh\left(\frac{z}{L_w}\right)\right),$$

we can estimate ΔS as

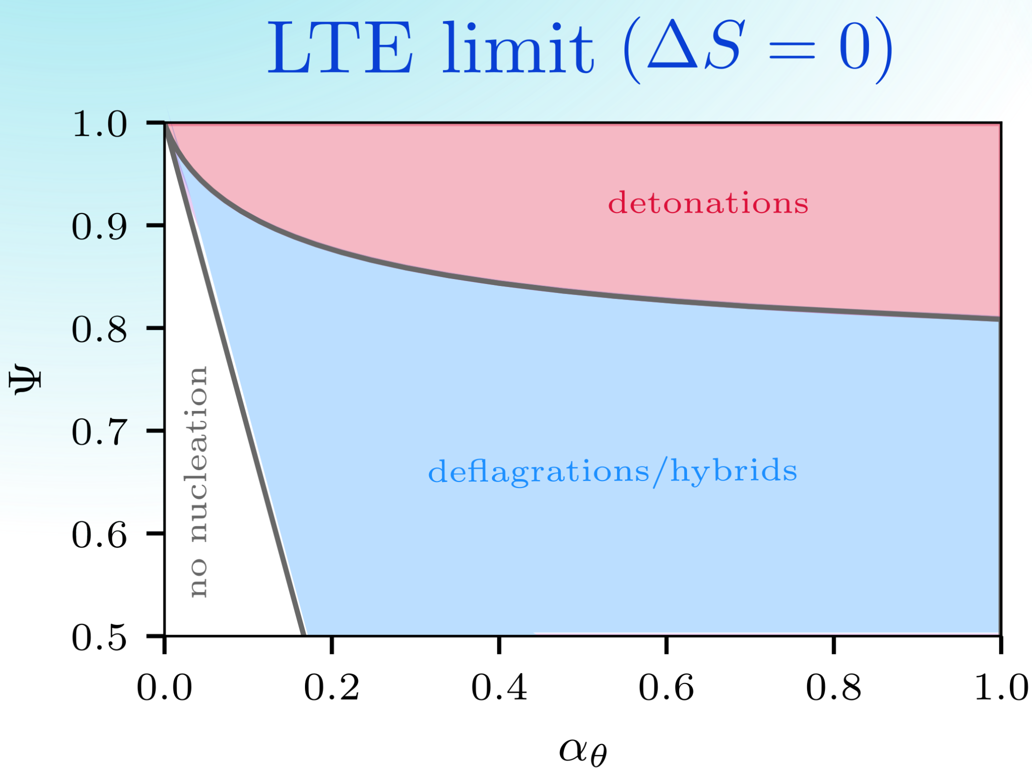
$$\Delta S \approx \tilde{\eta} \frac{\omega_+}{T_+} \gamma_+^2 v_+^2,$$

$$\tilde{\eta} = \frac{\eta\, v_0^2}{3\, \omega_+ L_w} \approx \frac{\eta\, v_0^2}{3\, \omega_s(T_n) L_w}.$$

The new matching codition is

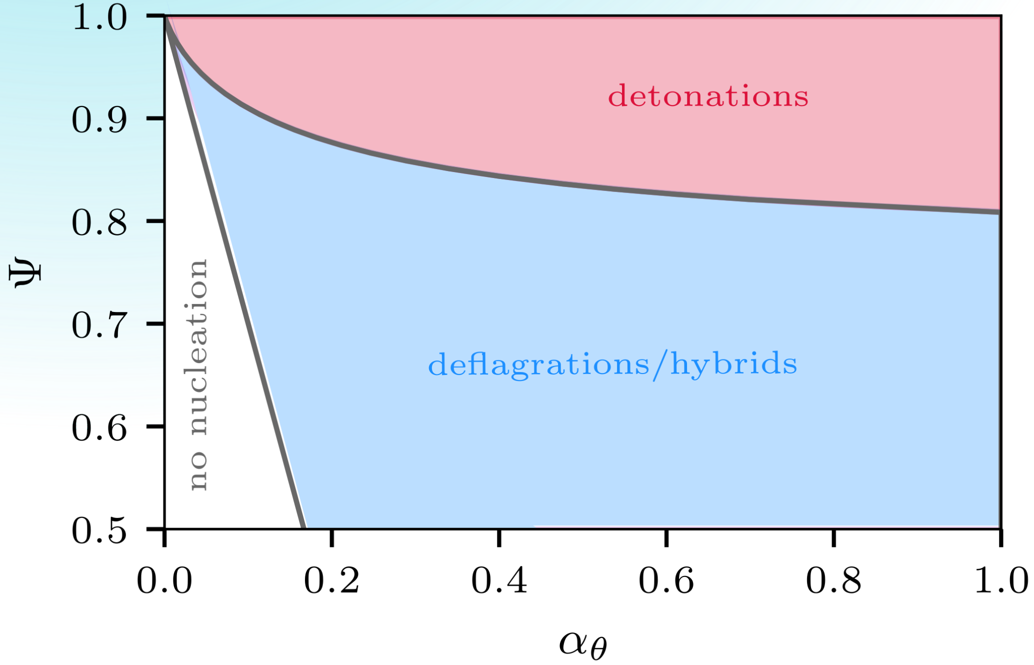
$$s_- \gamma_- v_- - s_+ \gamma_+ v_+ = \tilde{\eta} \frac{\omega_+}{T_+} \gamma_+^2 v_+^2.$$

Implications for LTE limit

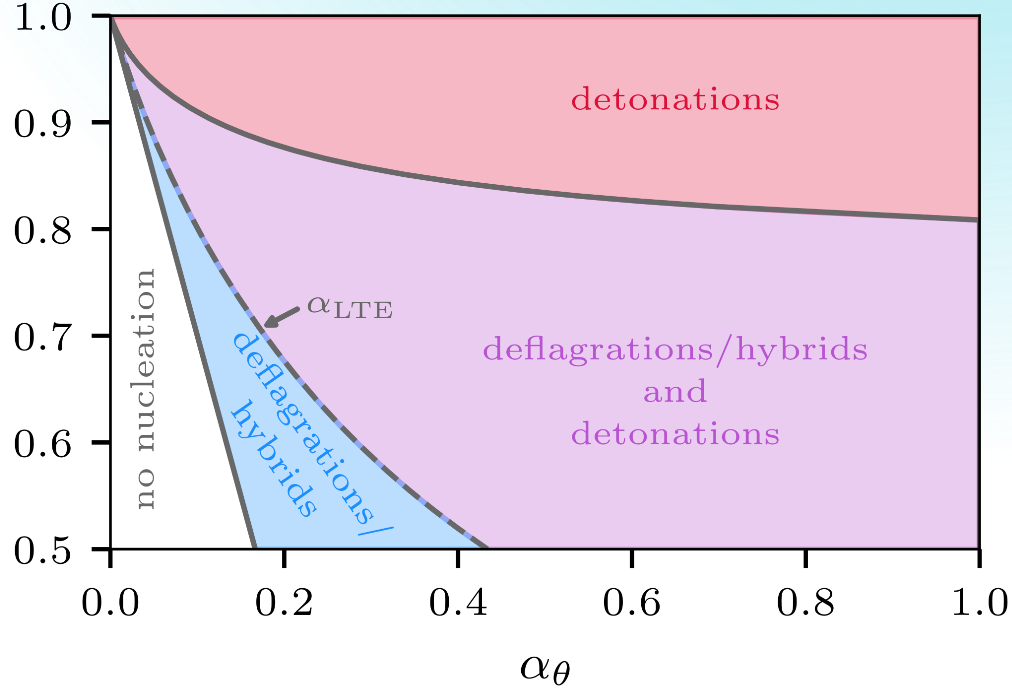


Implications for LTE

LTE limit $(\Delta S = 0)$



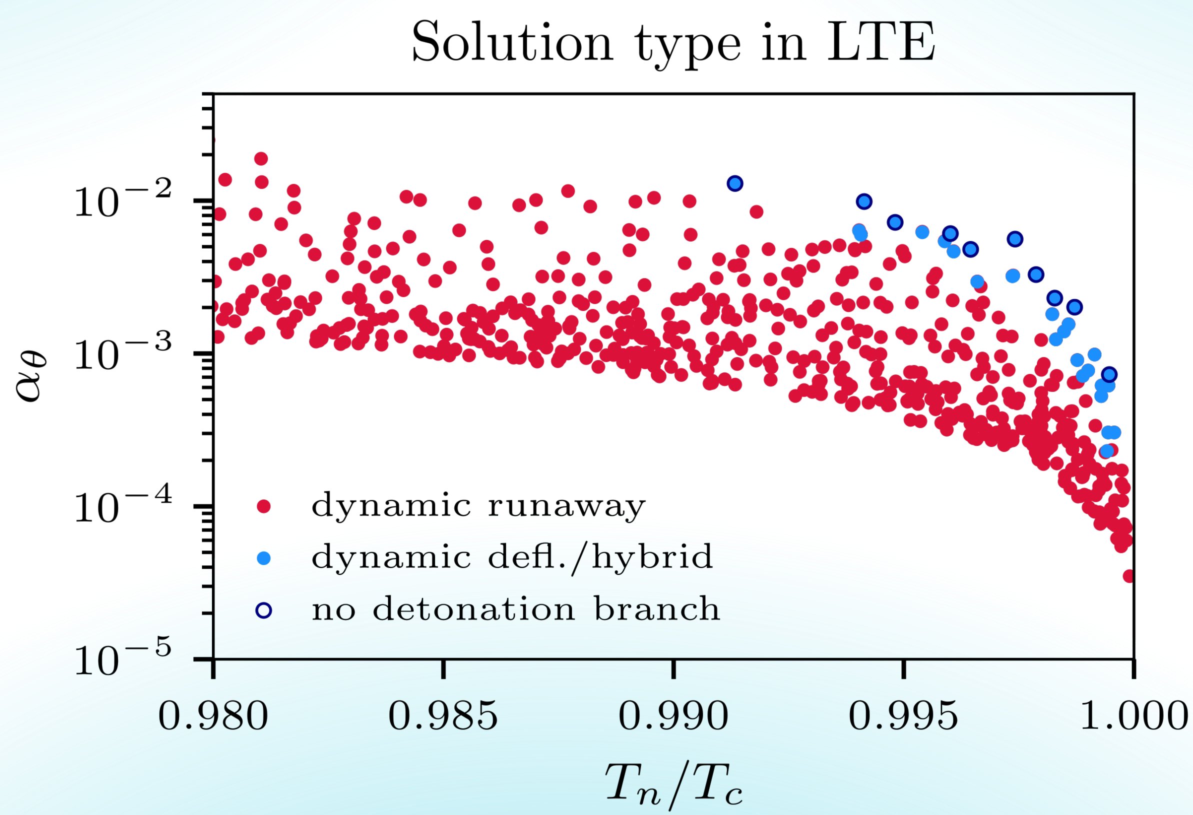
LTE limit $(\tilde{\eta} \to 0)$



LTE selection rule vs real-time simulations

$$\alpha_{\rm LTE} < \alpha_\theta \quad \longleftarrow \quad (\alpha_\theta, \Psi_n) \quad \longrightarrow \quad \alpha_\theta < \alpha_{\rm LTE}$$

Runaway detonation — Deflagration/hybrid



Summary

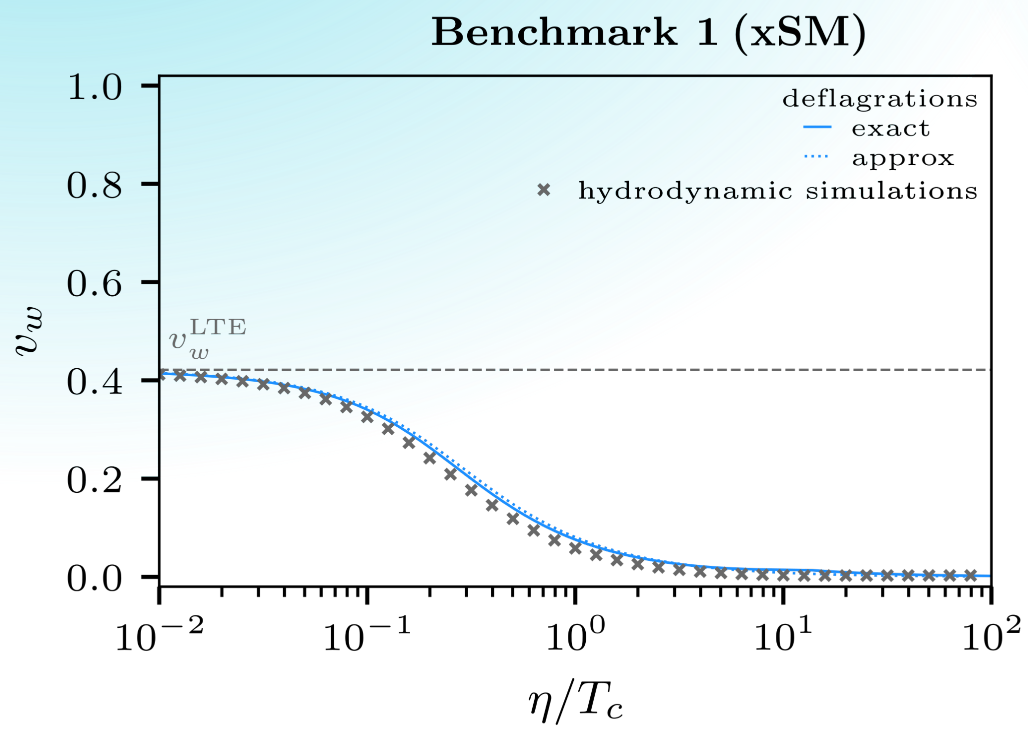
• Quantitative study of first-order phase transitions requires the computation of ξ_w.

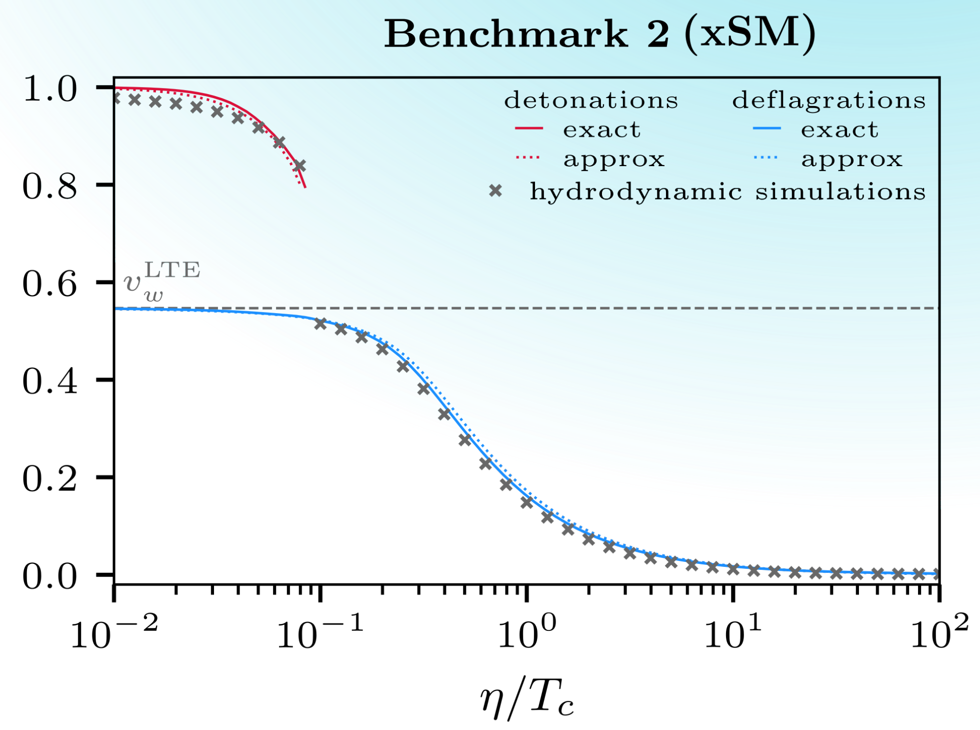
• The stationary wall velocity is determined by the entropy balance across the wall.

• In some cases, multiple stationary states can coexist, and one must perform real-time simulations to determine ξ_w.

• In the local thermal equilibrium limit, the system preferentially converges to the fastest available solution.
The slow deflagration/hybrid shock waves form if no stable detonation branch exists.

Cross-check with the real-time lattice simulations

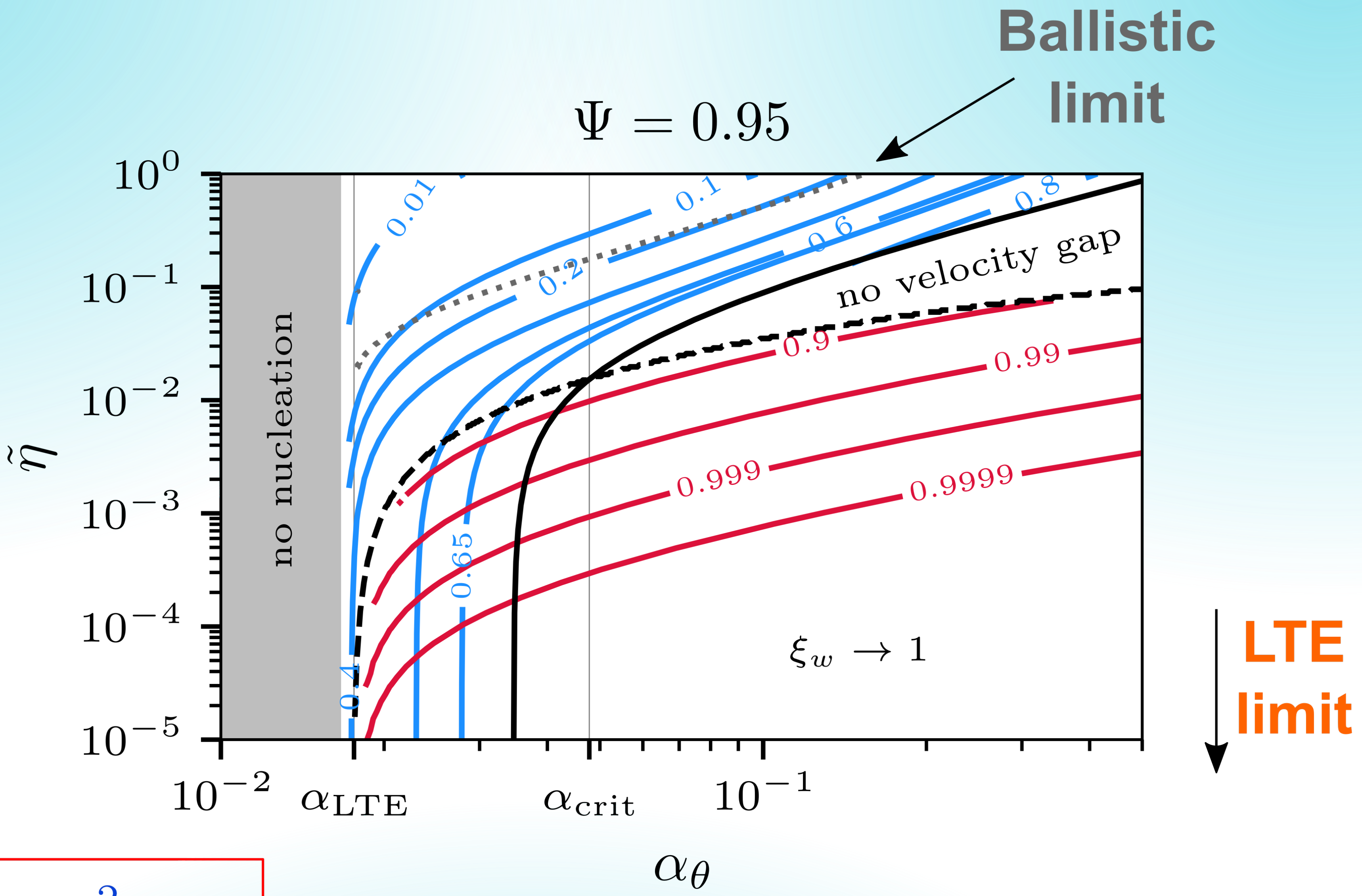




Only deflagration/hybrid branch in LTE

Deflagration/hybrid and detonation branch coexist in LTE

Wall velocity beyond LTE



$$\tilde{\eta} \approx \frac{\eta\, v_0^2}{3\,\omega_s(T_n) L_w}.$$

Effective dissipation model

Dissipative friction is often modelled with ansatz

$$\mathcal{F} = \eta\, u^\mu \partial_\mu \phi,$$

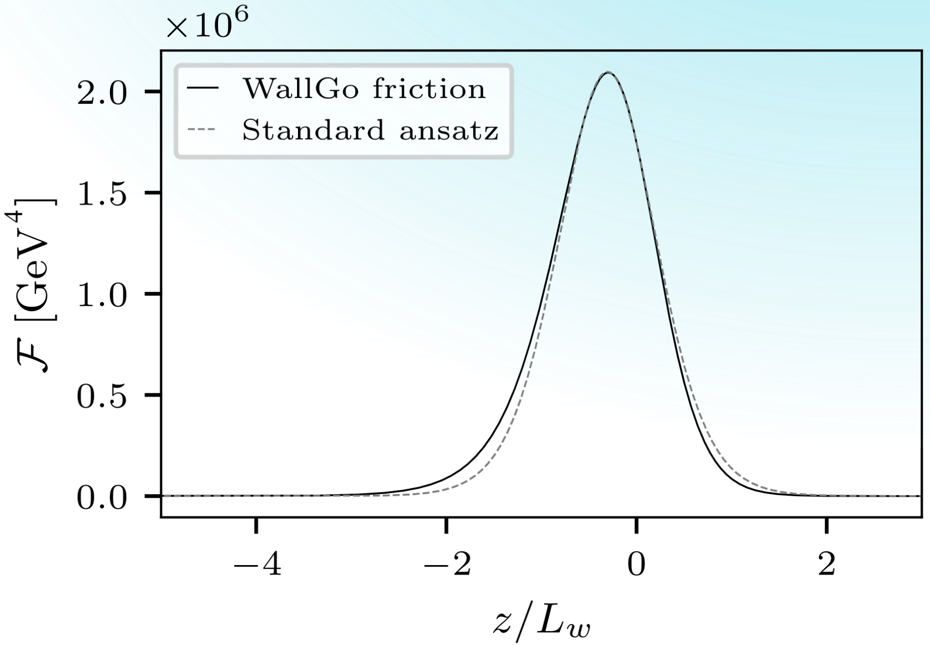
where η is a constant. Assuming that

$$\partial_\mu T^{\mu\nu}_{\text{fl.}} = -\partial_\mu T^{\mu\nu}_{\phi},$$

one obtains the entropy rate

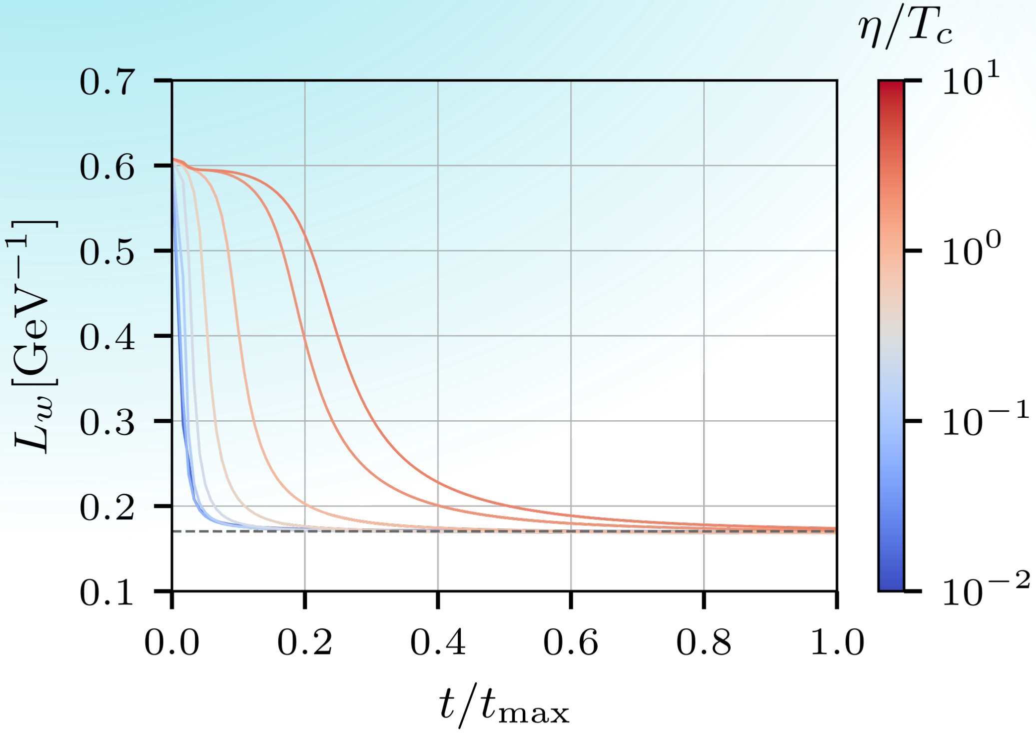
$$\partial_\mu (s u^\mu) = \frac{\eta}{T}\,(u^\mu \partial_\mu \phi)^2.$$

Cross-check with `WallGo`[3]

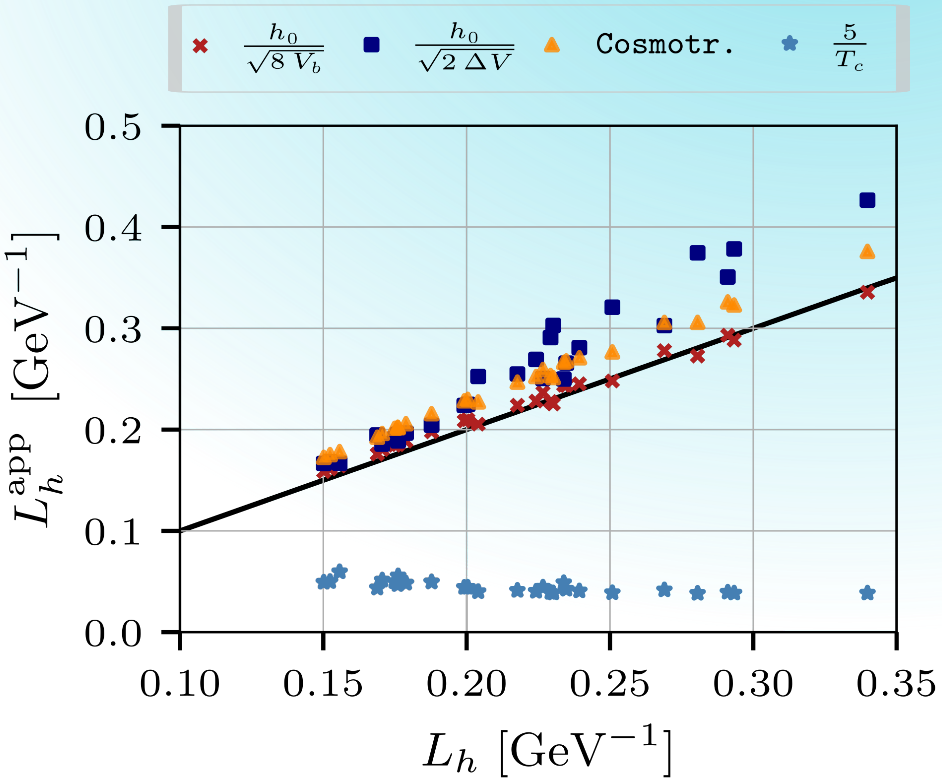


[3] A. Ekstedt et al., *How fast does the WallGo? A package for computing velocities in first-order phase transitions*, arXive: 2411.04970.

Estimation of the Wall Width



Evolution of the wall width L_w in the real-time hydrodynamic simulations



Different analytical estimations of L_w in LTE limit

Scalar field dynamics

The EOM of the scalar field in the bubble wall reads

$$\Box\phi + \frac{\partial V(\phi)}{\partial\phi} + \sum_i \frac{dm_i^2}{d\phi}\int\frac{d^3\vec{p}}{(2\pi)^3 2E_i}(f_{\mathrm{eq.}} + \delta f_i) = 0$$

$$\Box\phi + \frac{\partial V_{\mathrm{eff}}(\phi, T)}{\partial\phi} + \sum_i \frac{dm_i^2}{d\phi}\int\frac{d^3\vec{p}}{(2\pi)^3 2E_i}\delta f_i = 0$$

$$\downarrow \int_{\mathrm{Wall}} dz \frac{d\phi}{dz}$$

Backreaction | Dissipative friction

$$\int_{\mathrm{Wall}} dz \frac{\partial V_{\mathrm{eff}}}{\partial T}\frac{dT}{dz} + \sum_i \int d\phi \frac{dm_i^2}{d\phi}\int\frac{d^3\vec{p}}{(2\pi)^3 2E_i}\delta f_i = \Delta V_{\mathrm{eff}}$$

Equilibrium interactions

The equations of state relate plasma to scalar field

$$p = -V_{\rm eff}(\phi, T),$$

which also implies

$$\omega = -T\frac{dV_{\rm eff}(\phi, T)}{dT},$$

$$e = V_{\rm eff}(\phi, T) - T\frac{dV_{\rm eff}(\phi, T)}{dT}.$$

Stress-energy tensors read

$$T^{\mu\nu}_{\rm fl.} = \omega u^\mu u^\nu + p g^{\mu\nu},$$

$$T^{\mu\nu}_{\phi} = \partial^\mu\phi\partial^\nu\phi - \frac{1}{2}g^{\mu\nu}\partial^\alpha\phi\partial_\alpha\phi.$$

The energy-momentum conservation

$$\partial_\mu T^{\mu\nu}_\phi = -\partial_\mu T^{\mu\nu}_{\rm fl.} = \frac{\partial V_T}{\partial\phi}\partial^\nu\phi,$$

$$V_T(\phi, T) = V_{\rm eff}(\phi, T) - V(\phi).$$

Common temperature couples background field and plasama!

Out-of-equilibrium interactions

The δf^a can be computed by solving Boltzmann equations

$$\mathcal{L}_{\text{Luv.}}[f^a_{\text{eq.}} + \delta f^a] = -\mathcal{C}[\delta f^b]$$

where $\mathcal{C}$ is a collision term and $\mathcal{L}_{\text{Luv.}} = p^\mu \partial_\mu + \frac{1}{2}(\partial_\mu m_a^2)\partial_{p^\mu}$.

Exact solutions:

- Local thermal equilibrium $(\mathcal{C} \to \infty,\ \delta f^a = 0)$
- Ballistic limit $(C \to 0,\ \delta f^a \gg f^a_{\text{eq.}})$

Approximate methods:
Fluid ansatz, relaxation time approxmation, ect.

Nucleation

New phase bubbles nucleate at a rate

$$\Gamma = A(T)\,\exp(-\tfrac{S_3}{T}),$$

where

$$A(T) = T^4 \left(\tfrac{S_3}{2\pi T}\right)^{\frac{3}{2}},$$

while S_3 is an $\mathbb{O}(3)$-symmetric solution to equation of motion (EOM).

Nucleation criterium:

$$\Gamma(T_n)/H^4 \approx 1.$$