

# Nano-Hertz gravitational waves and sub-GeV dark matter from a classically conformal phase transition

Jonas Matuszak

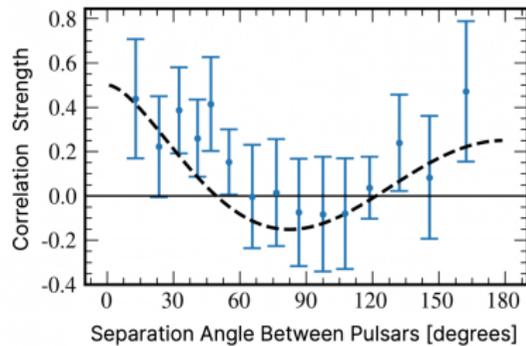
Based on [arxiv:2502.19478](https://arxiv.org/abs/2502.19478)

with Sowmiya Balan, Torsten Bringmann, Felix Kahlhöfer, Carlo Tasillo

24. September 2025

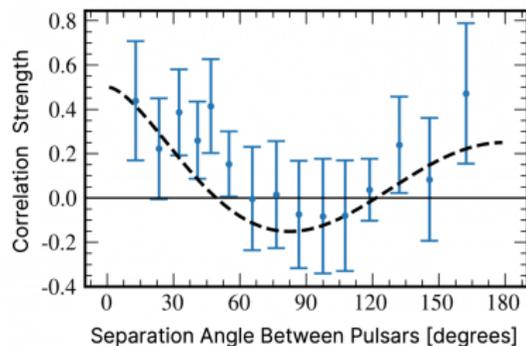
# Pulsar Timing Arrays and the GW Background

NANOGrav 15 Year Data Release

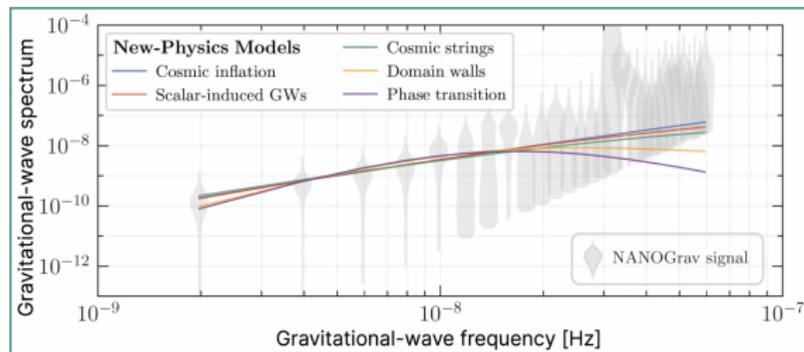


# Pulsar Timing Arrays and the GW Background

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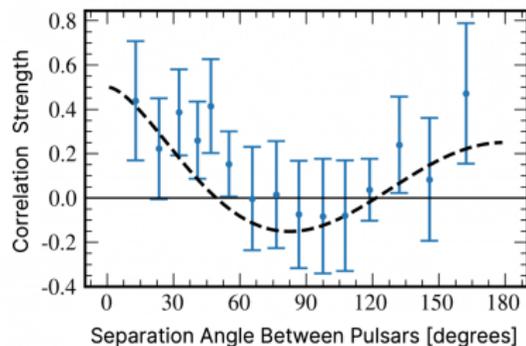
- Pulsar timing arrays found evidence of a stochastic GW background
- Astrophysical source: inspiralling black hole binaries
- Data allows for BSM sources of signal



Figures: [arxiv:2306.16213](https://arxiv.org/abs/2306.16213), [arxiv:2306.16219](https://arxiv.org/abs/2306.16219)

# Pulsar Timing Arrays and the GW Background

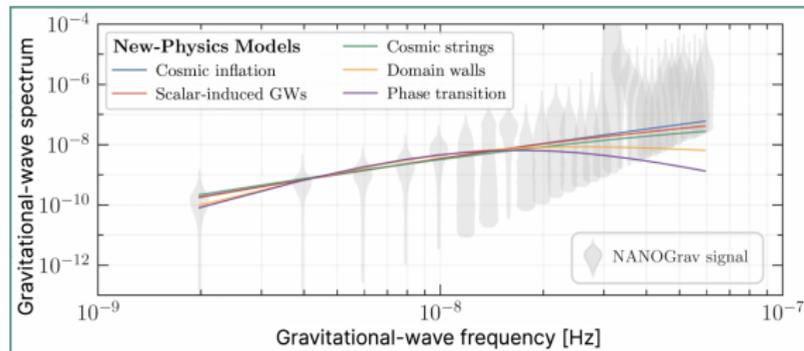
NANOGrav 15 year data release



## Question:

Can a new scalar field give us a GW signal *and* dark matter from a first-order phase transition?

- Pulsar timing arrays found evidence of a stochastic GW background
- Astrophysical source: inspiralling black hole binaries
- Data allows for BSM sources of signal



Figures: [arxiv:2306.16213](https://arxiv.org/abs/2306.16213), [arxiv:2306.16219](https://arxiv.org/abs/2306.16219)

# Dark Sector Phase Transition Recipe

$U(1)'$  dark sector



## Ingredients:

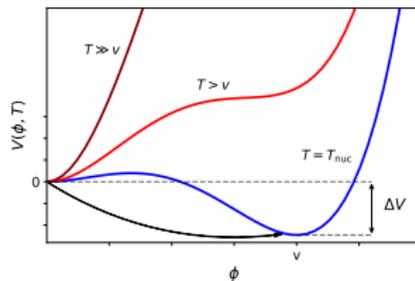
- Dark Higgs:  $\Phi$
- Dark photon:  $A'_\mu$
- Dark matter (fermion):  $\chi$
- Scalar potential:  $V(\Phi, T)$

## Mix to get the Lagrangian

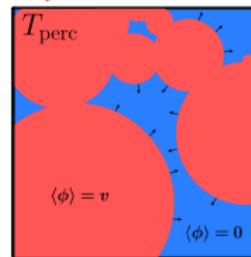
$$\mathcal{L}_{\text{DS}} = |D_\mu \Phi|^2 - \frac{1}{4} A_{\mu\nu}^{\prime 2} + i\chi_L \not{D} \chi_L + i\chi_R \not{D} \chi_R - y\Phi \chi_L^\dagger \chi_R + \text{h.c.} - V(\Phi)$$

■ Cook at high  $T \gg v_\phi$

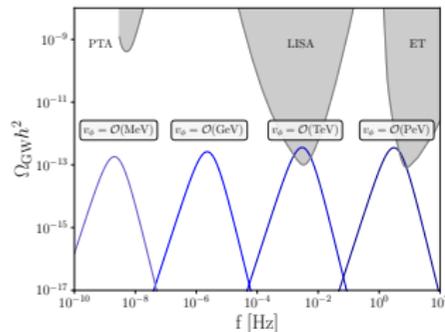
■ Let cool until symmetry breaks  $\langle \phi \rangle \neq 0$   
 $\Rightarrow$  Mass generation + stochastic GW signal



Symmetry breaking



Bubble collision & Sound Waves

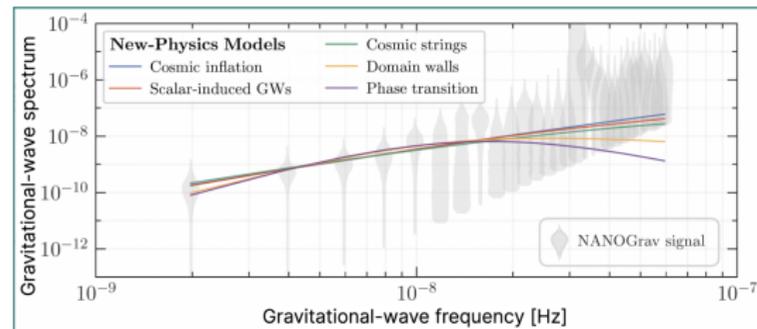


# Fitting the PTA Gravitational Wave Signal

## Nano Hertz Frequencies

### PTA signal:

- Frequency at nHz
- Large amplitude:  $h^2\Omega_{\text{PTA}} \sim 10^{-8}$



# Fitting the PTA Gravitational Wave Signal

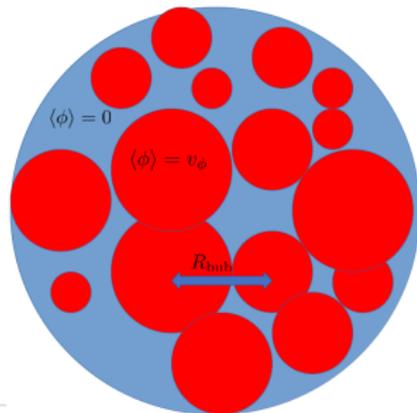
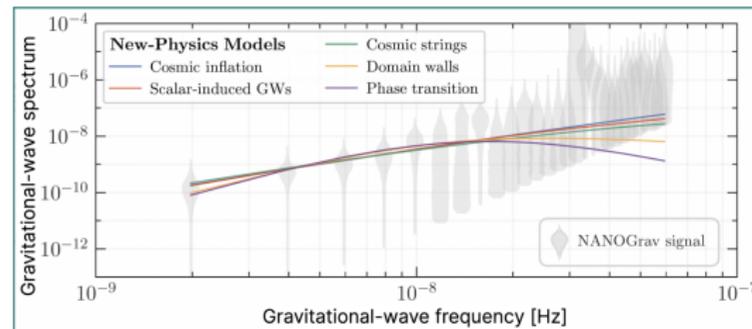
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### GW frequencies from a FOPT:

- Set by the characteristic length scale:  $R_*$



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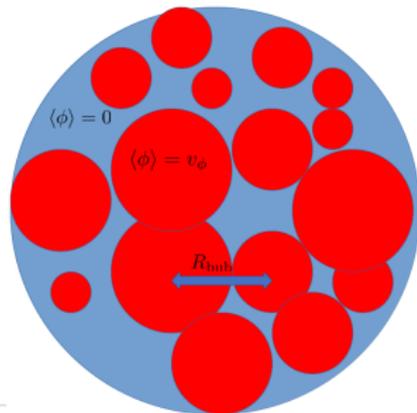
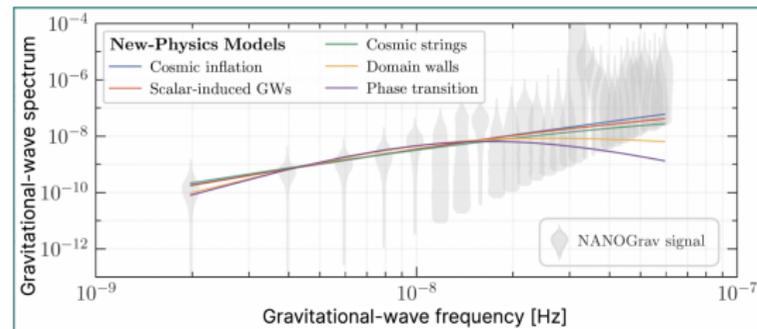
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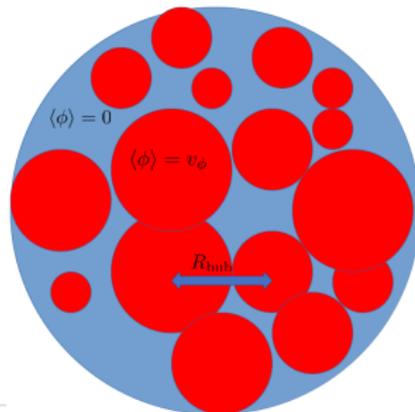
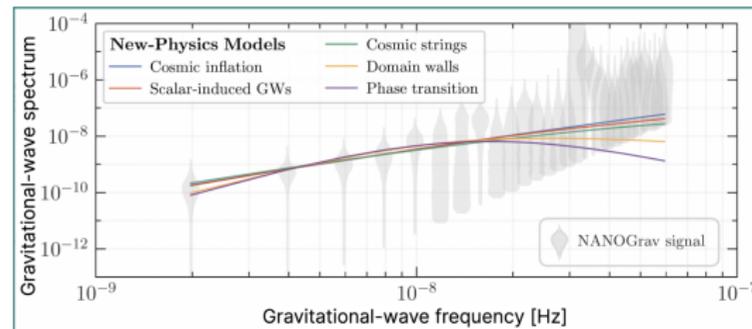
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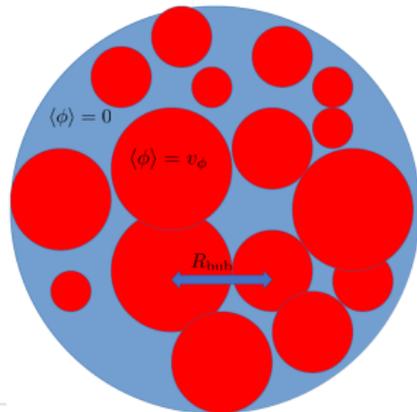
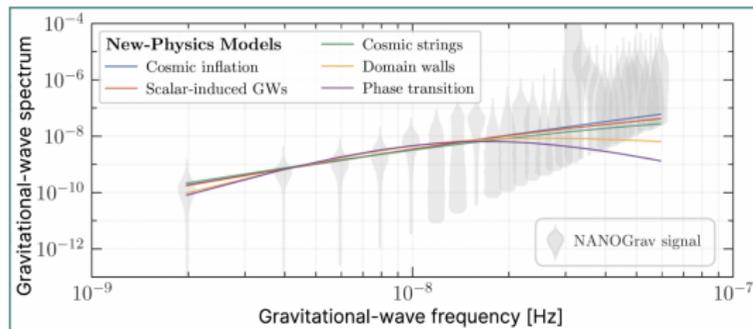
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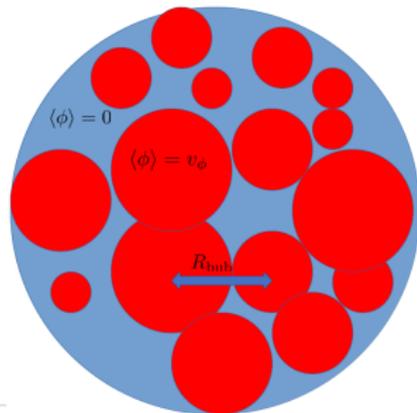
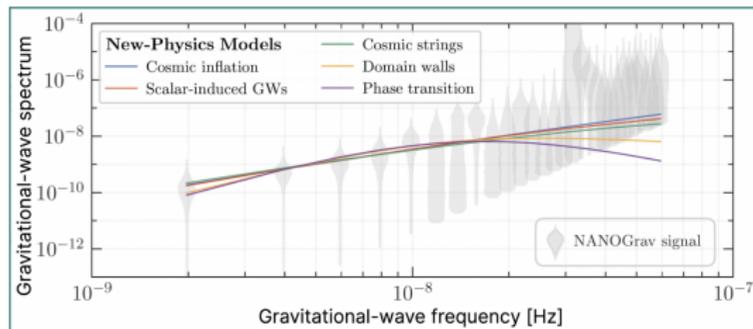
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### Sub-GeV dark matter

PT around  $\mathcal{O}(\text{MeV})$  to fit the PTA data requires a DS mass scale  $\mathcal{O}(\text{MeV})$



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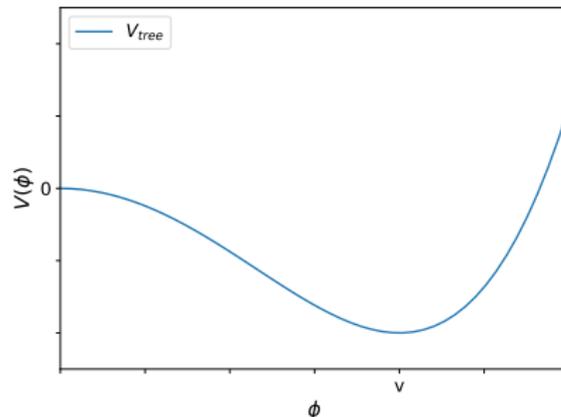
## Amplitude

### GW amplitude scaling

$$h^2 \Omega_{\text{GW}} \propto \underbrace{\left( \frac{\Delta V}{\Delta V + \rho_{\text{rad}}} \right)^2}_{\equiv \text{kin. energy fraction}} \underbrace{(R_* H)^2}_{\equiv \text{mean bubble sep.}}$$

### Potential:

$$V(\phi) = -\mu^2 |\phi|^2 + \lambda |\phi|^4$$



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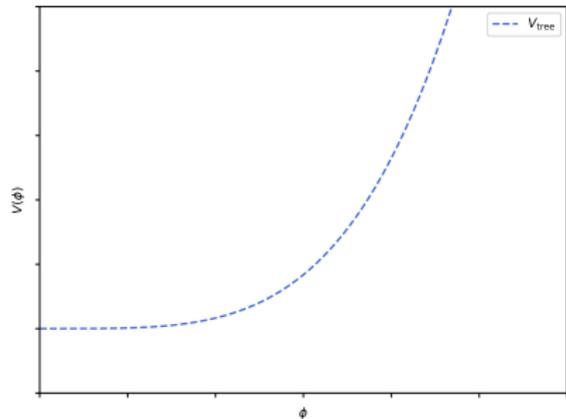
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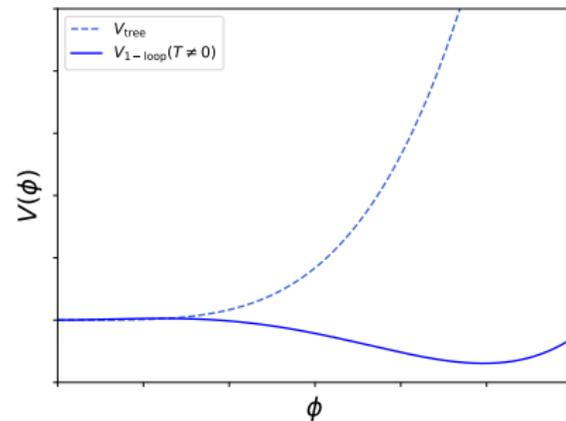
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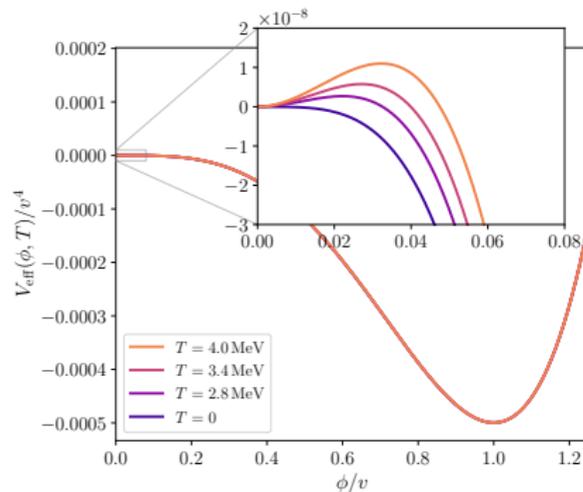
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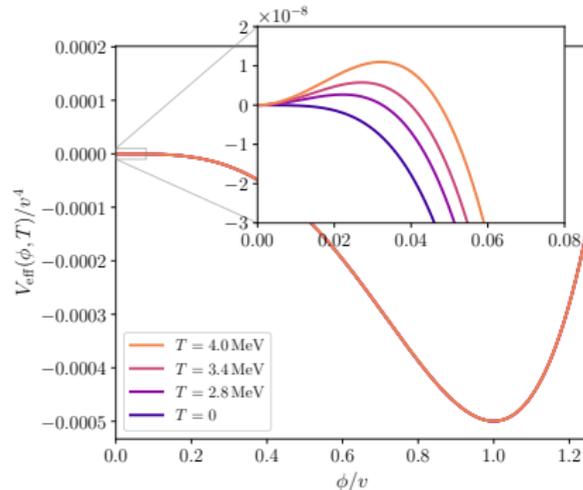
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## Classically conformal symmetry

Classical conformal symmetry gives strong supercooling and large GW signal.

# Relic Abundance

## Freeze-out for Sub-GeV DM

### BBN & CMB

How to get rid of DS the energy density after a PT around  $\mathcal{O}(10)$  MeV and produce  $h^2\Omega_{\text{DM}} \simeq 0.12$ ?

### What portal to the SM can we have?

- Couplings allowed by symmetry

$$\mathcal{L}_{\text{int}} = \lambda_{h\phi} |H|^2 |\Phi|^2 + \frac{\kappa}{\cos 2\theta_w} F'_{\mu\nu} B^{\mu\nu}$$

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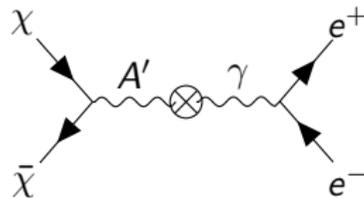
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### Effective coupling

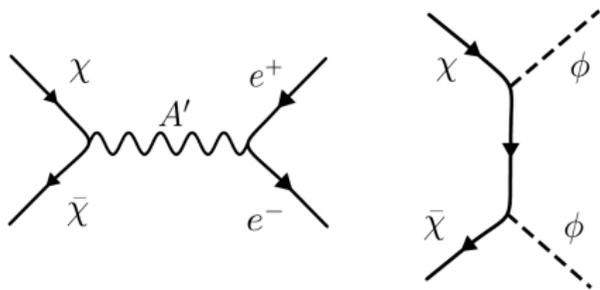


- Collider constraints  $\kappa \lesssim 10^{-3}$
- First freeze-out of DS
- Then freeze-out of DM within DS

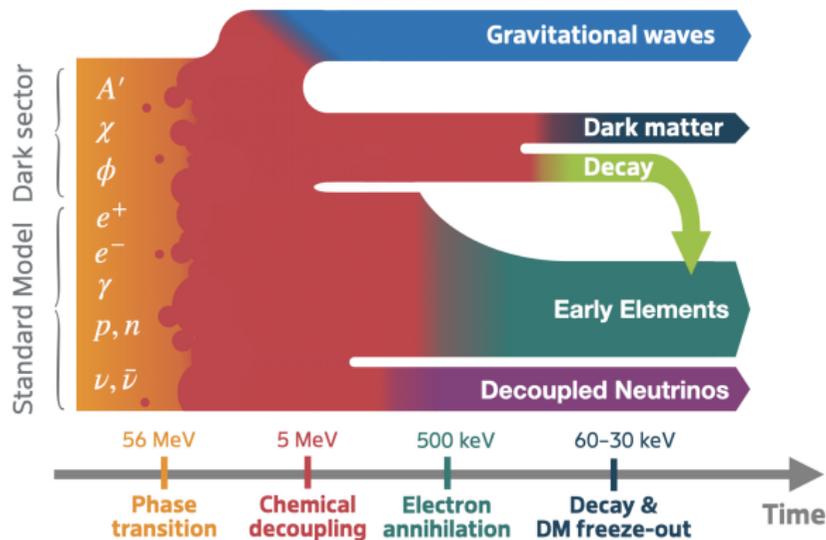
# Thermal History

## Thermal history of a benchmark point

$$\mu_\chi = \mu_\phi = 0$$



- After phase transition

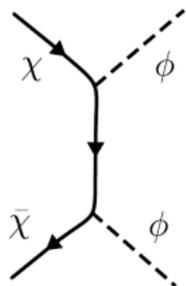
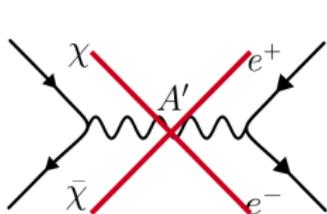


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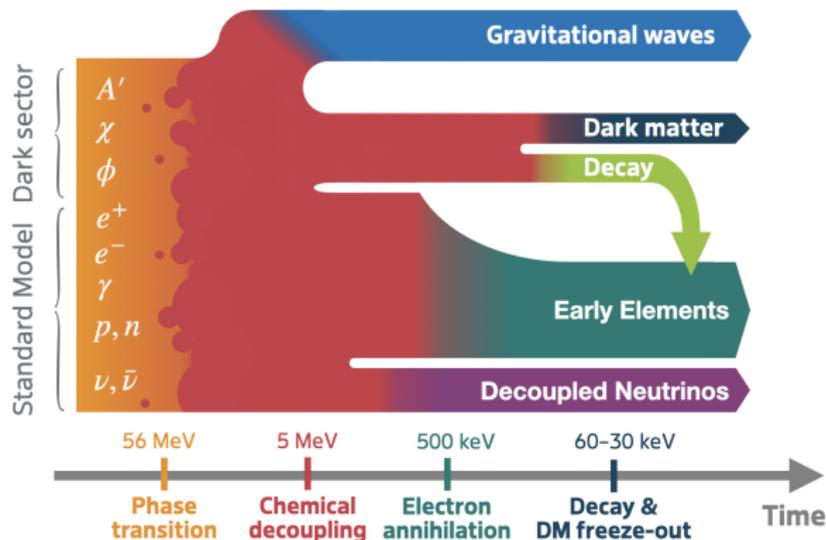
## Thermal history of a benchmark point

$$\mu_\chi = \mu_\phi \neq 0$$

$$2Y_\chi + Y_\phi = \text{const}$$

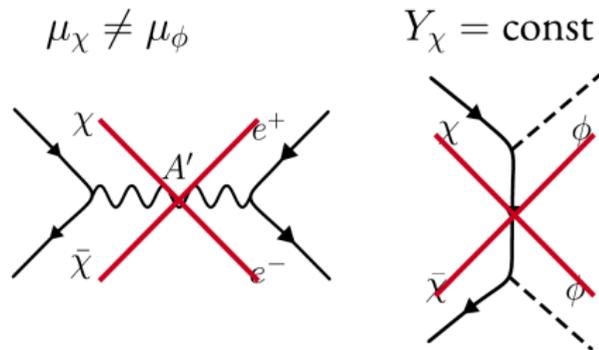


- DS decoupling

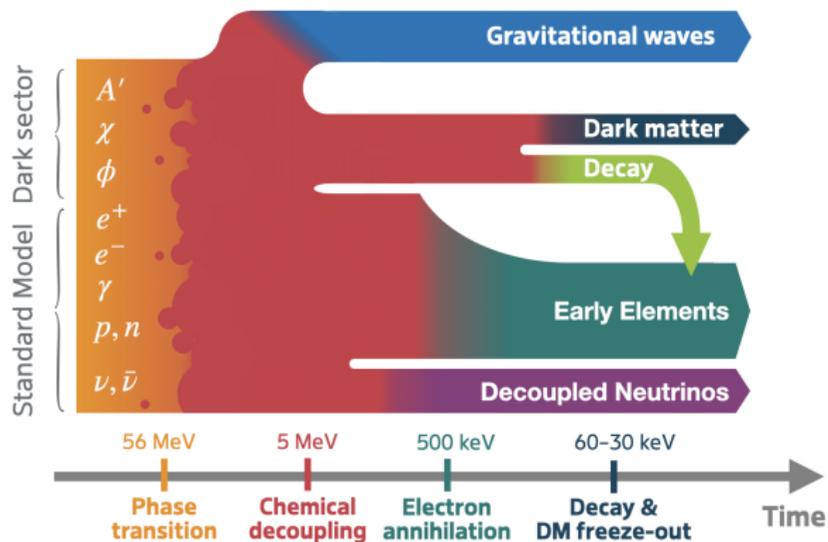


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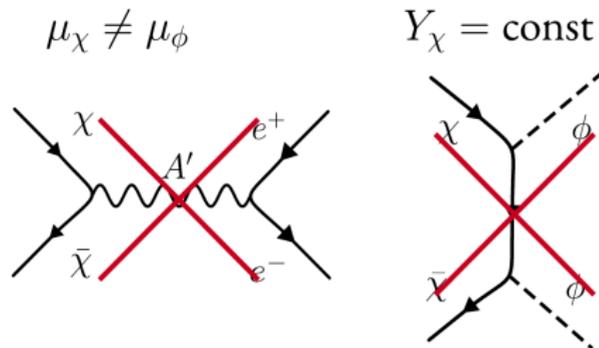


- DM freeze-out
- Decay of  $\phi$  during/after BBN



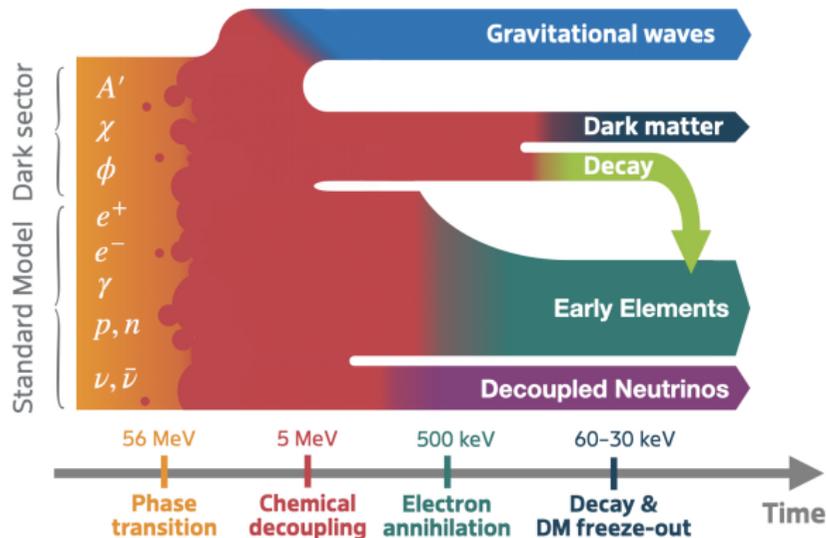
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## Thermal history of a benchmark point



## Cosmological constraints

Dark Higgs  $\phi$  decays around BBN. Can we avoid the cosmological and collider constraints?



# Cosmological, Astrophysical and Collider Constraints

Is this model consistent with data?

## Cosmological constraints:

- NANOGrav 15 year dataset: likelihood for SMBHB + FOPT GW signal  
PTARCADE (CEFFYL), TRANSITIONLISTENER (COSMOTRANSITIONS) [arxiv:2109.06208](https://arxiv.org/abs/2109.06208)  
[arxiv:2306.16377](https://arxiv.org/abs/2306.16377)
- Relic density calculation:  
DARKSUSY [arxiv:1802.03399](https://arxiv.org/abs/1802.03399)
- BBN: Light element abundances  $^4\text{He}$  and D  
ALTERBBN [arxiv:1806.11095](https://arxiv.org/abs/1806.11095)
- CMB constraints:
  - If semi-relativistic around neutrino decoupling increase  $N_{\text{eff}}$
  - DM annihilation into  $e^+e^-$  raise  $T_\gamma$  and decrease  $N_{\text{eff}}$

Likelihood from Planck collaboration:  $N_{\text{eff}} = 2.99 \pm 0.17$  (BAO+lensing+TT,TE,EE+lowE)

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## Astrophysical constraints:

- Bullet cluster constraints:  $\sigma_0/m_{\text{DM}} < 1.25 \text{ cm}^2\text{g}^{-1}$
- X-Ray:  $\chi\bar{\chi} \rightarrow A'$  p-wave suppressed  $\Rightarrow$  no constraints

## Accelerator constraints:

- Beam dump experiments: MiniBooNE, LSND  
Missing energy searches through  $\pi^0, \eta \rightarrow \gamma + A', A' \rightarrow \chi\bar{\chi}$
- NA64: dark photon production through  $e^- Z \rightarrow e^- Z A'$  and  $A' \rightarrow \chi\bar{\chi}$
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## Global fit with GAMBIT

- Parameters:  $m_{\text{DM}}, m_{A'}, \text{vev}, \kappa (\gamma_{\text{SMBHB}}, A_{\text{SMBHB}})$
- Combined likelihood:

$$\mathcal{L}_{\text{tot}} = \mathcal{L}_{\text{PTA}} \times \mathcal{L}_{\Omega_{\text{DM}} h^2} \times \mathcal{L}_{\text{BBN,CMB}} \times \mathcal{L}_{\text{BC}} \times \mathcal{L}_{\text{Coll}}$$



[arxiv:1705.07908](https://arxiv.org/abs/1705.07908)

# Calculation of the GW Spectrum

## TransitionListener v2.0

### Work on a new code based on

COSMOTRANSITION [arxiv:1109.4189](https://arxiv.org/abs/1109.4189)

- Using best practices for computation of GW signal
  - Tracking of false vacuum fraction  $P(T)$
  - Computation of  $T_{\text{perc}}, T_{\text{reh}}$
  - Computation of  $v_{\text{wall}}$  (LTE)
  - Calculation of  $R_* H$  from bubble density
  - Latest GW templates [[arxiv:2403.03723](https://arxiv.org/abs/2403.03723)]
- Interface to PTARCADE for the PTA likelihoods
- Can handle strong supercooling

### Phase transition temperature

- Typically:

$$\frac{S_3(T_{\text{nuc}})}{T_{\text{nuc}}} \approx 140$$

- Here track true vacuum fraction:

$$P(t) = \exp\left(-\frac{4\pi}{3} \int_{t_c}^t dt' \Gamma(t') v_w^3 (t - t')^3\right)$$

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### Bubble wall velocity

- Typically:

$$v_w = \text{const.}$$

- Here compute  $v_w$  in the LTE approximation

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### Transition speed/characteristic length scale

- Typically:

$$\frac{\beta}{H} \approx T \frac{d}{dT} \left( \frac{S_3}{T} \right)$$

- Here compute

$$\frac{\beta}{H} = (8\pi)^{1/3} \frac{\max(c_s, v_w)}{P(T_*)^{1/3} R_* H}, \quad R_* = n_{\text{bub}.*}^{-1/3}$$

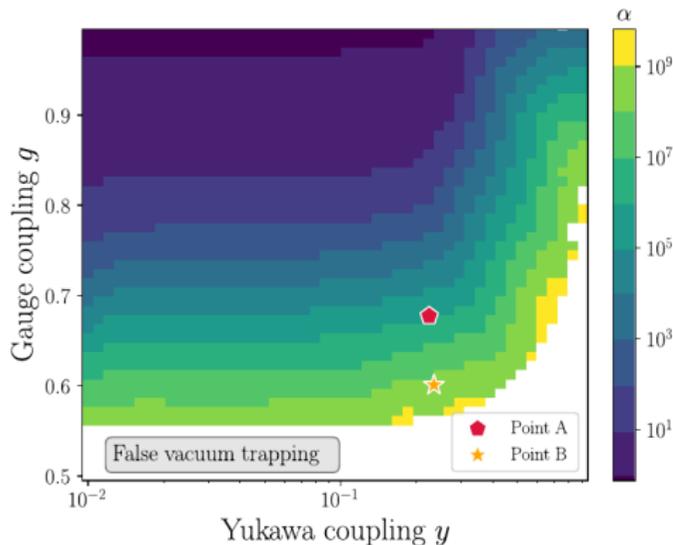
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  - Computation of  $T_{\text{perc}}, T_{\text{reh}}$
  - Computation of  $v_{\text{wall}}$  (LTE)
  - Calculation of  $R_*H$  from bubble density
  - Latest GW templates [[arxiv:2403.03723](https://arxiv.org/abs/2403.03723)]
- Interface to PTARCADE for the PTA likelihoods
- Can handle strong supercooling



# Calculation of the GW Spectrum

## TransitionListener v2.0

### Work on a new code based on

COSMOTRANSITION [arxiv:1109.4189](https://arxiv.org/abs/1109.4189)

- Using best practices for computation of GW signal
  - Tracking of false vacuum fraction  $P(T)$
  - Computation of  $T_{\text{perc}}, T_{\text{reh}}$
  - Computation of  $v_{\text{wall}}$  (LTE)
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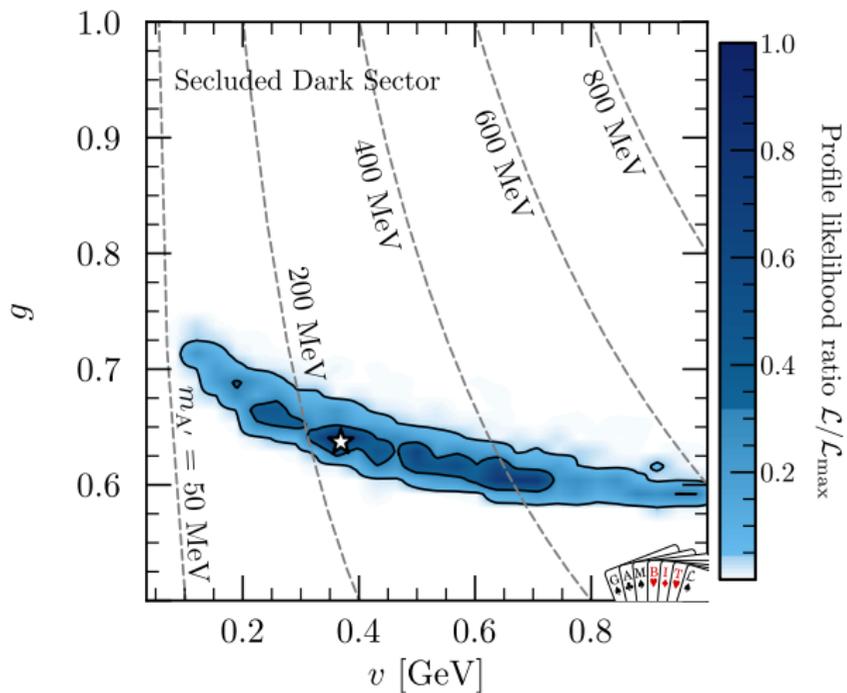
Upcoming code release [Ongoing work with Carlo Tasillo]

Implemented in a new version of TRANSITIONLISTENER (v2), which is based on COSMOTRANSITION. Soon\* to be released.

\*Fall 2025

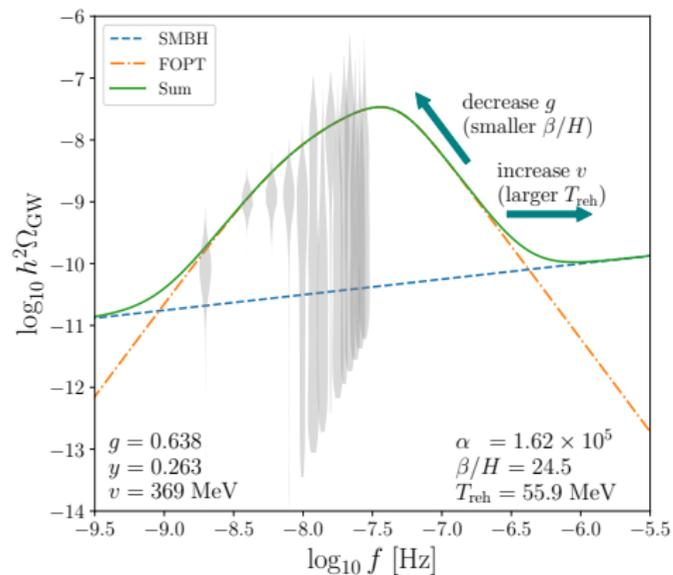
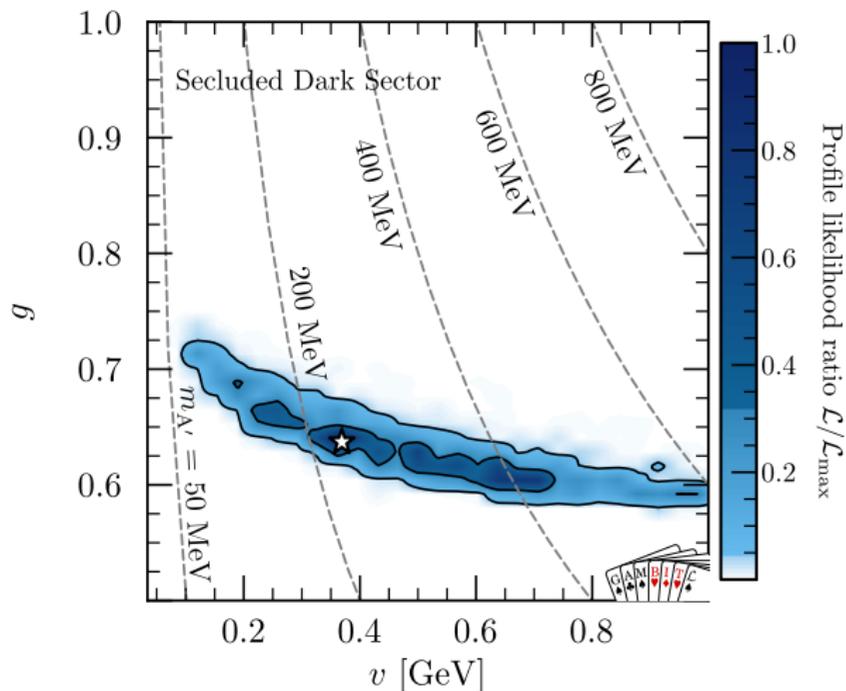
# Global Scans

## Gravitational Wave Signal



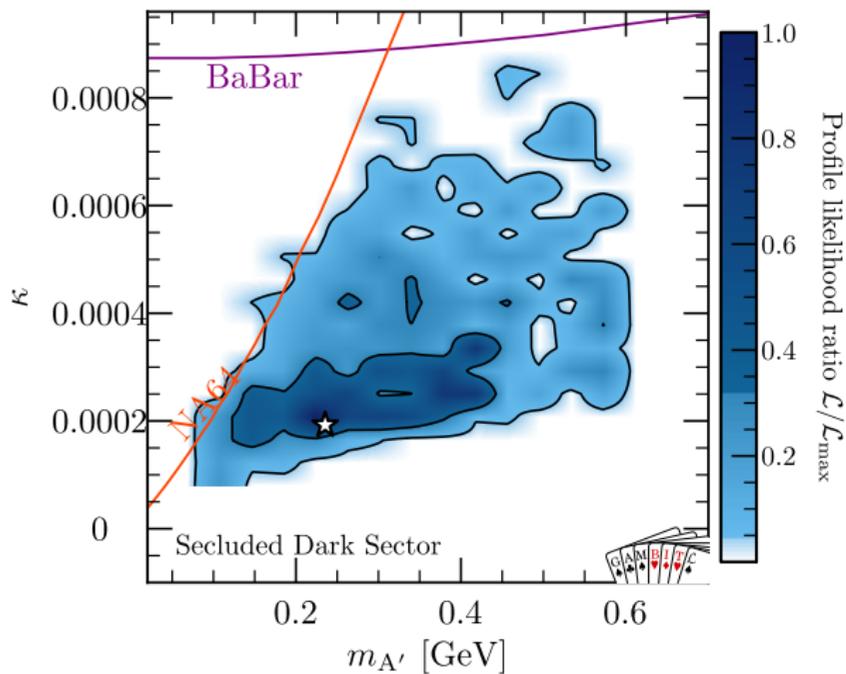
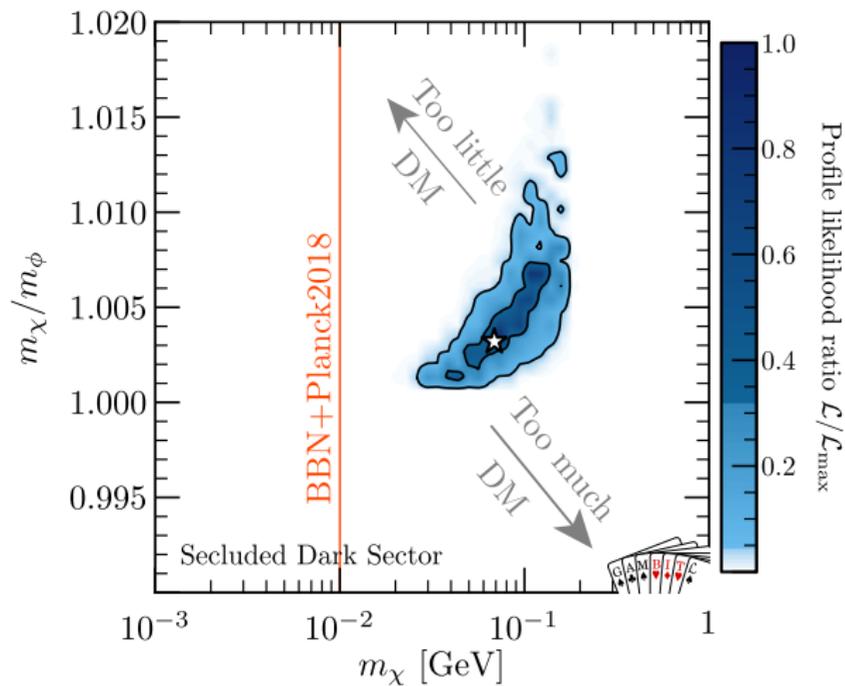
# Global Scans

## Gravitational Wave Signal



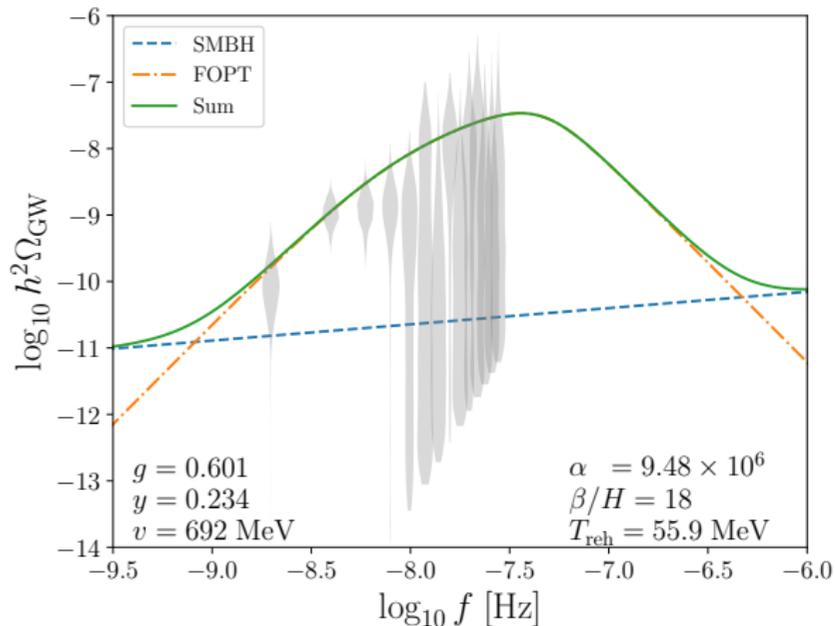
# Global Scans

## Relic Density and Collider Constraints



# Conclusion

- Dark sector phase transition can fit NANOGrav data
- At the same time produce Sub-GeV dark matter, evading current constraints
- Model testable with future experiments (LDMX)
- Upcoming code release



# Backup Slides

# Coupled Dark Sector

## Standard freeze-out

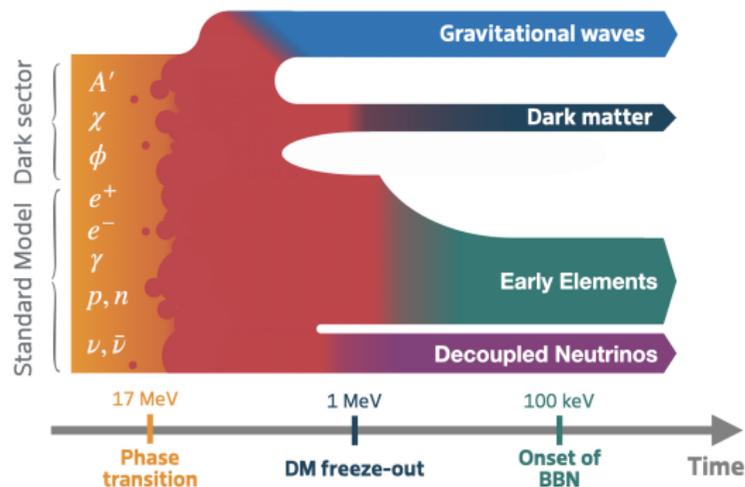
- Assume chemical equilibrium between SM and DS during freeze-out
- Generated through e.g. heavy vector like leptons, charged under SM and DS gauge groups
- Generates effective interaction

$$\mathcal{L}_{\text{int}} \supset \frac{1}{\Lambda^2} \phi^\dagger \phi \bar{l}_L e_R H + \text{h.c.}$$

- After electroweak symmetry breaking

$$\mathcal{L}_{\text{eff. int}} = \frac{v_h v_\phi}{2\Lambda^2} \phi \bar{e} e$$

- Standard freeze out of DM  $\chi$  from SM-DS bath



# GW spectrum

Template from [arxiv:2403.03723](https://arxiv.org/abs/2403.03723)

$$h^2 \Omega_{\text{gw}}^{\text{PT}}(f) = \mathcal{R} h^2 A_{\text{sw}} K^2 \mathcal{Y}_{\text{sw}}(RH_*) \tilde{S}(f),$$

Spectral shape:  $S(f) = N \left(\frac{f}{f_1}\right)^3 \left[1 + \left(\frac{f}{f_1}\right)^2\right]^{-1} \left[1 + \left(\frac{f}{f_2}\right)^4\right]^{-1}$ , with the normalisation

$$\tilde{S}(f) = \frac{1}{\pi} \left( \sqrt{2} + \frac{2 f_2 / f_1}{1 + f_2^2 / f_1^2} \right) \frac{S(f)}{S(f_2)}.$$

Frequencies breaks  $f_1 \simeq 0.2 \left(\frac{H_{*,0}}{RH_*}\right)$ ,  $f_2 \simeq 0.5 \Delta_w^{-1} \left(\frac{H_{*,0}}{RH_*}\right)$ .

Sound shell thickness  $\Delta_w = |v_w - c_s| / \max(v_w, c_s)$ . Frequency redshift factor is given by

$$H_{*,0} = 16.5 \text{ nHz} \left(\frac{T_{\text{reh}}}{100 \text{ MeV}}\right) \left(\frac{g_{\text{reh}}}{100}\right)^{1/2} \left(\frac{100}{h_{\text{reh}}}\right)^{1/3}.$$

Redshift of amplitude:  $\mathcal{R} h^2 = \left(\frac{a_{\text{reh}}}{a_0}\right)^4 \left(\frac{H_{\text{reh}}}{H_0}\right)^2 h^2 = \Omega_\gamma h^2 \left(\frac{h_0}{h_{\text{reh}}}\right)^{4/3} \frac{g_{\text{reh}}}{g_\gamma}$ ,

$$\mathcal{Y} = \min[1, \tau_{\text{sh}} H] \simeq \min\left[1, \frac{3.38}{\beta/H} \sqrt{\frac{1+\alpha}{\kappa_{\text{SW}} \alpha}}\right]$$

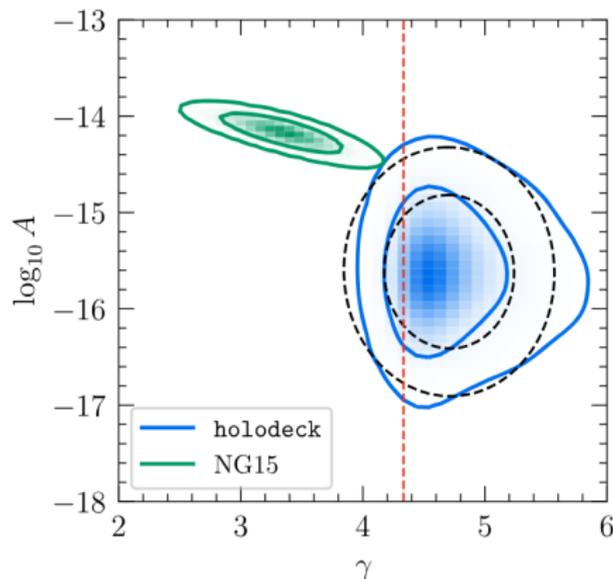
# SMBHB Signal

- Spectral shape:

$$\Phi_{\text{SMBHB}} = \frac{A_{\text{SMBHB}}^2}{12\pi} \frac{1}{T_{\text{obs}}} \left( \frac{f}{\text{yr}^{-1}} \right)^{-\gamma_{\text{SMBHB}}} \text{yr}^3$$

- Amplitude

$$\Omega_{\text{GW}} h^2 = 8\pi^4 f^5 \left( \frac{\text{Mpc}}{100} \right)^2 \frac{\Phi_{\text{SMBHB}}}{\Delta f}$$



# Dimensional Transmutation

Renormalised 1-loop effective potential:

$$V_{1\text{-loop}} = \frac{\lambda}{4} \phi^4 + \sum_a \frac{\eta_a g_a}{64\pi^2} m_a^4(\phi) \left[ \ln \frac{m_a^2(\phi)}{m_a^2(\Lambda)} - \frac{50}{12} \right].$$

Define  $\lambda$  at the vev of the theory  $\Lambda = \langle \phi \rangle = v_\phi$

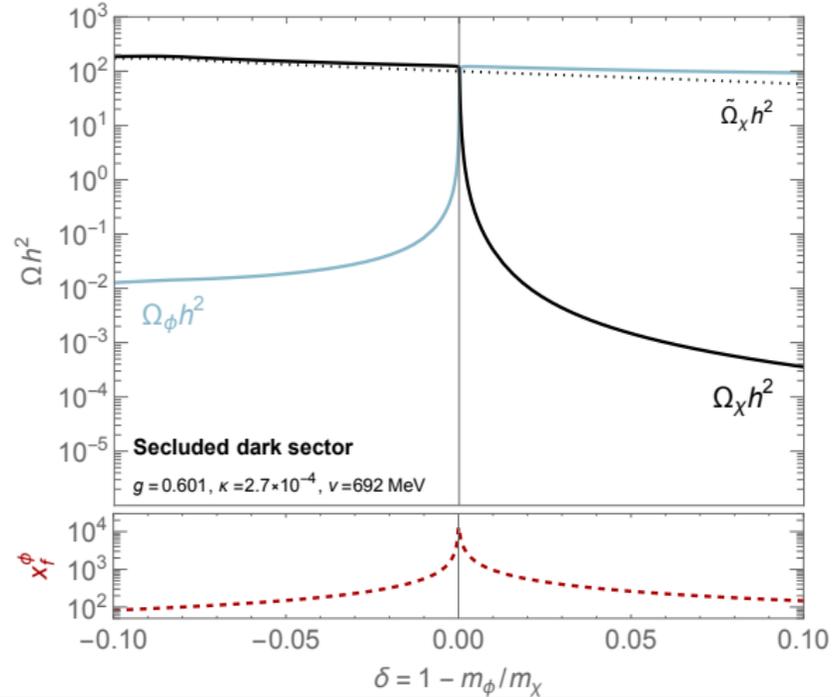
$$\left. \frac{\partial V}{\partial \phi} \right|_{\langle \phi \rangle} = \left( \lambda - \frac{32}{3} \sum_a \frac{\eta_a g_a}{64\pi^2} \lambda_a^4 \right) \langle \phi \rangle^3 = 0$$

For  $\lambda \ll g, y$ :

$$\lambda \approx \frac{32}{3} \sum_a \frac{\eta_a g_a}{64\pi^2} \lambda_a^4$$

$$\Rightarrow m_\phi = m_\phi(g, y) = \sqrt{3\lambda(g, y)} v_\phi$$

# Dark Matter Relic Density



# Dark Higgs Decays

