Bubble wall dynamics & the EW phase transition

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In collaboration with

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- arXiv:2504.21213
- to appear

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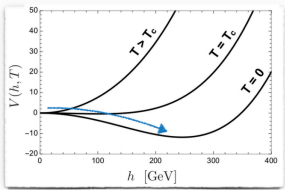
EW Phase transition in (B)SM

SM. EWPT is a crossover

Kajantie, Laine, Rummukainen, Shaposhnikov

Despite perturbation theory: Unreliable

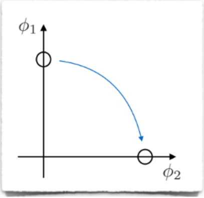


- h rolls down to non-trivial minimum for $T < T_c$
- No distinctive signature

BSM. Enlarged scalar sector:

Minima in various directions, barriers at tree-level → 1st-Order PT

- Nucleation of bubbles @ T_n & expansion



The dynamics can produce

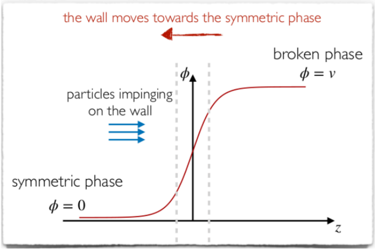
- Significant breaking of thermal eq. (baryogenesis)
- Collisions & turbulences in the plasma (GWs)

Experimental signatures crucially depend on bubble dynamics

Dynamics of the bubble wall

System: scalar fields ϕ + plasma

ϕ representative for the fields driving the PhT



- Expansion of the bubble wall in the false vacuum drives the plasma out of equilibrium
- Plasma back-reacts: interactions with PhT front produce a friction
- Balance between outward pressure and friction: steady-state regime with terminal velocity v_w

In the following we assume a planar wall and a steady-state regime

Set-up

For each particle species: $f(p,z) = \underbrace{f_v(p,z)}_{\text{LTE}} + \underbrace{\delta f(p,z)}_{\text{OOE}}$

Plasma described as mixture of three components

Assumption: $L_w \gg T^{-1}, q^{-1}$

1. Scalar fields driving the transition
2. Background of weakly coupled species: $\sim$ LTE
3. Strongly coupled species: OOE relevant

1. Scalar field EOMs

Moore, Prokopec

$$\Box\phi(z) - \partial_\phi V(\phi, T) = \sum_i N_i\, \partial_\phi m_i^2 \int \frac{d^3p}{(2\pi)^3 2E_p} \delta f_i$$

2. EM conservation for background $\rightarrow$ space-dependent profiles $T(z)$, $v_p(z)$

$$f_v = \frac{1}{e^{\beta(z)\gamma(z)(E - v_p(z)p_z)} \pm 1}$$

3. Boltzmann equation for out-of-equilibrium δf

$\mathcal{C}$ collision integral

$$\mathcal{L} \equiv \left(\frac{p_z}{E}\partial_z - \frac{(m^2)'}{2E}\partial_{p_z}\right)(f_v + \delta f) = -\mathcal{C}[f_v + \delta f]$$

Boltzmann equation and flow paths

Linearised eq $\mathcal{L}[\delta f] + \overline{\mathcal{C}}[\delta f] = -\mathcal{L}[f_v]$ Higher order subdominant

Integro-differential equation ... hard to solve

Solutions using moment expansion since '90s ... But several shortcomings ...

Moore, Prokopec Laurent, Cline / Dorsch, Huber, Konstandin / De Curtis, Delle Rose, Gil Muyor, Guiggiani, Panico

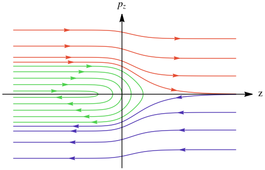
Recently: iterative (numerical) approach to get full solution

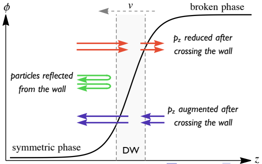
De Curtis, Delle Rose, Gil Muyor, Guiggiani, Panico

Along flow paths $p_\perp$ & $p_z^2 + m^2(z)$ are conserved

$$\mathcal{L} = \left(\frac{p_z}{E}\partial_z - \frac{(m^2(z))'}{2E}\partial_{p_z} \right) \quad \rightarrow \quad \frac{p_z}{E}\frac{d}{dz}$$

Trajectories in $(p_\perp, p_z, z)$ phase space in collisionless limit $C \to 0$





Collision integral & the iterative approach

Collision integral for $2 \to 2$ processes

$$\overline{\mathcal{C}}[\delta f] = \sum_i \frac{1}{4N_p E_p} \int \frac{d^3\mathbf{k}\, d^3\mathbf{p}'\, d^3\mathbf{k}'}{(2\pi)^5 2E_k 2E_{p'} 2E_{k'}} |\mathcal{M}_i|^2 \delta^4(p+k-p'-k') \overline{\mathcal{P}}[\delta f]$$

$$\overline{\mathcal{P}}[\delta f] = f_v(p) f_v(k) \left(1 \pm f_v(p')\right)\left(1 \pm f_v(k')\right) \sum_{q \in (p,\, k,\, p',\, k')} \frac{\mp \delta f(q)}{(f_v(q))'}$$

Collision integral splits in two pieces $\overline{\mathcal{C}}[\delta f] = c_1 \delta f(p) + \langle \delta f \rangle$

- $c_1 \delta f(p)$: p momentum **not** integrated
- $\langle \delta f \rangle$: Terms with $\delta f(q)$, q integrated $\qquad q = k,\, p',\, k'$

$\mathcal{L}[\delta f] + \overline{\mathcal{C}}[\delta f] = -\mathcal{L}[f_v]$ can be put in the form $\left(\frac{d}{dz} - \frac{\mathcal{Q}}{p_z}\right) \delta f = \mathcal{S}$

$\frac{\mathcal{Q}}{p_z} \leftrightarrow -c_1$ $\qquad$ $\mathcal{S} \leftrightarrow -(\langle \delta f \rangle + \mathcal{L}[f_v])$: corrections by iteration

$$\delta f = \left[B(p_\perp, p_z^2 + m^2(z)) + \int_{\bar{z}}^{z} dz' e^{-\mathcal{W}(z')} \mathcal{S} \right] \qquad \mathcal{W}(z) = \int^z dz' \frac{\mathcal{Q}}{p_z}$$

At step n: calculate $\mathcal{W}$, δf, then $\mathcal{S}$ to use in step $n+1 \to$ until convergence

Hydrodynamics of the plasma

Equations for the plasma from EMT conservation

$$T^{\mu\nu} = T^{\mu\nu}_{\phi} + T^{\mu\nu}_{pl} + T^{\mu\nu}_{out}, \qquad T^{\mu\nu}_{out} = \int \frac{d^3p}{(2\pi)^3} p^\mu p^\nu \delta f$$

$$T^{30} = w\gamma^2 v_p + T^{30}_{out} = c_1$$

$$T^{33} = \frac{(\partial_z\phi)^2}{2} - V(\phi, T) + w\gamma^2 v_p^2 + T^{33}_{out} = c_2$$

$w = T\partial_T V$ enthalpy, v_p plasma velocity, $\gamma = \left(\sqrt{1 - v_p^2}\right)^{-1}$

Constants $c_{1/2}$: boundaries for v_p and T $\qquad$ $+$ = in front of the wall, $-$ = behind the wall

Depend on combustion regime:
deflagration ($v_w < c_s$), hybrid ($c_s < v_w < v_J$), detonations ($v_w > v_J$)

Gyulassy, Kajantie, Kurki-Suonio, McLerran / Espinosa, Konstandin, No, Servant

In all cases, T_+ is given in terms of T_n

Scalar EOMs

2-step PhTs driven by two fields ($i = h, s$) $\quad (0,0) \to (0,\bar{s}) \to (\bar{h},0)$

$$E_i \equiv -\partial_z^2 \phi_i + \frac{\partial V(\phi_1, \ldots, \phi_n, T)}{\partial \phi_i} + F_i(z) = 0$$

with $F_i = \sum_j \frac{N_j}{2} \partial_i m_j^2 \int \frac{d^3p}{(2\pi)^3 E_p} \delta f_j$

Approximate solution: tanh ansatz

$$h(z) = \frac{h_0}{2}\left(1 + \tanh\left(\frac{z}{L_h}\right)\right) \qquad s(z) = \frac{s_0}{2}\left(1 - \tanh\left(\frac{z}{L_s} - \delta_s\right)\right)$$

$$h_0 : \partial_h V(h, 0, T_-) = 0 \qquad s_0 : \partial_s V(0, s, T_+) = 0$$

4 parameters to determine: $T_-\ (v_w),\ \delta_s,\ L_h,\ L_s$

Trade diff.eq. for four constraints eqs

$$P_i = \int dz\, E_i \phi_i' = 0 \qquad G_i = \int dz\, E_i \left(2\frac{\phi}{\phi_0} - 1\right) \phi_i' = 0$$

$$P_{tot} = P_h + P_s = \underbrace{\Delta V}_{\text{outward pressure}} \underbrace{- \int dz (\partial_T V) T' + \int dz\, F_{tot}(z)}_{\text{Friction: Hydrodynamic, Purely OOE}}$$

Model(s)

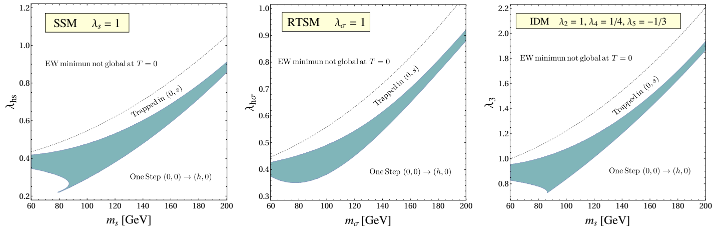
Step 0 of iterative procedure: Find LTE solution

* Sometimes, LTE truncation used as good approx ... we will see ...

- For LTE: we considered three models: singlet extension of SM (SSM), triplet extension of SM (RTSM), inert 2HDM (IDM), common tree-level potential

$$V_0(h,s) = \frac{\mu_h^2}{2}h^2 + \frac{\mu_s^2}{2}s^2 + \frac{\lambda_h}{4}h^4 + \frac{\lambda_s}{4}s^4 + \frac{\lambda_{hs}}{4}h^2 s^2$$

Preliminary: determine 2-step region

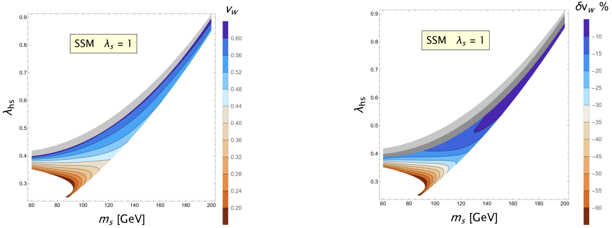


Results within LTE approx. are in arXiv:2504.21213

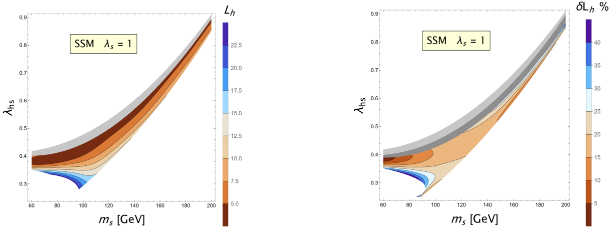
Out-of-equilibrium bubble dynamics

* (i) Only top OOE, (ii) leading-log terms , (iii) only 2 $\to$ 2 processes

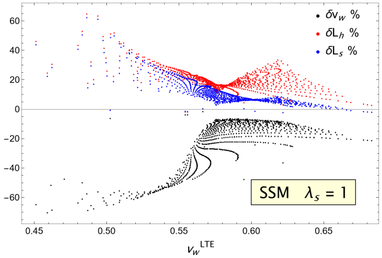
$$\delta x = \frac{x^{(\mathrm{OOE})} - x^{(\mathrm{LTE})}}{x^{(\mathrm{LTE})}}$$



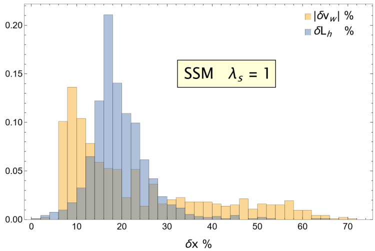
- OOE corrections can be quite large: $v_w \searrow$, $L_h \nearrow$
- Most of LTE ultra-relativistic detonations $\to$ deflagrations

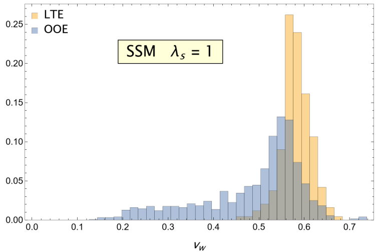


OOE vs LTE

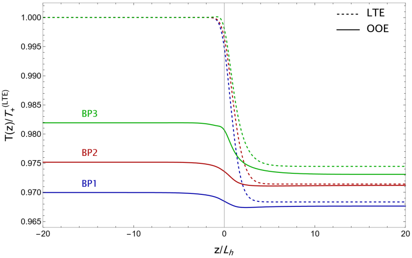


- v_w corrections larger for smaller v_w^{LTE}; persisten tail with $|\delta v_w| > 30\%$ correction
- δL_h more evenly distributed around 20%
- v_w distribution much larger with OOE than in LTE

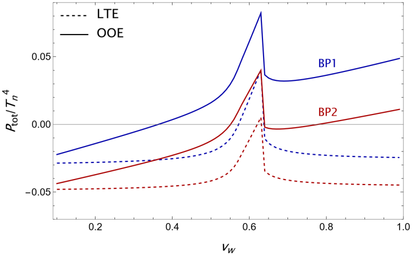




Out-of-equilibrium profiles and friction



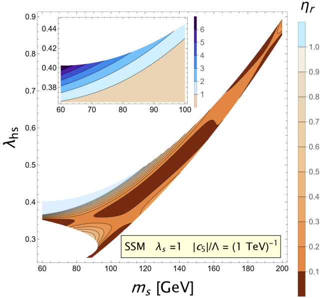
- Temperature gradient smaller when including OOE: T' **not** only source of friction anymore
- $P_{tot}(v_w)$ allows to better understand the impact of OOE

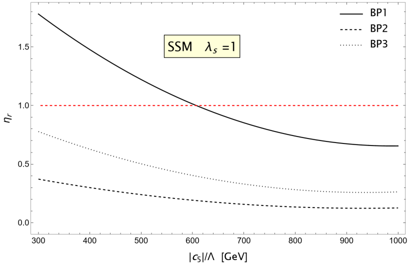


Application: Electroweak baryogenesis

Crucial ingredient : CP-violating interactions with the wall

- Simplest scenario: dim-5 operator $s\bar{Q}_L H t_R \to \frac{y_t}{\sqrt{2}} h(z) \bar{t}_L \left(1 + i c_5 \frac{s(z)}{\Lambda}\right) t_R +$ h.c.

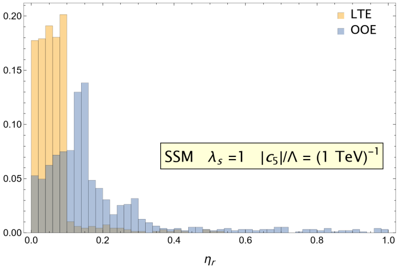


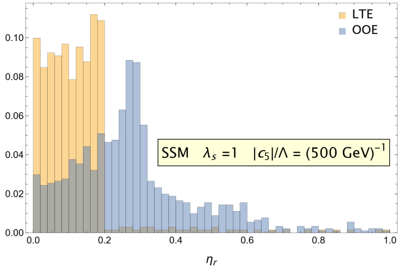


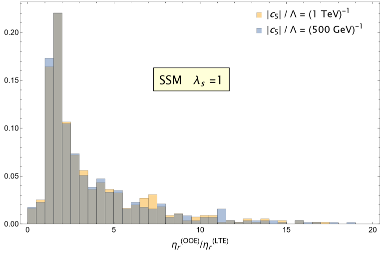
Important: our results take into account the dependence on **all** the parameters: static (T_n) & dynamical $(v_w,\ L_h,\ \delta_s,\ L_s)$

* We (i) verified dim-5 operator has negligible impact on dynamics & (ii) follow [Cline, Kainulainen '20] to solve for CP-odd perturbations

EWBG with out-of-equilibrium contributions







OOE contributions:
(i) decrease v_w, (ii) increase L_h, (iii) modify profiles ...
→ and altogether facilitate BAU!

OOE + decrease of c_5/Λ can lead to correct amount of BAU

m_s (GeV)	λ_{hs}	$\mathrm{BAU_{LTE}}$	$\mathrm{BAU_{full}}$
65	0.36	0.23 - 0.47	0.15 - 0.31
78	0.38	0.42 - 0.85	0.82 - 1.63
106	0.4	0.01 - 0.02	0.11 - 0.21
178	0.73	0.07 - 0.14	0.14 - 0.28

Summary

- Propose new method to directly solve the Boltzmann equation
- Two pillars: flow paths & spectral decomposition
- We solve the steady-state dynamics both in LTE and with full OOE contributions and perform surveys of models' parameter space
- We find that OOE contributions can have a large impact on v_w, the widths, the profiles
- As an application, we show OOE have a large impact in the prediction of the BAU