3HDM based on CP symmetry of order 4: the story so far

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Based on: Zhao, Ivanov, Pasechnik, Zhang, JHEP 04 (2023) 116 = 2302.03094
Liu, Ivanov, Gonçalves, JHEP 02 (2025) 069 = 2409.05992
Zhao, Ivanov, 2507.03320

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N-Higgs-doublet models

Higgses can come in generations $\to$ NHDMs [T.D.Lee 1973, Weinberg 1976, . . .]

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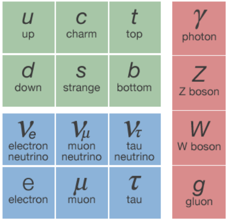

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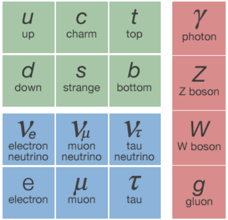
Multi-Higgs-doublet models



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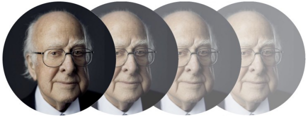

3HDM vs 2HDM

2HDMs: thousands of papers [Branco et al, 1106.0034].

What novelties can 3HDMs bring? [Ivanov, 1702.03776]

- More options for model-building (scalar and fermion) ⇒ richer pheno!
  - new options for *CP* violation [Branco, Gerard, Grimus, 1984];
  - an exotic *CP* symmetry of order 4 [Ivanov, Silva, 2015];
  - combining features of 2HDM: NFC + CPV [Weinberg, 1976; Branco, 1979], scalar DM + CPV [Grzadkowski et al, 2009].
  - astroparticle consequences: various dark sectors [Cordero et al, 2017; Dey et al, 2024; Kuncinas, Osland, Rebelo, 2024]; new options for baryon asymmetry [Davoudiasl, Lewis, Sullivan, 2019]; multi-step phase transitions [Athron et al, 2305.02357; Yang, Ivanov, 2024].
- Many symmetry options tested starting from late 1970's [many classic papers].

Symmetries = the single most powerful novelty of the 3HDMs.

CP4 3HDM

A dilemma in symmetry-based multi-Higgs model building:

- Large symmetry groups → very few free parameters, nicely calculable, very predictive, but conflicts experiment.
- Small symmetry groups → many free parameters, compatible with experiment but not quite predictive.

I will show a peculiar model based on three Higgs doublets (3HDM) which

- assumes very little: the minimal model realizing a particular symmetry;
- this symmetry is unusual: CP-symmetry of order 4 (CP4);
- remarkable connections between the scalar and Yukawa sectors → predictions.

A good balance between minimality, predictive power, and theoretical appeal.

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CP4 3HDM

CP4 $=$ CP symmetry of order 4 $=$ you need to apply it four times to get the identity.

Higher order CP

In QFT, CP is not uniquely defined *a priori*.

- phase factors $\phi(\vec{r}, t) \xrightarrow{CP} e^{i\alpha}\phi^*(-\vec{r}, t)$ [Feinberg, Weinberg, 1959].
- With N scalar fields ϕ_i, the general CP transformation is $\phi_i \xrightarrow{CP} X_{ij}\phi_j^*$, $X \in U(N)$.
- If $\mathcal{L}$ is invariant under CP with any X, it is explicitly CP-conserving [Grimus, Rebelo, 1997; Branco, Lavoura, Silva, 1999].

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- If $\mathcal{L}$ is invariant under CP with any X, it is explicitly CP-conserving [Grimus, Rebelo, 1997; Branco, Lavoura, Silva, 1999].
- Squaring general CP $\to$ a family transformation:

$$\phi_i \xrightarrow{CP} X_{ij}\phi_j^* \xrightarrow{CP} X_{ij}(X_{jk}^*\phi_k) = (XX^*)_{ik}\phi_k \,.$$

It can happen that $(CP)^2 = XX^* \neq \mathbb{I}$ but $(CP)^k = \mathbb{I}$ for $k > 2$.

CP-symmetry can be of a higher order $k > 2$.

The usual CP is of order 2. The exotic CP with $k = 4$ is denoted CP4.

CP4 3HDM

What is the minimal NHDM with CP4? [Ivanov, Silva, 1512.09276]

Consider the 3HDM with $V = V_0 + V_1$ (notation: $i \equiv \phi_i$), where

$$\begin{aligned} V_0 &= -m_{11}^2(1^\dagger 1) - m_{22}^2(2^\dagger 2 + 3^\dagger 3) + \lambda_1 (1^\dagger 1)^2 + \lambda_2\left[(2^\dagger 2)^2 + (3^\dagger 3)^2\right] \\ &\quad + \lambda_3 (1^\dagger 1)(2^\dagger 2 + 3^\dagger 3) + \lambda_3'(2^\dagger 2)(3^\dagger 3) + \lambda_4\left[(1^\dagger 2)(2^\dagger 1) + (1^\dagger 3)(3^\dagger 1)\right] + \lambda_4'(2^\dagger 3)(3^\dagger 2)\,, \end{aligned}$$

with all parameters real, and

$$V_1 = \lambda_5 (3^\dagger 1)(2^\dagger 1) + \frac{\lambda_6}{2}\left[(2^\dagger 1)^2 - (3^\dagger 1)^2\right] + \lambda_8 (2^\dagger 3)^2 + \lambda_9 (2^\dagger 3)\left[(2^\dagger 2) - (3^\dagger 3)\right] + h.c.$$

with real $\lambda_{5,6}$ and complex $\lambda_{8,9}$. It is invariant under CP4 $\phi_i \xrightarrow{CP} X_{ij}\phi_j^*$ with

$$X = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & i \\ 0 & -i & 0 \end{pmatrix} \quad \Rightarrow \quad XX^* = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}.$$

CP4-symmetric quark sector

Extending CP4 to the Yukawa sector: $\psi_a \to Y_{ab}\psi_b^{CP}$, where $\psi^{CP} = \gamma^0 C \bar{\psi}^T$.

$$-\mathcal{L}_Y = \bar{q}_L \Gamma_i d_R \phi_i + \bar{q}_L \Delta_i u_R \tilde{\phi}_i + h.c.$$

is invariant under CP4 if

$$(Y^L)^\dagger \Gamma_i Y^{d_R} X_{ij} = \Gamma_j^*\,, \quad (Y^L)^\dagger \Delta_i Y^{u_R} X_{ij}^* = \Delta_j^*\,.$$

Only X_{ij} is known; Γ_i, Δ_i, Y^L, Y^{d_R}, Y^{u_R} are to be found.

Solved in Ferreira et al, 1711.02042 $\to$ only four options for Γ_i and for Δ_i exist.

CP4-symmetric quark sector

case A: $\Gamma_1 \simeq$ arbitrary real matrix, $\Gamma_{2,3} = 0$.

case B_1

$$\Gamma_1 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ g_{31} & g_{31}^* & g_{33} \end{pmatrix}, \quad \Gamma_2 = \begin{pmatrix} g_{11} & g_{12} & g_{13} \\ g_{21} & g_{22} & g_{23} \\ 0 & 0 & 0 \end{pmatrix}, \quad \Gamma_3 = \begin{pmatrix} -g_{22}^* & -g_{21}^* & -g_{23}^* \\ g_{12}^* & g_{11}^* & g_{13}^* \\ 0 & 0 & 0 \end{pmatrix}.$$

case B_2

$$\Gamma_1 = \begin{pmatrix} 0 & 0 & g_{13} \\ 0 & 0 & g_{13}^* \\ 0 & 0 & g_{33} \end{pmatrix}, \quad \Gamma_2 = \begin{pmatrix} g_{11} & g_{12} & 0 \\ g_{21} & g_{22} & 0 \\ g_{31} & g_{32} & 0 \end{pmatrix}, \quad \Gamma_3 = \begin{pmatrix} g_{22}^* & -g_{21}^* & 0 \\ g_{12}^* & -g_{11}^* & 0 \\ g_{32}^* & -g_{31}^* & 0 \end{pmatrix}.$$

case B_3

$$\Gamma_1 = \begin{pmatrix} g_{11} & g_{12} & 0 \\ -g_{12}^* & g_{11}^* & 0 \\ 0 & 0 & g_{33} \end{pmatrix}, \quad \Gamma_2 = \begin{pmatrix} 0 & 0 & g_{13} \\ 0 & 0 & g_{23} \\ g_{31} & g_{32} & 0 \end{pmatrix}, \quad \Gamma_3 = \begin{pmatrix} 0 & 0 & -g_{23}^* \\ 0 & 0 & g_{13}^* \\ g_{32}^* & -g_{31}^* & 0 \end{pmatrix}.$$

CP4-symmetric quark sector

When combining up and down quarks, need to match α_L: 8 combinations.

$$(A^{down}, A^{up})\,, \quad (A^{down}, B_2^{up})\,, \quad (B_2^{down}, A^{up})\,, \quad (B_2^{down}, B_2^{up})\,,$$
$$(B_1^{down}, B_1^{up})\,, \quad (B_1^{down}, B_3^{up})\,, \quad (B_3^{down}, B_1^{up})\,, \quad (B_3^{down}, B_3^{up})\,.$$

- case (A, A) implies a real CKM $\to$ disregarded.
- cases B_1, B_2, B_3: quark mass matrices

$$M_d = \frac{1}{\sqrt{2}} \sum \Gamma_i v_i\,, \quad M_u = \frac{1}{\sqrt{2}} \sum \Delta_i v_i^*\,.$$

all v_i must be nonzero to avoid mass-degenerate quarks.

FCNC in CP4 3HDM

- Tree-level flavor-changing neutral couplings (FCNC) are a generic feature of multi-Higgs models.
- Unsuppressed FCNCs conflict meson oscillation parameters $\rightarrow$ need to be eliminated or suppressed (recent review: Sher, 2207.06771).

What's the status of FCNCs in the CP4 3HDM?

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What's the status of FCNCs in the CP4 3HDM?

- CP4 leads to remarkably tight connections between the Yukawa and scalar sectors
  $\to$ no built-in suppression of FCNC.
- Avoiding FCNC from h_{125} via the scalar alignment condition: $m_{11}^2 = m_{22}^2$.
- But then the additional neutral Higgses may exhibit significant FCNCs.

Must be explored in a full scan of the parameter space.

FCNC in CP4 3HDM

- Ferreira et al, 1711.02042: the first pheno scan (theory constraints, EWPT, fermion masses and mixing, meson oscillations) $\to$ many benchmark points found.
- Ruled out in Ivanov, Obodenko, 2104.11440 based on the LHC Run 2 charged Higgs searches.
- Need to repeat the scan — but do it in a smart way.

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- Need to repeat the scan — but do it in a smart way.
- The usual scanning procedure

$$\text{random seed point in } \Gamma_i,\, \Delta_i \quad \Rightarrow \quad \text{fit } m_q,\, \text{CKM}$$

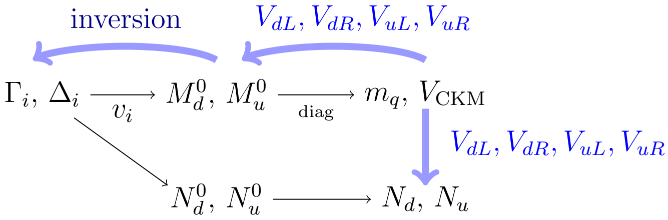
  is computer time consuming: many trial points are thrown away.
- A more efficient scanning procedure is needed:

$$\text{start with } m_q,\, \text{CKM} \quad \Rightarrow \quad \text{reconstruct } \Gamma_i,\, \Delta_i$$

  If this inversion is feasible, every trial point will give a viable model.

Scanning CP4 3HDM Yukawa sector

In Zhao et al, 2302.03094, we constructed an inversion procedure.



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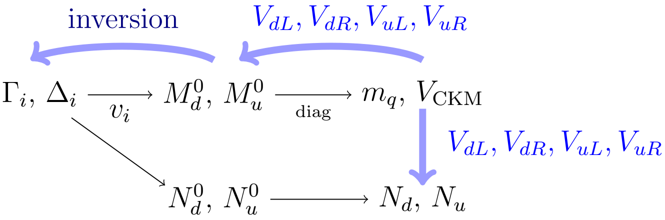
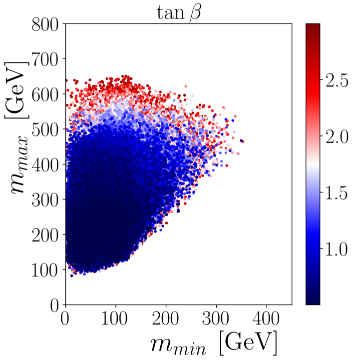


We expressed the FCNC matrices N_d, N_u via physical quark observables and quark rotation parameters.

Scanning CP4 3HDM scalar sector

In Liu, Ivanov, Gonçalves, 2409.05992, we considered the scalar sector and constructed an "observables-first" scanning procedure:

> the input is not λ_i but the properties of vevs, h_{125}, and charged Higgses.



Some observations:

- no decoupling limit; all $m_i < 700$ GeV;
- two regimes: light Higgses ($m_{min} < m_t$) or moderately heavy Higgses $\sim$ 300–500 GeV;
- typical vevs: $v_1 \sim v_2 \sim v_3$; no strong hierarchy.

FCNC in CP4 3HDM

Reminder: there are 8 Yukawa sectors compatible with CP4:

$$(A^{down}, A^{up})\,, \quad (A^{down}, B_2^{up})\,, \quad (B_2^{down}, A^{up})\,, \quad (B_2^{down}, B_2^{up})\,,$$
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All of them can be fitted to quark masses and mixing. But what about FCNC?

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In Zhao et al, 2302.03094 and 2507.03320, using the Yukawa inversion procedure and scalar sector scan, we explored FCNC consequences in each of these scenarios.

- Meson oscillation parameters for K, B, B_s, and D-meson oscillations.
- Diagonal and off-diagonal top quark observables: $t \to h_{\text{SM}} u_i$, $\kappa_t = (t\bar{t}h_{\text{SM}})/(t\bar{t}h)_{\rm SM}$, Γ_t.
- For a new light Higgs H or $H^\pm$ ($m < m_t$), new top decay channels:

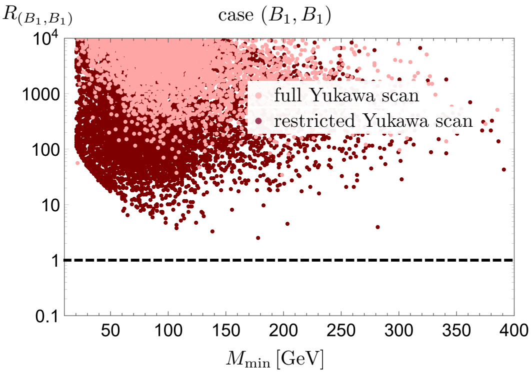
$$t \to H u_i \quad \text{or} \quad t \to H^+ d_i\,,$$

  with subsequent quark pair decays of H or $H^\pm$ (lots of recent LHC searches).

FCNC in CP4 3HDM → a unique surviving Yukawa sector

~~(A^{down}, A^{up})~~, (A^{down}, B_2^{up}), ~~(B_2^{down}, A^{up})~~, ~~(B_2^{down}, B_2^{up})~~,

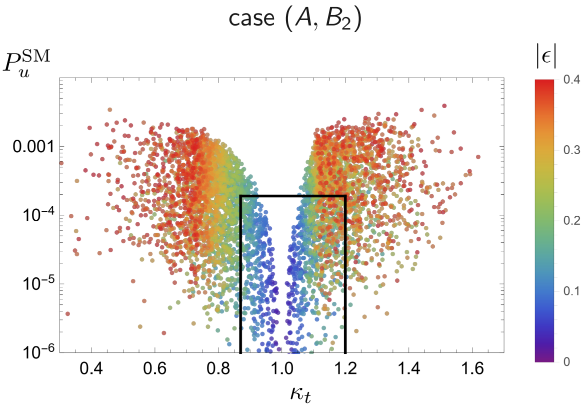
~~(B_1^{down}, B_1^{up})~~, ~~(B_1^{down}, B_3^{up})~~, ~~(B_3^{down}, B_1^{up})~~, ~~(B_3^{down}, B_3^{up})~~.



Example: (B_1, B_1)

- checked Δm_K, $|\epsilon_K|$, Δm_B, Δm_{B_s}, Δm_D;
- computed ratios R_i, e.g. $R(\Delta m_D) = \Delta m_D/(\Delta m_D)_{\rm exp.}$
- define $R_{(B_1,B_1)} = \max(R_i)$.
- A viable model must have $R_{(B_1,B_1)} < 1$.

FCNC in CP4 3HDM → a unique surviving Yukawa sector



- $P_u^{\rm SM} = {\rm Br}(t \to uh_{SM}) < 1.9 \times 10^{-4}$ [CMS, 2021].
- $P_c^{\rm SM}$ is even smaller in CP4 3HDM.
- $\kappa_t \in (0.87, 1.20)$ [ATLAS, 2022].
- ϵ is the misalignment angle:

$$\cos\epsilon = \kappa_V = \frac{g(h_{SM}VV)}{g_{SM}(h_{SM}VV)}\,.$$

- $\cos\epsilon < 0.98$ (or $|\epsilon| > 0.2$) disfavored; a bound stronger than $h_{SM} \to VV^*$.

FCNC in CP4 3HDM → a unique surviving Yukawa sector

The unique surviving CP4 Yukawa scenario: (A, B_2).

- It survives all the above constraints despite the absence of decoupling and the unavoidable FCNC in the up-sector.
- Can accommodate a neutral or charged Higgs lighter than the top quark → benchmark points.

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- It survives all the above constraints despite the absence of decoupling and the unavoidable FCNC in the up-sector.
- Can accommodate a neutral or charged Higgs lighter than the top quark → benchmark points.
- However, not all the observables have been checked!

Suspense

What's the eventual fate of the CP4 3HDM?

Is it already ruled out? by which observable?

Is it still viable and can be further tested?

Conclusions

- CP4 3HDM, the minimal model based on CP symmetry of order 4 (CP4) and no accidental symmetries.
- CP4 can be extended to the Yukawa sector $\to$ characteristic flavor sector with peculiar FCNC!
- Remarkably, out of 8 possible CP4 invariant Yukawa sectors, only one scenario survives meson oscillation checks: (A, B_2). This scenario seems robust and even allows for new Higgses lighter than top quark.
- An intriguing, well-shaped model with peculiar off-diagonal couplings emerges from a single symmetry requirement $\to$ much remains to be studied!

Tired of 2HDM and singlets? Try CP4 3HDM

- based on a single symmetry assumption,
- quite predictive with rich phenomenology,
- analytical insights guide numerical exploration.