

# RGE effects on new physics search via gravitational waves

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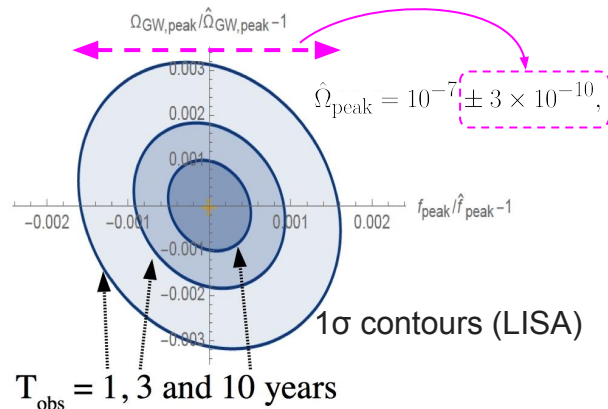
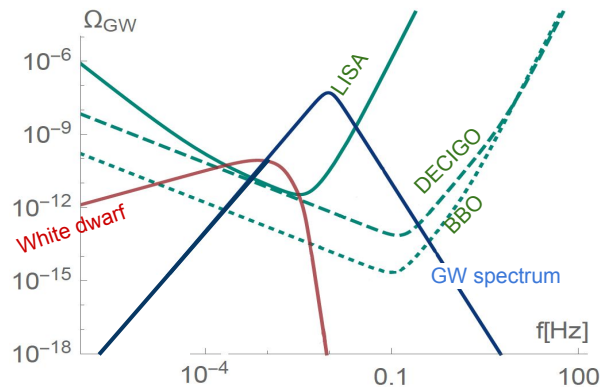
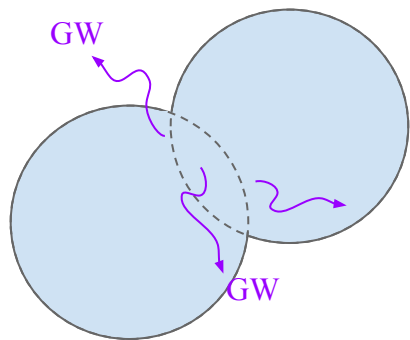
[K. H., Daiki Ueda, Phys.Rev.D 107 (2023) 9, 095022 ]

[K. H., Daiki Ueda, arXiv: 2505.13074, JHEP09(2025)094] 1

# Introduction

- ★ The gravitational waves (GWs) are produced by the first-order phase transition (FOPT).

GW observations can be used to explore the new physics (NP) effects for the first-order phase transition.



$$\hat{\Omega}_{\text{peak}} = 10^{-7}, \hat{f}_{\text{peak}} = 10^{-2} \text{ Hz}$$

We can quantitatively discuss how precisely obtain the NP parameter at the GW observation by the Fisher matrix analysis.

# Introduction

GW spectra depends on the new physics effects.

**Expected uncertainties for GW spectra**

$$\hat{\Omega}_{\text{peak}} = 10^{-7} \pm 3 \times 10^{-10}$$

$$\hat{f}_{\text{peak}} = 10^{-2} \pm 1.5 \times 10^{-5} \text{ Hz}$$



Error  
propagation

**Expected uncertainties for NP effects**

$$\lambda_{\text{new}} = 0.5 \pm 0.05$$

$$M_{\text{new}} = 300 \pm 3 \text{ GeV}$$

\* Example

**We can evaluate the width of confidence interval of the NP effects by GW observation.**

K. H., R. Jinno, M. Kakizaki, S. Kanemura, T. Takahashi and M. Takimoto, PRD 99 (2019) no.7, 075011,  
K. H., Daiki Ueda, Phys.Rev.D 107 (2023) 9, 095022

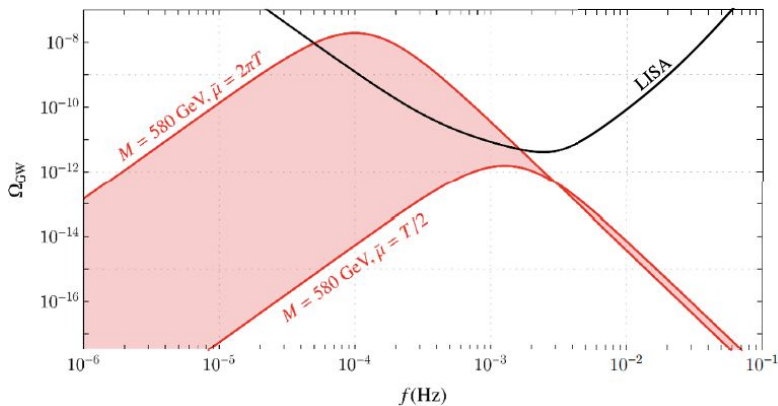
# Theoretical uncertainties

★ However, the GW spectra have some theoretical uncertainties...

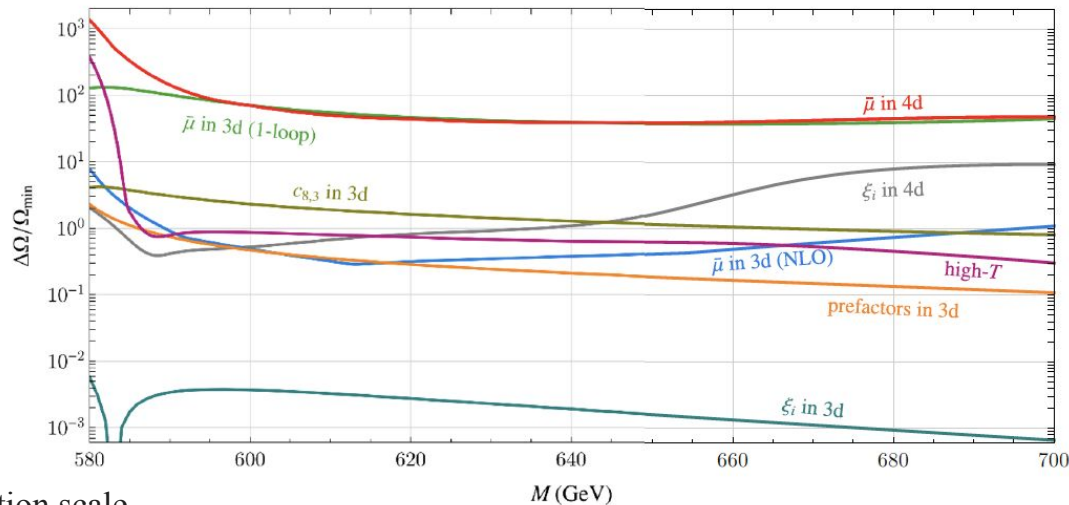
[D. Croon, O. Gould, P. Schicho, T. V. I. Tenkanen and G. White, JHEP 04 (2021), 055]

| $\Delta\Omega_{\text{GW}}/\Omega_{\text{GW}}$ | 4d approach                   | 3d approach                   |
|-----------------------------------------------|-------------------------------|-------------------------------|
| RG scale dependence                           | $\mathcal{O}(10^2 - 10^3)$    | $\mathcal{O}(10^0 - 10^1)$    |
| Gauge dependence                              | $\mathcal{O}(10^1)$           | $\mathcal{O}(10^{-3})$        |
| High- $T$ approximation                       | $\mathcal{O}(10^{-1} - 10^0)$ | $\mathcal{O}(10^0 - 10^2)$    |
| Higher loop orders                            | unknown                       | $\mathcal{O}(10^0 - 10^1)$    |
| Nucleation corrections                        | unknown                       | $\mathcal{O}(10^{-1} - 10^0)$ |
| Nonperturbative corrections                   | unknown                       | unknown                       |

Unphysical scale dependence: 4d approach without RGE-running



$$V_{\text{tree}}(\phi) = \mu_h^2 \phi^\dagger \phi + \lambda (\phi^\dagger \phi)^2 + \frac{1}{M^2} (\phi^\dagger \phi)^3 \quad \bar{\mu} : \text{Renormalization scale}$$



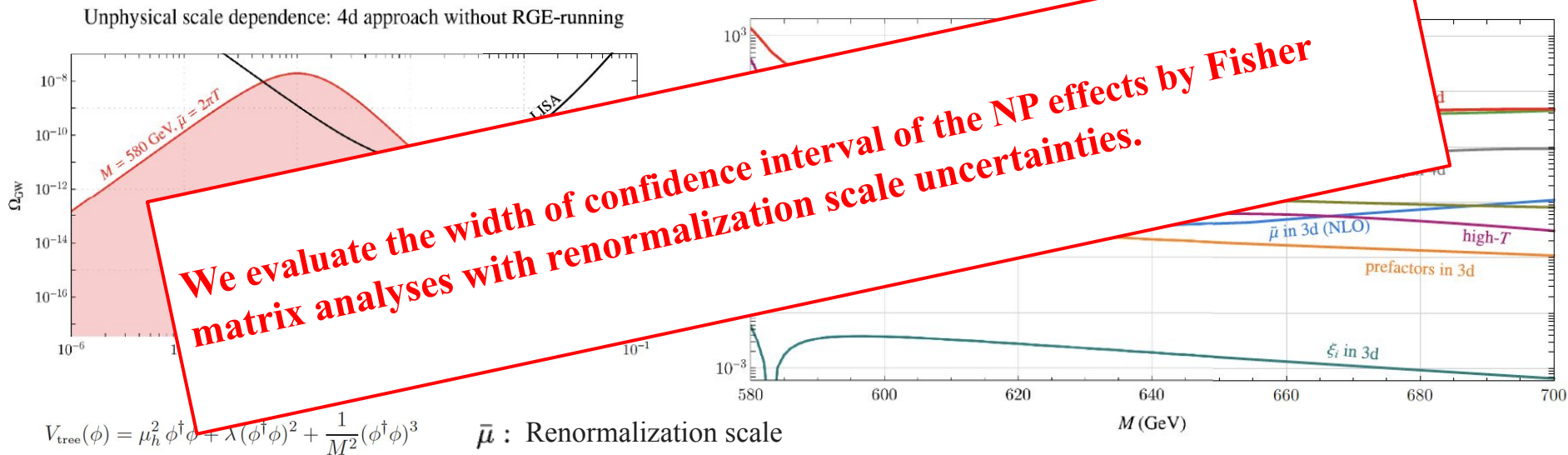
The uncertainties have been highlighted using the Standard Model Effective Field Theory (SMEFT) as a benchmark.

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| Nucleation corrections                        |                               | $\mathcal{O}(10^{-1} - 10^0)$ |
| Nonperturbative                               |                               | unknown                       |



The uncertainties have been highlighted using the Standard Model Effective Field Theory (SMEFT) as a benchmark.

# Benchmark model (SMEFT)

- SMEFT** {
- The first-order phase transition
  - Theoretical uncertainties

We adopt the SMEFT as a benchmark.

$$\mathcal{L}_{\text{SMEFT}} = \mathcal{L}_{\text{SM}} + \sum_i C_i \mathcal{O}_i$$

These operators can describe various NP effects  
in a model-independent manner.

| $X^3$                       |                                                           | $H^6$ and $H^4 D^2$       |                                                                 | $\psi^2 H^3 + \text{h.c.}$ |                                                                                 |
|-----------------------------|-----------------------------------------------------------|---------------------------|-----------------------------------------------------------------|----------------------------|---------------------------------------------------------------------------------|
| $\mathcal{O}_W$             | $\epsilon^{IJK} W_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$ | $\mathcal{O}_H$           | $(H^\dagger H)^3$                                               | $\mathcal{O}_{uH}$         | $(H^\dagger H)(\bar{q}_p u_r \tilde{H})$                                        |
|                             |                                                           | $\mathcal{O}_{H\Box}$     | $(H^\dagger H)\Box(H^\dagger H)$                                |                            |                                                                                 |
|                             |                                                           | $\mathcal{O}_{HD}$        | $(H^\dagger D_\mu H)^* (H^\dagger D_\mu H)$                     |                            |                                                                                 |
| $X^2 H^2$                   |                                                           | $\psi^2 XH + \text{h.c.}$ |                                                                 | $\psi^2 H^2 D$             |                                                                                 |
| $\mathcal{O}_{HG}$          | $H^\dagger H G_{\mu\nu}^A G^{A\mu\nu}$                    | $\mathcal{O}_{uG}$        | $(\bar{q}_p \sigma^{\mu\nu} T^A u_r) \tilde{H} G_{\mu\nu}^A$    | $\mathcal{O}_{Hl}^{(3)}$   | $(H^\dagger i \overleftrightarrow{D}_\mu^I H)(\bar{l}_p \tau^I \gamma^\mu l_r)$ |
| $\mathcal{O}_{H\tilde{G}}$  | $H^\dagger H \tilde{G}_{\mu\nu}^A G^{A\mu\nu}$            | $\mathcal{O}_{uW}$        | $(\bar{q}_p \sigma^{\mu\nu} u_r) \tau^I \tilde{H} W_{\mu\nu}^I$ | $\mathcal{O}_{Hq}^{(1)}$   | $(H^\dagger i \overleftrightarrow{D}_\mu^I H)(\bar{q}_p \gamma^\mu q_r)$        |
| $\mathcal{O}_{HW}$          | $H^\dagger H W_{\mu\nu}^I W^{I\mu\nu}$                    | $\mathcal{O}_{uB}$        | $(\bar{q}_p \sigma^{\mu\nu} u_r) \tilde{H} B_{\mu\nu}$          | $\mathcal{O}_{Hq}^{(3)}$   | $(H^\dagger i \overleftrightarrow{D}_\mu^I H)(\bar{q}_p \tau^I \gamma^\mu q_r)$ |
| $\mathcal{O}_{H\tilde{W}}$  | $H^\dagger H \tilde{W}_{\mu\nu}^I W^{I\mu\nu}$            |                           |                                                                 | $\mathcal{O}_{Hu}$         | $(H^\dagger i \overleftrightarrow{D}_\mu^I H)(\bar{u}_p \gamma^\mu u_r)$        |
| $\mathcal{O}_{HB}$          | $H^\dagger H B_{\mu\nu} B^{\mu\nu}$                       |                           |                                                                 |                            |                                                                                 |
| $\mathcal{O}_{H\tilde{B}}$  | $H^\dagger H \tilde{B}_{\mu\nu} B^{\mu\nu}$               |                           |                                                                 |                            |                                                                                 |
| $\mathcal{O}_{HWB}$         | $H^\dagger \tau^I H W_{\mu\nu}^I B^{\mu\nu}$              |                           |                                                                 |                            |                                                                                 |
| $\mathcal{O}_{H\tilde{W}B}$ | $H^\dagger \tau^I H \tilde{W}_{\mu\nu}^I B^{\mu\nu}$      |                           |                                                                 |                            |                                                                                 |
| $(\bar{L}L)(\bar{L}L)$      |                                                           | $(\bar{L}L)(\bar{R}R)$    |                                                                 |                            |                                                                                 |
| $\mathcal{O}_{ll}$          | $(\bar{l}_p \gamma_\mu l_r)(\bar{l}_s \gamma^\mu l_t)$    | $\mathcal{O}_{qu}^{(1)}$  | $(\bar{q}_p \gamma_\mu q_r)(\bar{u}_s \gamma^\mu u_t)$          |                            |                                                                                 |
|                             |                                                           | $\mathcal{O}_{qu}^{(8)}$  | $(\bar{q}_p \gamma_\mu T^A q_r)(\bar{u}_s \gamma^\mu T^A u_t)$  |                            |                                                                                 |

# Benchmark model (SMEFT)

- ★ The GW spectra is evaluated by the effective potential at typical scales of the FOPT.

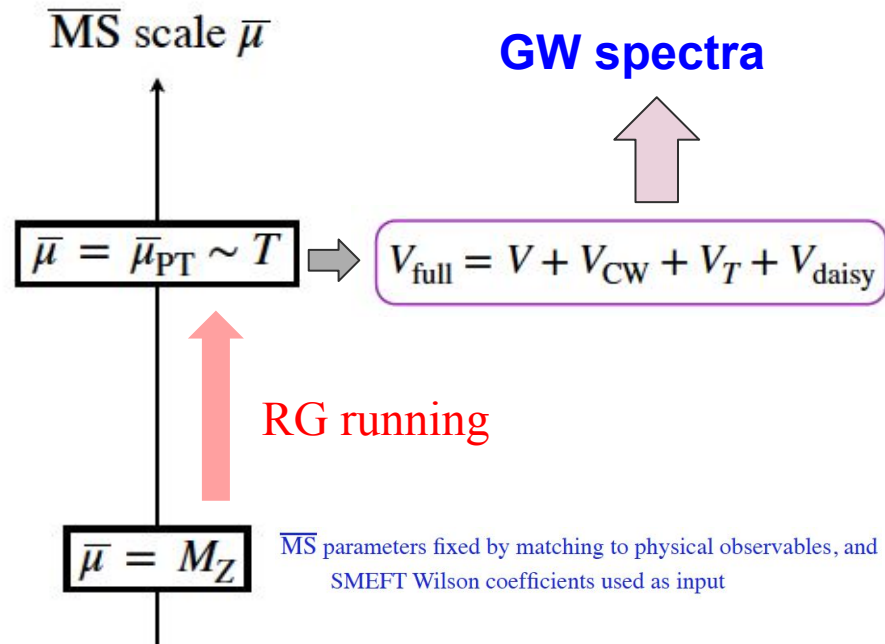
The vast number of SMEFT operators affect the potential.

| $X^3$       |                                                                    | $H^6$ and $H^4 D^2$ |                                                                | $\psi^2 H^3$ + h.c. |                                                      |
|-------------|--------------------------------------------------------------------|---------------------|----------------------------------------------------------------|---------------------|------------------------------------------------------|
| $C_W$       | $(\ell^\dagger \bar{L}^c W_\mu^c W_\mu^c W_\mu^c)$                 | $C_H$               | $(H^\dagger H)^3$                                              | $C_{uH}$            | $(H^\dagger H)(\bar{q}_L u_R)$                       |
|             |                                                                    | $C_{HD}$            | $(H^\dagger H) \Box (H^\dagger H)$                             |                     |                                                      |
|             |                                                                    |                     | $(H^\dagger D_\mu H)^* (H^\dagger D_\mu H)$                    |                     |                                                      |
| $X^2 H^2$   |                                                                    | $\psi^2 X H$ + h.c. |                                                                | $\psi^2 H^2 D$      |                                                      |
| $C_{H\Box}$ | $H^\dagger H \Box G_{\mu\nu}^a G^{\mu\nu a}$                       | $C_{u\Box}$         | $(\bar{q}_L \gamma^\mu T^a u_R) \Box G_{\mu\nu}^a$             | $C_{H\Box}^{(1)}$   | $(H^\dagger \Box H)(\bar{\ell}_L \gamma^\mu \ell_R)$ |
| $C_{H\Box}$ | $H^\dagger H \Box G_{\mu\nu}^a G^{\mu\nu a}$                       | $C_{u\Box}$         | $(\bar{q}_L \gamma^\mu u_R) \gamma^\nu \Box W_{\mu\nu}^c$      | $C_{H\Box}^{(2)}$   | $(H^\dagger \Box H)(\bar{q}_L \gamma^\mu q_R)$       |
| $C_{HW}$    | $H^\dagger H W_{\mu\nu}^c W^{\mu\nu c}$                            | $C_{uB}$            | $(\bar{q}_L \gamma^\mu u_R) \Box B_{\mu\nu}$                   | $C_{H\Box}^{(3)}$   | $(H^\dagger \Box H)(\bar{q}_L \gamma^\mu q_R)$       |
| $C_{HW}$    | $H^\dagger H \Box W_{\mu\nu}^c W^{\mu\nu c}$                       |                     |                                                                | $C_{H\Box}^{(4)}$   | $(H^\dagger \Box H)(\bar{q}_L \gamma^\mu q_R)$       |
| $C_{HW}$    | $H^\dagger H \Box W_{\mu\nu}^c W^{\mu\nu c}$                       |                     |                                                                |                     |                                                      |
| $C_{HB}$    | $H^\dagger H B_{\mu\nu} B^{\mu\nu}$                                |                     |                                                                |                     |                                                      |
| $C_{HB}$    | $H^\dagger H \Box B_{\mu\nu} B^{\mu\nu}$                           |                     |                                                                |                     |                                                      |
| $C_{HWB}$   | $H^\dagger H W_{\mu\nu}^c B^{\mu\nu}$                              |                     |                                                                |                     |                                                      |
| $C_{HWB}$   | $H^\dagger H \Box W_{\mu\nu}^c B^{\mu\nu}$                         |                     |                                                                |                     |                                                      |
| $(LL)(LL)$  |                                                                    | $(LL)(RR)$          |                                                                |                     |                                                      |
| $C_{ll}$    | $(\bar{\ell}_L \gamma_\mu \ell_L)(\bar{\ell}_L \gamma^\mu \ell_L)$ | $C_{ll}^{(1)}$      | $(\bar{q}_L \gamma_\mu q_L)(\bar{u}_L \gamma^\mu u_L)$         |                     |                                                      |
|             |                                                                    | $C_{ll}^{(2)}$      | $(\bar{q}_L \gamma_\mu T^a q_L)(\bar{u}_L \gamma^\mu T^a u_L)$ |                     |                                                      |

over 20 operators

- ★ In this work, we compare the GW spectra predicted by the effective potential at two different renormalization scales:

$$\bar{\mu}_{PT} = 2\pi T_n \text{ and } T_n/2$$



It is the source of  
first-order PT

# The GW spectra

- ★ If the value of  $|C_H|$  is  $1/\Lambda^2 = 1/(600 \text{ GeV})^2$ ,  
the GW spectra are given by right figure.

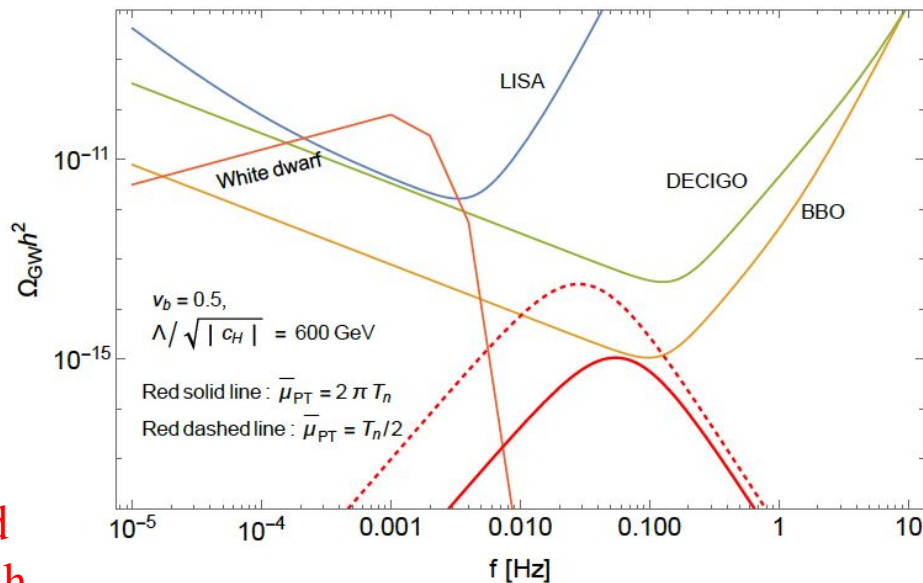
(All others are set to zero)

It is difficult to determine the  
value of  $C_H$  by only an  
observation of GW spectrum.



We assume that the value of  $C_H$  is provided  
by future collider experiments, e.g., FCC-hh.

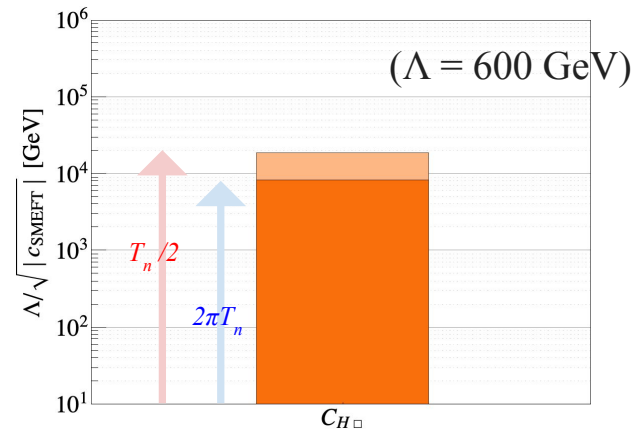
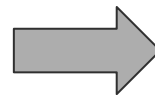
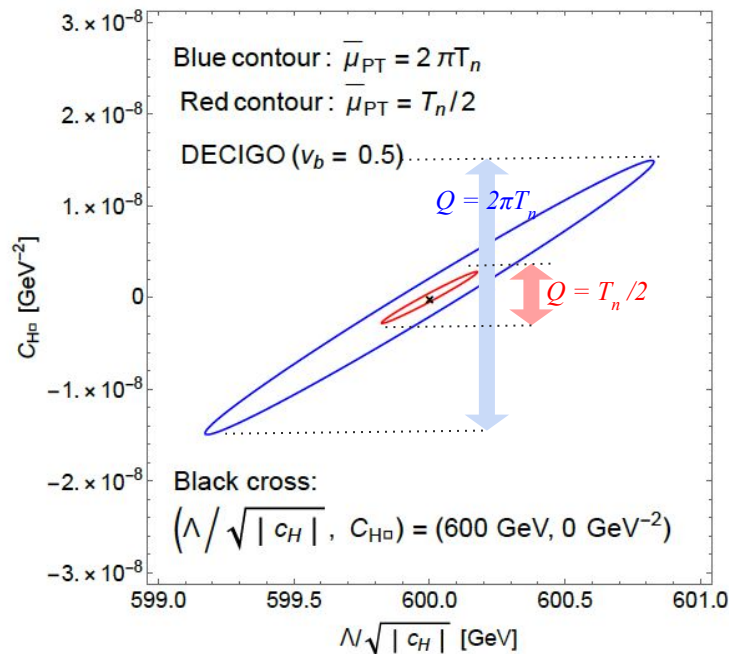
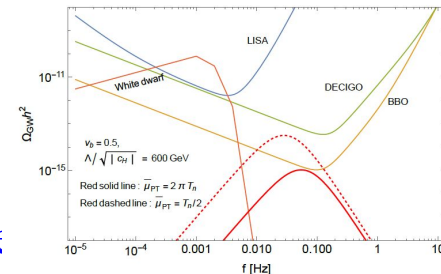
This  $\Lambda$  value is defined at Z boson mass scale.





# The results

★  $1/\Lambda^2 = 1/(600 \text{ GeV})^2$ ,  $C_{H\Box}$ ,  $v_b = 0.5$ , and DECIGO experiment case

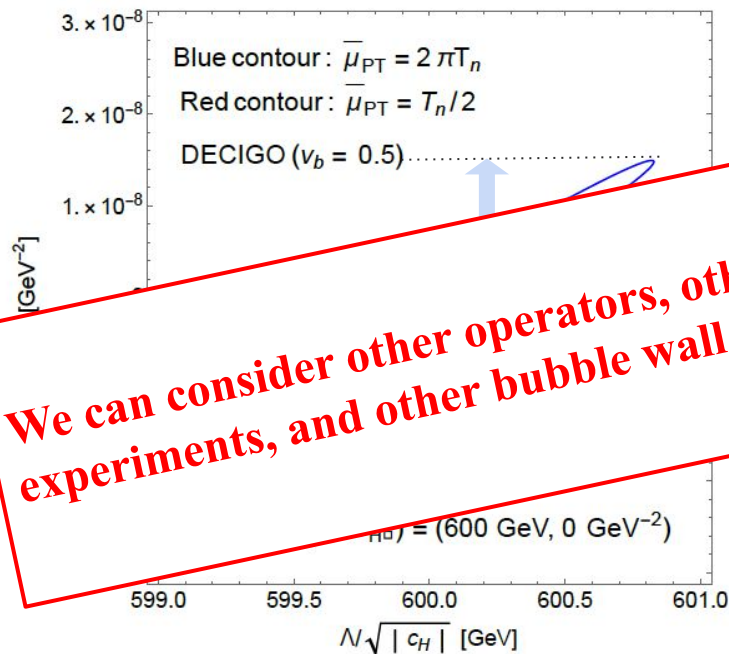
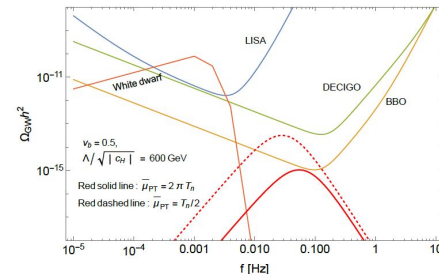


[K. H., Daiki Ueda, arXiv: 2505.13074, JHEP09(2025)094]

We also performed the analysis for different values of  $\Lambda$ , operators, velocities, and experiments.

# The results

★  $1/\Lambda^2 = 1/(600 \text{ GeV})^2$ ,  $C_{H\Box}$ ,  $v_b = 0.5$ , and DECIGO experiment case



**We can consider other operators, other value of  $\Lambda$ , other GW experiments, and other bubble wall velocity cases!!!!**



[K. H., Daiki Ueda, arXiv: 2505.13074, JHEP09(2025)094]

We also performed the analysis for different values of  $\Lambda$ , operators, velocities, and experiments.

# Results(DECIGO)

DECIGO:  $\Lambda/\sqrt{|c_H|}$  [GeV] for RGE scale  $\bar{\mu}_{PT} = 2\pi T_n$

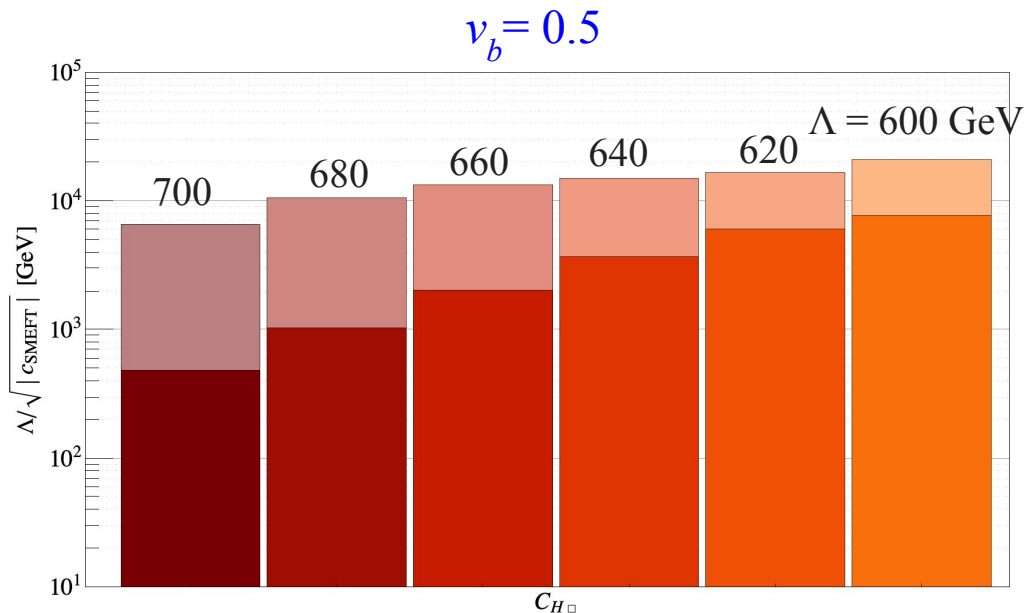
700 GeV 680 GeV 660 GeV  
640 GeV 620 GeV 600 GeV

DECIGO:  $\Lambda/\sqrt{|c_H|}$  [GeV] for RGE scale  $\bar{\mu}_{PT} = T_n/2$

700 GeV 680 GeV 660 GeV  
640 GeV 620 GeV 600 GeV

The dark- and light-colored bands represent the choices  $\mu_{PT} = 2\pi T_n$  and  $T_n/2$ , respectively,

Within each shade, different colors indicate different central values of  $|c_H|$ .



[K. H., Daiki Ueda, arXiv: 2505.13074, JHEP09(2025)094]

# Results(DECIGO and BBO)

DECIGO:  $\Lambda/\sqrt{|c_H|}$  [GeV] for RGE scale  $\bar{\mu}_{PT} = 2\pi T_n$

700 GeV 680 GeV 660 GeV  
640 GeV 620 GeV 600 GeV

DECIGO:  $\Lambda/\sqrt{|c_H|}$  [GeV] for RGE scale  $\bar{\mu}_{PT} = T_n/2$

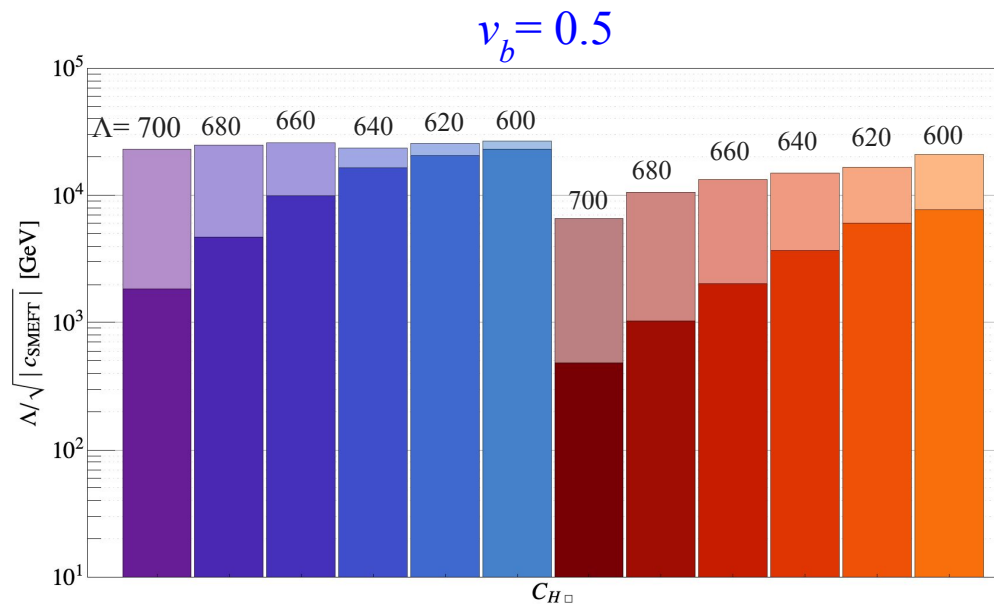
700 GeV 680 GeV 660 GeV  
640 GeV 620 GeV 600 GeV

BBO:  $\Lambda/\sqrt{|c_H|}$  [GeV] for RGE scale  $\bar{\mu}_{PT} = 2\pi T_n$

700 GeV 680 GeV 660 GeV  
640 GeV 620 GeV 600 GeV

BBO:  $\Lambda/\sqrt{|c_H|}$  [GeV] for RGE scale  $\bar{\mu}_{PT} = T_n/2$

700 GeV 680 GeV 660 GeV  
640 GeV 620 GeV 600 GeV

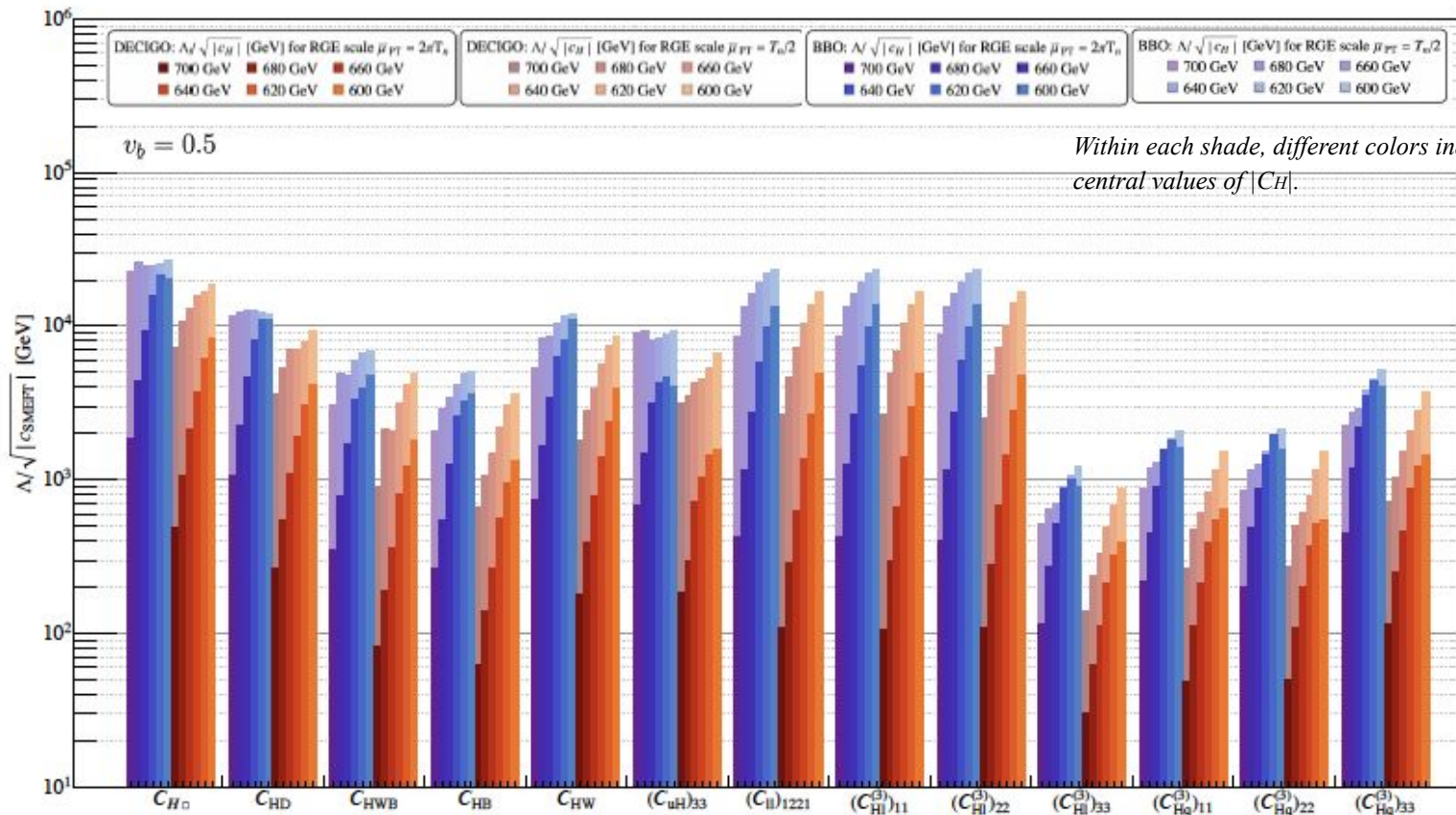


[K. H., Daiki Ueda, arXiv: 2505.13074, JHEP09(2025)094]

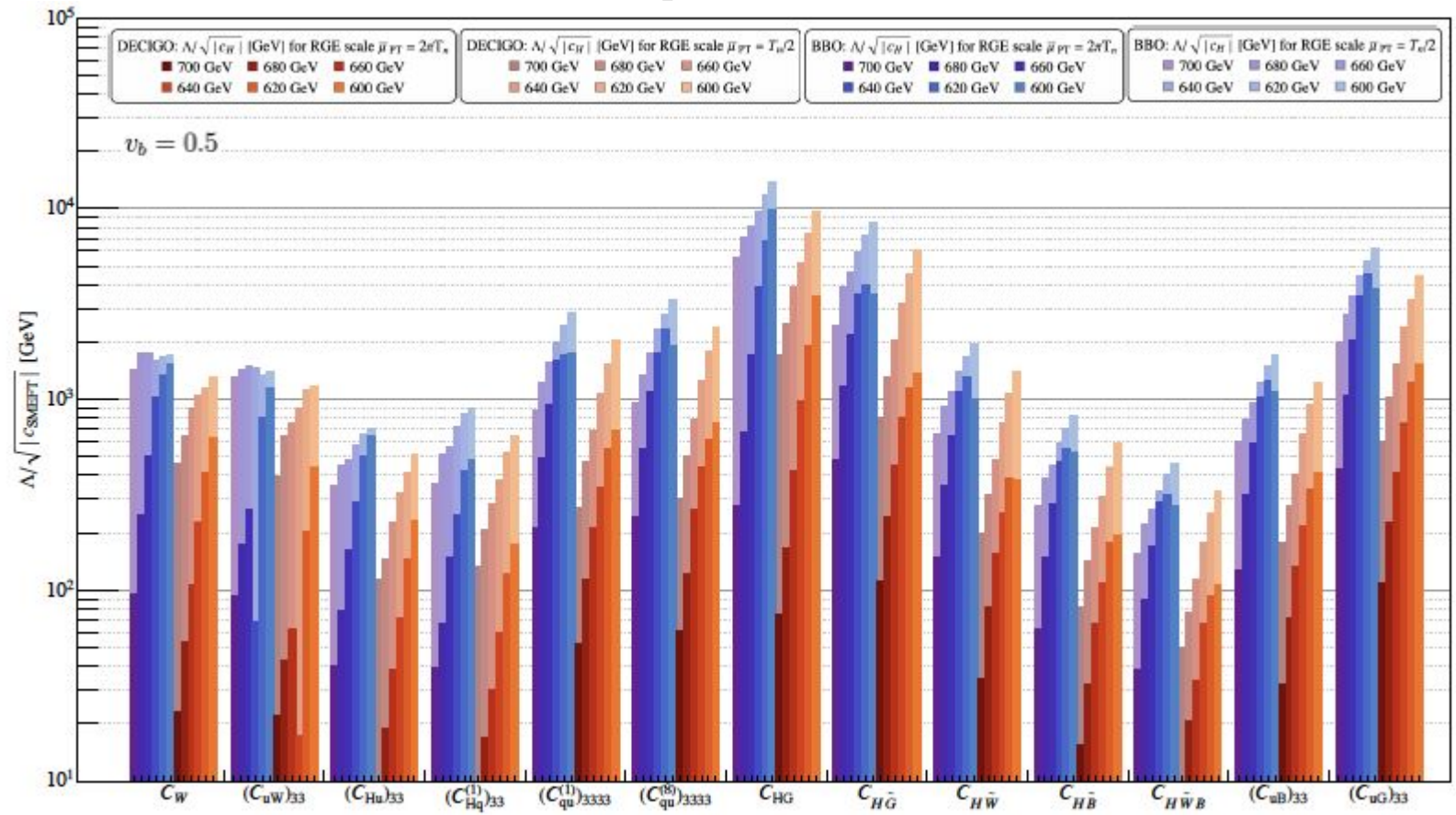
# Other operators

The dark- and light-colored bands represent the choices  $\mu_{PT} = 2\pi T_n$  and  $T_n/2$ , respectively,

BBO (bluish bands) and DECIGO (reddish bands)

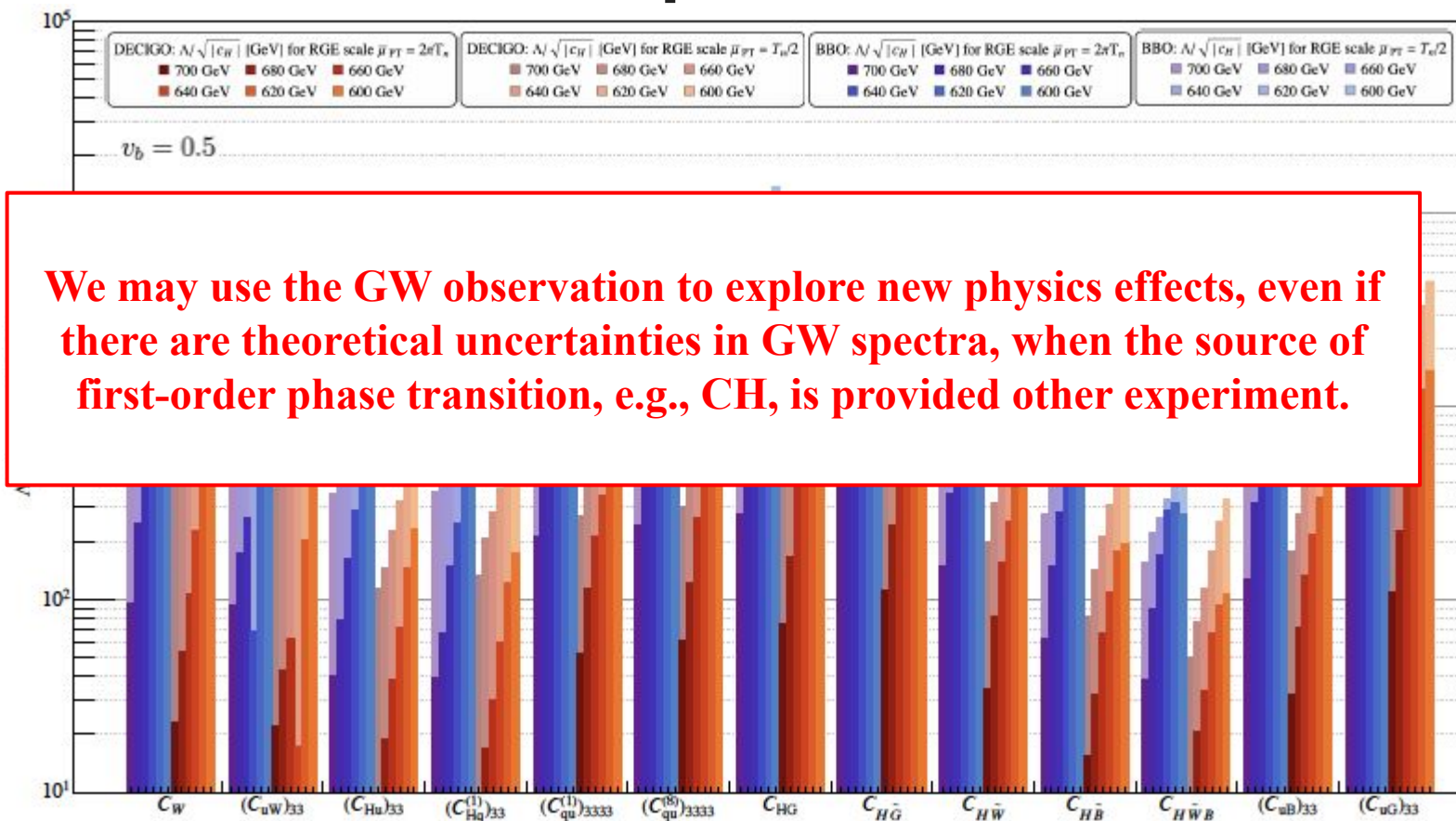


# Other operators



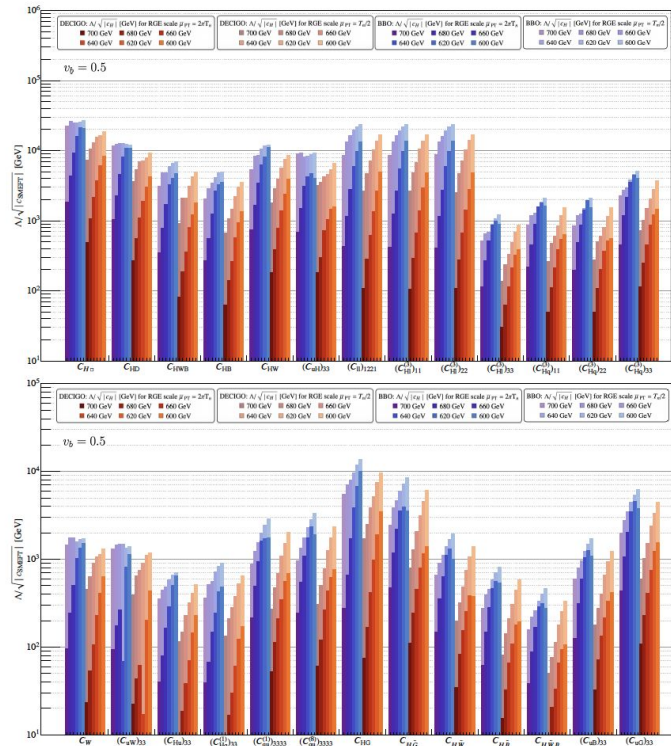


# Other operators



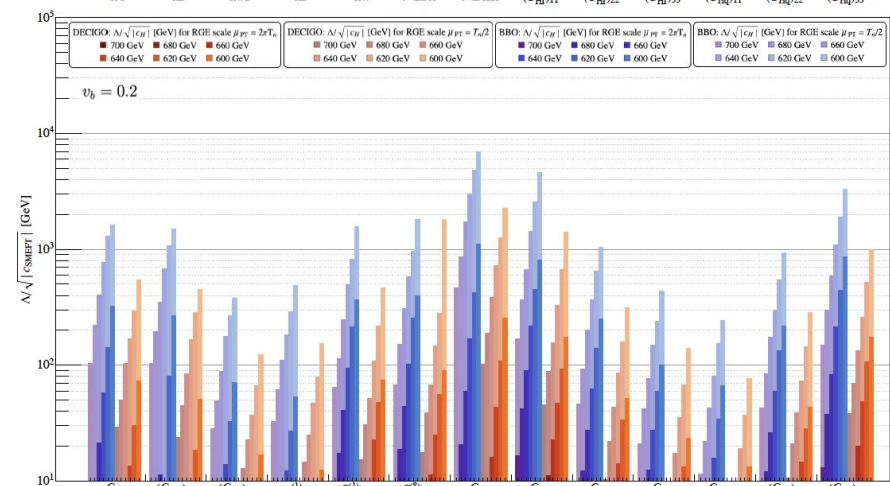
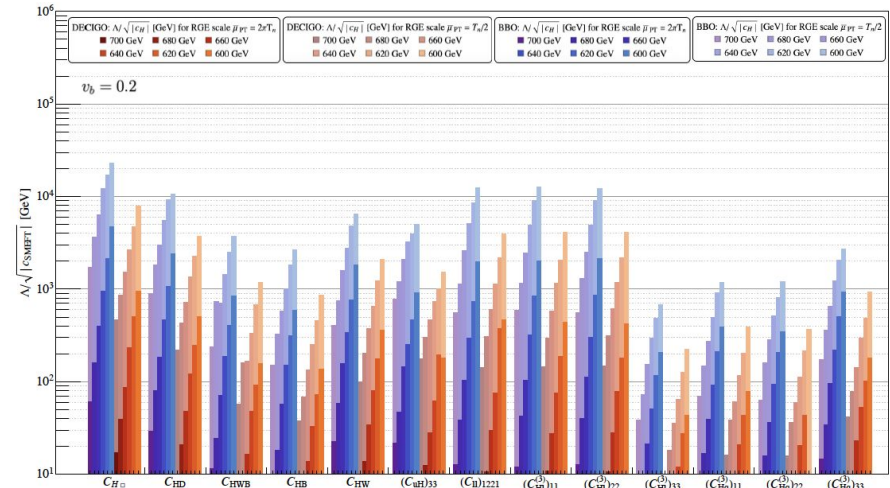
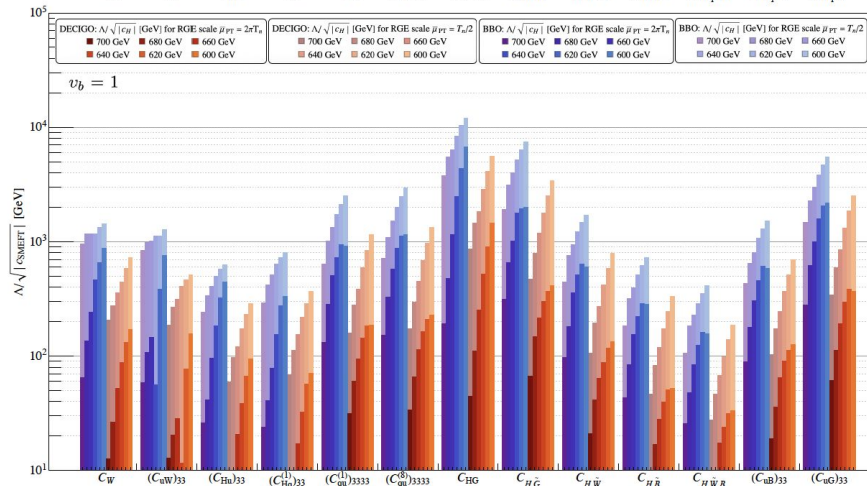
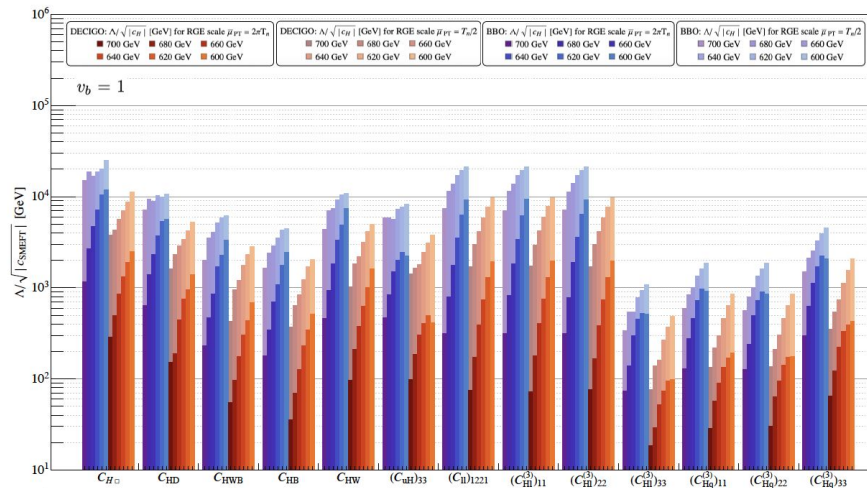
# Summary

- ★ The spectra of GW from first-order phase transition depends on the NP effects.
  - We can use the GW observation experiments to explore the NP effects.
- ★ The GW observations can remain sensitive to various new physics effects, even in the presence of renormalization scale uncertainties, provided that the source of first-order phase transition is precisely measured, e. g., by future collider experiments.

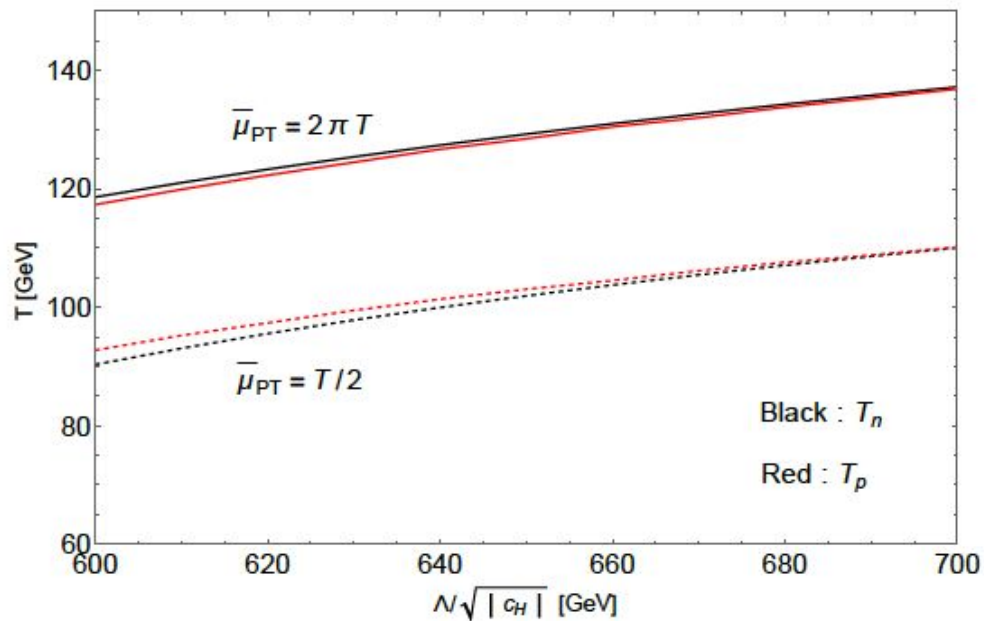
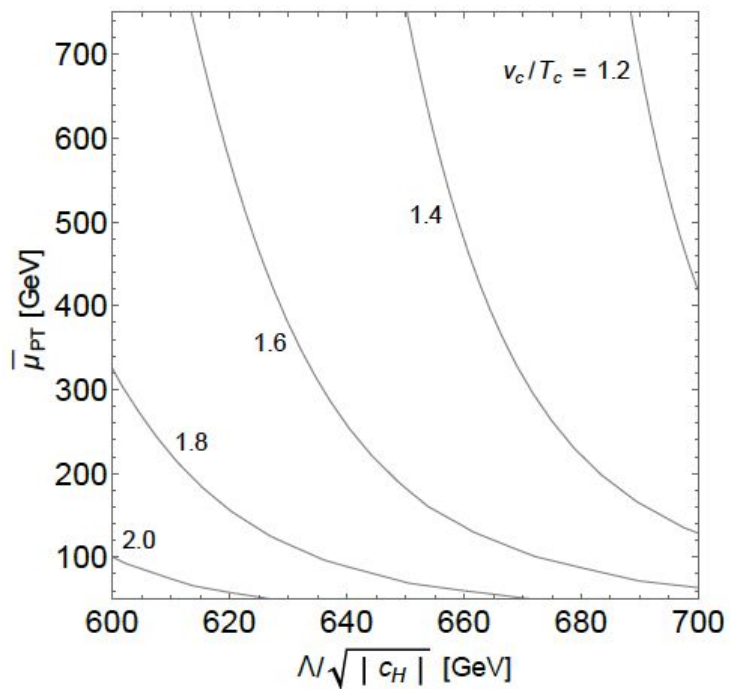




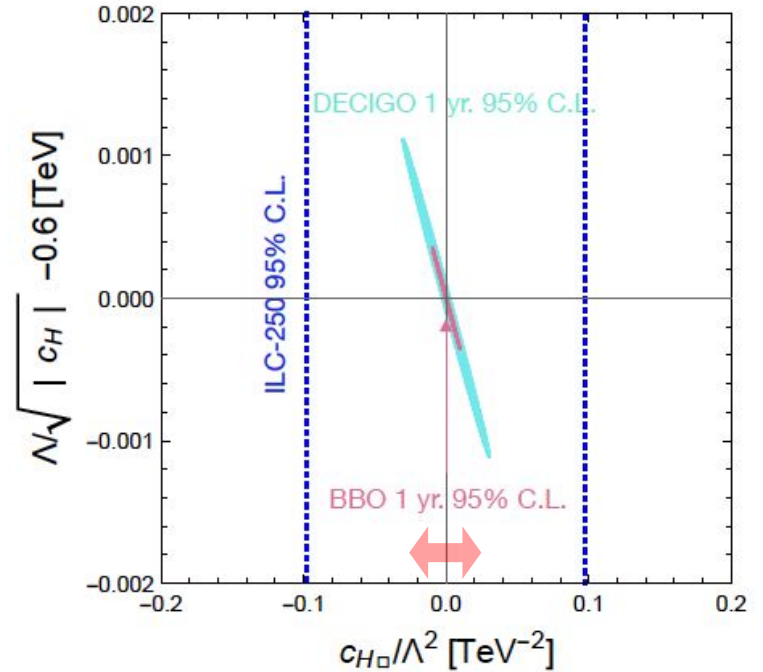
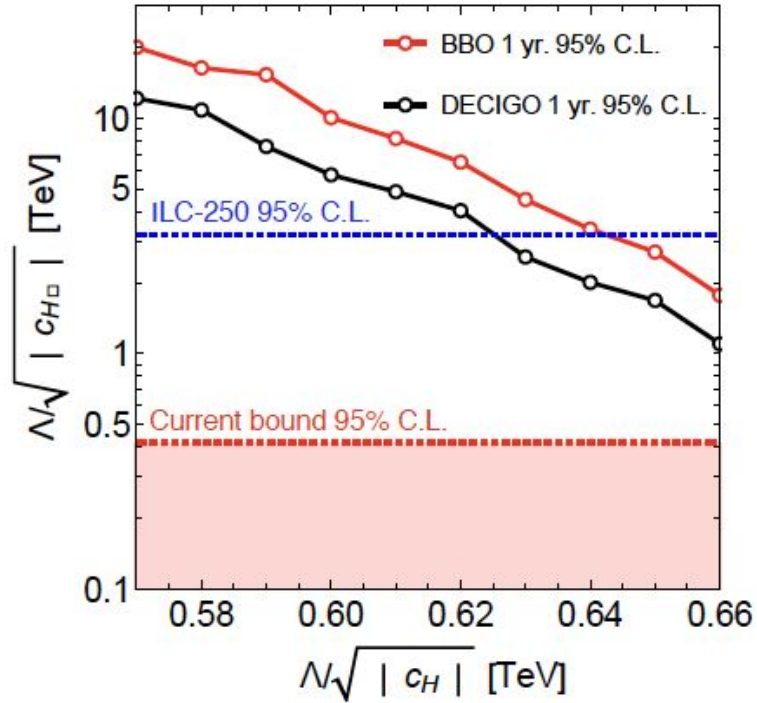
# Backup



# The results



# The results

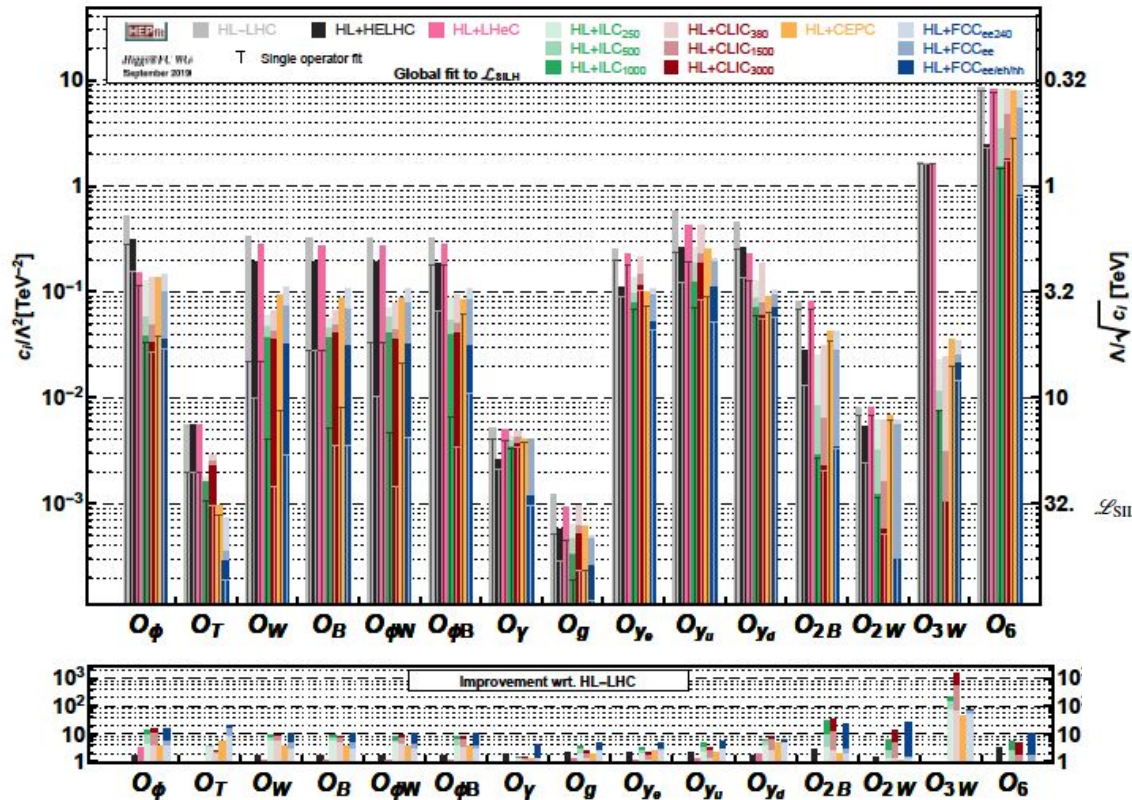


[K. H., Daiki Ueda, Phys.Rev.D 107 (2023) 9, 095022 ]

# EFT (dim. 6)

1 $\sigma$

J. de Blas, M. Cepeda, J. D'Hondt, R. K. Ellis, C. Grojean, B. Heinemann, F. Maltoni, A. Nisati, E. Petit and R. Rattazzi, et al., JHEP 01 (2020), 139 [arXiv:1905.03764 [hep-ph]].



$$\begin{aligned} \mathcal{L}_{\text{SILH}} = & \frac{c_\phi}{\Lambda^2} \frac{1}{2} \partial_\mu (\phi^\dagger \phi) \partial^\mu (\phi^\dagger \phi) + \frac{c_T}{\Lambda^2} \frac{1}{2} (\phi^\dagger \overleftrightarrow{D}_\mu \phi) (\phi^\dagger \overleftrightarrow{D}^\mu \phi) - \frac{c_6}{\Lambda^2} \lambda (\phi^\dagger \phi)^3 + \left( \frac{c_{Y_f}}{\Lambda^2} y_{ij}^f \phi^\dagger \phi \bar{\psi}_{Li} \phi \psi_{Rj} \right. \\ & + \frac{c_W}{\Lambda^2} \frac{ig}{2} (\phi^\dagger \overleftrightarrow{D}_\mu \phi) D_\nu W^{a\mu\nu} + \frac{c_B}{\Lambda^2} \frac{ig'}{2} (\phi^\dagger \overleftrightarrow{D}_\mu \phi) \partial_\nu B^{\mu\nu} + \frac{c_W}{\Lambda^2} ig D_\mu \phi^\dagger \sigma_a D_\nu \phi W^{a\mu\nu} + \frac{c_Y}{\Lambda^2} g' \phi^\dagger \phi B^{\mu\nu} B_{\mu\nu} + \frac{c_g}{\Lambda^2} g_s^2 \phi^\dagger \phi G^{A\mu\nu} G_{\mu\nu}^A \\ & - \frac{c_{2W}}{\Lambda^2} \frac{g^2}{2} (D^\mu W_{\mu\nu}^a) (D_\rho W^{a\rho\nu}) - \frac{c_{2B}}{\Lambda^2} \frac{g'^2}{2} (\partial^\mu B_{\mu\nu}) (\partial_\rho B^{\rho\nu}) - \frac{c_{2G}}{\Lambda^2} \frac{g_s^2}{2} (D^\mu G_{\mu\nu}^A) (D_\rho G^{A\rho\nu}) \\ & \left. + \frac{c_{3W}}{\Lambda^2} g^3 \epsilon_{abc} W_\mu^a W_\nu^b W_\rho^c \mu + \frac{c_{3G}}{\Lambda^2} g_s^3 f_{ABC} G_\mu^A G_\nu^B G_\rho^C \mu \right), \end{aligned}$$