Unifying Seesaw and Radiative Neutrino Mass Mechanisms with A_4 Symmetry

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Based on 2310.20681, 2311.15997, 2406.17861

NARODOWE CENTRUM NAUKI

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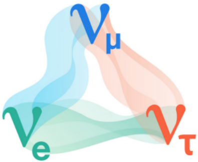
24.09.2025

Neutrino mixing



OSCILLATING

Neutrinos come in three types, called flavors. There are electron neutrinos, muon neutrinos and tau neutrinos. One of the strangest aspects of neutrinos is that they don't pick just one flavor and stick to it. They oscillate between all three.

LIGHTWEIGHT

Neutrinos weigh almost nothing, and they travel close to the speed of light. Neutrino masses are so small that so far no experiment has succeeded in measuring them. The masses of other fundamental particles come from the Higgs field, but neutrinos might get their masses another way.

Neutrino parameters and the known unknowns

- Neutrinos are special!! It's flavor and mass eigenstates are related by :

$$|\nu_e, \nu_\mu, \nu_\tau\rangle^T_{\text{flavor}} = U_{\alpha i}|\nu_1, \nu_2, \nu_3\rangle^T_{\text{mass}}$$

- Pontecorvo-Maki-Nakagawa-Sakata parametrization: $\boxed{U_{\alpha i} = U_{PMNS}}$

$$U_{PMNS} = \begin{bmatrix} C_{12}C_{13} & S_{12}C_{13} & S_{13}e^{-i\delta} \\ -S_{12}C_{23} - C_{12}S_{13}S_{23}e^{i\delta} & C_{12}C_{23} - S_{12}S_{13}S_{23}e^{i\delta} & C_{13}S_{23} \\ S_{12}S_{23} - C_{12}S_{13}C_{23}e^{i\delta} & -C_{12}S_{23} - S_{12}S_{13}C_{23}e^{i\delta} & C_{13}C_{23} \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & e^{i\alpha_{21}/2} & 0 \\ 0 & 0 & e^{i\alpha_{31}/2} \end{bmatrix}$$

here $C_{ij} = \cos\theta_{ij}$ and $S_{ij} = \sin\theta_{ij}$.

- Large Lepton Mixings

$$|U_{PMNS}| \sim \begin{pmatrix} 0.79 - 0.86 & 0.50 - 0.61 & 0.14 - 0.16 \\ 0.24 - 0.52 & 0.44 - 0.69 & 0.63 - 0.79 \\ 0.26 - 0.52 & 0.47 - 0.71 & 0.60 - 0.77 \end{pmatrix}$$

- Small Quark Mixings

$$|V_{CKM}| \sim \begin{pmatrix} 0.9745 - 0.9757 & 0.219 - 0.224 & 0.002 - 0.005 \\ 0.218 - 0.224 & 0.9736 - 0.9750 & 0.036 - 0.046 \\ 0.004 - 0.014 & 0.034 - 0.046 & 0.9989 - 0.9993 \end{pmatrix}$$

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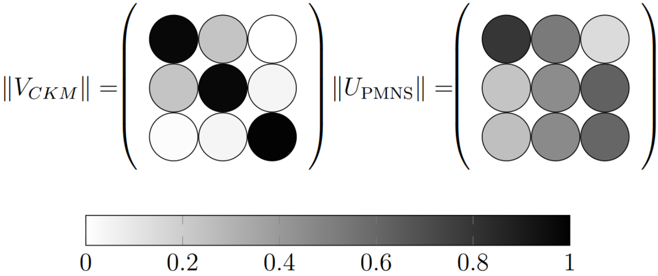

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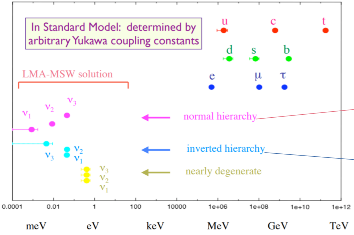

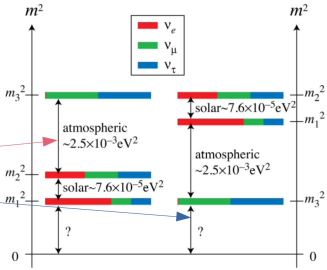

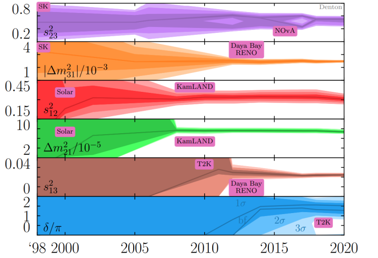

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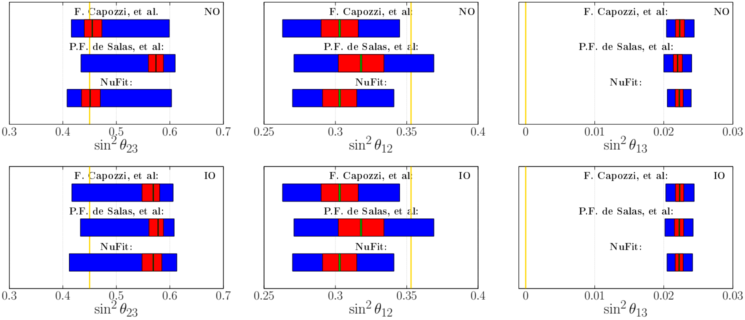
$\theta_{13}/^\circ$	$8.57^{+0.13}_{-0.12}$	$8.20 \to 8.97$	$8.60^{+0.12}_{-0.12}$	$8.24 \to 8.98$
$\delta_{CP}/^\circ$	194^{+52}_{-25}	$105 \to 405$	287^{+27}_{-32}	$192 \to 361$
$\frac{\Delta m^2_{21}}{10^{-5}\ \text{eV}^2}$	$7.42^{+0.21}_{-0.20}$	$6.82 \to 8.04$	$7.42^{+0.21}_{-0.20}$	$6.82 \to 8.04$
$\frac{\Delta m^2_{3\ell}}{10^{-3}\ \text{eV}^2}$	$+2.515^{+0.028}_{-0.028}$	$+2.431 \to +2.599$	$-2.498^{+0.028}_{-0.029}$	$-2.584 \to -2.413$

'Big' Data

Parameter	Ordering	NuFit 5.2		de Salas et al.		Capozzi et al.	
		bf$\pm 1\sigma$	3σ range	bf$\pm 1\sigma$	3σ range	bf$\pm 1\sigma$	3σ range
$\sin^2\theta_{12}/10^{-1}$	NO, IO	$3.03^{+0.12}_{-0.12}$	$2.70 - 3.41$	$3.18^{+0.16}_{-0.16}$	$2.71 - 3.69$	$3.03^{+0.13}_{-0.13}$	$2.63 - 3.45$
$\sin^2\theta_{23}/10^{-1}$	NO	$4.51^{+0.19}_{-0.16}$	$4.08 - 6.03$	$5.74^{+0.14}_{-0.14}$	$4.34 - 6.10$	$4.55^{+0.18}_{-0.15}$	$4.16 - 5.99$
	IO	$5.69^{+0.16}_{-0.21}$	$4.12 - 6.13$	$5.78^{+0.10}_{-0.17}$	$4.33 - 6.08$	$5.69^{+0.12}_{-0.21}$	$4.17 - 6.06$
$\sin^2\theta_{13}/10^{-2}$	NO	$2.225^{+0.056}_{-0.059}$	$2.052 - 2.398$	$2.200^{+0.069}_{-0.062}$	$2.000 - 2.405$	$2.23^{+0.07}_{-0.06}$	$2.04 - 2.44$
	IO	$2.223^{+0.058}_{-0.058}$	$2.048 - 2.416$	$2.225^{+0.064}_{-0.070}$	$2.018 - 2.424$	$2.23^{+0.06}_{-0.06}$	$2.03 - 2.45$
δ/π	NO	$1.29^{+0.20}_{-0.14}$	$0.80 - 1.94$	$1.08^{+0.13}_{-0.12}$	$0.71 - 1.99$	$1.24^{+0.18}_{-0.13}$	$0.77 - 1.97$
	IO	$1.53^{+0.12}_{-0.16}$	$1.08 - 1.91$	$1.58^{+0.15}_{-0.16}$	$1.11 - 1.96$	$1.52^{+0.15}_{-0.11}$	$1.07 - 1.90$
$\Delta m^2_{21}/10^{-5}\text{eV}^2$	NO, IO	$7.41^{+0.21}_{-0.20}$	$6.82 - 8.03$	$7.50^{+0.22}_{-0.20}$	$6.94 - 8.14$	$7.36^{+0.16}_{-0.15}$	$6.93 - 7.93$
$\left\|\Delta m^2_{atm}\right\|/10^{-3}\text{eV}^2$	NO	$2.507^{+0.026}_{-0.027}$	$2.427 - 2.590$	$2.55^{+0.02}_{-0.03}$	$2.47 - 2.63$	$2.485^{+0.023}_{-0.031}$	$2.401 - 2.565$
	IO	$2.486^{+0.028}_{-0.025}$	$2.406 - 2.570$	$2.45^{+0.02}_{-0.03}$	$2.37 - 2.53$	$2.455^{+0.030}_{-0.025}$	$2.376 - 2.541$



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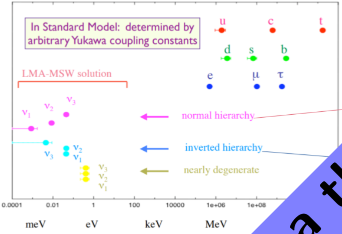
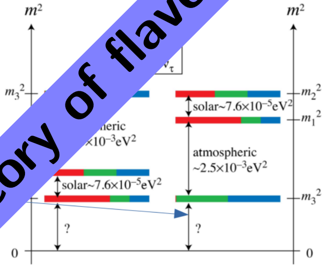
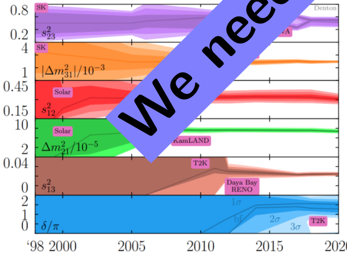




We need a theory of flavor!!



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Neutrino Mixing: General Frameworks

Anarchy

- Neutrino mixing anarchy is the hypothesis that the leptonic mixing matrix can be described as the result of a random draw from an unbiased distribution of unitary 3×3 matrices.
- Random analysis without imposing prior theories or symmetries on the mass and mixing matrices.
- This hypothesis does not make any correlation among the neutrino masses and mixing parameters

de Gouvea, Haba, Hall, Murayama : 9911341, 0009174, 1204.1249

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Texture

- More specific studies with imposed mass or mixing textures for which models with underlying symmetries can be sought.
- It's an intermediate approach
- Some texture zeros of neutrino mass matrices can be eliminated.

Alejandro Ibarra, Graham Ross: Phys.Lett.B 2003

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- The symmetry-based approach to explain the non-trivial mixing in the lepton sector known as family symmetry or horizontal symmetry

Reviews: Tanimoto 1003.3552, Altarelli, Feruglio 1002.0211, King 1301.1340, Chauhan *et.al*. 2310.20681 (BK).

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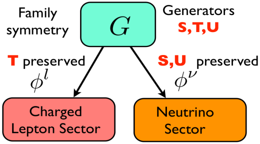
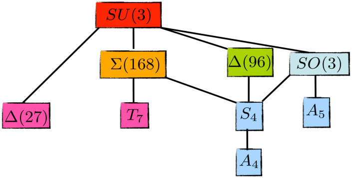
- Flavor from Fractal Mass Chains, Ibarra, Singh, Vempati 2509.04811

General Framework: Symmetry based approach

- Fundamental symmetry in the lepton sector can easily explain the origin of neutrino mixing which is considerably different from quark mixing.
- Incidentally, both Abelian or non-Abelian family symmetries have potential to shade light on the Yukawa couplings.
- The Abelian symmetries (such as Froggatt-Nielsen symmetry) only points towards a hierarchical structure of the Yukawa couplings.
- Non-Abelian symmetries are more equipped to explain the non-hierarchical structures of the observed lepton mixing as observed by the oscillation experiments.





S. F. King 1301.1340

$\boxed{G_f \rightarrow G_e, G_\nu}$ typically, $G_e = Z_3$ and $G_\nu = Z_2 \times Z_2$.

Flavor symmetries, why?

$$U_{PMNS} = \begin{pmatrix} C_{12}C_{13} & S_{12}C_{13} & S_{13}e^{-i\delta} \\ -S_{12}C_{23} - C_{12}S_{13}S_{23}e^{i\delta} & C_{12}C_{23} - S_{12}S_{13}S_{23}e^{i\delta} & C_{13}S_{23} \\ S_{12}S_{23} - C_{12}S_{13}C_{23}e^{i\delta} & -C_{12}S_{23} - S_{12}S_{13}C_{23}e^{i\delta} & C_{13}C_{23} \end{pmatrix}$$

$\Downarrow$

(Prior to 2012)

$$\boxed{s_{23} = 1/\sqrt{2}\ (\theta_{23} = 45^\circ)\ \text{and}\ \theta_{13} = 0}$$

$\Downarrow$

$$U_0 = \begin{pmatrix} c_{12} & s_{12} & 0 \\ -\frac{s_{12}}{\sqrt{2}} & \frac{c_{12}}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ -\frac{s_{12}}{\sqrt{2}} & \frac{c_{12}}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}.$$

$\theta_{12} = 45^\circ (s_{12} = 1/\sqrt{2})$ Bimaximal Mixing

$\theta_{12} = 35.26^\circ (s_{12} = 1/\sqrt{3})$ Tribimaximal Mixing

$\theta_{12} = 31.7^\circ$ Golden Ratio Mixing

$\theta_{12} = 30^\circ (s_{12} = 1/2)$ Hexagonal Mixing

$$U_0 = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \\ \frac{1}{2} & -\frac{1}{2} & \frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} \frac{\varphi}{\sqrt{2+\varphi}} & \frac{1}{\sqrt{2+\varphi}} & 0 \\ \frac{-1}{\sqrt{4+2\varphi}} & \frac{\varphi}{\sqrt{4+2\varphi}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{4+2\varphi}} & \frac{-\varphi}{\sqrt{4+2\varphi}} & \frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} \sqrt{\frac{3}{4}} & \frac{1}{2} & 0 \\ -\frac{1}{2\sqrt{2}} & \frac{\sqrt{3}}{2\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{2\sqrt{2}} & \frac{\sqrt{3}}{2\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

Fukugita, Tanimoto, Yanagida PRD98; Harrison Perkins, Scott PLB02; Dutta,Ramond NPB03; Rodejohann et. al. EPJC10

(GR: $\tan\theta_{12} = 1/\phi$ where $\phi = (1 + \sqrt{5})/2$)

Flavor symmetries, why?

Simple example: $\mu - \tau$ permutation symmetry and TBM

$$m_\nu = U_0^\star \,\mathrm{diag}(m_1, m_2, m_3) U_0^\dagger ,$$

such a mixing matrices can easily diagonalize a $\mu - \tau$ symmetric (transformations $\nu_e \to \nu_e$, $\nu_\mu \to \nu_\tau$, $\nu_\tau \to \nu_\mu$ under which the neutrino mass term remains unchanged) neutrino mass matrix of the form

$$m_\nu = \begin{pmatrix} A & B & B \\ B & C & D \\ B & D & C \end{pmatrix},$$

With $A + B = C + D$ this matrix yields tribimaximal mixing pattern where $s_{12} = 1/\sqrt{3}$ *i.e.*, $\theta_{12} = 35.26^\circ$

- Compatible Mixing Matrix :

$$U_{\rm TB} \simeq \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

Non-zero θ_{13}

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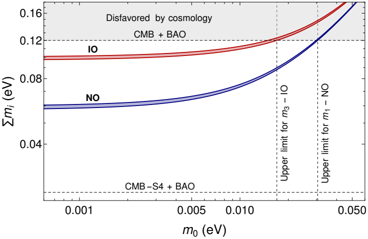
Bimaximal Mixing | Tribimaximal Mixing | Golden Ratio Mixing | Hexagonal Mixing

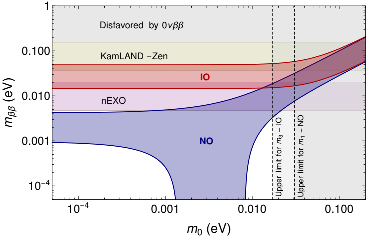
$$U_0 = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \\ \frac{1}{2} & -\frac{1}{2} & \frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} \frac{\varphi}{\sqrt{2+\varphi}} & \frac{1}{\sqrt{2+\varphi}} & 0 \\ \frac{-1}{\sqrt{4+2\varphi}} & \frac{\varphi}{\sqrt{4+2\varphi}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{4+2\varphi}} & \frac{-\varphi}{\sqrt{4+2\varphi}} & \frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} \sqrt{\frac{3}{4}} & \frac{1}{2} & 0 \\ -\frac{1}{2\sqrt{2}} & \frac{\sqrt{3}}{2\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{2\sqrt{2}} & \frac{\sqrt{3}}{2\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

Decendents of fixed pattern mixing schemes

Dirac or Majorana Particle??

Neutrino Mass : Cosmology to $0\nu\beta\beta$



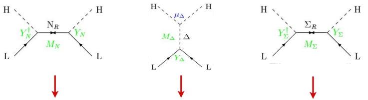


- Absolute neutrino mass : $m_\nu^2 < 0.9$ eV^2 (The KATRIN Collaboration 2022)

Neutrino Mass Generation

Seesaw frameworks

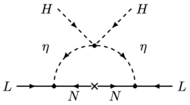
Right-handed singlet: (type-I seesaw)	Scalar triplet: (type-II seesaw)	Fermion triplet: (type-III seesaw)



$m_\nu = Y_N^T \frac{1}{M_N} Y_N v^2$ $\quad$ $m_\nu = Y_\Delta \frac{\mu_\Delta}{M_\Delta^2} v^2$ $\quad$ $m_\nu = Y_\Sigma^T \frac{1}{M_\Sigma} Y_\Sigma v^2$

- **Type-I Seesaw, Type-II Seesaw, Type-III Seesaw, etc.:** Minkowski 77; Gellman, Ramond, Slansky 80; Glashow, Yanagida 79; Mohapatra, Senjanovic 80; Lazarides, Shafi; Schechter, Valle 81; Schechter, Valle 80; Mohapatra, Senjanovic 81; Lazarides, Shafi, Wetterich 81; Mohapatra Valle 86; Foot, Lew, He, Joshi 89; Ma 98; Bajc, Senjanovic 07....

Radiative neutrino mass

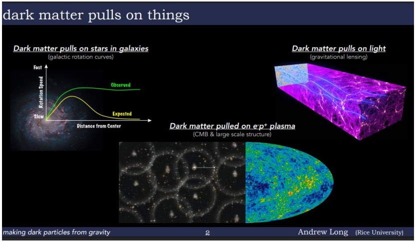


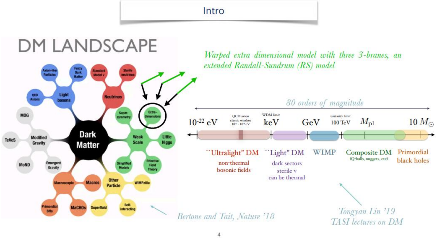
- **Radiative models, started in 80s:** Zee 80, Cheng, Li 80; Zee 86; Babu 88; Babu, Ma, Valle, 02; Ma 06;
- **For a review of radiative models:** Cai, Herrero-Garcia, Schmidt, Vicente, Volkas 17;

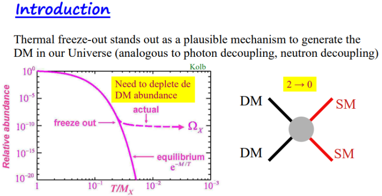
Hybrid Scenarios?? $\Rightarrow$ This talk

Non-exhaustive list for Majorana mass + Dirac counterparts

Dark Matter



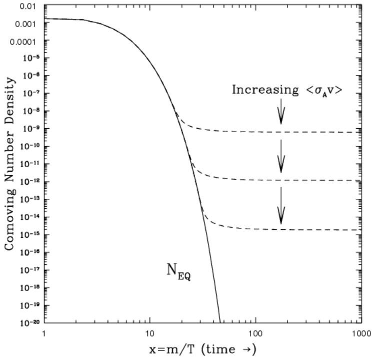




Salars 2025 : A. Hryczuk, F. Koutroulis, A. Long, A. Ibarra, R. Kolb......

Dark Matter: WIMP Miracle

- Early Universe: DM in thermal equilibrium with the Standard Model.
- Due to the expansion of the Universe DM particles fall out of chemical equilibrium and cannot annihilate anymore.
- A relic density of DM is obtained which remains constant $\Rightarrow$ Collisionless cold WIMP Dark Matter
- $\Omega_{\rm DM} h^2 \approx \frac{3\times 10^{-27} cm^3 s^{-1}}{<\sigma v>}$

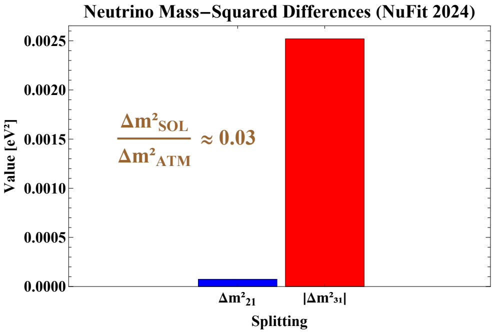


WIMP DM typically requires: $<\sigma v>$ few $\sim 10^{-27} cm^3 s^{-1} \Rightarrow$ GeV to TeV masses, $\mathcal{O}(1)$ couplings DM-SM

$$\Omega_{\rm DM} h^2 \simeq 0.12$$

talk by Rojalin Padhan

Hybrid Mass Mechanisms: Why?

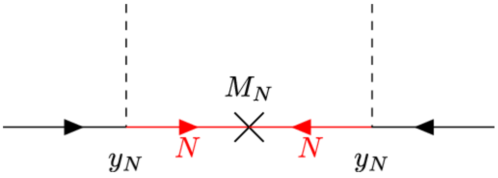


- Ratio of solar to atmospheric mass difference :

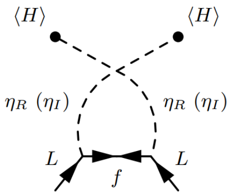
$$r = \Delta m^2_{21}/|\Delta m^2_{31}| \quad \sim \quad 0.03$$

- Two different mass scales that might originate from entirely separate mechanisms !!

Seesaw Vs. Scotogenic Mechanisms



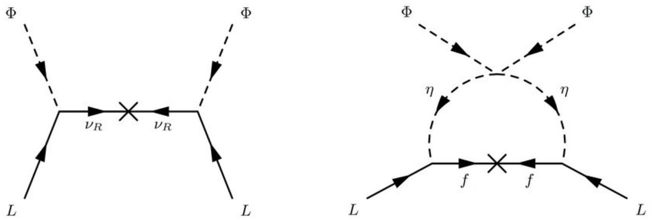
Type-I Seesaw, neutrino mass $\Rightarrow m_\nu = \frac{y_N^2 v^2}{M_N}$ +RHNs



- 'Scotos' : Dark
- f, η : $\mathcal{Z}_2$ odd

Scotogenic mass $\Rightarrow m_\nu = \mathcal{F}(m_{\eta_R}, m_{\eta_I}, M_f) M_f Y_f^i Y_f^j$ $+f, \eta$

Scoto-Seesaw Mechanisms: ν mass hierarchy and DM



- Minimal scoto-seesaw scenario: SM $+\ N, f, \eta$

Rojas, Srivastava, Valle 1807.11447

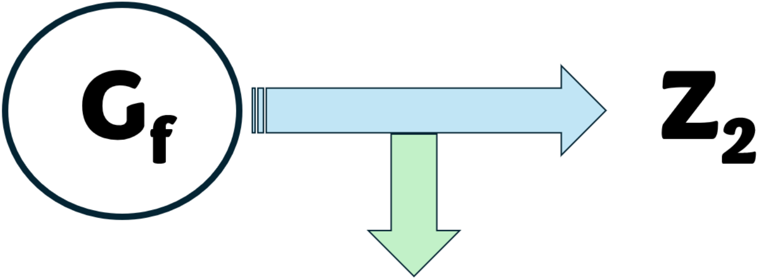
$$\mathrm{M}_\nu = \frac{v^2}{M_N} Y_N^i Y_N^j + \mathcal{F}(m_{\eta_R}, m_{\eta_I}, M_f) M_f Y_f^i Y_f^j$$

$$\frac{\Delta m^2_{\rm SOL}}{\Delta m^2_{\rm ATM}} \sim \left(\frac{1}{32\pi^2}\right)^2 \lambda_5^2 \left(\frac{M_N M_f}{M_f^2 - m_\eta^{(R)2}}\right)^2 \left(\frac{\mathbb{Y}^2_{(f)}}{\mathbb{Y}^2_{(N)}}\right)^2$$

$$M_N \sim 10^{12}\,\mathrm{GeV},\ M_f \sim 10^4\,\mathrm{GeV},\ m_\eta^{(R)} \sim 10^3\,\mathrm{GeV},\ \mathbb{Y}_{(N)} \sim 10^{-1},\ \mathbb{Y}_{(f)} \sim 10^{-4}$$

- Seesaw Analog : Sequential dominance: Antusch, King 0405272

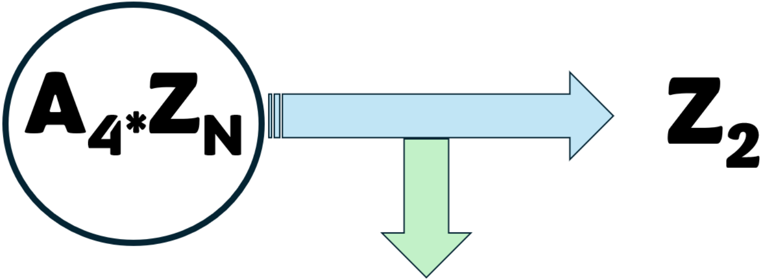
DM & Neutrino Mixing: Flavor symmetry approach



Leptonic Flavor Structure, Neutrino Mixing

low energy signatures

DM & Neutrino Mixing: Flavor symmetry approach



Leptonic Flavor Structure, Neutrino Mixing

low energy signatures

Flavor symmetric scoto-seesaw: TM mixing

$$U_{TBM} = \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}, \quad U_{\rm PMNS} \simeq \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \epsilon \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}}(?) \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}}(?) \end{pmatrix}$$

$$|U_{\rm TM_1}| = \begin{pmatrix} \frac{2}{\sqrt{6}} & * & * \\ \frac{1}{\sqrt{6}} & * & * \\ \frac{1}{\sqrt{6}} & * & * \end{pmatrix} \qquad |U_{\rm TM_2}| = \begin{pmatrix} * & \frac{1}{\sqrt{3}} & * \\ * & \frac{1}{\sqrt{3}} & * \\ * & \frac{1}{\sqrt{3}} & * \end{pmatrix},$$

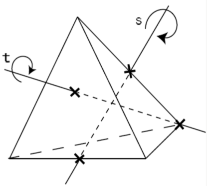
- If S_4 is considered to be broken spontaneously into $Z_3 = \{1, T, T^2\}$ (for the charged lepton sector) $Z_2 = \{1, SU\}$ (for the neutrino sector) such that it satisfies : $[T, M_\ell^\dagger M_\ell] = [SU, M_\nu] = 0$

$$U_{\rm TM_1} = \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{c_\theta}{\sqrt{3}} & \frac{s_\theta}{\sqrt{3}} e^{-i\gamma} \\ -\frac{1}{\sqrt{6}} & \frac{c_\theta}{\sqrt{3}} - \frac{s_\theta}{\sqrt{2}} e^{i\gamma} & -\frac{s_\theta}{\sqrt{3}} e^{-i\gamma} - \frac{c_\theta}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{c_\theta}{\sqrt{3}} - \frac{s}{\sqrt{2}} e^{i\gamma} & -\frac{s_\theta}{\sqrt{3}} e^{-i\gamma} + \frac{c_\theta}{\sqrt{2}} \end{pmatrix}, \; U_{\rm TM_2} = \begin{pmatrix} \frac{2c_\theta}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{2s_\theta}{\sqrt{6}} e^{-i\gamma} \\ -\frac{c_\theta}{\sqrt{6}} + \frac{s}{\sqrt{2}} e^{i\gamma} & \frac{1}{\sqrt{3}} & -\frac{s_\theta}{\sqrt{3}} e^{-i\gamma} - \frac{c_\theta}{\sqrt{2}} \\ -\frac{c_\theta}{\sqrt{6}} + \frac{s}{\sqrt{2}} e^{i\gamma} & \frac{1}{\sqrt{3}} & -\frac{s_\theta}{\sqrt{3}} e^{-i\gamma} + \frac{c_\theta}{\sqrt{2}} \end{pmatrix}$$

Flavor symmetric scoto-seesaw:

Standard Model with A4 discrete flavor symmetry

- A_4 is considered to be a favored symmetry in the neutrino sector
- Even permutation of 4 objects/invariant group of a tetrahedron
- Minimal group which contains 3 dim. representation (can accommodate three flavors of leptons)
- Product rule: $3 \otimes 3 = 1 \oplus 1' \oplus 1'' \oplus 3_A \oplus 3_S$
- $1 \otimes 1 = 1,\ 1' \otimes 1' = 1'', 1' \otimes 1'' = 1$
  $1'' \otimes 1'' = 1'$ etc



Flavor symmetric scoto-seesaw : TM$_1$ mixing

- L, ϕ_a and $\phi_s \to A_4$ triplets; H, $N_R \to A_4$ singlets
- A_4 multiplication rules: If we have two triplets (a_1, a_2, a_3) and (b_1, b_2, b_3), their products are given by $\Rightarrow 3 \otimes 3 = 1 + 1' + 1'' + 3_A + 3_S$

$$\begin{aligned} 1 &\sim a_1 b_1 + a_2 b_3 + a_3 b_2 , 1' \sim a_3 b_3 + a_1 b_2 + a_2 b_1 , 1'' \sim a_2 b_2 + a_3 b_1 + a_1 b_3 , \\ 3_S &\sim \begin{bmatrix} 2a_1 b_1 - a_2 b_3 - a_3 b_2 \\ 2a_3 b_3 - a_1 b_2 - a_2 b_1 \\ 2a_2 b_2 - a_1 b_3 - a_3 b_1 \end{bmatrix}, 3_A \sim \begin{bmatrix} a_2 b_3 - a_3 b_2 \\ a_1 b_2 - a_2 b_1 \\ a_3 b_1 - a_1 b_3 \end{bmatrix}. \end{aligned}$$

- Contributions to the neutrino mass: Ganguly, Gluza, BK, Mahapatra, 2311.15997

$$\begin{aligned} \mathcal{L} &= \frac{y_N}{\Lambda}(\bar{L}\phi_s)\tilde{H}N_R + \frac{1}{2}M_N \bar{N}_R^c N_R + \frac{y_s}{\Lambda^2}(\bar{L}\phi_a)\xi i\sigma_2 \eta^* f + \frac{1}{2}M_f \bar{f}^c f + h.c., \\ M_\nu &= -\frac{v^2}{M_N} Y_N^i Y_N^j + \mathcal{F}(m_{\eta_R}, m_{\eta_I}, M_f) M_f Y_f^i Y_f^j \end{aligned}$$

- flavon fields get VEVs along $\langle \phi_s \rangle = (0, -v_s, v_s)$, $\langle \phi_a \rangle = (2v_a, v_a, v_a)$

$$\frac{y_N}{\Lambda}(\bar{L}\phi_s)_1 \tilde{H}N_R = \frac{y_N}{\Lambda}(\bar{L_1}\phi_{s1} + \bar{L_2}\phi_{s3} + \bar{L_3}\phi_{s2})_1 \tilde{H}N_R = \frac{y_N}{\Lambda}(0 - \bar{L_2}v_s + \bar{L_3}v_s)_1 \tilde{H}N_R$$

Flavor symmetric scoto-seesaw : TM_1 mixing

$$Y_N = (Y_N^e, Y_N^\mu, Y_N^\tau)^T = (0, y_N \frac{v_s}{\Lambda}, -y_N \frac{v_s}{\Lambda})^T, ;$$

$$Y_F = (Y_F^e, Y_F^\mu, Y_F^\tau)^T = (y_s \frac{v_\xi}{\Lambda}\frac{v_a}{\Lambda}, y_s \frac{v_\xi}{\Lambda}\frac{2v_a}{\Lambda}, 0)^T$$

- Light neutrino mass matrix :

$$M_\nu = \begin{pmatrix} b & 2b & 0 \\ 2b & -a+4b & a \\ 0 & a & -a \end{pmatrix},$$

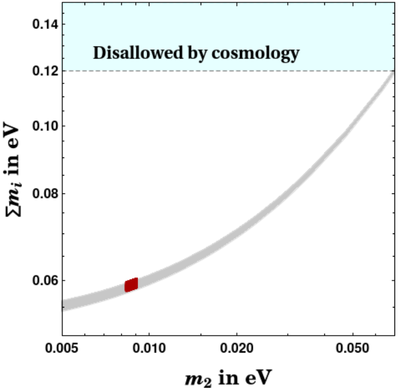
$$a = y_N^2 \frac{v^2}{M_N}\frac{v_s^2}{\Lambda^2}, b = y_s^2 \frac{v_\xi^2}{\Lambda^2}\frac{v_a^2}{\Lambda^2}\mathcal{F}(m_{\eta_R}, m_{\eta_I}, M_f)M_f$$

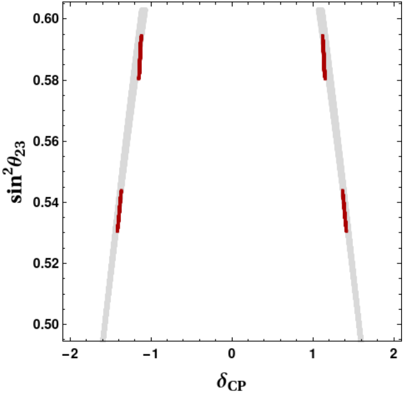
- Diagonalizing matrix:

$$U_\nu = \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{\cos\theta}{\sqrt{3}} & \frac{e^{-i\psi}\sin\theta}{\sqrt{3}} \\ -\frac{1}{\sqrt{6}} & \frac{\cos\theta}{\sqrt{3}} + \frac{e^{i\psi}\sin\theta}{\sqrt{2}} & -\frac{\cos\theta}{\sqrt{2}} + \frac{e^{-i\psi}\sin\theta}{\sqrt{3}} \\ -\frac{1}{\sqrt{6}} & \frac{\cos\theta}{\sqrt{3}} - \frac{e^{i\psi}\sin\theta}{\sqrt{2}} & \frac{\cos\theta}{\sqrt{2}} + \frac{e^{-i\psi}\sin\theta}{\sqrt{3}} \end{pmatrix} U_m$$

$$\sin\theta_{13} e^{-i\delta_{\rm CP}} = \frac{e^{-i\psi}\sin\theta}{\sqrt{3}}, \quad \sin^2\theta_{12} = 1 - \frac{2}{3-\sin^2\theta}, \quad \sin^2\theta_{23} = \frac{1}{2}\Big(1 - \frac{\sqrt{6}\sin 2\theta\cos\psi}{3-\sin^2\theta}\Big).$$

Flavor symmetric scoto-seesaw : TM_1 mixing





m_2 (meV)	m_3 (meV)	$\sum m_i$ (meV)	$m_{\beta\beta}$ (meV)
8.3 – 9.0	49.7 – 51.3	58.0 – 60.3	1.61 – 3.85

Flavor symmetric scoto-seesaw : TM_1 mixing

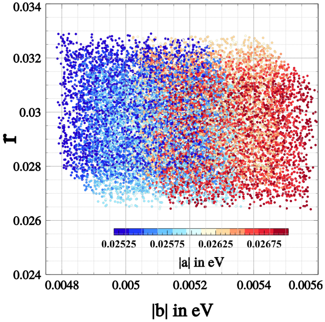
- Mass eigenvalues:

Ganguly, Gluza, BK, Mahapatra, 2311.15997

$$\widetilde{m}_1 = 0,$$
$$\widetilde{m}_2 = \frac{1}{2}\Big(-2a + 5b - \sqrt{4a^2 + 4ab + 25b^2}\Big),$$
$$\widetilde{m}_3 = \frac{1}{2}\Big(-2a + 5b + \sqrt{4a^2 + 4ab + 25b^2}\Big).$$

- Ratio of the solar to atmospheric mass-squared differences:

$$r \sim \frac{m_2^2}{m_3^2}$$



Flavor symmetric scoto-seesaw : TM_2 mixing

Type-I Seesaw contribution:

Ganguly, Gluza, BK 2209.08610

$$\mathcal{L}_{\rm TREE} = \frac{y_{N_1}}{\Lambda}(\bar{L}\phi_s)\tilde{H}N_{R_1} + \frac{y_{N_2}}{\Lambda}(\bar{L}\phi_a)\tilde{H}N_{R_2} + \frac{1}{2}M_{N_1}\bar{N}^c_{R_1}N_{R_1} + \frac{1}{2}M_{N_2}\bar{N}^c_{R_2}N_{R_2} + h.c.,$$

Scotogenic contribution:

$$\begin{aligned}
\mathcal{L}_{\rm LOOP} &= \frac{y_s}{\Lambda^2}(\bar{L}\phi_s)\xi i\sigma_2\eta^* f + \frac{1}{2}M_f\bar{f}^c f + h.c.,\\
(M_\nu)_{\rm LOOP} &= \mathcal{F}(m_{\eta_R}, m_{\eta_I}, M_f)M_f Y_f^i Y_f^j.\\
Y_F &= (Y_F^e, Y_F^\mu, Y_F^\tau)^T = (y_s\frac{v_s}{\Lambda}\frac{v_\xi}{\Lambda}, 0, -y_s\frac{v_s}{\Lambda}\frac{v_\xi}{\Lambda})^T.
\end{aligned}$$

- **Effective neutrino mass matrix:**

$$\begin{aligned}
M_\nu &= -M_D M_R^{-1}M_D^T + (M_\nu)_{\rm LOOP}\\
&= (M_\nu)_{\rm TREE} + (M_\nu)_{\rm LOOP}\\
&= \begin{pmatrix} -B+C & -B & -B-C \\ -B & -A-B & A-B \\ -B-C & A-B & -A-B+C \end{pmatrix}.
\end{aligned}$$

- **After rotation by TBM matrix:**

$$\begin{aligned}
M'_\nu &= U_{TB}^T M_\nu U_{TB}\\
&= \frac{1}{2}\begin{pmatrix} 3C & 0 & -\sqrt{3}C \\ 0 & -6B & 0 \\ -\sqrt{3}C & 0 & -4A+C \end{pmatrix},
\end{aligned}$$

Flavor symmetric scoto-seesaw : TM_2 mixing

- Effective neutrino mixing matrix (TM_2 mixing):

$$U_\nu = \begin{pmatrix} \sqrt{\frac{2}{3}}\cos\theta & \frac{1}{\sqrt{3}} & \sqrt{\frac{2}{3}}e^{i\phi}\sin\theta \\ -\frac{\cos\theta}{\sqrt{6}} + \frac{e^{i\phi}\sin\theta}{\sqrt{2}} & \frac{1}{\sqrt{3}} & -\frac{\cos\theta}{\sqrt{2}} - \frac{e^{i\phi}\sin\theta}{\sqrt{6}} \\ -\frac{\cos\theta}{\sqrt{6}} - \frac{e^{i\phi}\sin\theta}{\sqrt{2}} & \frac{1}{\sqrt{3}} & \frac{\cos\theta}{\sqrt{2}} - \frac{e^{i\phi}\sin\theta}{\sqrt{6}} \end{pmatrix} U_m.$$

- Corelations:

$$\tan\phi = \frac{\alpha\sin\phi_{AC}}{1-\alpha\cos\phi_{AC}}, \quad \tan 2\theta = \frac{\sqrt{3}}{\cos\phi + 2\alpha\cos(\phi_{AC}+\phi)}.$$

- Comparing with U_{PMNS}:

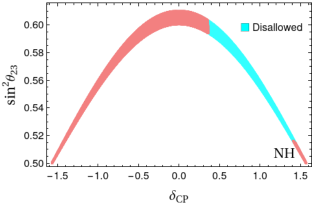
$$\sin\theta_{13}e^{-i\delta_{\rm CP}} = \sqrt{\frac{2}{3}}e^{-i\phi}\sin\theta, \quad \tan^2\theta_{12} = \frac{1}{2-3\sin^2\theta_{13}},$$

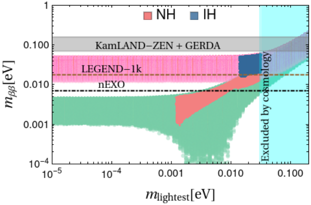
$$\tan^2\theta_{23} = \frac{\left(1+\frac{\sin\theta_{13}\cos\phi}{\sqrt{2-3\sin^2\theta_{13}}}\right)^2 + \frac{\sin^2\theta_{13}\sin^2\phi}{(2-3\sin^2\theta_{13})}}{\left(1-\frac{\sin\theta_{13}\cos\phi}{\sqrt{2-3\sin^2\theta_{13}}}\right)^2 + \frac{\sin^2\theta_{13}\sin^2\phi}{(2-3\sin^2\theta_{13})}}.$$

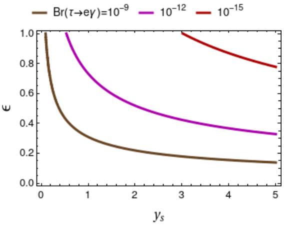
For more see 2209.08610

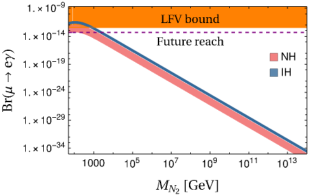
Flavor symmetric scoto-seesaw : TM_2 mixing

- Predictions:





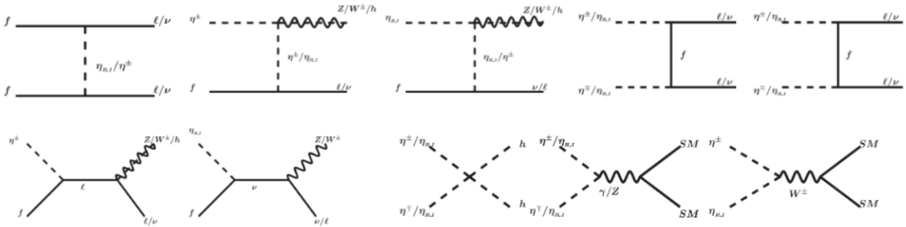


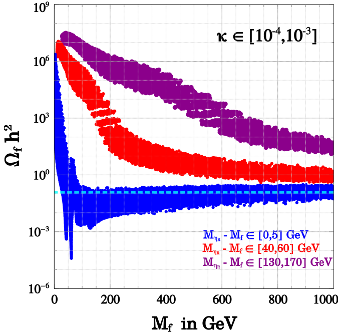


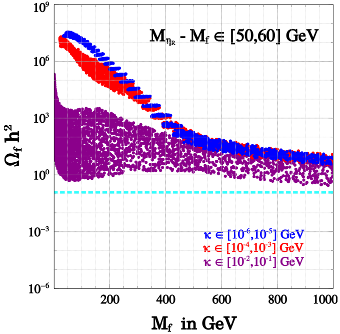
For more see 2209.08610

FSS_1 phenomenology: dark matter

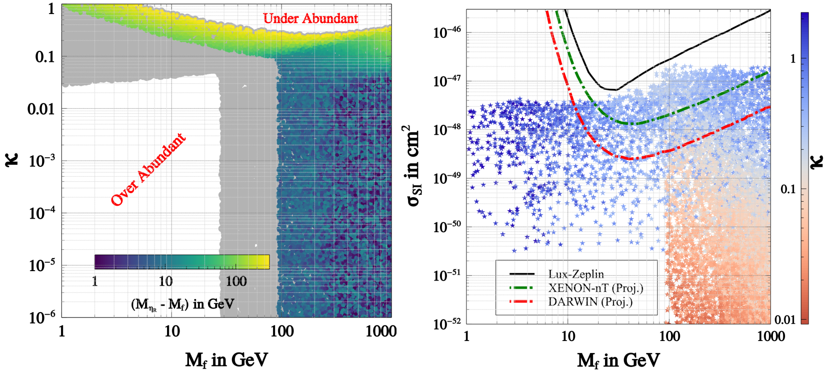
- 2 viable DM candidates ⇒ the **lightest neutral scalar** (Mandal, Srivastava, Valle, 2104.13401)
  ⇒ the **singlet fermion** (Ganguly, Gluza, BK, Mahapatra 2311.15997).







FSS_1 phenomenology: dark matter

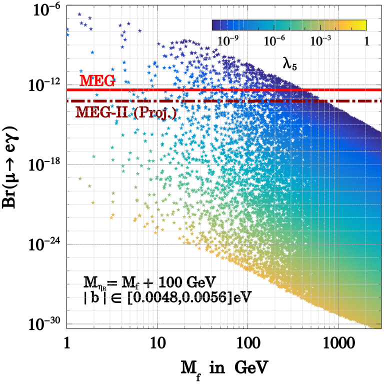


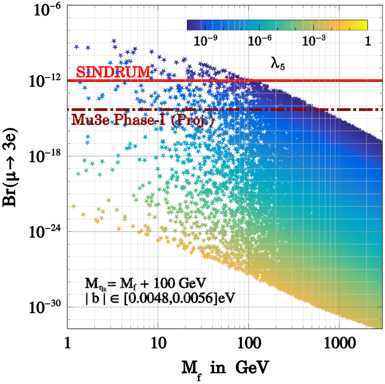
- Neutrino mixing dependence of dark matter phenomenology :

$$\Rightarrow \kappa^2 = \frac{|b|}{\mathcal{F}(M_{\eta_R}, M_{\eta_I}, M_f) M_f}$$

FSS_1 phenomenology: Lepton Flavor Violation

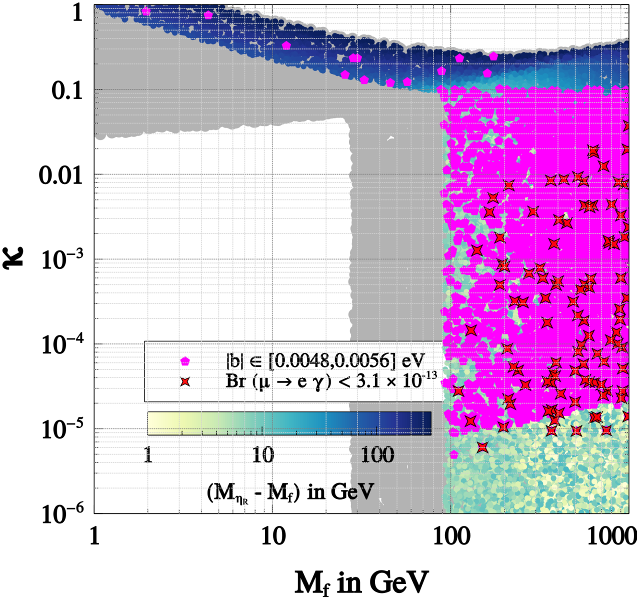
Decay Modes	Scotogenic contribution	Seesaw Contribution	Remarks
$\mu \to e\gamma$	✓	✗	$Y_N^e = 0$
$\tau \to e\gamma$	✗	✗	$Y_F^\tau = 0, Y_N^e = 0$
$\tau \to \mu\gamma$	✗	✓	$Y_F^\tau = 0$
$\mu \to 3e$	✓	✗	$Y_N^e = 0$
$\tau \to 3e$	✗	✗	$Y_F^\tau = 0, Y_N^e = 0$
$\tau \to 3\mu$	✗	✓	$Y_F^\tau = 0$



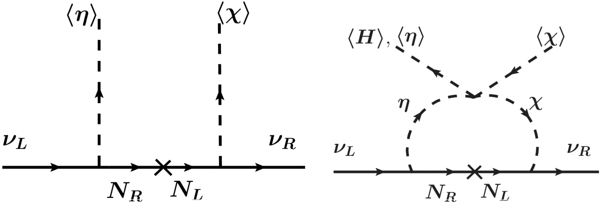


λ_5 is the coupling for the interaction $(H^\dagger \eta)(H^\dagger \eta)$

FSS_1 phenomenology: Summary



Dirac Neutrinos



$$A_4 \otimes Z_N \rightarrow Z_2$$

Borah, Das, BK, Mahapatra 2406.17861

Conclusion

- Is there any guiding principle behind the observed pattern of lepton mixing?
- (Discrete) flavor symmetry is one such potential candidate.
- Tiny neutrino mass may originate from hybrid scoto-seesaw scenarios, explaining the hierarchy of the mass scales involved in neutrino oscillation
- Flavor symmetric breaking into a remnant Z_2 symmetry may explain leptonic flavor structure and DM stability
- Possible frameworks : FSS_1 for TM_1 mixing and FSS_2 for TM_2 mixing.
- Rich phenomenology : $h \to \gamma\gamma$, potential DM candidates, LFV decays.....
- Not covered: Collider prospect of BSM states, leptogenesis, CP properties of heavy neutrinos

Thank you for your attention!!

• Multiplication Rules:

It has four irreducible representations: three one-dimensional and one three dimensional which are denoted by $\mathbf{1}, \mathbf{1}', \mathbf{1}''$ and $\mathbf{3}$ respectively. The multiplication rules of the irreducible representations are given by

$$\mathbf{1} \otimes \mathbf{1} = \mathbf{1}, \mathbf{1}' \otimes \mathbf{1}' = \mathbf{1}'', \mathbf{1}' \otimes \mathbf{1}'' = \mathbf{1}, \mathbf{1}'' \otimes \mathbf{1}'' = \mathbf{1}', \mathbf{3} \otimes \mathbf{3} = \mathbf{1} + \mathbf{1}' + \mathbf{1}'' + \mathbf{3_a} + \mathbf{3_s} \tag{2}$$

where **a** and **s** in the subscript corresponds to anti-symmetric and symmetric parts respectively. Now, if we have two triplets as $A = (a_1, a_2, a_3)^T$ and $B = (b_1, b_2, b_3)^T$ respectively, their direct product can be decomposed into the direct sum mentioned above. The product rule for this two triplets in the S diagonal basis[1] can be written as

$$(A \times B)_{\mathbf{1}} \backsim a_1 b_1 + a_2 b_2 + a_3 b_3, \tag{3}$$
$$(A \times B)_{\mathbf{1}'} \backsim a_1 b_1 + \omega^2 a_2 b_2 + \omega a_3 b_3, \tag{4}$$
$$(A \times B)_{\mathbf{1}''} \backsim a_1 b_1 + \omega a_2 b_2 + \omega^2 a_3 b_3, \tag{5}$$
$$(A \times B)_{\mathbf{3_s}} \backsim (a_2 b_3 + a_3 b_2, a_3 b_1 + a_1 b_3, a_1 b_2 + a_2 b_1), \tag{6}$$
$$(A \times B)_{\mathbf{3_a}} \backsim (a_2 b_3 - a_3 b_2, a_3 b_1 - a_1 b_3, a_1 b_2 - a_2 b_1), \tag{7}$$

here ω $(= e^{2i\pi/3})$ is the cube root of unity

[1] Here S is a 3×3 diagonal generator of A_4.