NICPB KBFI

CONNECTING SCIENCES

$\tilde{\xi}$-attractors in metric-affine gravity

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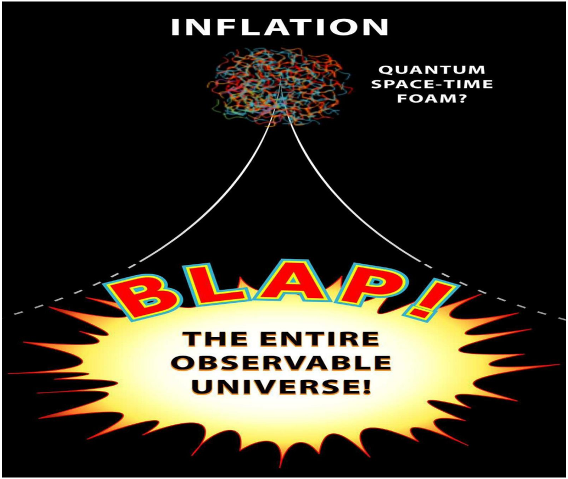
based on
JCAP 04 (2025) 084
(arXiv:2411.08031)

Euroopa Liit
Euroopa Sotsiaalfond

Eesti tuleviku heaks

• Inflation •



- Solution for:
  - ◇ horizon problem
  - ◇ flatness problem
- Realizable with a CC:

$$\sqrt{-g}\mathcal{L} = \sqrt{-g}\left[\frac{M_P^2 R}{2} - \Lambda^4\right]$$

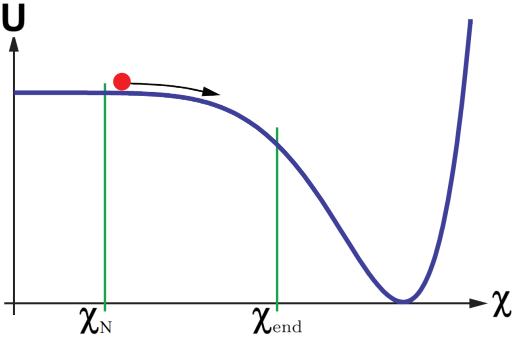
$$\Downarrow$$

FRW: $ds^2 = -dt^2 + a(t)^2 d\mathbf{x}^2$

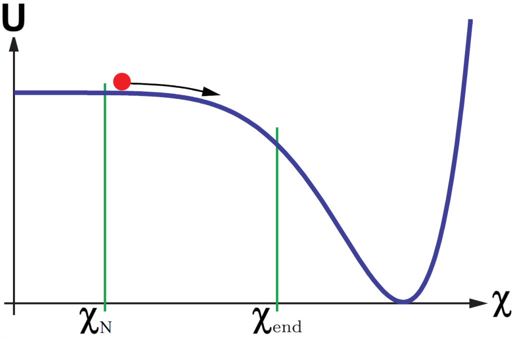
EoM $\rightarrow$ $a(t) \sim e^{\frac{\Lambda^2}{M_P} t}$

- PROBLEM: it never stops!
- SOLVED: inflaton χ with quasi-flat $U(\chi)$

• Slow-roll & inflationary observables •



• Slow-roll & inflationary observables •

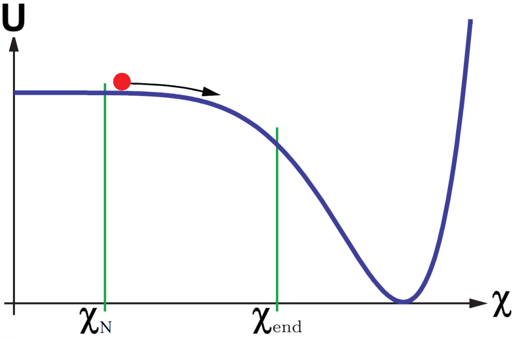


- (potential) SR conditions:

$$\epsilon_U(\chi) = \frac{M_P^2}{2}\left(\frac{U'(\chi)}{U(\chi)}\right)^2 \ll 1 \qquad U(\chi)' = \frac{\partial U}{\partial \chi}$$

$$\eta_U(\chi) = M_P^2 \frac{U''(\chi)}{U(\chi)} \Rightarrow |\eta_U| \ll 1$$

• Slow-roll & inflationary observables •

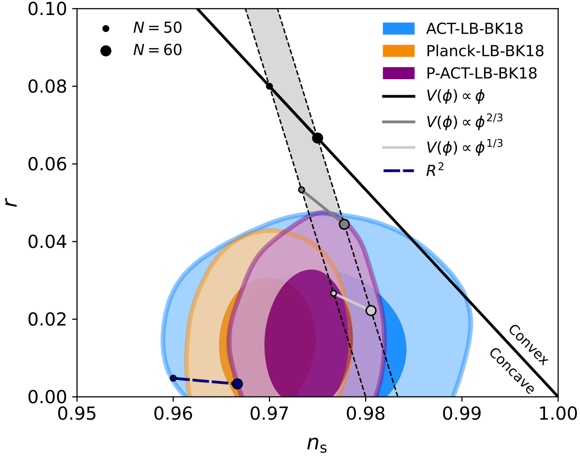


- (potential) SR approx. $\Rightarrow$ observables from U and $\frac{\partial^n U}{\partial \chi^n}$

$$N_e = \frac{1}{M_P^2}\int_{\chi_{\rm end}}^{\chi_N} \mathrm{d}\chi \frac{U(\chi)}{U'(\chi)} \qquad r = 16\epsilon_U(\chi_N)$$

$$A_{\rm s} = \frac{1}{24\pi^2 M_P^4}\frac{U(\chi_N)}{\epsilon_U(\chi_N)} \qquad n_{\rm s} = 1 + 2\eta_U(\chi_N) - 6\epsilon_U(\chi_N)$$

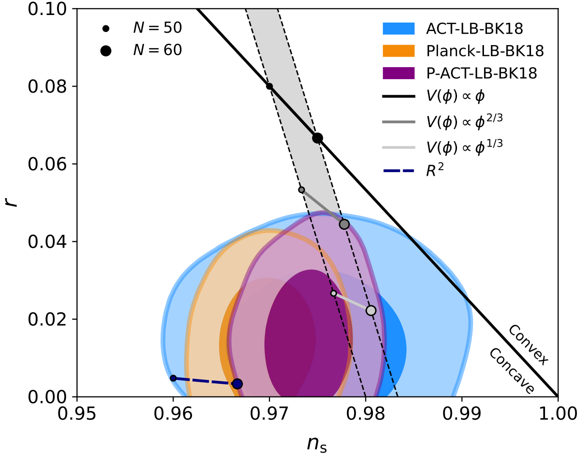
• Inflation & Data •



ACT Collaboration 2503.14454

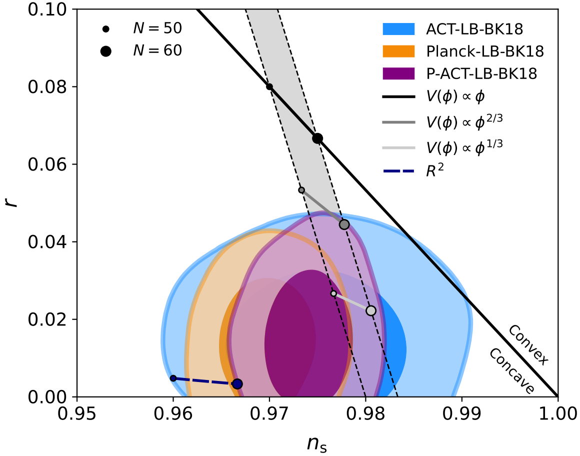
- substantial shift towards higher n_s values

• Inflation & Data •



ACT Collaboration 2503.14454

- substantial shift towards higher n_s values
- O(100) articles (I am guilty twice 😅)



ACT
Collaboration
2503.14454

- substantial shift towards higher n_s values
- O(100) articles → but not this talk...

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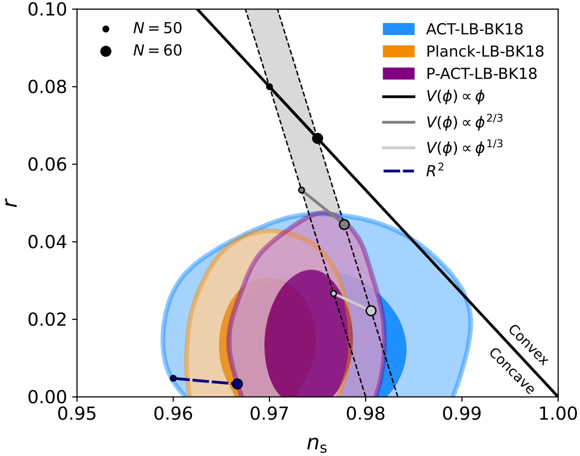

ACT Collaboration 2503.14454

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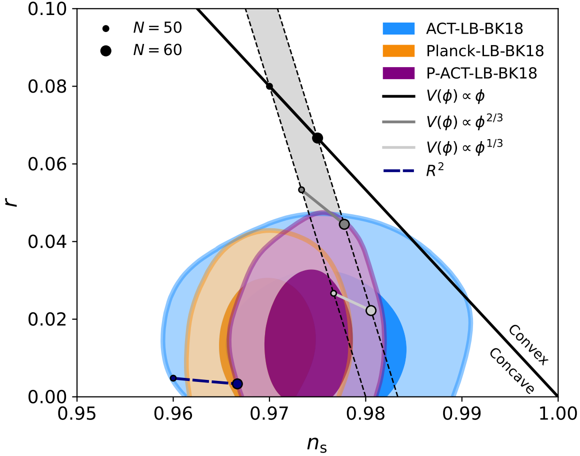

ACT Collaboration 2503.14454

- substantial shift towards higher n_s values
- O(100) articles → but not this talk... actually just a bit
- we need to be cautious: M. Ferreira et al., 2507.12459

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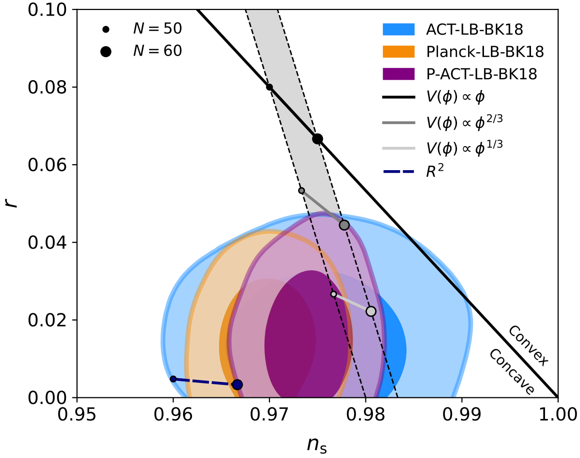

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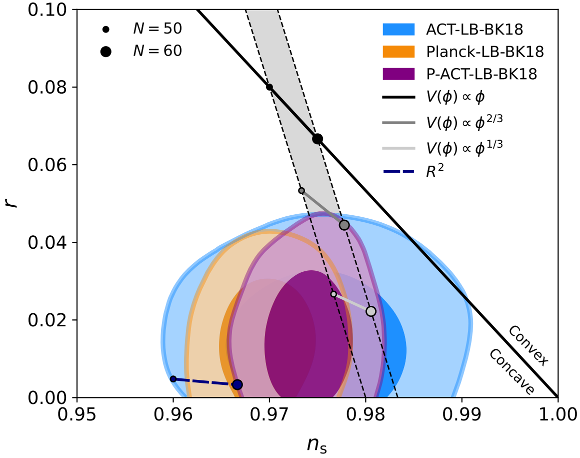
- R^2 disfavored at $N_e = 50$

• Inflation & Data •



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- R^2 disfavored at $N_e = 50$
- (quasi-)FLAT CONCAVE potentials are strongly FAVORED!!!

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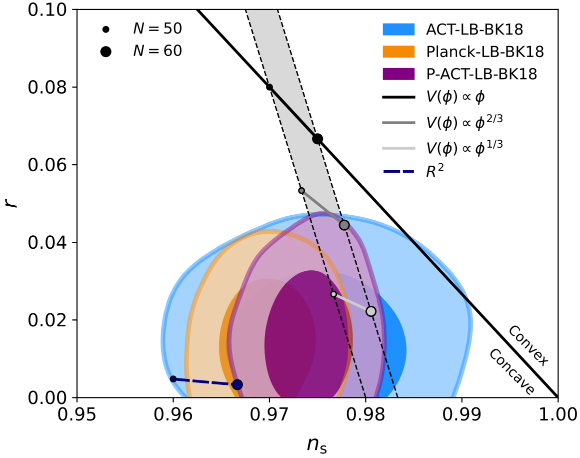

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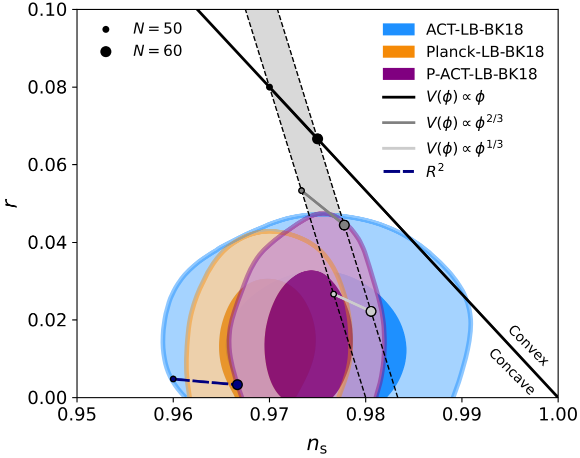
- R^2 disfavored at $N_e = 50$
- (quasi-)FLAT CONCAVE potentials are strongly FAVORED!!!
- many ways to achieve it → see Kallosh & Linde talks



ACT Collaboration 2503.14454

- R^2 disfavored at $N_e = 50$
- (quasi-)FLAT CONCAVE potentials are strongly FAVORED!!!
- many ways to achieve it → now MAG

• MAG preliminaries •

MAG: spacetime described by (e.g. Koivisto & al 0509422,1903.06830):

- the metric tensor: $g_{\mu\nu}$
- the affine connection: $\mathcal{A}^{\lambda}_{\alpha\beta}$

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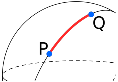

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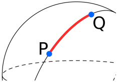

- the affine connection: $\mathcal{A}^{\lambda}_{\alpha\beta} \rightarrow$ parallel transport

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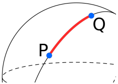
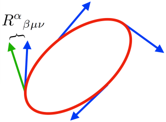


- the affine connection: $\mathcal{A}^{\lambda}_{\alpha\beta} \rightarrow$ parallel transport

If Einsteinian gravity

$$\left\{{}^{\alpha}{}_{\mu\nu}\right\} = \frac{1}{2} g^{\alpha\lambda}\left(g_{\lambda\nu,\mu} + g_{\mu\lambda,\nu} - g_{\mu\nu,\lambda}\right) \quad \text{(Levi-Civita)}$$

$$\Gamma^{\alpha}{}_{\mu\nu} = \left\{{}^{\alpha}{}_{\mu\nu}\right\}$$



curvature

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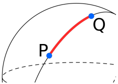
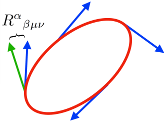


- the affine connection: $\mathcal{A}^{\lambda}_{\alpha\beta} \rightarrow$ parallel transport

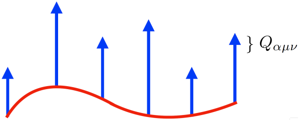
If non-minimal theory of gravity:

$$\left\{{}^{\alpha}{}_{\mu\nu}\right\} = \frac{1}{2} g^{\alpha\lambda}\left(g_{\lambda\nu,\mu} + g_{\mu\lambda,\nu} - g_{\mu\nu,\lambda}\right) \quad \text{(Levi-Civita)}$$

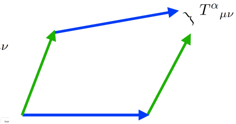
$$\Gamma^{\alpha}{}_{\mu\nu} = \left\{{}^{\alpha}{}_{\mu\nu}\right\} + L^{\alpha}{}_{\mu\nu} + K^{\alpha}{}_{\mu\nu} \qquad \text{unless } K = 0, L = 0 \text{ by hand}$$



curvature



non-metricity



torsion

• MAG preliminaries •

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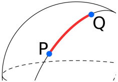
- the metric tensor: $g_{\mu\nu} \to$ distance

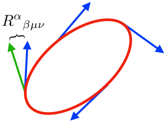


- the affine connection: $\mathcal{A}^{\lambda}_{\alpha\beta} \to$ parallel transport

If non-minimal theory of gravity:

$$\left\{{}^{\alpha}{}_{\mu\nu}\right\} = \frac{1}{2} g^{\alpha\lambda}\left(g_{\lambda\nu,\mu} + g_{\mu\lambda,\nu} - g_{\mu\nu,\lambda}\right) \quad \text{(Levi-Civita)}$$

$$\Gamma^{\alpha}{}_{\mu\nu} = \left\{{}^{\alpha}{}_{\mu\nu}\right\} + L^{\alpha}{}_{\mu\nu} + K^{\alpha}{}_{\mu\nu} \qquad \to \tilde{\mathcal{R}} = g_{\alpha\mu}\epsilon^{\mu\beta\gamma\delta}\mathcal{R}^{\alpha}{}_{\beta\gamma\delta}$$



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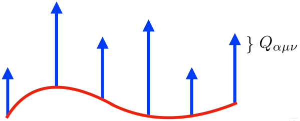

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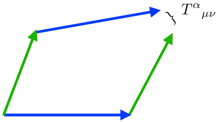

torsion

• MAG preliminaries •

- generic affine connection

$$\Gamma^{\alpha}{}_{\mu\nu} = \left\{{}^{\alpha}{}_{\mu\nu}\right\} + L^{\alpha}{}_{\mu\nu} + K^{\alpha}{}_{\mu\nu}$$

- curvature:

$$\mathcal{R}^{\alpha}{}_{\beta\gamma\delta} = \partial_{\gamma}\Gamma^{\alpha}{}_{\delta\beta} - \partial_{\delta}\Gamma^{\alpha}{}_{\gamma\beta} + \Gamma^{\alpha}{}_{\gamma\mu}\Gamma^{\mu}{}_{\delta\beta} - \Gamma^{\alpha}{}_{\delta\mu}\Gamma^{\mu}{}_{\gamma\beta} \neq 0$$

- non-metricity: $Q_{\alpha\mu\nu} = \nabla_{\alpha} g_{\mu\nu} \neq 0$
- torsion: $T^{\alpha}{}_{\mu\nu} = 2\Gamma^{\alpha}{}_{[\mu\nu]} \neq 0$

$$\text{disformation: } L^{\alpha}{}_{\mu\nu} = \frac{1}{2} Q^{\alpha}{}_{\mu\nu} - Q_{(\mu}{}^{\alpha}{}_{\nu)} \neq 0$$

$$\text{contortion: } K^{\alpha}{}_{\mu\nu} = \frac{1}{2} T^{\alpha}{}_{\mu\nu} + T_{(\mu}{}^{\alpha}{}_{\nu)} \neq 0 \Leftrightarrow \Gamma^{\alpha}{}_{\mu\nu} \neq \Gamma^{\alpha}{}_{\nu\mu}$$

- Holst invariant: $\boxed{\tilde{\mathcal{R}} = g_{\alpha\mu}\epsilon^{\mu\beta\gamma\delta}\mathcal{R}^{\alpha}{}_{\beta\gamma\delta}} \neq 0$

• MAG action •

- starting Jordan frame action with $Q_{\alpha\mu\nu}$, $T^{\alpha}_{\ \mu\nu} \neq 0$

$$S = \int d^4x \sqrt{-g_J} \left[\frac{M_P^2}{2} \left(f(\phi)\mathcal{R}_J + \tilde{f}(\phi)\tilde{\mathcal{R}}_J \right) - \frac{\partial_\mu \phi \, \partial^\mu \phi}{2} - V(\phi) \right]$$

- $f(\phi) > 0 \leftarrow$ nmc with $\mathcal{R}$
- $\tilde{f}(\phi) \gtreqless 0 \leftarrow$ nmc with $\tilde{\mathcal{R}}$

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- starting Jordan frame action with $Q_{\alpha\mu\nu}$, $T^{\alpha}_{\ \mu\nu} \neq 0$

$$S = \int d^4x \sqrt{-g_J} \left[\frac{M_P^2}{2} \left(f(\phi)\mathcal{R}_J + \tilde{f}(\phi)\tilde{\mathcal{R}}_J \right) - \frac{\partial_\mu\phi\,\partial^\mu\phi}{2} - V(\phi) \right]$$

  - $f(\phi) > 0 \leftarrow$ nmc with $\mathcal{R}$
  - $\tilde{f}(\phi) \gtreqless 0 \leftarrow$ nmc with $\tilde{\mathcal{R}}$

- simplest setup ("straight from" $\mathcal{R}_{\mu\nu\alpha\beta}$) with Q and T
  - without new physical dof's in addition to $h_{\mu\nu}$, $\phi \rightarrow$ no $\tilde{\mathcal{R}}^2$
  - without $(\partial\phi)^4$ terms $\rightarrow$ no $\mathcal{R}^2$

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- starting Jordan frame action with $Q_{\alpha\mu\nu}$, $T^{\alpha}_{\ \mu\nu} \neq 0$

$$S = \int d^4x \sqrt{-g_J} \left[\frac{M_P^2}{2} \left(f(\phi) \mathcal{R}_J + \tilde{f}(\phi) \tilde{\mathcal{R}}_J \right) - \frac{\partial_\mu \phi \, \partial^\mu \phi}{2} - V(\phi) \right]$$

– $f(\phi) > 0 \leftarrow$ nmc with $\mathcal{R}$

– $\tilde{f}(\phi) \gtreqless 0 \leftarrow$ nmc with $\tilde{\mathcal{R}}$

- simplest setup ("straight from" $\mathcal{R}_{\mu\nu\alpha\beta}$) with Q and T
  - without new physical dof's in addition to $h_{\mu\nu}$, $\phi \rightarrow$ no $\tilde{\mathcal{R}}^2$
  - without $(\partial\phi)^4$ terms $\rightarrow$ no $\mathcal{R}^2$

- see also
  - Langvik et al., 2007.12595 ($\phi \rightarrow H$)
  - Shaposhnikov et al., 2007.14978 ($\phi \rightarrow H$)
  - Annala, Barker, Gialamas, He, Iosifidis, Karam, Koivisto, Marzo, Räsänen, Rigouzzo, Rubio, Salvio, Tomberg, Zell and many more

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• MAG action •

- starting Jordan frame action with $Q_{\alpha\mu\nu}$, $T^{\alpha}_{\ \mu\nu} \neq 0$

$$S = \int d^4x\sqrt{-g_J}\left[\frac{M_P^2}{2}\left(f(\phi)\mathcal{R}_J + \tilde{f}(\phi)\tilde{\mathcal{R}}_J\right) - \frac{\partial_\mu\phi\,\partial^\mu\phi}{2} - V(\phi)\right]$$

  - $f(\phi) > 0 \leftarrow$ nmc with $\mathcal{R}$
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- simplest setup ("straight from" $\mathcal{R}_{\mu\nu\alpha\beta}$) with Q and T
  - without new physical dof's in addition to $h_{\mu\nu}$, ϕ → no $\tilde{\mathcal{R}}^2$
  - without $(\partial\phi)^4$ terms → no $\mathcal{R}^2$

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  - Langvik et al., 2007.12595 ($\phi \to H$)
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  - in case your name is not mentioned

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- starting Jordan frame action with $Q_{\alpha\mu\nu}$, $T^{\alpha}_{\ \mu\nu} \neq 0$

$$S = \int d^4x \sqrt{-g_J} \left[\frac{M_P^2}{2} \left(f(\phi) \mathcal{R}_J + \tilde{f}(\phi) \tilde{\mathcal{R}}_J \right) - \frac{\partial_\mu \phi \, \partial^\mu \phi}{2} - V(\phi) \right]$$

- it is possible to integrate out the $\tilde{\mathcal{R}}$ term

- obtain an equivalent theory with $Q_{\alpha\mu\nu} \neq 0$, $T^{\alpha}_{\ \mu\nu} = 0$

$$S = \int d^4x \sqrt{-g_J} \left[\frac{M_P^2}{2} f(\phi) \mathcal{R}_J - \left[1 + \frac{6 M_P^2 \left[f' \tilde{f} - f \tilde{f}' \right]^2}{f \left[f^2 + 4 \tilde{f}^2 \right]} \right] \frac{\partial_\mu \phi \, \partial^\mu \phi}{2} - V \right]$$

- the effect of $T^{\alpha}_{\ \mu\nu} \neq 0$ is moved to the inflaton kinetic term

• MAG action •

- starting Jordan frame action with $Q_{\alpha\mu\nu} \neq 0$, $T^{\alpha}_{\ \mu\nu} = 0$

$$S = \int d^4x\sqrt{-g_J}\left[\frac{M_P^2}{2}f(\phi)\mathcal{R}_J - \left[1 + \frac{6M_P^2\left[f'\tilde{f} - f\tilde{f}'\right]^2}{f\left[f^2 + 4\tilde{f}^2\right]}\right]\frac{\partial_\mu\phi\,\partial^\mu\phi}{2} - V\right]$$

- move to the Einstein frame so that $Q_{\alpha\mu\nu} = T^{\alpha}_{\ \mu\nu} = 0$

$$\boxed{S = \int d^4x\sqrt{-g_E}\left[\frac{M_P^2}{2}R_E - \frac{\partial_\mu\chi\,\partial^\mu\chi}{2} - U(\chi)\right]}$$

$$\begin{cases} \frac{d\chi}{d\phi} = \sqrt{\frac{1}{f}\left[1 + \frac{6M_P^2(f'\tilde{f} - f\tilde{f}')^2}{f(f^2+4\tilde{f}^2)}\right]} \\ U(\chi) = \frac{V(\phi(\chi))}{f^2(\phi(\chi))} \end{cases}$$

- the effects of $Q_{\alpha\mu\nu}$, $T^{\alpha}_{\ \mu\nu} \neq 0$ are now moved to $(\partial\chi)^2$ and U

• MAG action •

- starting Jordan frame action with $Q_{\alpha\mu\nu} \neq 0,\ T^{\alpha}_{\ \mu\nu} = 0$

$$S = \int d^4x\sqrt{-g_J}\left[\frac{M_P^2}{2}f(\phi)\mathcal{R}_J - \left[1 + \frac{6M_P^2\left[f'\tilde{f} - f\tilde{f}'\right]^2}{f\left[f^2+4\tilde{f}^2\right]}\right]\frac{\partial_\mu\phi\,\partial^\mu\phi}{2} - V\right]$$

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$$\boxed{S = \int d^4x\sqrt{-g_E}\left[\frac{M_P^2}{2}R_E - \frac{\partial_\mu\chi\,\partial^\mu\chi}{2} - U(\chi)\right]}$$

$$\xrightarrow[\text{with } f=1]{\text{today only}} \begin{cases} \frac{d\chi}{d\phi} = \sqrt{1 + \frac{6M_P^2(\tilde{f}')^2}{(1+4\tilde{f}^2)}} \\ U(\chi) = V(\phi(\chi)) \end{cases}$$

- the effects of $T^{\alpha}_{\ \mu\nu} \neq 0$ are now moved to $(\partial\chi)^2$

- Jordan frame action with $T^{\alpha}_{\ \mu\nu} \neq 0$

$$S = \int d^4x \sqrt{-g} \left[\frac{M_P^2}{2} \left(f(\phi)\mathcal{R} + \tilde{f}(\phi)\tilde{\mathcal{R}} \right) - \frac{\partial_\mu \phi \, \partial^\mu \phi}{2} - V(\phi) \right]$$

- Jordan frame action with $T^{\alpha}_{\ \mu\nu} \neq 0$

$$S = \int d^4x\sqrt{-g}\left[\frac{M_P^2}{2}\left(\ \mathcal{R}\ + \tilde{f}(\phi)\tilde{\mathcal{R}}\right) - \frac{\partial_\mu\phi\,\partial^\mu\phi}{2} - V(\phi)\right]$$

$f(\phi) = 1$

N.B. symmetry: $\tilde{f} \to -\tilde{f}$

- Einstein frame action

$$S = \int d^4x\sqrt{-g_E}\left[\frac{M_P^2}{2}R_E - \left[1 + \frac{6M_P^2\left(\tilde{f}'\right)^2}{\left(1+4\tilde{f}^2\right)}\right]\frac{\partial_\mu\phi\,\partial^\mu\phi}{2} - V(\phi)\right]$$

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$$f(\phi) = 1$$
$$\tilde{f}(\phi) = \tilde{f}_0^2 + \tilde{\xi}\,\Omega(\phi)$$

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• $\tilde{\xi}$-attractors •

- Jordan frame action with $T^{\alpha}_{\ \mu\nu} \neq 0$

$$S = \int d^4x\sqrt{-g}\left[\frac{M_P^2}{2}\left(\mathcal{R} + \tilde{f}(\phi)\tilde{\mathcal{R}}\right) - \frac{\partial_\mu\phi\,\partial^\mu\phi}{2} - V(\phi)\right]$$

$$f(\phi) = 1$$
$$\tilde{f}(\phi) = \tilde{f}_0^2 + \tilde{\xi}\,\Omega(\phi)$$
$$V(\phi) = \Lambda^4\,\Omega(\phi)^2$$

N.B. symmetry: $\tilde{f} \to -\tilde{f}$

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$$S = \int d^4x\sqrt{-g_E}\left[\frac{M_P^2}{2}R_E - \left[1 + \frac{6M_P^2\left(\tilde{f}'\right)^2}{\left(1+4\tilde{f}^2\right)}\right]\frac{\partial_\mu\phi\,\partial^\mu\phi}{2} - V(\phi)\right]$$

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$$S = \int d^4x\sqrt{-g}\left[\frac{M_P^2}{2}\left(\mathcal{R} + \tilde{f}(\phi)\tilde{\mathcal{R}}\right) - \frac{\partial_\mu\phi\,\partial^\mu\phi}{2} - V(\phi)\right]$$

$$f(\phi) = 1$$
$$\tilde{f}(\phi) = \tilde{f}_0^2 + \tilde{\xi}\,\Omega(\phi)$$
$$V(\phi) = \Lambda^4\,\Omega(\phi)^2$$

N.B. symmetry: $\tilde{f} \to -\tilde{f}$

$\Rightarrow$ $\tilde{f}_0^2, \Omega \geq 0,\ \tilde{\xi} \gtreqless 0$

- Einstein frame action

$$S = \int d^4x\sqrt{-g_E}\left[\frac{M_P^2}{2}R_E - \left[1 + \frac{6M_P^2\left(\tilde{f}'\right)^2}{\left(1+4\tilde{f}^2\right)}\right]\frac{\partial_\mu\phi\,\partial^\mu\phi}{2} - V(\phi)\right]$$

• $\tilde{\xi}$-attractors •

$$U(\chi) = \Lambda^4\, \Omega(\phi(\chi))^2$$

$$\left(\frac{d\chi}{d\phi}\right)^2 = k(\phi) = 1 + \frac{6\,\tilde{\xi}^2\,[\Omega(\phi)']^2}{1 + 4\left(\tilde{f}_0{}^2 + \tilde{\xi}\Omega(\phi)\right)^2}$$

$$U(\chi) = \Lambda^4\, \Omega(\phi(\chi))^2$$

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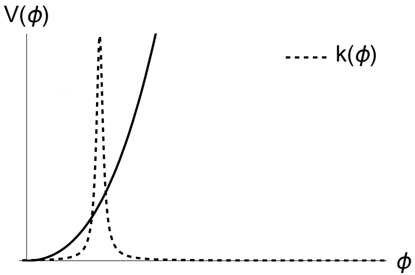
- $k(\phi)$ with a pronounced peak



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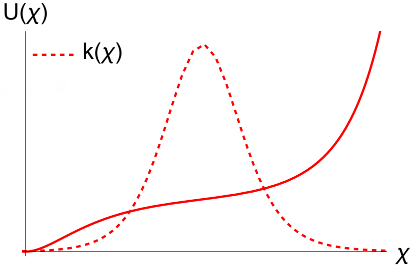
- $k(\phi)$ with a pronounced peak $\rightarrow$ $U(\chi)$ exhibits a flat region



INFLECTION POINT!!!

$$U(\chi) = \Lambda^4\, \Omega(\phi(\chi))^2$$

$$\left(\frac{d\chi}{d\phi}\right)^2 = k(\phi) = 1 + \frac{6\,\tilde{\xi}^2\,[\Omega(\phi)']^2}{1 + 4\left(\tilde{f}_0^{\,2} + \tilde{\xi}\Omega(\phi)\right)^2} \simeq \frac{6\,\tilde{\xi}^2\,[\Omega(\phi)']^2}{1 + 4\left(\tilde{f}_0^{\,2} + \tilde{\xi}\Omega(\phi)\right)^2}$$

- neglecting the "1+" term, we get

$$\chi \simeq -\sqrt{\frac{3}{2}} M_P \left\{ \text{arcsinh}\left[2\left(\tilde{f}_0^{\,2} + \tilde{\xi}\Omega(\phi)\right)\right] - \text{arcsinh}\left(2\tilde{f}_0^{\,2}\right)\right\}$$

$$U(\chi) = \Lambda^4\,\Omega(\phi(\chi))^2$$

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- which can be inverted in function of $\Omega(\phi)$, allowing us to write

$$U(\chi) \simeq \frac{\Lambda^4}{4\tilde{\xi}^2}\left\{\sinh\left[\frac{\sqrt{\frac{2}{3}}\chi}{M_P} - \text{arcsinh}\left(2\tilde{f}_0^{\,2}\right)\right] + 2\tilde{f}_0^{\,2}\right\}^2$$

- cf. Salvio 2207.08830, $\mathcal{L} = \frac{M_P^2}{2}\left(\mathcal{R} + \tilde{f}_0^2\tilde{\mathcal{R}}\right) + c\tilde{\mathcal{R}}^2,\ c = \tilde{\xi}^2\left(\frac{M_P}{4\Lambda}\right)^4$

$$U(\chi) = \Lambda^4\, \Omega(\phi(\chi))^2$$

$$\left(\frac{d\chi}{d\phi}\right)^2 = k(\phi) = 1 + \frac{6\,\tilde{\xi}^2\,[\Omega(\phi)']^2}{1 + 4\left(\tilde{f}_0^{\,2} + \tilde{\xi}\Omega(\phi)\right)^2} \simeq \frac{6\,\tilde{\xi}^2\,[\Omega(\phi)']^2}{1 + 4\left(\tilde{f}_0^{\,2} + \tilde{\xi}\Omega(\phi)\right)^2}$$

- allowing us to write

$$U(\chi) \simeq \frac{\Lambda^4}{4\tilde{\xi}^2}\left\{\sinh\left[\frac{\sqrt{\frac{2}{3}}\chi}{M_P} - \text{arcsinh}\left(2\tilde{f}_0^{\,2}\right)\right] + 2\tilde{f}_0^{\,2}\right\}^2$$

- using the properties of the hyperbolic functions

$$U(\chi)_{\tilde{\mathcal{R}}^2} \simeq \frac{\Lambda^4}{4\tilde{\xi}^2}\left\{\sqrt{1 + 4\tilde{f}_0^{\,4}}\sinh\left(\frac{\sqrt{\frac{2}{3}}\chi}{M_P}\right) - 2\tilde{f}_0^{\,2}\cosh\left(\frac{\sqrt{\frac{2}{3}}\chi}{M_P}\right) + 2\tilde{f}_0^{\,2}\right\}^2$$

• $\tilde{\xi}$-attractors •

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- taking the $\tilde{f}_0 \to \infty$, we obtain the Starobinsky potential

$$U(\chi)_{R^2} \simeq \frac{\Lambda^4\tilde{f}_0^{\,4}}{\tilde{\xi}^2}\left(1 - e^{-\sqrt{\frac{2}{3}}\frac{\chi}{M_P}}\right)^2$$

not in 2207.08830!

• $\tilde{\xi}$-attractors •

$$U(\chi) = \Lambda^4\, \Omega(\phi(\chi))^2$$

$$\left(\frac{d\chi}{d\phi}\right)^2 = k(\phi) = 1 + \frac{6\,\tilde{\xi}^2\,[\Omega(\phi)']^2}{1+4\left(\tilde{f}_0^{\,2} + \tilde{\xi}\Omega(\phi)\right)^2} \simeq \frac{6\,\tilde{\xi}^2\,[\Omega(\phi)']^2}{1+4\left(\tilde{f}_0^{\,2} + \tilde{\xi}\Omega(\phi)\right)^2}$$

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but in He et al., 2402.05358

KBFI NICPB

• Trial example: setup •

$$\Omega(\phi)^2 = \left(\frac{\phi}{M_P}\right)^n$$

$$\left.\begin{array}{l} n \text{ even: } \phi \to -\phi \\ n \text{ odd: } \phi > 0 \end{array}\right\} \Rightarrow \boxed{\phi > 0}$$

$$V(\phi) = \Lambda^4 \left(\frac{\phi}{M_P}\right)^n \qquad \tilde{f}(\phi) = \left[\tilde{f}_0^2 + \tilde{\xi}\left(\frac{\phi}{M_P}\right)^{\frac{n}{2}}\right] \qquad \boxed{\tilde{f}_0^2 > 0, \tilde{\xi} \gtreqless 0}$$

$$k(\phi) = 1 + \frac{6M_P^2\left(\tilde{f}'\right)^2}{\left(1 + 4\tilde{f}^2\right)} \gg 1 \Rightarrow \cancel{1+}\frac{\text{BIG}}{\text{small}} \Rightarrow$$

- $|f'| \gg 1 \Rightarrow |\tilde{\xi}|, \phi/M_P \gg 1$
- $\tilde{f} \sim 0 \Rightarrow \boxed{\tilde{\xi} < 0}$

see also AR & al. 2403.18004, 2412.17738; He & al., 2504.16069

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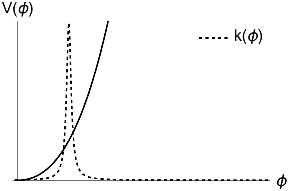
Trial example: $k(\phi)$

- $$k(\phi) = 1 + \frac{3\, n^2\, \tilde{\xi}^2 \left(\frac{\phi}{M_P}\right)^{n-2}}{2\left[1 + 4\left(\tilde{f}_0^{\,2} + \tilde{\xi}\left(\frac{\phi}{M_P}\right)^{n/2}\right)^2\right]}$$



- $$\phi_{\text{peak}} = \left(\frac{\Delta}{\tilde{\xi}}\right)^{2/n} M_P \qquad \Delta = \frac{\tilde{f}_0^{\,2}}{4}\left(n - 4 - \sqrt{n^2 + \frac{2(n-2)}{\tilde{f}_0^{\,4}}}\right)$$

- $\Delta < 0$ & $\tilde{\xi} < 0 \rightarrow$ OK!

- $$k(\phi_{\text{peak}}) = 1 + \left|\tilde{\xi}\right|^{4/n} \frac{3n^2 \Delta^{2-\frac{4}{n}}}{2\left(4\left(\tilde{f}_0^{\,2} + \Delta\right)^2 + 1\right)} \gg 1, \text{ if } |\tilde{\xi}| \gg 1$$

- $$\tilde{f}_0 \gg 1 \Rightarrow \Delta \approx -\tilde{f}_0^2 \Rightarrow \phi_{\text{peak}} \approx \left(\frac{\tilde{f}_0^2}{|\tilde{\xi}|}\right)^{2/n} M_P \Rightarrow \tilde{f}(\phi_{\text{peak}}) \approx 0$$

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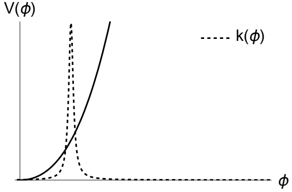
- $k(\phi) = 1 + \dfrac{3\, n^2\, \tilde{\xi}^2 \left(\frac{\phi}{M_P}\right)^{n-2}}{2\left[1 + 4\left(\tilde{f}_0^{\,2} + \tilde{\xi}\left(\frac{\phi}{M_P}\right)^{n/2}\right)^2\right]}$



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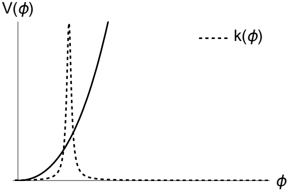

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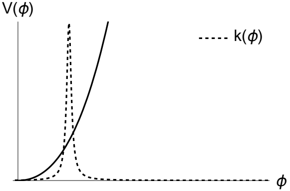
- $\Delta < 0$ & $\tilde{\xi} < 0 \rightarrow$ OK!

- $k(\phi_{\text{peak}}) = 1 + \left|\tilde{\xi}\right|^{4/n} \dfrac{3n^2\Delta^{2-\frac{4}{n}}}{2\left(4\left(\tilde{f}_0^{\,2}+\Delta\right)^2+1\right)} \gg 1$, if $|\tilde{\xi}| \gg 1$

- $\tilde{f}_0 \gg 1 \Rightarrow \Delta \approx -\tilde{f}_0^2 \Rightarrow \phi_{\text{peak}} \approx \left(\dfrac{\tilde{f}_0^2}{|\tilde{\xi}|}\right)^{2/n} M_P \Rightarrow \tilde{f}(\phi_{\text{peak}}) \approx 0$

Trial example: $k(\phi)$

- $k(\phi) = 1 + \dfrac{3\, n^2\, \tilde{\xi}^2 \left(\frac{\phi}{M_P}\right)^{n-2}}{2\left[1 + 4\left(\tilde{f}_0^{\,2} + \tilde{\xi}\left(\frac{\phi}{M_P}\right)^{n/2}\right)^2\right]}$



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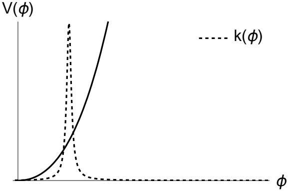
• Trial example: $k(\phi)$ •

- $k(\phi) = 1 + \dfrac{3\, n^2\, \tilde{\xi}^2 \left(\frac{\phi}{M_P}\right)^{n-2}}{2\left[1 + 4\left(\tilde{f}_0^{\,2} + \tilde{\xi}\left(\frac{\phi}{M_P}\right)^{n/2}\right)^2\right]}$



- $\phi_{\text{peak}} = \left(\dfrac{\Delta}{\tilde{\xi}}\right)^{2/n} M_P \qquad \Delta = \dfrac{\tilde{f}_0^{\,2}}{4}\left(n - 4 - \sqrt{n^2 + \dfrac{2(n-2)}{\tilde{f}_0^{\,4}}}\right)$
- $\Delta < 0 \;\&\; \tilde{\xi} < 0 \rightarrow$ OK!
- $k(\phi_{\text{peak}}) = 1 + \left|\tilde{\xi}\right|^{4/n} \dfrac{3n^2\Delta^{2-\frac{4}{n}}}{2\left(4\left(\tilde{f}_0^{\,2} + \Delta\right)^2 + 1\right)} \gg 1,$ if $|\tilde{\xi}| \gg 1$
- $\tilde{f}_0 \gg 1 \Rightarrow \Delta \approx -\tilde{f}_0^2 \Rightarrow \phi_{\text{peak}} \approx \left(\dfrac{\tilde{f}_0^2}{|\tilde{\xi}|}\right)^{2/n} M_P \Rightarrow \tilde{f}(\phi_{\text{peak}}) \approx 0$

• Inflationary results •

- trial example: $\Omega(\phi)^2 = \left(\frac{\phi}{M_P}\right)^n$

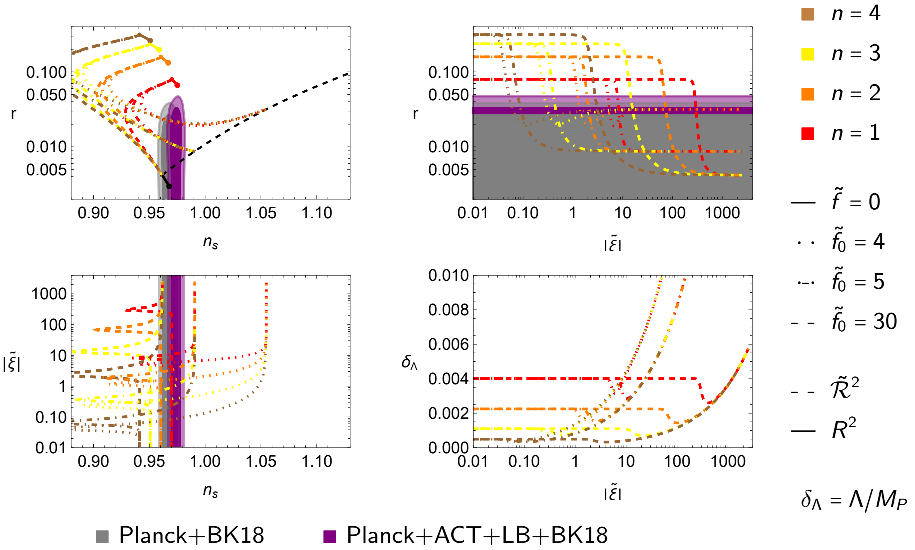
$$V(\phi) = \Lambda^4 \left(\frac{\phi}{M_P}\right)^n \qquad \tilde{f}(\phi) = \left[\tilde{f}_0^2 + \tilde{\xi}\left(\frac{\phi}{M_P}\right)^{\frac{n}{2}}\right] \qquad \boxed{\tilde{f}_0^2 > 0, \tilde{\xi} < 0}$$

- $\tilde{\xi} \to -\infty \Rightarrow \tilde{\mathcal{R}}^2$ limit

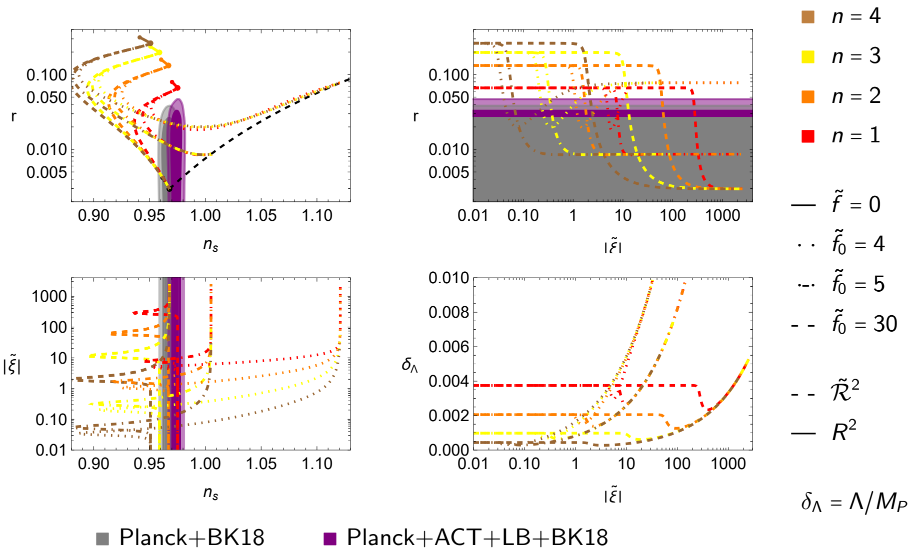
$$\begin{aligned} r &\approx \frac{12}{N_e^2}\left(1 + \frac{4N_e^2}{27\tilde{f}_0^4}\right), \\ n_s &\approx 1 - \frac{2}{N_e}\left(1 - \frac{4N_e^2}{27\tilde{f}_0^4}\right), \\ A_s &\approx \frac{\Lambda^4}{\tilde{\xi}^2 M_P^4}\frac{\tilde{f}_0^4 N_e^2}{72\pi^2}\left(1 - \frac{4N_e^2}{27\tilde{f}_0^4}\right), \end{aligned}$$

- $\tilde{f}_0 \to \infty \Rightarrow R^2$

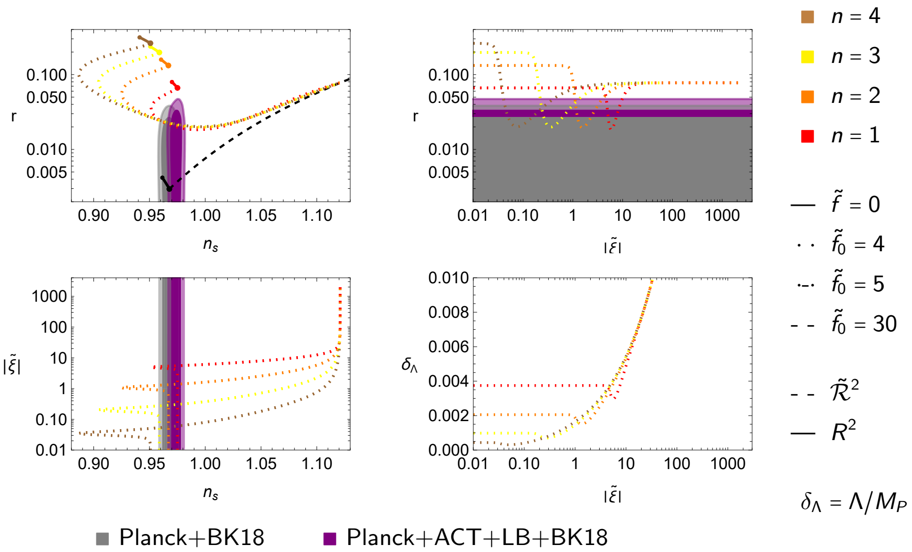
$\tilde{\xi} < 0 \quad N_e = 50$



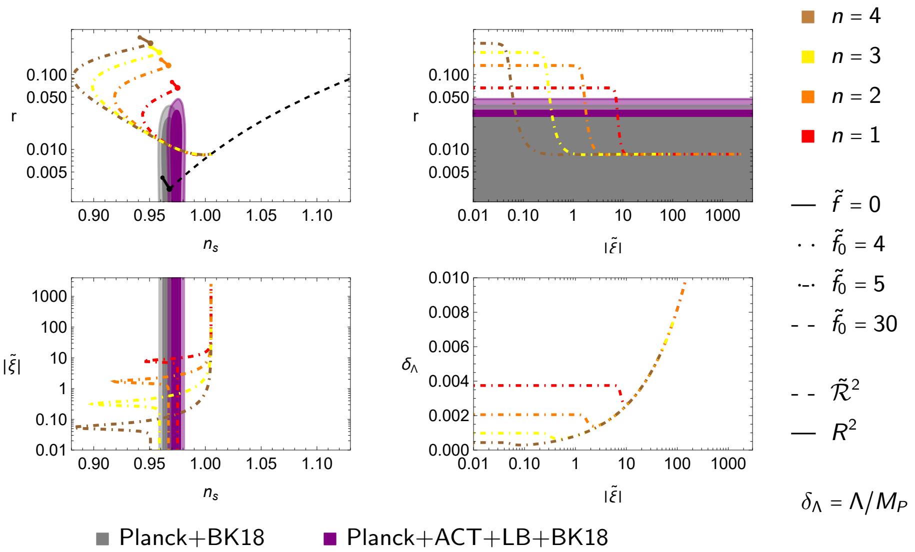
• $\tilde{\xi} < 0 \quad N_e = 60$ •



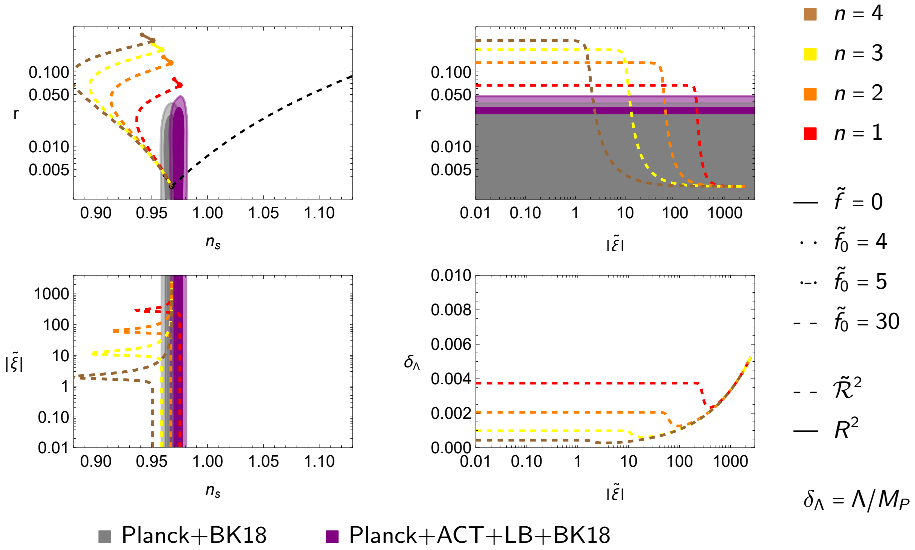
• $\tilde{\xi} < 0 \quad N_e = 60$ •



• $\tilde{\xi} < 0 \quad N_e = 60$ •



• $\tilde{\xi} < 0$ $N_e = 60$ •



Summary & Conclusions

- Data require us to build quasi-flat concave potentials
- MAG gives us an interesting framework to achieve that
- allowing for torsion
  - new nmc: $\tilde{f}(\phi)\tilde{\mathcal{R}}$
  - inflection point inflation setup → today $\tilde{\xi}$ attractors
    - ◇ strong coupling limit I: $\tilde{\mathcal{R}}^2$
    - ◇ strong coupling limit II: R^2
    - ◇ OK with ACT away from strong coupling limit II
- our study
  - agrees with Langvik et al., 2007.12595
  - cannot be compared with Shaposhnikov et al., 2007.14978 (only $\tilde{\xi} > 0$, if interested see also Gialamas & AR, 2412.17738)

Grazie! - Thank you! - Aitäh!

BACKUP SLIDES

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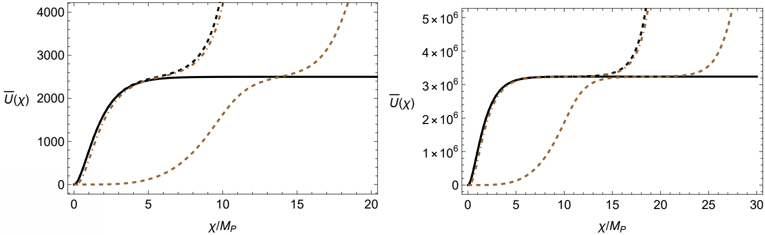
• MAG Dictionary •

- non-metricity: $Q_{\alpha\mu\nu} \equiv \nabla_\alpha g_{\mu\nu}$
- torsion: $T^\alpha{}_{\mu\nu} \equiv 2\Gamma^\alpha{}_{[\mu\nu]}$
- Levi-Civita connection: $\left\{{\alpha \atop \mu\nu}\right\} = \frac{1}{2} g^{\alpha\lambda}\left(g_{\lambda\nu,\mu} + g_{\mu\lambda,\nu} - g_{\mu\nu,\lambda}\right)$
- generic connection: $\Gamma^\alpha{}_{\mu\nu} = \left\{{\alpha \atop \mu\nu}\right\} + L^\alpha{}_{\mu\nu} + K^\alpha{}_{\mu\nu}$
- contortion: $L^\alpha{}_{\mu\nu} = \frac{1}{2} T^\alpha{}_{\mu\nu} + T_{(\mu}{}^{\alpha}{}_{\nu)}$
- disformation: $K^\alpha{}_{\mu\nu} = \frac{1}{2} Q^\alpha{}_{\mu\nu} - Q_{(\mu}{}^{\alpha}{}_{\nu)}$
- distortion: $L^\alpha{}_{\mu\nu} + K^\alpha{}_{\mu\nu}$

• Levi-Civita •

- Levi-Civita tensor: $\epsilon_{\alpha\beta\gamma\delta} = \sqrt{-g}\,\mathring{\varepsilon}_{\alpha\beta\gamma\delta}$
- Levi-Civita symbol: $\mathring{\varepsilon}_{\alpha\beta\gamma\delta}$ with $\mathring{\varepsilon}_{0123} = 1$
- $\mathring{\varepsilon}^{\alpha\beta\gamma\delta} = \mathrm{sign}(g)\mathring{\varepsilon}_{\alpha\beta\gamma\delta} = -\mathring{\varepsilon}_{\alpha\beta\gamma\delta}$
  (eq. for the components, not tensorial!!!)

• Einstein frame potential •



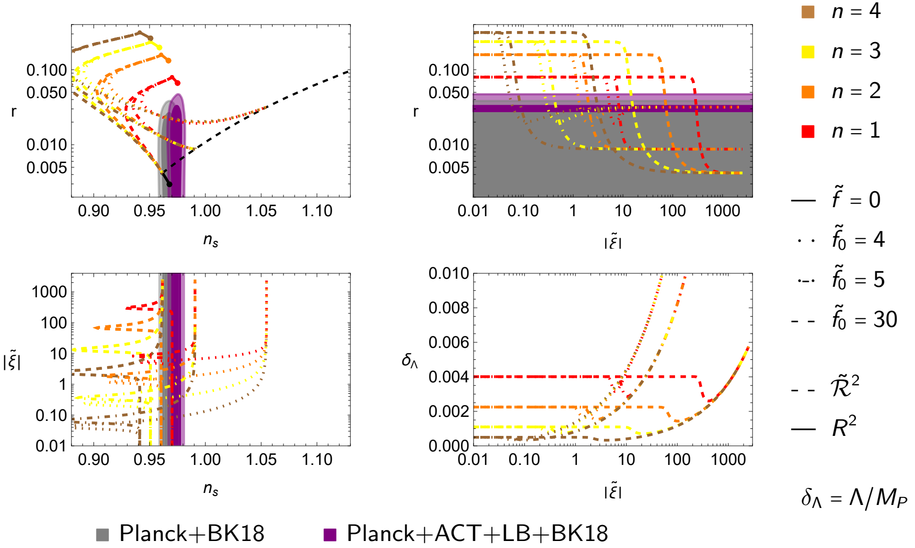
$\bar{U}(\chi)$ vs. χ/M_P for $n = 4$ (brown)

(a) $\tilde{f}_0 = 5$, $|\tilde{\xi}| \simeq 0.22$ (dashed) and $|\tilde{\xi}| \simeq 46.8$ (dot-dashed)

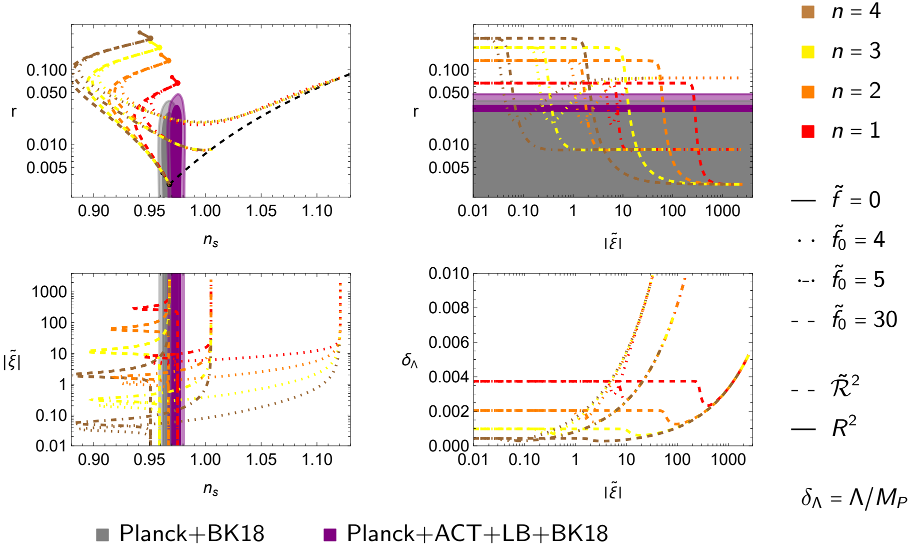
(b) $\tilde{f}_0 = 30$, $|\tilde{\xi}| \simeq 7.24$ (dashed) and $|\tilde{\xi}| \simeq 1.66 \times 10^3$ (dot-dashed)

- $\bar{U}(\chi)_{\tilde{\mathcal{R}}^2}$ (black, dashed)
- $\bar{U}(\chi)_{R^2}$ (black, continuous).

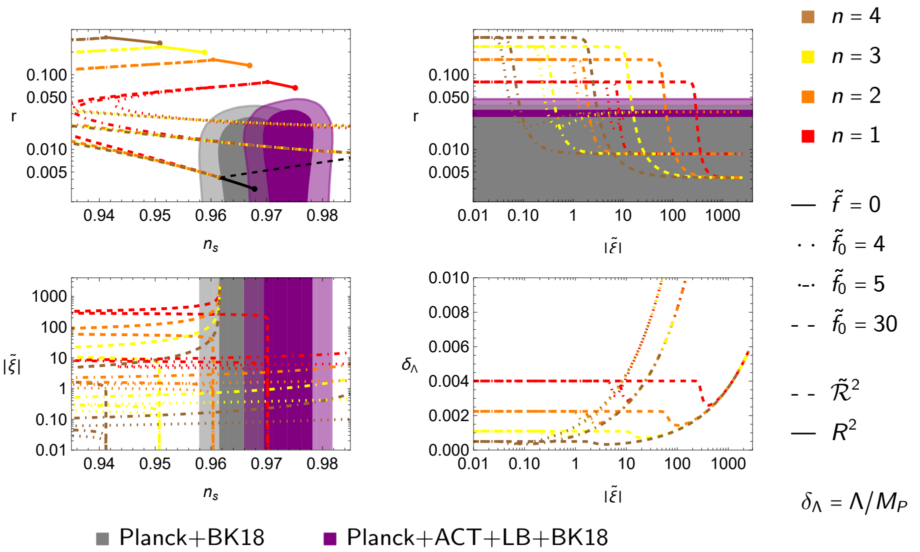
• $\tilde{\xi} < 0$ $N_e = 50$ •



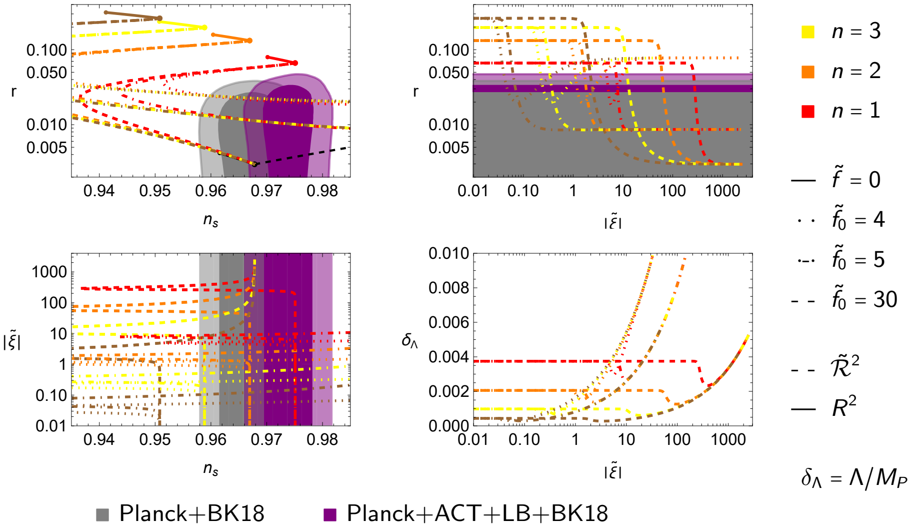
• $\tilde{\xi} < 0 \quad N_e = 60$ •



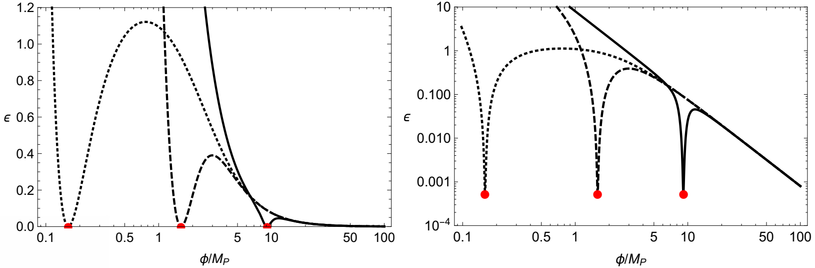
• $\tilde{\xi} < 0$ $N_e = 50$: zoom •



• $\tilde{\xi} < 0 \quad N_e = 60$: **zoom** •



• Validity of the solution I •



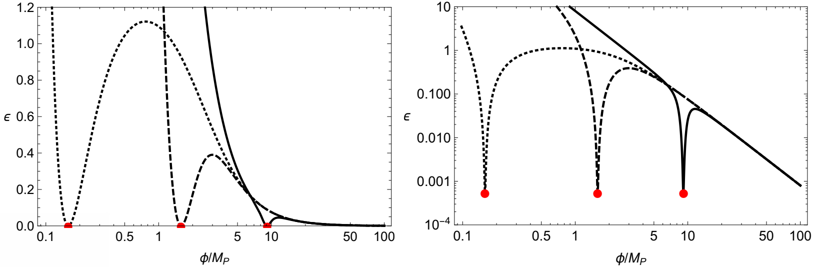
$\epsilon(\phi)$ for $n = 4$, $\tilde{f}_0 = 5$ and $|\tilde{\xi}| = 0.3$ (continuous), $|\tilde{\xi}| = 10$ (dashed) and $|\tilde{\xi}| = 1000$ (dotted). The red bullets represents the corresponding $(\phi_N, \epsilon(\phi_N))$

- when $\phi \gtrsim 20 M_P$, all the three lines converge to the same behaviour. This happens because in the $\phi \to +\infty$ limit, the kinetic function behaves like

$$k(\phi) \simeq 1 + 3\left[\frac{\Omega(\phi)'}{\Omega(\phi)}\right]^2 = 1 + \frac{3\,n^2}{8}\left(\frac{M_P}{\phi}\right)^2 = 1 + 6\left(\frac{M_P}{\phi}\right)^2$$

where we have used $n = 4$. Note that both the $\tilde{f}_0$ and $\tilde{\xi}$ contributions are absent. Moreover for $\phi \gg M_P$, $k(\phi) \approx 1$ and the setup behaves like standard monomial inflation.

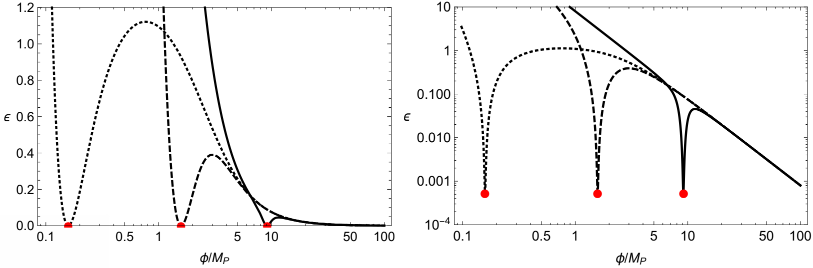
• Validity of the solution I •



$\epsilon(\phi)$ for $n = 4$, $\tilde{f}_0 = 5$ and $|\tilde{\xi}| = 0.3$ (continuous), $|\tilde{\xi}| = 10$ (dashed) and $|\tilde{\xi}| = 1000$ (dotted). The red bullets represents the corresponding $(\phi_N, \epsilon(\phi_N))$

- ϵ always develops a local maximum in ϕ_{max} and a local minimum in ϕ_{min} with $\phi_N \simeq \phi_{\text{min}}$. On one hand $\epsilon(\phi_N)$ is always the same as expected, since we chose the points in the asymptotic limit of r. On the other hand, with $|\tilde{\xi}|$ increasing, ϕ_{max} decreases and $\epsilon(\phi_{\text{max}})$ increases. In particular for $|\tilde{\xi}| = 1000$, we have $\epsilon(\phi_{\text{max}}) > 1$. This implies having two different regions available for inflation: one before (very fine-tuned) and one after ϕ_{max} (complete different predictions) $\Rightarrow$ reject

• Validity of the solution I •



$\epsilon(\phi)$ for $n = 4$, $\tilde{f}_0 = 5$ and $|\tilde{\xi}| = 0.3$ (continuous), $|\tilde{\xi}| = 10$ (dashed) and $|\tilde{\xi}| = 1000$ (dotted). The red bullets represents the corresponding $(\phi_N, \epsilon(\phi_N))$

- Doing all the math to solve $\epsilon(\phi_{\max}) < 1$, we obtain the rough upper bound

$$|\tilde{\xi}| < \tilde{\xi}_{\max} = 2\left[\left(\frac{32}{3}\right)^n n^{-3n}(n+4)^{n+4}\right]^{1/4} \tilde{f}_0^{\,2} \simeq \begin{cases} 27.02\tilde{f}_0^{\,2}\,, & n=1 \\ 33.94\tilde{f}_0^{\,2}\,, & n=2 \\ 30.02\tilde{f}_0^{\,2}\,, & n=3 \\ 21.33\tilde{f}_0^{\,2}\,, & n=4 \end{cases}$$

therefore both strong coupling limits are reached before the issue appears.

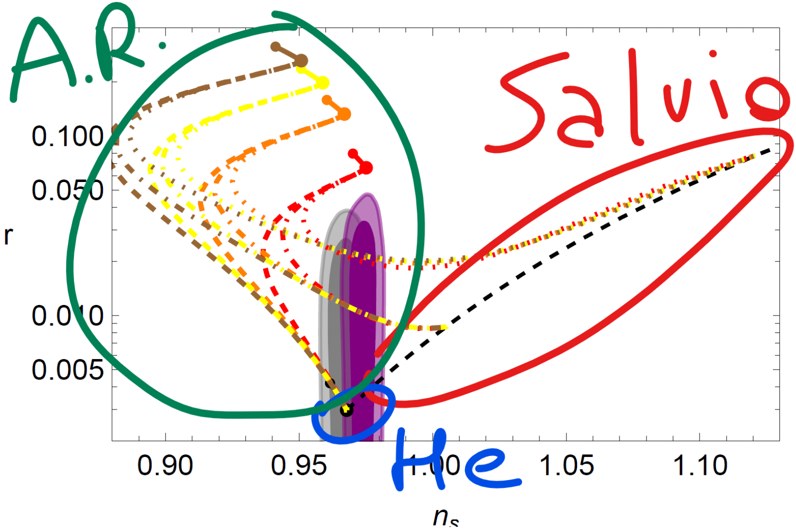
• Validity of the solution II •

We comment on the eventual impact of the non-minimal function $f(\phi)$. Such a contribution should imply a loss of attractor behaviour. On the other hand, $\xi \neq 0$ becomes relevant at big ϕ values, while our relevant inflationary region is at small ϕ's (see peak eq.). Therefore, the attractor behaviour of our model will be not spoiled if the condition $\xi\Omega(\phi_N) \ll 1$ is satisfied. We can estimate such an upper bound as

$$\xi \ll \left(\frac{M_P}{\phi_N}\right)^{n/2} \simeq \frac{\tilde{\xi}}{\tilde{f}_0{}^2} \lesssim 2\left[\left(\frac{32}{3}\right)^n n^{-3n}(n+4)^{n+4}\right]^{1/4} \simeq \begin{cases} 27.02\,, & n=1 \\ 33.94\,, & n=2 \\ 30.02\,, & n=3 \\ 21.33\,, & n=4 \end{cases} ,$$

W can expect that the attractor behaviour described before for $1 \leq n \leq 4$ will not be compromised by the presence of a $f(\phi) \neq 1$, as long as $\xi \lesssim 0.1$.

• Results •



• More on the inflection point •

Equation for the inflection point:

$$U''(\chi_{\text{flex}}) = 0$$

In terms of ϕ (when $f = 1$), can be written as

$$\frac{1}{2}\frac{k'(\phi_{\text{flex}})}{k(\phi_{\text{flex}})} = \frac{V''(\phi_{\text{flex}})}{V'(\phi_{\text{flex}})}$$

using $V \propto \phi^n$, becomes

$$\frac{M_P}{2}\frac{k'(\phi_{\text{flex}})}{k(\phi_{\text{flex}})} = (n-1)\frac{M_P}{\phi_{\text{flex}}}$$

- $n = 1 \Rightarrow k'(\phi_{\text{flex}}) = 0 \Rightarrow \phi_{\text{flex}} = \phi_{\text{peak}}$
- other n's
  - $\phi_{\text{flex}} \gg M_P \Rightarrow \phi_{\text{flex}} \simeq \phi_{\text{peak}}$
  - $\phi_{\text{flex}} \ll M_P \Rightarrow$ still possible $\phi_{\text{flex}} \simeq \phi_{\text{peak}}$ as both close to 0

• Trial example: setup •

$$\Omega(\phi)^2 = \left(\frac{\phi}{M_P}\right)^n$$

$$\left.\begin{array}{ll} n \text{ even: } & \phi \to -\phi \\ n \text{ odd: } & \phi > 0 \end{array}\right\} \Rightarrow \boxed{\phi > 0}$$

$$V(\phi) = \Lambda^4 \left(\frac{\phi}{M_P}\right)^n \qquad \tilde{f}(\phi) = \left[\tilde{f}_0^2 + \tilde{\xi}\left(\frac{\phi}{M_P}\right)^{\frac{n}{2}}\right] \qquad \boxed{\tilde{f}_0^2 > 0, \tilde{\xi} \gtreqless 0}$$

$$k(\phi) = 1 + \frac{3\, n^2\, \tilde{\xi}^2 \left(\frac{\phi}{M_P}\right)^{n-2}}{2\left[1 + 4\left(\tilde{f}_0^{\,2} + \tilde{\xi}\left(\frac{\phi}{M_P}\right)^{n/2}\right)^2\right]} \gg 1 \Rightarrow \cancel{1+} \frac{\text{BIG}}{\text{small}}$$

$$\Rightarrow \left[1 + 4\left(\tilde{f}_0^{\,2} + \tilde{\xi}\left(\frac{\phi}{M_P}\right)^{n/2}\right)^2\right] \sim 1 \Rightarrow \tilde{f}_0^{\,2} + \tilde{\xi}\left(\frac{\phi}{M_P}\right)^{n/2} \sim 0 \Rightarrow \boxed{\tilde{\xi} < 0}$$

• Trial example: setup •

$$\Omega(\phi)^2 = \left(\frac{\phi}{M_P}\right)^n$$

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