Anti-de Sitter Wormholes as seeds for “Higgs” Inflation

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Based on: P. Betzios, **I.D.G.**, O. Papadoulaki, Phys.Rev.D (2025), 2412.03639

Scalars 2025, University of Warsaw, Faculty of Physics, September 24, 2025

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Inflation

FRW metric:

$$ds^2 = -dt^2 + a^2(t)\left(\frac{dr^2}{1-kr^2} + r^2 d\theta^2 + r^2 \sin^2\theta d\varphi^2\right)$$

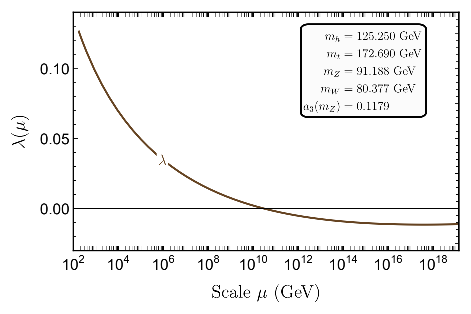
- Inflation is a theory of exponential expansion of space in the early universe, i.e. $a \sim e^{Ht}$.
- $\sim 10^{-36} - 10^{-32}$ seconds after the Big Bang.
- Solves the **horizon** and **flatness** problems.
- It can also provide a mechanism for the generation of the perturbations that have resulted in the anisotropies observed in the CMB.

Higgs inflation and metastability

potential $V(\phi) \sim \frac{\lambda(\mu)}{4}\phi^4 + \cdots$ nonminimal couplings $\xi\phi^2 R$

(F. Bezrukov, M. Shaposhnikov, 2007) **In one-loop:**

$$(4\pi)^2 \frac{\mathrm{d}\lambda(\mu)}{\mathrm{d}\ln\mu} = -6y_t^4 + \frac{27}{200}g_1^4 + \frac{9}{8}g_2^4 + \frac{9}{20}g_1^2 g_2^2 + 24\lambda^2 + \lambda\left(12y_t^2 - 9g_2^2 + \frac{9g_1^2}{5}\right)$$



- (J.R. Espinosa, M. Quiros, 1995)
- (V. Branchina, E. Messina, 2013)
- (F. Bezrukov, J. Rubio, M. Shaposhnikov, 2014)
- (F. Bezrukov, M. Shaposhnikov, 2014)
- (Buttazzo, Degrassi, Giardino, Giudice, Sala, Salvio, Strumia, 2013)
- ... many others

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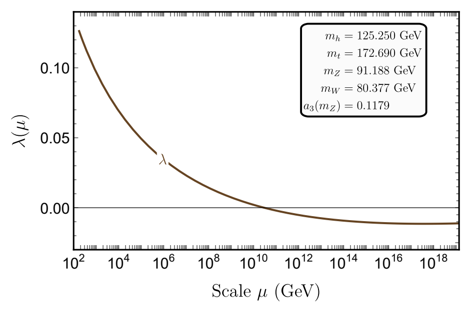
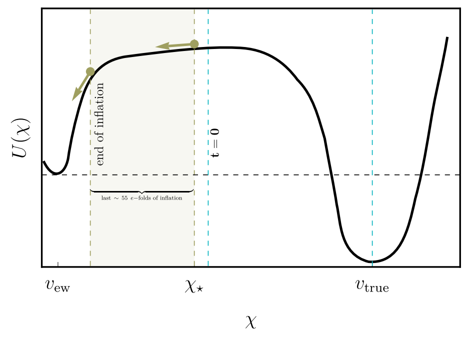


- As $m_t \uparrow$ the yukawa coupling y_t becomes larger.
- So, heavy top-quark masses (m_t) makes the quartic coupling $\lambda(\mu)$ negative at large energies.

Metastable potential

- Assume that we are in the metastability regime with a negative minimum above the inflationary scale



Some String/SUSY realizations

- (S. Kachru, R. Kallosh, A. Linde, S. Trivedi, 2003)
- (C. Burgess, R. Kallosh, F. Quevedo, 2003)
- (R. Kallosh, A. Linde, 2004)
- (S. AbdusSalam, C. Hughes, F. Quevedo, A. Schachner, 2025)
- ... others

- The precise shape of the potential at high energies does depend on the UV completion
- In order to have successful Inflation, we need as initial condition to start high up in the hilltop (a generic issue of inflationary models)

Pre-inflationary/Initial condition issues

Pertinent Question

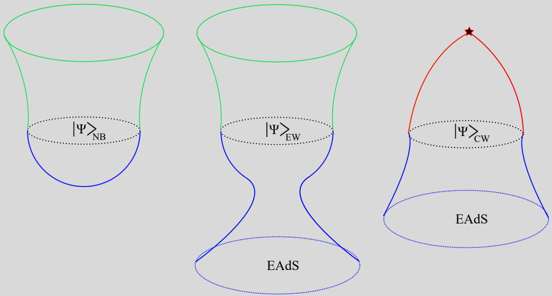
- What gave rise to the initial conditions/state of inflation?
- Why to start high up in the inflaton potential?

I will show that it is possible to obtain a semi-classical understanding for the ``birth/nucleation of the Universe" (i.e. Lorentzian Inflationary evolution that starts high up in the inflaton potential, even if the global minimum is AdS)

(P. Betzios, O. Papadoulaki, Phys.Rev.Lett., 2024)

Euclidean geometries that prepare initial states

- Euclidean geometries have interesting connections to Lorentzian geometries upon analytic continuation
- Euclidean geometries with Z_2 reflection symmetry can be sliced in half to define initial $\tau = t = 0$ states/wavefunctions of the Lorentzian evolution
- By cutting it in half we can "glue" to it an expanding Lorentzian Universe



- I will focus in the **middle** picture: A ``wineglass´´ (half) - wormhole

(P. Betzios, O. Papadoulaki, Phys.Rev.Lett., 2024)

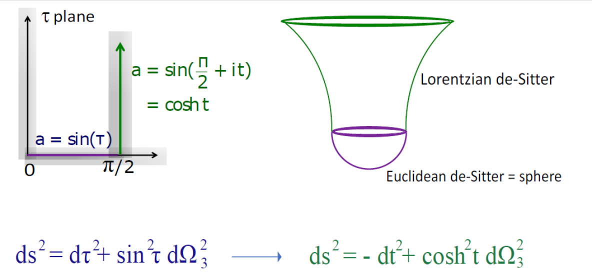
A brief look at the No-boundary proposal

Consider the Einstein Hilbert action with positive cosmological constant

$$S = \frac{M_{\rm Pl}^2}{2} \int \mathrm{d}^4 x (R - 2\Lambda)$$

that admits an empty de Sitter solution.

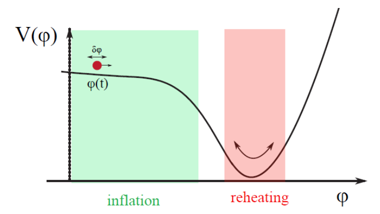
♣ The (Hartle, Hawking Phys.Rev.D, 1983) proposal classically describes a (complex) metric - half of Euclidean de-Sitter glued to half of Lorentzian de-Sitter



(Jean-Luc Lehners, review, 2023)

An exponential (hierarchy) problem

Remember the current cosmological constant problem $\frac{M_{\rm Pl}^4}{V_{\rm vac}} \simeq 10^{120}$



There is an exponentially worse problem with the No Boundary proposal

$$P_{NB}(\phi) = |\Psi|^2 \simeq e^{-S_E(\phi)} \simeq e^{M_{\rm Pl}^4/V(\phi)}$$

- It gives an overwhelming probability ($P_{NB} \gg 1$) for an empty cold universe, with the smallest allowed number for the cosmological constant.
- The issue stems from the fact that the on-shell action for the positively curved Euclidean de-Sitter is negative.

Wineglass AdS wormholes

- We shall call our geometries wineglass AdS (half) wormholes.
- Their defining properties: They should asymptote to a EAdS space: $a(\tau \to \pm\infty) \sim \exp(H_{\rm AdS}|\tau|)$ and

$$a''(0) < 0\,, \;\; a'(0) = 0\,, \;\; a(0) = a_{\rm max}\,, \;\; \phi'(0) = 0$$

  so that $a_{\rm max}$ is a local maximum of the scale factor (in Euclidean)
- These are also good initial conditions for a subsequent inflationary Lorentzian evolution (since $t = i\tau \Rightarrow \dot{a}(0) = \dot{\phi}(0) = 0\,,\; \ddot{a}(0) > 0$)
- To obtain such solutions we need:
  1. A scalar potential that takes both positive and negative values
  2. some form of negative Euclidean energy that supports their throat from collapsing

Models for wineglass AdS wormholes

- Consider a general GR-inflaton-radiation-matter action

$$\mathcal{S} = \int \mathrm{d}^4x\sqrt{g_E}\left(-\frac{1}{2\kappa}R + \frac{1}{2}(\partial_\mu\chi)^2 + U(\chi) + \mathcal{L}_{\rm rad} + \mathcal{L}_{\rm axion}\right)$$

and the spherically symmetric and homogeneous ansatze

$$\mathrm{d}s^2 = \mathrm{d}\tau^2 + a^2(\tau)\mathrm{d}\Omega_3^2\,, \quad \phi = \phi(\tau)\,.$$

♣ The equations of motion read

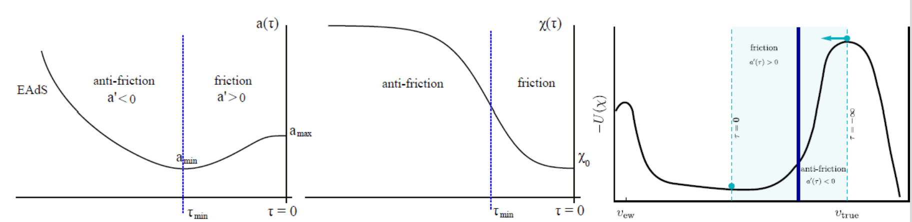
$$\frac{a'^2}{a^2} - \frac{1}{a^2} + \frac{\kappa}{3}\left(U(\chi) - \frac{\chi'^2}{2}\right) - \frac{\tilde{\rho}^E_{\rm rad}}{a^4} - \frac{\rho^E_{\rm axion}}{a^6} = 0$$

$$\chi'' + 3\frac{a'\chi'}{a} - \partial_\chi U(\chi) = 0$$

- Wineglass Wormholes can be supported by axions (Betzios, Papadoulaki, 2024) or magnetic radiation (Betzios, IDG, Papadoulaki, 2025)
- Magnetic radiation leads to $\rho^E_{\rm rad} = T^{E\ \ 0}_{\rm rad\ 0} = \frac{1}{2}(E^2 - B^2) = 3\tilde{\rho}^E_{\rm rad}/(\kappa a^4)$

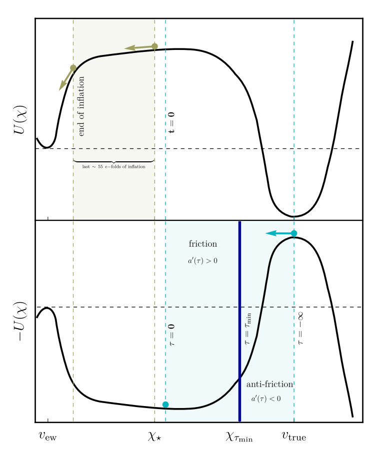
Models for wineglass AdS wormholes

- Gravity wants to shrink the scale factor $(-1/a^2)$, while axions/magnetic fluxes try to expand it $(-\rho^E_{\rm axion}/a^6\ ,\ -\tilde{\rho}^E_{\rm rad}/a^4\ ,\ \ \tilde{\rho}^E_{\rm rad} < 0)$
- The (Euclidean) EOM for the scalar field describes a particle moving in the potential $-U(\chi)$ with an (anti)-friction term $3a'\chi'/a$



- The Euclidean manifold initially shrinks $(a' < 0$-gravitational dominance) and then expands ($a' > 0$-flux dominance) causing the χ particle to first accelerate ($a' < 0$-anti-friction) and then to stop ($a' > 0$-friction) at χ_0.

Subsequent Lorentzian evolution



- The Euclidean trajectory describes the nucleation of the Universe at $\chi_0 = \chi(\tau = 0)$, high up in the potential with $\dot{a}(0) = \dot{\chi}(0) = 0$. It then follows the slow-roll trajectory to the vacuum
- It predicts a dominant magnetic radiation and/or axionic component in the very early Universe (we give precise bounds in the papers)
- They both get diluted to an enormous degree during inflation

Evading the issue of the No-boundary proposal

- We compute the probability and compare with the No-Boundary proposal ($P = |\Psi|^2 \simeq e^{-\mathcal{S}_E}$)
  $\Rightarrow$ evaluate the Euclidean wormhole on-shell action

$$\mathcal{S}_E = 4\pi^2 \int_{\mathrm{UV}}^{0} \mathrm{d}\tau \left(\frac{-\tilde{\rho}^E_{\mathrm{rad}}}{a} \text{ or } \frac{-\rho^E_{\mathrm{axion}}}{a^3} - a^3 V(\phi) \right) + \mathcal{S}^{\mathrm{UV}}_{\mathrm{GH}} + \mathcal{S}^{\mathrm{UV}}_{\mathrm{c.t.}}$$

- The EAdS UV boundary contains the Gibbons-Hawking $\mathcal{S}^{\mathrm{UV}}_{\mathrm{GH}}$ as well as boundary counterterms $\mathcal{S}^{\mathrm{UV}}_{\mathrm{c.t.}}$
- Either numerically or analytically using thin wall approximations one typically finds a positive on-shell action for the wormhole
- Due to the AdS asymptotics we have a well defined probability ($P \simeq e^{-\mathcal{S}_E} < 1$) and the issue of the No-boundary proposal can be evaded: The Universe prefers to nucleate high up in the potential and then follows the slow roll trajectory

Summary

- This is a new type of wavefunction for the universe computed from the gravitational path integral, with asymptotically EAdS boundary conditions
- In the semiclassical limit, it describes a Euclidean AdS (half)-wormhole geometry. If the scale factor acquires a local maximum at the surface of reflection (Z_2) symmetry, it gives rise to an expanding universe upon analytic continuation to Lorentzian signature
- This proposal can be realised with a non-trivial scalar potential $V(\phi)$ that takes both positive and negative values
- Some initial flux is needed (e.g. radiation or axions)
- This proposal evades some issues of the No-boundary proposal, leading to a well defined probability $P \simeq e^{-\mathcal{S}_E} < 1$.

Thank you!

BACKUP SLIDES

In Minkowski spacetime $(-,+,+,+)$, the action of a single particle is

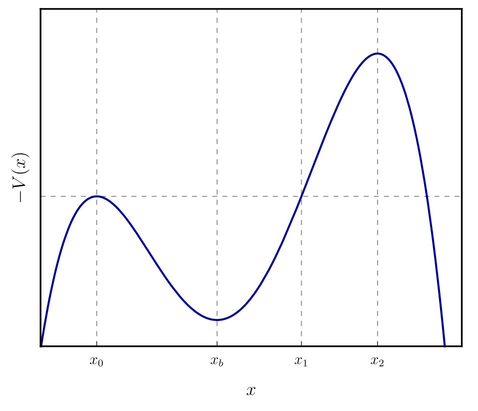
$$S_M = \int dt \left(\frac{1}{2} m \left(\frac{dx}{dt} \right)^2 - V(x) \right) .$$

Suppose that we change variables to an imaginary time $\tau = it$, we have

$$S_M = i \int d\tau \left(\frac{1}{2} m \left(\frac{dx}{d\tau} \right)^2 + V(x) \right) = iS_E \, ,$$

where S_E is the Euclidean action.

BACKUP SLIDES



Euclidean action

- $S = \int d\tau \left(\frac{m}{2} \left(\frac{dx}{d\tau} \right)^2 + V(x) \right)$
- EOM: $m \frac{d^2 x}{d\tau^2} - \frac{\partial V}{\partial x} = 0$
- Integral: $\frac{m}{2} \left(\frac{dx}{d\tau} \right)^2 - V(x) = 0$
- Bounce solution (x_B): $x_0 \to x_1 \to x_0$ at $\tau = -\infty \to 0 \to +\infty$
- $S(x_B) = \int_{-\infty}^{+\infty} d\tau \left(\frac{m}{2} \left(\frac{dx_B}{d\tau} \right)^2 + V(x_B) \right)$

$$= 2 \int_{-\infty}^{0} 2V(x_B) d\tau$$

$$= 2 \int_{x_0}^{x_1} \sqrt{2mV(x_B)} dx_B$$

$$d\tau = \sqrt{\frac{m}{2V(x_b)}} dx_B$$

Same as before

BACKUP SLIDES

The Euclidean Einstein-scalar action is (Betzios, Papadoulaki, 2024)

$$S_E = \int d^4x \sqrt{g_E} \left(-\frac{1}{2\kappa} R + \frac{1}{2} (\partial_\mu \chi)^2 + V(\chi) + \frac{1}{12 f_\alpha^2} H_{\mu\nu\rho} H^{\mu\nu\rho} \right) , \quad (1)$$

to which we have added an additional axionic contribution (f_α is the axion coupling constant and $H_{\mu\nu\rho}$ its three-form field strength). Assuming the simplest spherically symmetric and homogeneous ansatze (q is a constant axion charge)

$$ds^2 = d\tau^2 + a^2(\tau) d\Omega_3^2 , \quad \chi(\tau) , \quad H_{ijk} = q \epsilon_{ijk} , \quad (2)$$

we find the set of equations of motion ($Q^2 \equiv q^2/2f_\alpha^2$)

$$\frac{2a''}{a} + \frac{a'^2}{a^2} - \frac{1}{a^2} + \kappa \left(V(\chi) + \frac{\chi'^2}{2} \right) - \frac{\kappa Q^2}{a^6} = 0 , \quad (3)$$

$$\frac{a'^2}{a^2} - \frac{1}{a^2} + \frac{\kappa}{3} \left(V(\chi) - \frac{\chi'^2}{2} \right) + \frac{\kappa Q^2}{3a^6} = 0 , \quad (4)$$

$$\chi'' + 3 \frac{a' \chi'}{a} - \frac{dV}{d\chi} = 0 . \quad (5)$$

BACKUP SLIDES

In the semi-classical (WKB) approximation, the wavefunctions are found to behave in the oscillatory (Lorentzian) region as follows

$$\Psi_{NB}(A,\tilde{\chi}) \simeq P_{NB}^{1/2}\, Re\left(e^{iS_L(A,\tilde{\chi})}\right), \quad P_{NB} = e^{-S_E(\tilde{\chi})}, \tag{6}$$

$$S_E(\tilde{\chi}) = -\frac{8\pi^2}{\kappa \tilde{V}(\tilde{\chi})}, \quad S_L(A,\tilde{\chi}) \simeq \frac{8\pi^2(e^{2A}\tilde{V}(\tilde{\chi})-1)^{3/2}}{\kappa \tilde{V}(\tilde{\chi})}. \tag{7}$$

- These formulae are valid assuming a slowly varying scalar potential.
- The oscillating part is related to the Lorentzian on-shell action S_L and the no-boundary wavefunction is real
- The prefactor $P_{NB}^{1/2}$ is determined by a WKB procedure and depends exponentially on the Euclidean on-shell action
- The probability measure is then computed from $P = |\Psi|^2$ and one observes that the Hartle-Hawking proposal exponentially peaks at the smallest (positive) value of the potential.

(Lehners, 2023, Maldacena 1998, 2024)

BACKUP SLIDES

At the time $\tau = t = 0$, where the Lorentzian evolution begins, the field is already positioned in the plateau region, allowing us to approximate

$$U_0 \simeq U(\chi_\star) \simeq (1.0 - 1.2) \times 10^{-10} M_{\rm Pl}^4 \,, \tag{8}$$

- Bound on radiation $0 > \rho^E_{\mathrm{rad}\star} > -(2.5 - 3.0) \times 10^{-11} M_{\rm Pl}^4$
- Assuming that no fine-tuning takes place, but rather a small asymmetry between the magnitudes of the magnetic and electric components, the scale of the magnetic field density at the beginning of inflation is $B_\star^2 \simeq 10^{-11} M_{\rm Pl}^4$
- $\dfrac{a_\star}{a_{\rm end}} = e^{-N_\star} \simeq 10^{-96} - 10^{-105} \Rightarrow B_{\rm end} = B_{\rm reh} \simeq 1000 - 0.1\, G$
- Possible galactic dynamo mechanisms that can sustain the currently observed magnetic fields require a minimum ``seed'' field of the order of $B_{\rm seed} \simeq 10^{-12} - 10^{-22}\, G \xrightarrow{a_0/a_{\rm end} \simeq 10^{29}} B_{\rm reh} \simeq 10^{35}\, G$

BACKUP SLIDES

We first assume an analytic ansatz for the scale factor comprised out of two overlapping regions

$$a(\tau) = \begin{cases} \dfrac{1}{H_{AdS}} \cosh\left(H_{AdS}(\tau - \tau_{min})\right) + c_{AdS}\,, & \text{for } -\infty < \tau < \tau_{\min} \quad \textbf{(I)} \\ a_{min} + \dfrac{a''_{min}}{2}(\tau - \tau_{min})^2\,, & \text{for } \tau \simeq \tau_{\min} \quad \textbf{(II)} \\ \dfrac{1}{H_{dS}} \cos\left(H_{dS}\tau\right) + c_{dS}\,, & \text{for } \tau_{\min} < \tau \leq 0\,, \quad \textbf{(III)} \end{cases}$$

where $0 > c_{AdS} > -1/H_{AdS}$ and $c_{dS} > 1/H_{dS}$.

- Region **(I)** corresponds to the asymptotic approximately AdS part of the wineglass wormhole geometry
- Region **(II)** merely serves as a matching intermediate region in order to guarantee the smooth overlap between the **(I)** $-$ **(III)** patches.
- Region **(III)** corresponds to the approximately dS throat region

BACKUP SLIDES

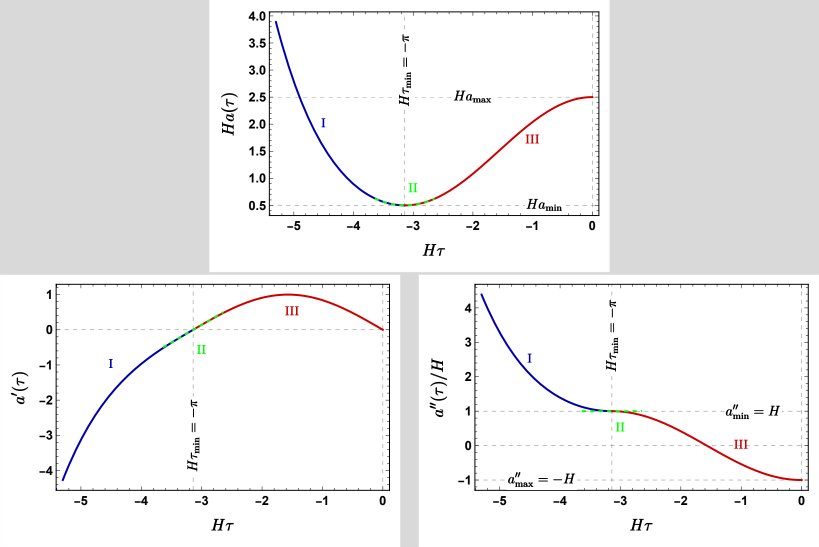
In particular matching the geometries we obtain the conditions

$$H_{AdS} = H_{dS} \equiv H\,, \qquad \text{and} \qquad c_{dS} = c_{AdS} + \frac{2}{H}\,. \tag{9}$$

Note also that $\tau_{min} = -\pi/H$, $a_{min} = c_{dS} - \frac{1}{H}$, $a_{max} = c_{dS} + \frac{1}{H}$, and $a''_{min} = -a''_{max} = H$.
For convenience, we introduce the parameter ϵ by assuming

$$c_{dS} = \frac{\epsilon}{H}\,, \qquad \text{and} \qquad c_{AdS} = \frac{\epsilon - 2}{H}\,, \qquad \text{with} \qquad 1 < \epsilon < 2\,. \tag{10}$$

BACKUP SLIDES



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