Bubble wall dynamics from non-equilibrium QFT

Based on 2504.13725

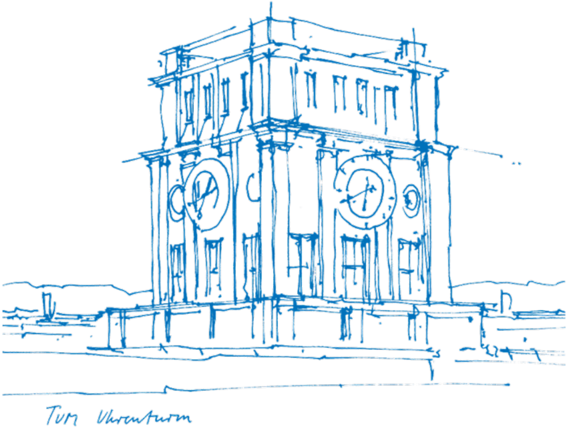
with W.Ai, B.Garbrecht, C.Tamarit, M.Vanvlasselaer

Matthias Carosi

Theoretical Physics of the Early Universe
TUM School of Natural Sciences
Technical University of Munich

Gravitational Waves as Probes of Physics Beyond the Standard Model

26/06/2025



Outline

Driving pressure vs. friction

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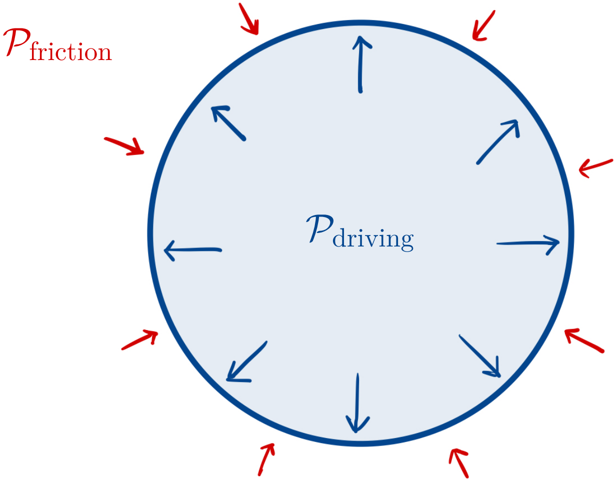

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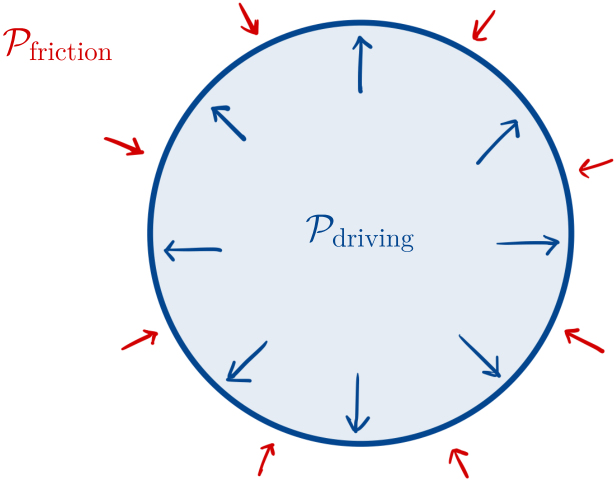

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$$\mathcal{P}_{\text{friction}} = \mathcal{P}_{\text{driving}}$$

the system reaches a *steady state*

$\Longrightarrow$ terminal wall velocity

$\equiv v_w$

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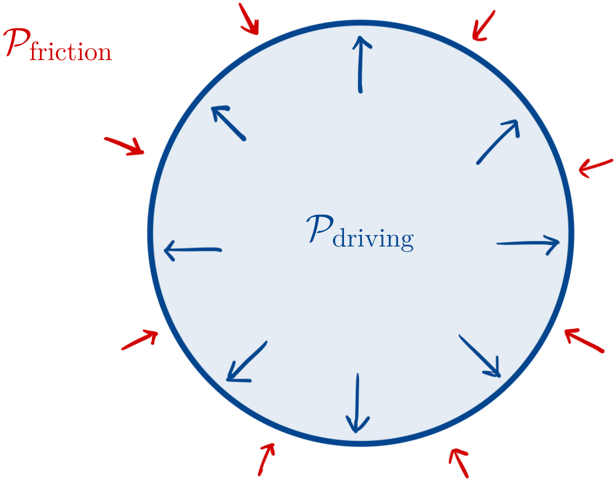

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Goal: identify v_w from the steady state condition.

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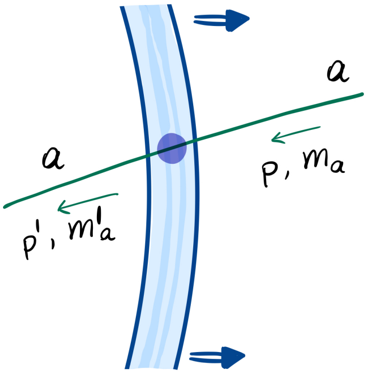

Mass gain

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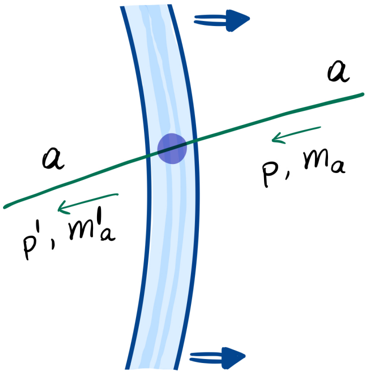

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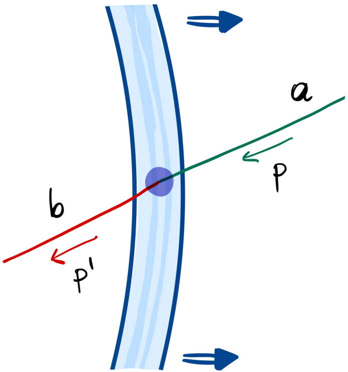

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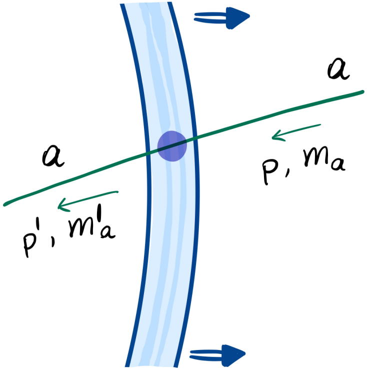

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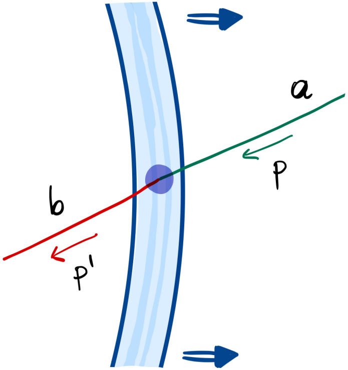

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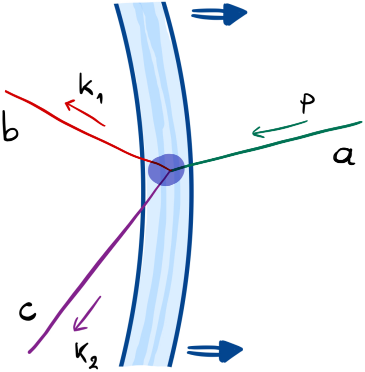

Particle production

Current status

Two main approaches exist for studying the dynamics of a single bubble

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Kinetic picture
[Moore and Propokopec '95]

Set of dynamical equations

$$\begin{cases} \Box\varphi + V'(\varphi) + \sum_i \frac{\mathrm{d}m_i^2}{\mathrm{d}\varphi} \int_{\mathbf{p}} f_i(\mathbf{p}, x) = \ 0 \\ \frac{\mathrm{d}f_i}{\mathrm{d}t} = \ -\mathcal{C}[f] \end{cases}$$

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$$\mathcal{P}_{\text{kick}} = \sum_{i,X} \int_{\mathbf{p}} 2p^z \, \mathrm{d}\mathbb{P}_{i\to X}(\mathbf{p}) \, f_i(\mathbf{p}) \, \Delta p^z_{i\to X}$$

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Goal: extend the kinetic picture to capture all the microphysics

Outline

1 The dynamics of a single bubble

2 The language of non-equilibrium QFT: CTP and 2PI
- Brief review of the CTP formalism
- Introducing the 2PI effective action
- The full dynamical equations
- Identifying sources of friction

3 Example: friction from pair production

4 Conclusions and outlook

The tools of non-equilibrium QFT

real time correlators $\Longrightarrow$ CTP formalism

dynamical equations $\Longrightarrow$ 2PI effective action

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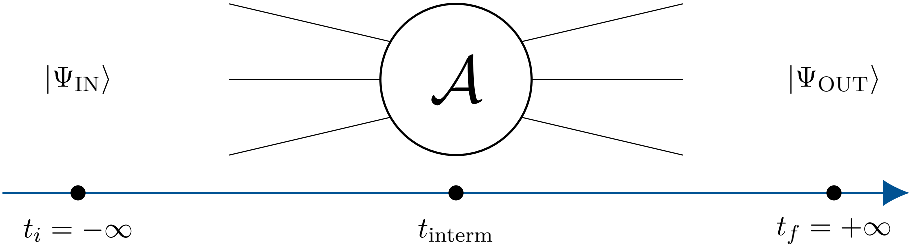

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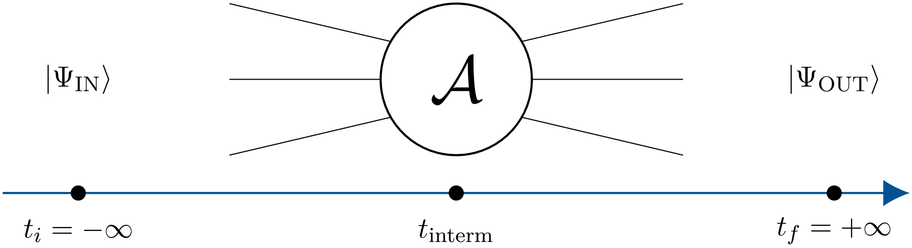

Using it, we compute transition amplitudes between **asymptotic states**

$$\mathcal{A} = \langle\Psi_{\mathrm{OUT}}|\, \mathcal{O}(\hat{\phi})\, |\Psi_{\mathrm{IN}}\rangle = \mathcal{N} \int [\mathcal{D}\phi]\, \Psi^*_{\mathrm{OUT}}(\phi)\mathcal{O}(\phi)\Psi_{\mathrm{IN}}(\phi) e^{iS[\phi]}$$

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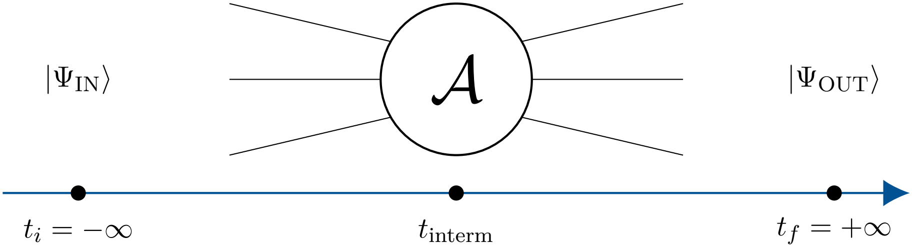

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But how can we compute time (and space) dependent correlators?

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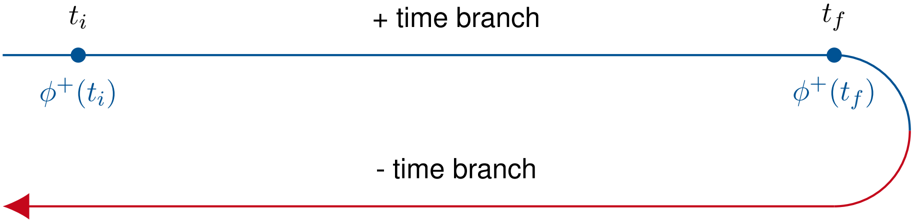

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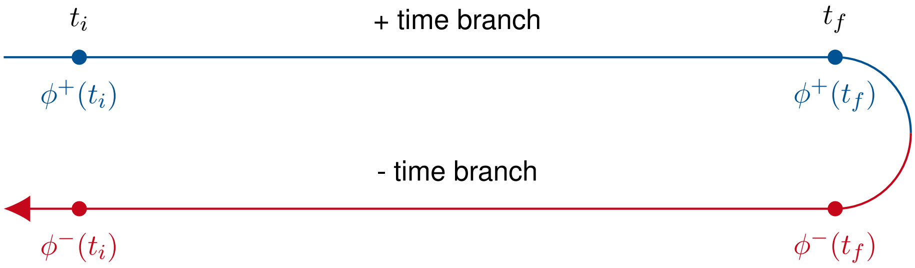

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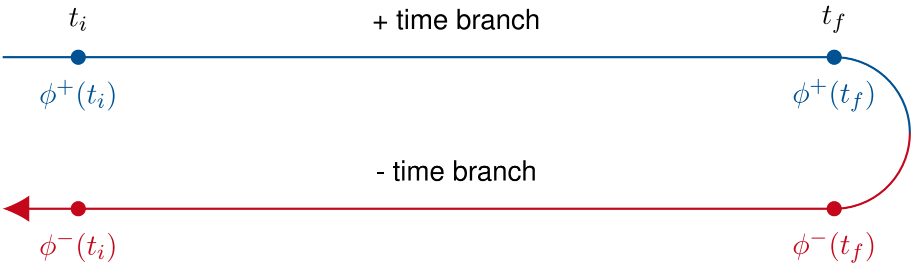

We introduce the label $\pm$ for the time branch, double our degrees of freedom, and can now use all the tools from the path integral formalism.

The tools of non-equilibrium QFT

real time correlators $\Longrightarrow$ CTP formalism ✓

dynamical equations $\Longrightarrow$ 2PI effective action

Introducing the 2PI effective action

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Equations for the one- and two-point functions are then easily generated

$$\boxed{\frac{\delta\Gamma_{2\mathrm{PI}}}{\delta\varphi(x)} = 0 \qquad\qquad \frac{\delta\Gamma_{2\mathrm{PI}}}{\delta\Delta(x,y)} = 0}$$

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2PI effective action $+$ CTP formalism $\Longrightarrow$ out-of-equilibrium dynamical equations

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Doing a loop expansion of the effective action

$$\Gamma_{\mathrm{2PI}}[\varphi^a, \Delta^{ab}] = S[\varphi^+] - S[\varphi^-] + \frac{i}{2}\,\mathrm{Tr}\log\Delta^{-1} + \frac{i}{2}\,\mathrm{Tr}\,G_\varphi^{-1}\Delta + \Gamma_2[\varphi^a, \Delta^{ab}]$$

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and the trace runs over the CTP indices as well. The inverse tree-level propagator reads

$$G_\varphi^{ab,-1}(x,y) = i\delta^{(4)}(x-y)a\delta^{ab}\left(\Box + V''(\varphi^a)\right)$$

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All terms of loop order larger than two are inside Γ_2

$$\Gamma_2 \supset \text{ two-particle-irreducible vacuum diagrams with two or more loops}$$

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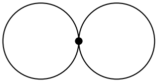
$$\mathcal{L}_{\mathrm{int}} = -\frac{\lambda}{4!}\phi^4 \longrightarrow i\Gamma_2 =$$



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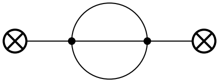

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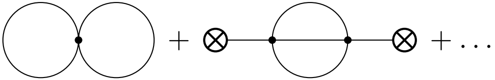

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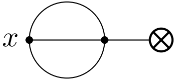

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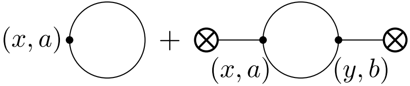

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Having solved for the two-point function in Wigner space at leading order in the gradients, we have the EoM for the bubble wall

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The bubble wall equation of motion

Having solved for the two-point function in Wigner space at leading order in the gradients, we have the EoM for the bubble wall

$$\Box\varphi(x) + V_0'(\varphi(x)) + \frac{1}{2}\frac{\mathrm{d}m_\varphi^2}{\mathrm{d}\varphi(x)}\int\frac{\mathrm{d}^4k}{(2\pi)^4}\overline{\Delta}^T(k,x) + \int \mathrm{d}^4y\,\Pi^R(x,y)\varphi(y) = 0$$

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$$\Pi^R(x,y) = -\frac{i\lambda^2}{3!}\left[(\Delta^T(x,x'))^3 - (\Delta^<(x,x'))^3\right]$$

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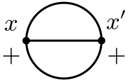

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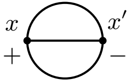
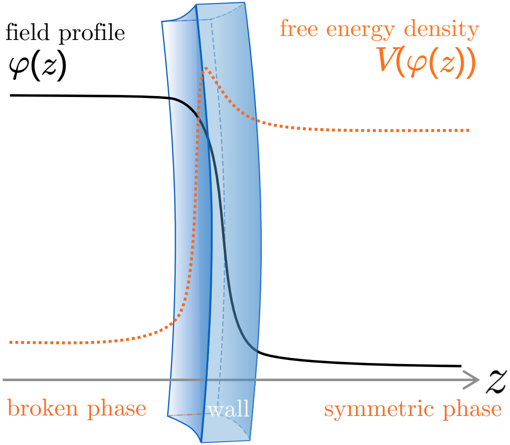


Identifying sources of friction

Identifying sources of friction

In the planar wall limit $\varphi(x) = \varphi(z)$, with the wall centered at $z = 0$ **in the wall frame**



Identifying sources of friction

In the planar wall limit $\varphi(x) = \varphi(z)$, with the wall centered at $z = 0$ **in the wall frame**

$$\frac{\mathrm{d}^2}{\mathrm{d}z^2}\varphi(z) + V'_{\text{eff}}(\varphi(z), T) + \frac{\mathrm{d}m_\varphi^2}{\mathrm{d}\varphi(z)} \int_{\mathbf{k}} \delta f(\mathbf{k}, z) + \int \mathrm{d}z' \, \pi^R(z, z')\varphi(z') \quad = 0$$

Identifying sources of friction

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$$\int_{-\delta}^{\delta} \mathrm{d}z \, \frac{\mathrm{d}}{\mathrm{d}z}\varphi(z) \left(\frac{\mathrm{d}^2}{\mathrm{d}z^2}\varphi(z) + V'_{\text{eff}}(\varphi(z), T) + \frac{\mathrm{d}m_\varphi^2}{\mathrm{d}\varphi(z)} \int_{\mathbf{k}} \delta f(\mathbf{k}, z) + \int \mathrm{d}z' \, \pi^R(z, z')\varphi(z') \right) = 0$$

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$$\begin{aligned} \Rightarrow \int_{-\delta}^{\delta} \mathrm{d}z \, \frac{\mathrm{d}}{\mathrm{d}z} \left[\frac{1}{2} \left(\frac{\mathrm{d}\varphi(z)}{\mathrm{d}z} \right)^2 + V_{\mathrm{eff}}(\varphi(z), T) \right] = \int_{-\delta}^{\delta} \mathrm{d}z \, \Bigg[& \frac{\partial V_{\mathrm{eff}}}{\partial T} \frac{\mathrm{d}T}{\mathrm{d}z} \\ & - \frac{\mathrm{d}m_\varphi^2}{\mathrm{d}z} \int_{\mathbf{k}} \delta f(\mathbf{k}, z) \\ & - \frac{\mathrm{d}\varphi(z)}{\mathrm{d}z} \int \mathrm{d}z' \, \pi^R(z, z')\varphi(z') \Bigg] \end{aligned}$$

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$$\Rightarrow \underbrace{\int_{-\delta}^{\delta} \mathrm{d}z \, \frac{\mathrm{d}}{\mathrm{d}z} \left[\cancel{\frac{1}{2} \left(\frac{\mathrm{d}\varphi(z)}{\mathrm{d}z} \right)^2} + V_{\text{eff}}(\varphi(z), T) \right]}_{\Delta V_{\text{eff}} \equiv \mathcal{P}_{\text{driving}}} = \int_{-\delta}^{\delta} \mathrm{d}z \left[\frac{\partial V_{\text{eff}}}{\partial T} \frac{\mathrm{d}T}{\mathrm{d}z} - \frac{\mathrm{d}m_\varphi^2}{\mathrm{d}z} \int_{\mathbf{k}} \delta f(\mathbf{k}, z) - \frac{\mathrm{d}\varphi(z)}{\mathrm{d}z} \int \mathrm{d}z' \, \pi^R(z, z')\varphi(z') \right]$$

Identifying sources of friction

In the planar wall limit $\varphi(x) = \varphi(z)$, with the wall centered at $z = 0$ **in the wall frame**

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$$- \frac{\mathrm{d}m_\varphi^2}{\mathrm{d}z} \int_{\mathbf{k}} \delta f(\mathbf{k}, z)$$

$$\left. - \frac{\mathrm{d}\varphi(z)}{\mathrm{d}z} \int \mathrm{d}z'\, \pi^R(z, z')\varphi(z') \right]$$

Identifying sources of friction

In the planar wall limit $\varphi(x) = \varphi(z)$, with the wall centered at $z = 0$ **in the wall frame**

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& - \frac{\mathrm{d}m_\varphi^2}{\mathrm{d}z} \int_{\mathbf{k}} \delta f(\mathbf{k}, z) && \equiv \mathcal{P}_{\text{dissipative}} \\
& - \frac{\mathrm{d}\varphi(z)}{\mathrm{d}z} \int \mathrm{d}z' \, \pi^R(z, z')\varphi(z') \Bigg]
\end{aligned}$$

Identifying sources of friction

In the planar wall limit $\varphi(x) = \varphi(z)$, with the wall centered at $z = 0$ **in the wall frame**

$$\int_{-\delta}^{\delta} \mathrm{d}z \, \frac{\mathrm{d}}{\mathrm{d}z}\varphi(z) \left(\frac{\mathrm{d}^2}{\mathrm{d}z^2}\varphi(z) + V'_{\text{eff}}(\varphi(z), T) + \frac{\mathrm{d}m_\varphi^2}{\mathrm{d}\varphi(z)} \int_{\mathbf{k}} \delta f(\mathbf{k}, z) + \int \mathrm{d}z' \, \pi^R(z, z')\varphi(z') \right) = 0$$

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& - \frac{\mathrm{d}\varphi(z)}{\mathrm{d}z} \int \mathrm{d}z' \, \pi^R(z, z')\varphi(z') \Bigg] && \equiv \mathcal{P}_{\text{vertex}}
\end{aligned}$$

Outline

The self-energy

The self-energy

At leading order in the gradient expansion

$$\mathcal{P}_{\text{vertex}} \equiv -\int \mathrm{d}z\mathrm{d}z' \, \frac{\mathrm{d}\varphi(z)}{\mathrm{d}z} \pi^R(z,z')\varphi(z') \simeq -\int \frac{\mathrm{d}q^z}{2\pi} i q^z \left|\tilde{\varphi}(q^z)\right|^2 \tilde{\pi}^R(-q^z)$$

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$$\left[\mathrm{Im}\tilde{\pi}^R(q) = -\mathrm{Im}\tilde{\pi}^R(-q)\,, \qquad \mathrm{Re}\tilde{\pi}^R(q) = \mathrm{Re}\tilde{\pi}^R(-q)\right] = -\int \frac{\mathrm{d}q^z}{2\pi} q^z \, |\tilde{\varphi}(q^z)|^2 \, \mathrm{Im}\tilde{\pi}^R(q^z)$$

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Introduce a heavy scalar field χ in the Lagrangian

$$\mathcal{L}_{\text{int}} \supset -\frac{g}{4}\phi^2\chi^2\,, \qquad m_\chi \gg m_\phi\,, T \implies f_\chi \sim 0$$

The self-energy

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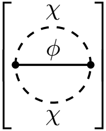
$$\mathrm{Im}\tilde{\pi}^R(q^z) \supset \mathrm{Im}$$



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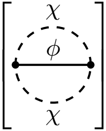

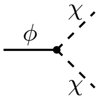

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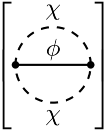
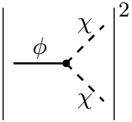
$\mathrm{Im}\tilde{\pi}^R(q^z) \supset \mathrm{Im}$ 

$=$ 

$\implies$ pair production!

The self-energy

The imaginary part of the self-energy is computed via CTP cutting rules

$$\mathrm{Im}\tilde{\pi}^R(q^z) = -\frac{i}{2}\left(\tilde{\pi}^>(q^z) - \tilde{\pi}^<(q^z)\right)$$

The self-energy

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$$\begin{aligned}\mathrm{Im}\tilde{\pi}^R(q^z) &= -\frac{i}{2}\left(\tilde{\pi}^>(q^z) - \tilde{\pi}^<(q^z)\right)\\ &\simeq \frac{g^2}{4}\int_{\mathbf{p},\mathbf{k}_1,\mathbf{k}_2}(2\pi)^3\delta^{(3)}(\mathbf{q}-\mathbf{p}+\mathbf{k}_1+\mathbf{k}_2)(2\pi)\delta(E^{(\phi)}_{\mathbf{p}} - E^{(\chi)}_{\mathbf{k}_1} - E^{(\chi)}_{\mathbf{k}_2})\\ &\qquad\qquad \times\left[f_\phi(\mathbf{p}) - f_\phi(-\mathbf{p})\right]\end{aligned}$$

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\begin{aligned}
\mathrm{Im}\tilde{\pi}^R(q^z) =& -\frac{i}{2}\left(\tilde{\pi}^>(q^z) - \tilde{\pi}^<(q^z)\right) \\
\simeq& \frac{g^2}{4}\int_{\mathbf{p},\mathbf{k}_1,\mathbf{k}_2} (2\pi)^3\delta^{(3)}(\underbrace{\mathbf{q}}_{=(0,0,q^z)} - \mathbf{p} + \mathbf{k}_1 + \mathbf{k}_2)(2\pi)\delta(E^{(\phi)}_{\mathbf{p}} - E^{(\chi)}_{\mathbf{k}_1} - E^{(\chi)}_{\mathbf{k}_2}) \\
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and the pressure due to pair production reads

$$\begin{aligned}\mathcal{P}_{\phi\to\chi\chi} = \frac{g^2}{2}\int_{\mathbf{p},\mathbf{k}_1,\mathbf{k}_2}&(2\pi)^2\delta^{(2)}(\mathbf{p}_\perp - \mathbf{k}_{1,\perp} - \mathbf{k}_{2,\perp})(2\pi)\delta(E^{(\phi)}_{\mathbf{p}} - E^{(\chi)}_{\mathbf{k}_1} - E^{(\chi)}_{\mathbf{k}_2}) \\ &\times f_\phi(\mathbf{p}) \quad \Delta p^z \quad |\tilde{\varphi}(\Delta p^z)|^2\end{aligned}$$

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&\times f_\phi(\mathbf{p}) \quad \Delta p^z \quad |\tilde{\varphi}(\Delta p^z)|^2
\end{aligned}
$$

density of incoming particles

The self-energy

The imaginary part of the self-energy is computed via CTP cutting rules

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\mathrm{Im}\tilde{\pi}^R(q^z) &= -\frac{i}{2}\left(\tilde{\pi}^>(q^z) - \tilde{\pi}^<(q^z)\right) \\
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&\times f_\phi(\mathbf{p}) \quad \Delta p^z \quad |\tilde{\varphi}(\Delta p^z)|^2
\end{aligned}
$$

density of incoming particles

momentum exchange

The self-energy

The imaginary part of the self-energy is computed via CTP cutting rules

$$\begin{aligned}\mathrm{Im}\tilde{\pi}^R(q^z) &= -\frac{i}{2}\left(\tilde{\pi}^>(q^z) - \tilde{\pi}^<(q^z)\right) \\ &\simeq \frac{g^2}{4}\int_{\mathbf{p},\mathbf{k}_1,\mathbf{k}_2}(2\pi)^3\delta^{(3)}(\mathbf{q}-\mathbf{p}+\mathbf{k}_1+\mathbf{k}_2)(2\pi)\delta(E_{\mathbf{p}}^{(\phi)}-E_{\mathbf{k}_1}^{(\chi)}-E_{\mathbf{k}_2}^{(\chi)}) \\ &\qquad\times\left[f_\phi(\mathbf{p})-f_\phi(-\mathbf{p})\right]\end{aligned}$$

and the pressure due to pair production reads

$$\begin{aligned}\mathcal{P}_{\phi\to\chi\chi} = \frac{g^2}{2}\int_{\mathbf{p},\mathbf{k}_1,\mathbf{k}_2}&(2\pi)^2\delta^{(2)}(\mathbf{p}_\perp-\mathbf{k}_{1,\perp}-\mathbf{k}_{2,\perp})(2\pi)\delta(E_{\mathbf{p}}^{(\phi)}-E_{\mathbf{k}_1}^{(\chi)}-E_{\mathbf{k}_2}^{(\chi)}) \\ &\times f_\phi(\mathbf{p})\quad\Delta p^z\quad|\tilde{\varphi}(\Delta p^z)|^2\end{aligned}$$

density of incoming particles

Fourier tf. of the wall

momentum exchange

The ultrarelativistic regime

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Analytic formula for an ultrarelativistic (tanh) wall in the limit of light ϕ-particles

$$\mathcal{P}_{\phi\to\chi\chi}^{\gamma_w\to\infty} \approx \frac{g^2 v_b^2 T^2}{24\times 32\pi^2}\log\left(\frac{\gamma_w T}{2\pi L_w m_\chi^2}\right)$$

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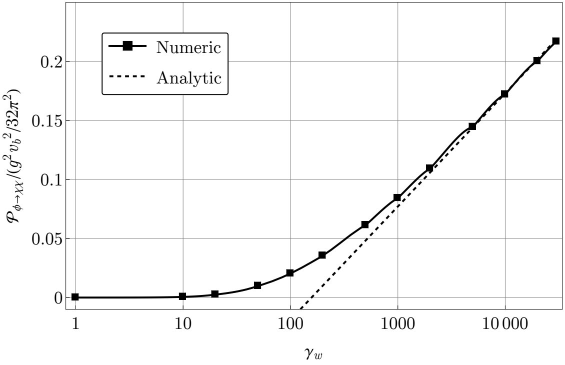

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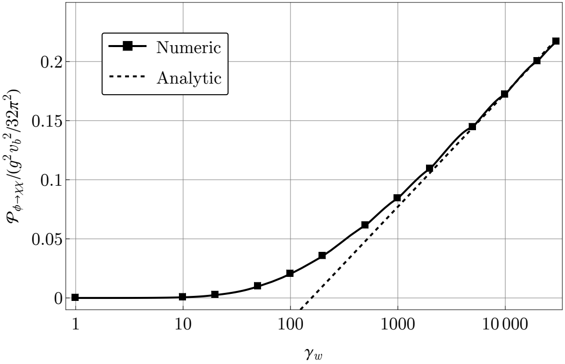

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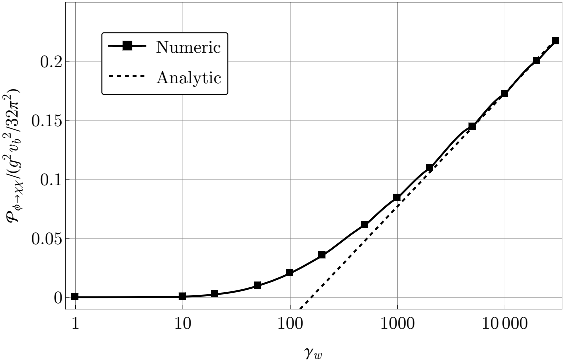
Similarly, we show in our work that particle mixing and transition radiation are also captured within this framework.



Outline

Conclusions and outlook [2504.13725]

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Future directions

- investigate further out-of-equilibrium effects affecting the wall expansion, such as gauge boson saturation
- study numerically the effect of so far overlooked quantum effects for intermediate wall velocities

BACK-UP SLIDES

The CTP formalism

Time ordering is replaced by ordering on the CTP $\mathcal{T} \to \mathcal{T}_{\mathrm{CTP}}$. We then have four two-point functions

$$\langle \mathcal{T}_{\mathrm{CTP}} \phi^+(x)\phi^+(y)\rangle = \langle \mathcal{T}\phi(x)\phi(y)\rangle = \Delta^T(x,y)$$

$$\langle \mathcal{T}_{\mathrm{CTP}} \phi^-(x)\phi^-(y)\rangle = \langle \overline{\mathcal{T}}\phi(x)\phi(y)\rangle = \Delta^{\overline{T}}(x,y)$$

$$\langle \mathcal{T}_{\mathrm{CTP}} \phi^+(x)\phi^-(y)\rangle = \langle \phi(y)\phi(x)\rangle = \Delta^<(x,y)$$

$$\langle \mathcal{T}_{\mathrm{CTP}} \phi^-(x)\phi^+(y)\rangle = \langle \phi(x)\phi(y)\rangle = \Delta^>(x,y)$$

We can then do perturbation theory, but in particular, we will be interested in generating equations of motion. For this, we work with the effective action.

The Wigner transform

To put the equations in a useful form, we go to Wigner space

$$\overline{\Delta}^{ab}(k,x) = \int \mathrm{d}^4 r \, e^{ik\cdot r} \Delta^{ab}\left(x + \frac{r}{2}, x - \frac{r}{2}\right)$$

Generally, the equations contain derivatives in x of all orders. To leading order in the derivative (or gradient) expansion we can solve for the two-point functions

$$\overline{\Delta}^{<}(k,x) = 2\pi\delta(k^2 - m^2)\left[\vartheta(k^0) f(\mathbf{k},x) + \vartheta(-k^0)(1 + f(-\mathbf{k},x))\right]$$

$$\overline{\Delta}^{>}(k,x) = 2\pi\delta(k^2 - m^2)\left[\vartheta(k^0)(1 + f(\mathbf{k},x)) + \vartheta(-k^0) f(-\mathbf{k},x)\right]$$

$$\overline{\Delta}^{T}(k,x) = \frac{\mathrm{i}}{k^2 - m^2 + \mathrm{i}\varepsilon} + 2\pi\delta(k^2 - m^2)\left[\vartheta(k^0) f(\mathbf{k},x) + \vartheta(-k^0) f(-\mathbf{k},x)\right]$$

$$\overline{\Delta}^{\overline{T}}(k,x) = -\frac{\mathrm{i}}{k^2 - m^2 - \mathrm{i}\varepsilon} + 2\pi\delta(k^2 - m^2)\left[\vartheta(k^0) f(\mathbf{k},x) + \vartheta(-k^0) f(-\mathbf{k},x)\right]$$

A comment on the gradient expansion

In our derivation, we made extensive use of the gradient expansion. What is the validity of this approximation?

$$\text{small field gradients} \equiv \frac{\nabla\varphi}{k} \ll 1$$

$$\nabla\varphi \sim \frac{1}{L_w}\,, \qquad L_w \equiv \text{wall width}$$

$$k \sim \gamma_w T \ \equiv \ \text{typical momentum of a particle } \textbf{in the wall frame}$$

$$\Longrightarrow \gamma_w T L_w \gg 1$$

The gradient expansion is valid if the wall is either **fast** or **thick**.For the numerical and analytical results, we assumed the plasma outside the bubble to be **in equilibrium**, which is once again only valid if the wall is very fast.

Mixing

Assume two mixing scalar species χ and s interacting through the background

$$\mathcal{L}_{\text{int}} \supset -\kappa\varphi\chi s\,, \qquad \text{and} \quad m_\chi \gg m_s$$

Particles χ are absent in the plasma but are generated via mixing as s-particles go through the wall. In the ultrarelativistic limit

$$\mathcal{P}_{s\to\chi}^{\gamma_w\to\infty} = \frac{2\kappa^2 v_b^2}{m_\chi^2}\frac{T^2}{24}$$