Gravitational Waves from a First-Order Phase Transition of the Inflaton

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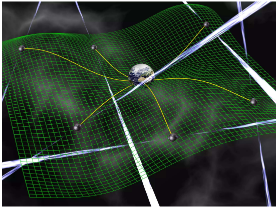
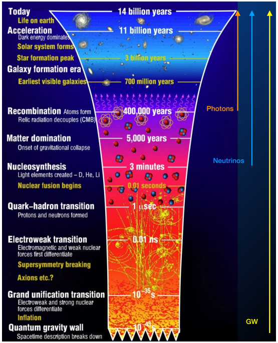
Based on JK, Seong Chan Park, Yeji Park, Juhoon Son, Liliana Velasco Sevilla,
JCAP **04** (2025) 053 [arXiv:2412.17278]

A Window to the Very Early Universe?

- Pulsar Timing Arrays
  ⇝ Evidence for Stochastic GW Background
  NANOGrav, ApJL **951** (2023)
  EPTA, InPTA, A&A **678** (2023)
  Parkes PTA, ApJL **951** (2023)
  CPTA, RAA **23** (2023)
- Lower frequency than LIGO/Virgo events
  ⇝ Mergers of supermassive black holes?
- More interesting: particle physics origin
  - First-Order Phase Transition (FOPT)
  - Cosmic strings or domain walls
  - Inflation



D. Champion/MPI for Radio Astronomy



Higgs Inflation

Bezrukov & Shaposhnikov, PLB **659** (2008)

$$\mathcal{L}_J = \sqrt{-g_J}\left[\frac{M_\mathrm{P}^2}{2}R_J + \xi\phi^\dagger\phi\, R_J + g_J^{\mu\nu}(D_\mu\phi)^\dagger(D_\nu\phi) - V_J(\phi) + \dots\right]$$

- J: Jordan frame
- R: Ricci scalar
- V_J: SM Higgs potential
- ξ: non-minimal coupling to gravity
  - Consistent with all symmetries
  - Required for renormalization in curved spacetime
  - Not necessarily small

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- Def.: conformal factor $\Omega^2(\phi) \equiv 1 + 2\xi \frac{\phi^\dagger\phi}{M_P^2}$

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Higgs Inflation

- Unitary gauge: $\phi = \begin{pmatrix} 0 \\ (\varphi + v)/\sqrt{2} \end{pmatrix}$
- Weyl transformation to Einstein frame: $g_J \to g_E \equiv \Omega^2(\varphi)\, g_J$
- Canonical normalization: $\chi = \int_0^\varphi \mathrm{d}\varphi \sqrt{\frac{3}{2}\frac{(\Omega^2{}_{,\varphi})^2}{\Omega^4} + \frac{1}{\Omega^2}}$

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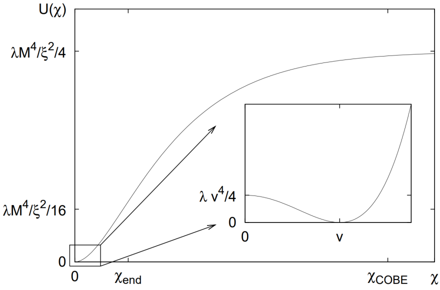
$$\mathcal{L}_E = \sqrt{-g_E}\left[\frac{M_\mathrm{P}^2}{2} R_E + \frac{1}{2} g_E^{\mu\nu} (\partial_\mu \chi)(\partial_\nu \chi) - V_E(\chi) + \dots\right]$$

- Minimal coupling to gravity but modified potential
- Potential flat for large field values $\rightsquigarrow$ slow-roll

Higgs Inflation



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Higgs Inflation in a Dark Sector

$$\mathcal{L}_J = \sqrt{-g_J}\left[\frac{M_P^2}{2}\,\Omega^2(\phi)\,R_J + g_J^{\mu\nu}(D_\mu\phi)^\dagger(D_\nu\phi) - \frac{1}{4}g_J^{\mu\rho}g_J^{\nu\sigma}X_{\mu\nu}X_{\rho\sigma} - V_J(\phi)\right]$$

$$V_J(\phi) = -\mu^2\phi^\dagger\phi + \lambda\,(\phi^\dagger\phi)^2 = \lambda\left(\phi^\dagger\phi - \frac{v^2}{2}\right)^2 + \text{const.}$$

- $U(1)_X$ gauge symmetry with gauge coupling g
- SM singlet scalar ϕ with $U(1)_X$ charge
  - Spontaneously breaks $U(1)_X \rightsquigarrow$ FOPT $\rightsquigarrow$ GW
  - Non-minimal coupling to gravity $\rightsquigarrow$ Higgs-inflation-like inflation

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- Optional: fermions with mass from Yukawa coupling
- Weak coupling to SM $\rightsquigarrow$ reheating

$$\mathcal{L}_\psi = \sqrt{-g_J}\sum_{i=1}^{n_\psi}\left[\overline{\psi}_{Li}i\not{D}\psi_{Li} + \overline{\psi}_{Ri}i\not{D}\psi_{Ri} - \frac{y}{2}\left(\overline{\psi}_{Li}\,\phi\,\psi_{Li}^c + \overline{\psi}_{Ri}\,\phi\,\psi_{Ri}^c + \text{h.c.}\right)\right]$$

Gravitational Waves from Phase Transition

Scalar potential with 1-loop and finite-temperature corrections

- $\rightsquigarrow$ Potential barrier around $T \sim v$
- $\rightsquigarrow$ FOPT possible

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GW spectrum determined by

- Nucleation temperature T_n
- $\alpha \leftrightsquigarrow$ strength of PT
- $\beta \leftrightsquigarrow$ duration

Calculated with help from `CosmoTransitions`

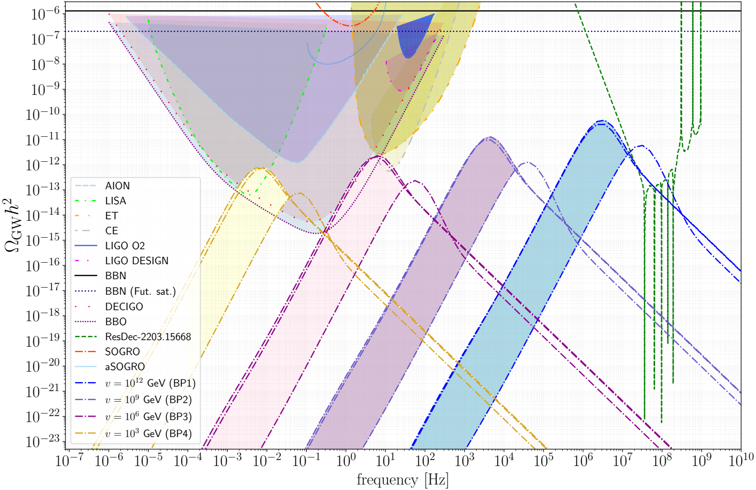
Wainwright, Comput. Phys. Commun. **183** (2012)

Effective Scalar Potential at Finite Temperature

$$
\begin{aligned}
V(h_c, T) &= V_{\text{tree}}(h_c) + V_{\text{1-loop}}(h_c) + V_{\text{th}}(h_c, T) \\
V_{\text{tree}}(h_c) &= -\frac{\mu^2}{2} h_c^2 + \frac{\lambda}{4} h_c^4 \\
V_{\text{1-loop}}(h_c) &= \sum_{i=h,\chi,g,f} \frac{n_i}{64\pi^2} m_i^4(h_c) \left[\ln \frac{|m_i^2(h_c)|}{v^2} - C_i \right] \\
V_{\text{th}}(h_c, T) &= \sum_{i=h,\chi,g} \frac{n_i}{2\pi^2} T^4 \, \operatorname{Re} J_b\!\left(\frac{m_i^2(h_c)}{T^2} \right) + \frac{n_f}{2\pi^2} T^4 \, J_f\!\left(\frac{m_f^2(h_c)}{T^2} \right) \\
m_h^2(h_c) &= -\mu^2 + 3\lambda h_c^2 \\
m_\chi^2(h_c) &= -\mu^2 + \lambda h_c^2 \\
m_g^2(h_c) &= g^2 h_c^2 \\
m_f^2(h_c) &= \frac{y^2}{2} h_c^2
\end{aligned}
$$

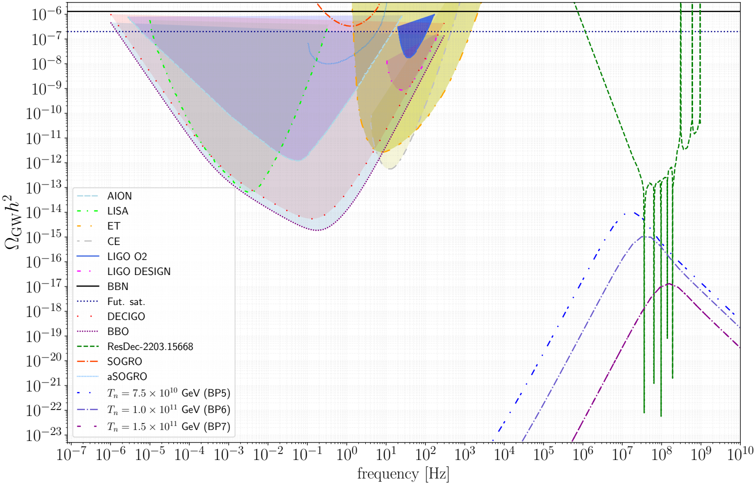
Gravitational Wave Spectrum I

No fermions, $g = 0.95$, $\lambda = 10^{-3}$
Bubble wall velocity $v_w = 1, v_{\rm det}, 0.1$

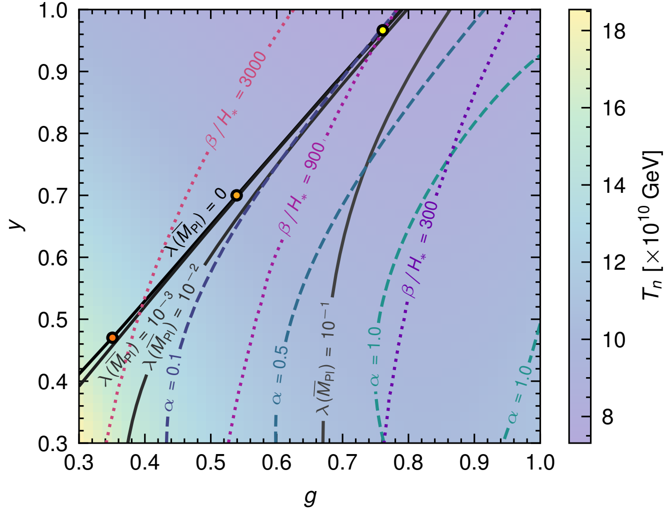


Gravitational Wave Spectrum II

1 fermion pair, $v_w = v_{\text{det}}$, $v = 10^{12}$ GeV, $\lambda = 10^{-3}$
$(g, y) \simeq (0.76, 0.97), (0.54, 0.70), (0.35, 0.47)$



Parameter Space with 1 Fermion Pair



Inflationary Observables

CMB measurements (pre-ACT)

Scalar power spectrum amplitude	$A_s = (2.098 \pm 0.023) \times 10^{-9}$
Scalar spectral index	$n_s = 0.9649 \pm 0.0042$
Tensor-to-scalar power ratio	$r < 0.036$ (95% CL)

Planck, A&A **641** (2020); BICEP/Keck, PRL **127** (2021)

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Model results (60 *e*-foldings)

- $n_s \simeq 0.965$
- $r \simeq 0.003$
- $A_s \simeq 5.1 \, \frac{\lambda(M_P)}{\xi^2} \quad \rightsquigarrow \quad \xi \simeq 5 \times 10^4 \sqrt{\lambda(M_P)}$

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	$g(M_P)$	$\lambda(M_P)$	$y(M_P)$	n_ψ	ξ
BP1	0.98	0.37	-	0	3×10^4
BP2	0.99	0.57	-	0	4×10^4
BP3	1.00	0.92	-	0	5×10^4
BP4	1.02	2.22	-	0	7×10^4
BP5	0.79	$\simeq 0$	1.10	1	$\mathcal{O}(1)$
BP6	0.55	$\simeq 0$	0.74	1	$\mathcal{O}(1)$
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- With fermions: renormalization group running allows $\xi \sim 1$
- Analogous to Critical Higgs Inflation
  Hamada et al., PRL **112** (2014), PRD **91** (2015)

Conclusions and Outlook

- Unified framework for inflation and first-order phase transition
- Dark $U(1)_X$ and non-minimal coupling to gravity
  $\rightsquigarrow$ gravitational waves and inflation from same scalar
- Peak GW frequencies between 10^{-2} Hz and 10^{8} Hz

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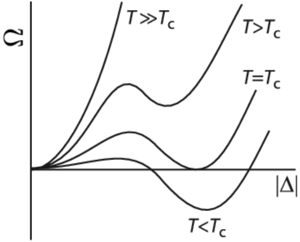
- Future improvements
  - Calculation of bubble wall velocity using `WallGo`
    Ekstedt et al., JHEP **04** (2025)
  - Inflaton coupling to SM $\rightsquigarrow$ reheating
  - Dark Matter
  - Different gauge groups

First-Order Phase Transitions



Kinnunen et al., Rep. Prog. Phys. **81** (2018)

1. High temperature: potential minimum at $\phi = 0$
2. $T < T_c$: deeper mimimum at $\phi = v$, separated by barrier
3. Tunneling $\rightsquigarrow$ bubbles of true vacuum
4. GW from expanding bubbles and surrounding plasma

Parameters Determining the GW Signal

- T_n: temperature at which probability of nucleating 1 bubble per horizon volume is ~ 1
- $\alpha = \frac{1}{\rho_{\text{rad}}} \left[\Delta V(h_c, T) - T \frac{dV(h_c,T)}{dT} \right] \Big|_{T=T_n}$
- $\frac{\beta}{H(T_n)} = T_n \frac{d}{dT} \frac{S_3}{T} \Big|_{T_n} \simeq \left(\frac{T_n}{T-T_n} \right) \left(\frac{S_3(T)}{T} - \frac{S_3(T_n)}{T_n} \right)$
  (T: arbitrary reference temperature not too far from T_n)
- Expressions modified for large supercooling

SM Extensions with First-Order Phase Transitions

- SM: no FO electroweak PT for Higgs mass $\gtrsim 70\,\text{GeV}$
  Kajantie et al., PRL **77** (1996); Karsch et al., NP Proc. Suppl. **53** (1997)
  Csikor et al., PRL **82** (1999)
- Most famous: SUSY with light stop
  Carena et al., PLB **380** (1996); Espinosa, NPB **475** (1996)
  Delepine et al., PLB **386** (1996); Cline & Kainulainen, NPB **482** (1996); ...
- Minimal from model building perspective: 2HDM
  Dorsch et al., JHEP **10** (2013); Basler et al., JHEP **02** (2017)
  Andersen et al., PRL **121** (2018); ...
- Minimal from EFT perspective: higher-dimensional operators
  Zhang, PRD **47** (1993); Grojean et al., PRD **71** (2005)
  Bödeker et al., JHEP **02** (2005); Chala et al., JHEP **07** (2018); ...
- Many recent works Review: Roshan & White, arXiv:2401.04388