PATHS THROUGH THE DARK: COMPARING APPROACHES FOR FOPTS

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In collaboration with Eric Madge, Maura E. Ramírez-Quezada and Pedro Schwaller

GW detection opens a window to study physics BSM

GWs down to the nHz regime

→ **PTA**: EPTA, NANOGrav, IPTA

- SMBHB
- New Physics
  - FOPTs
  - DW
  - CS
  - Axion dynamics

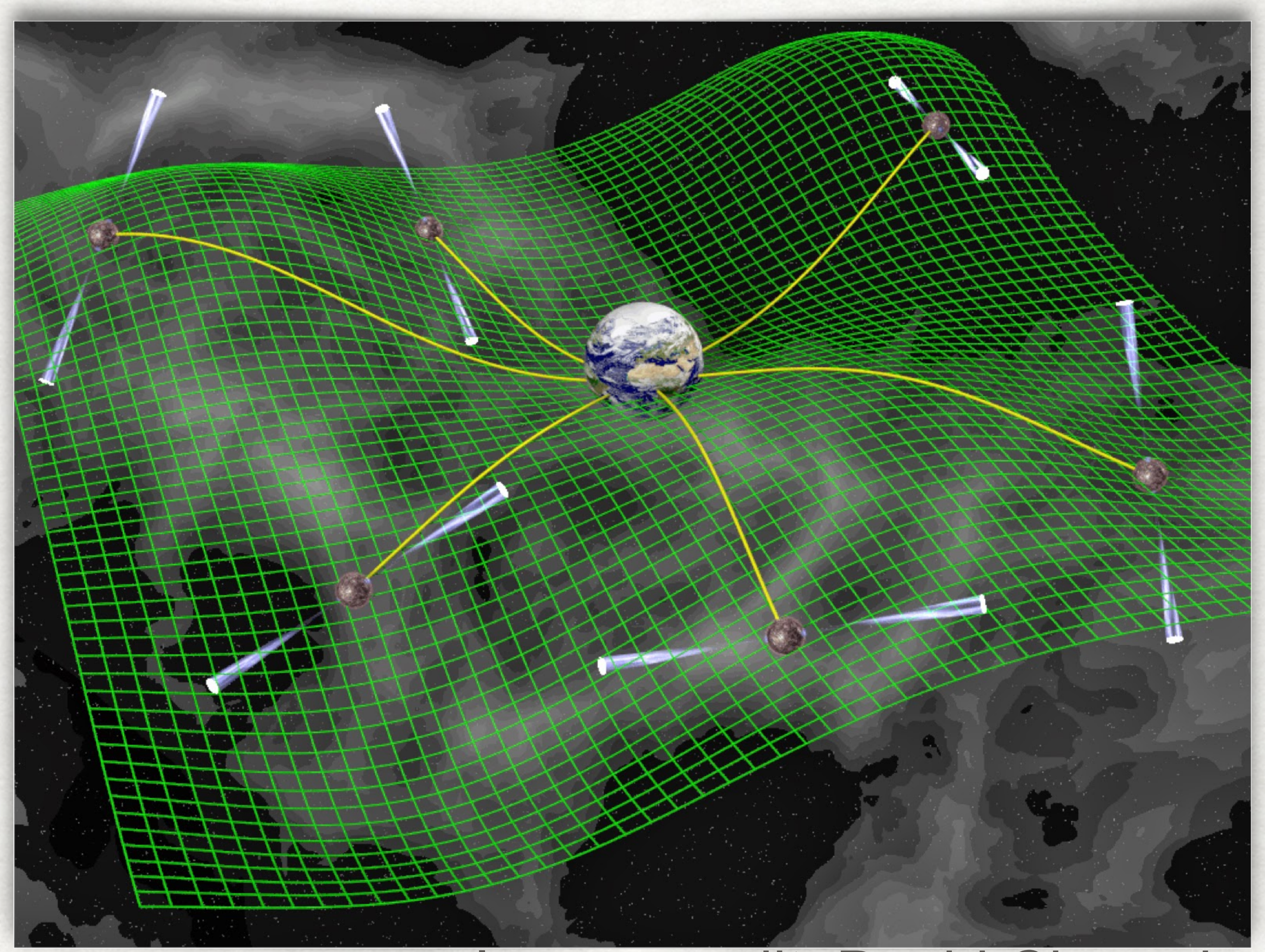

Image credit: David Champion

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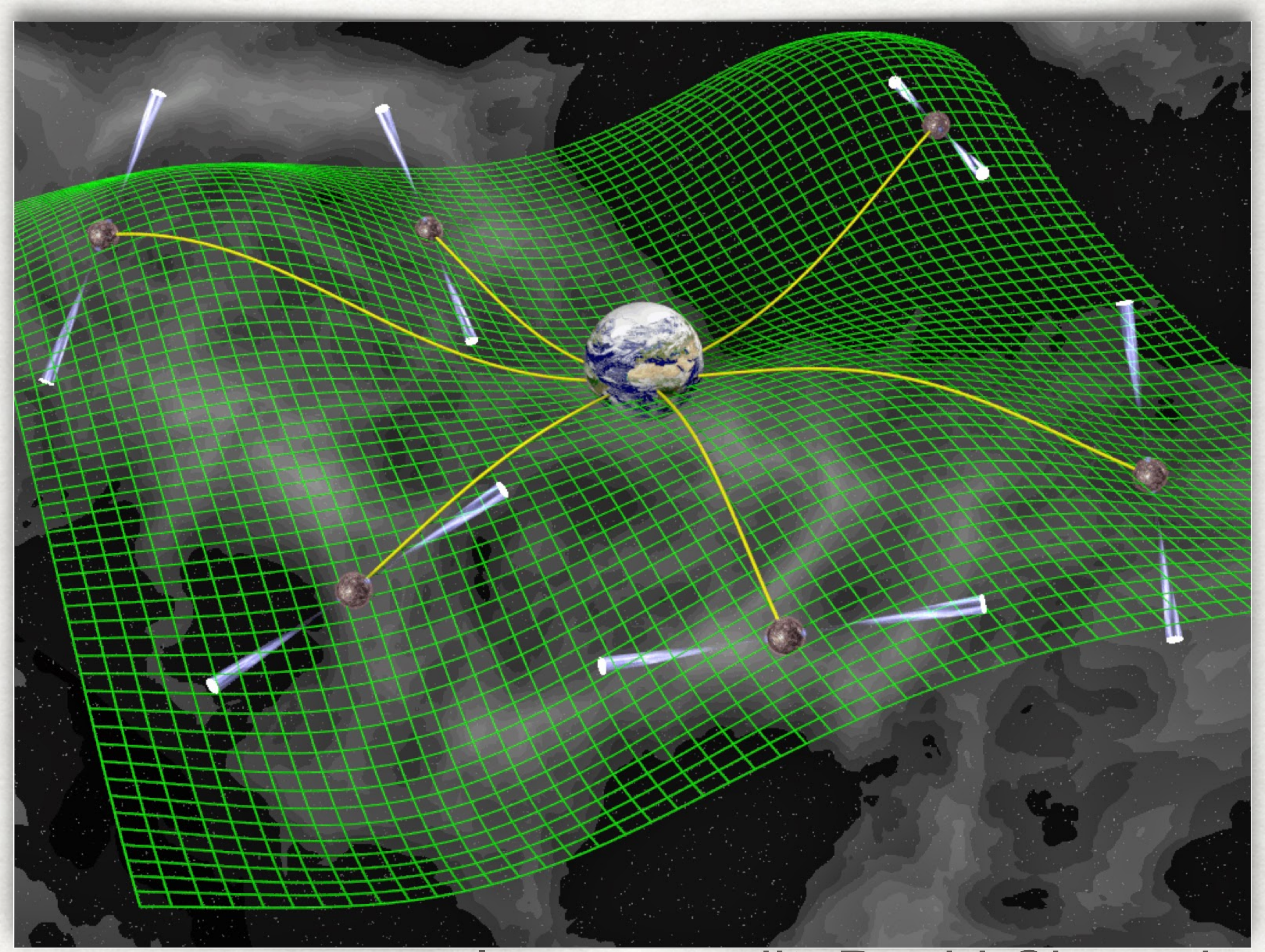

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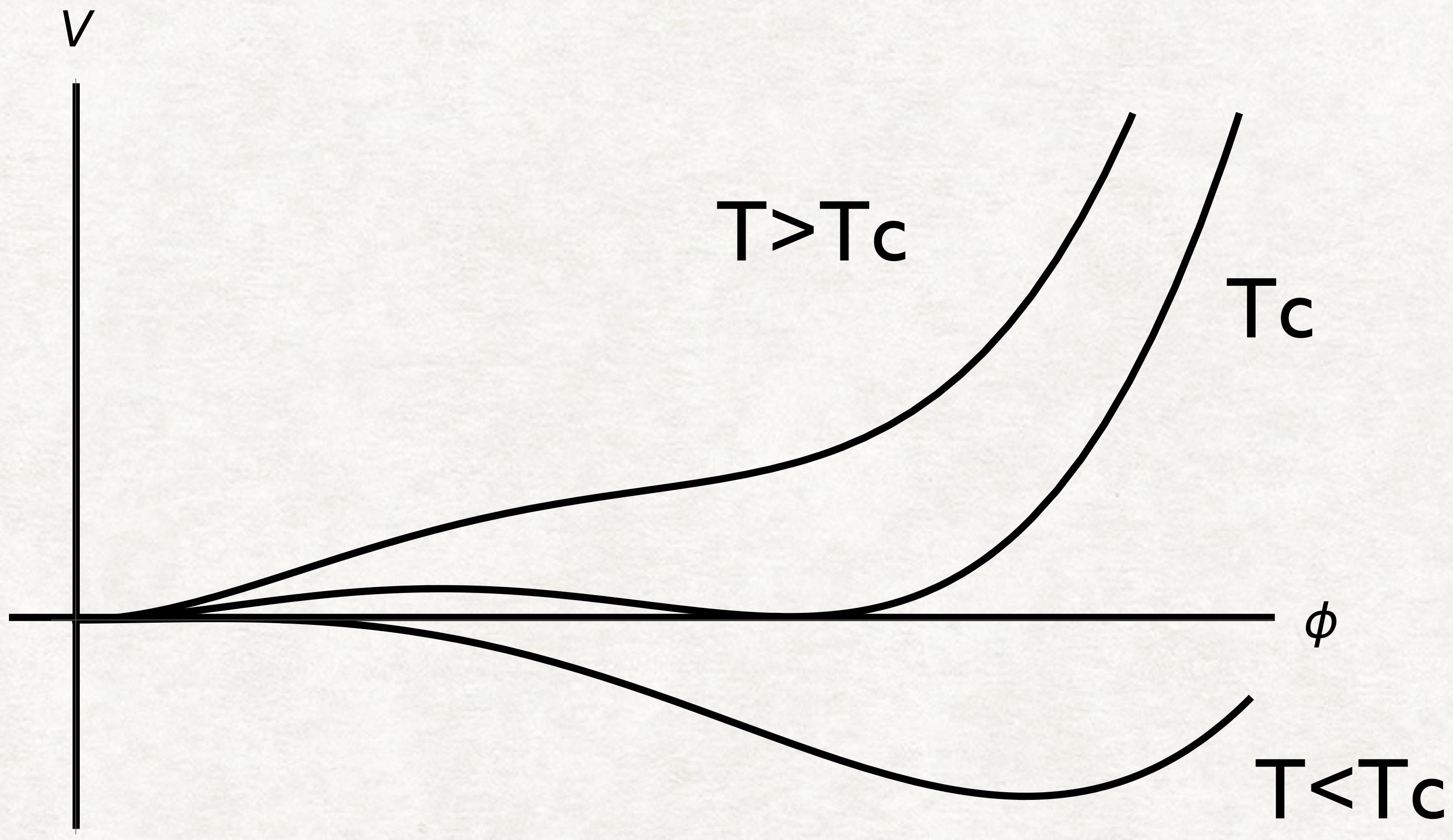


Supercooling: Delay of the FOPT below Tc

→ large release of latent heat α

PREVIOUS WORK

→ Supercooled FOPTs

Model: classically scale invariance +radiative breaking of conformal symmetry: **Coleman-Weinberg (CW).**

- Zero-T CW at 1-Loop

$$V_{CW} = \sum_i N_i \frac{m_i^4(\phi)}{64\pi^2}\left[\log\left(\frac{m_i^2(\phi)}{\mu_R^2}\right) - c_i\right]$$

- Finite Temperature:

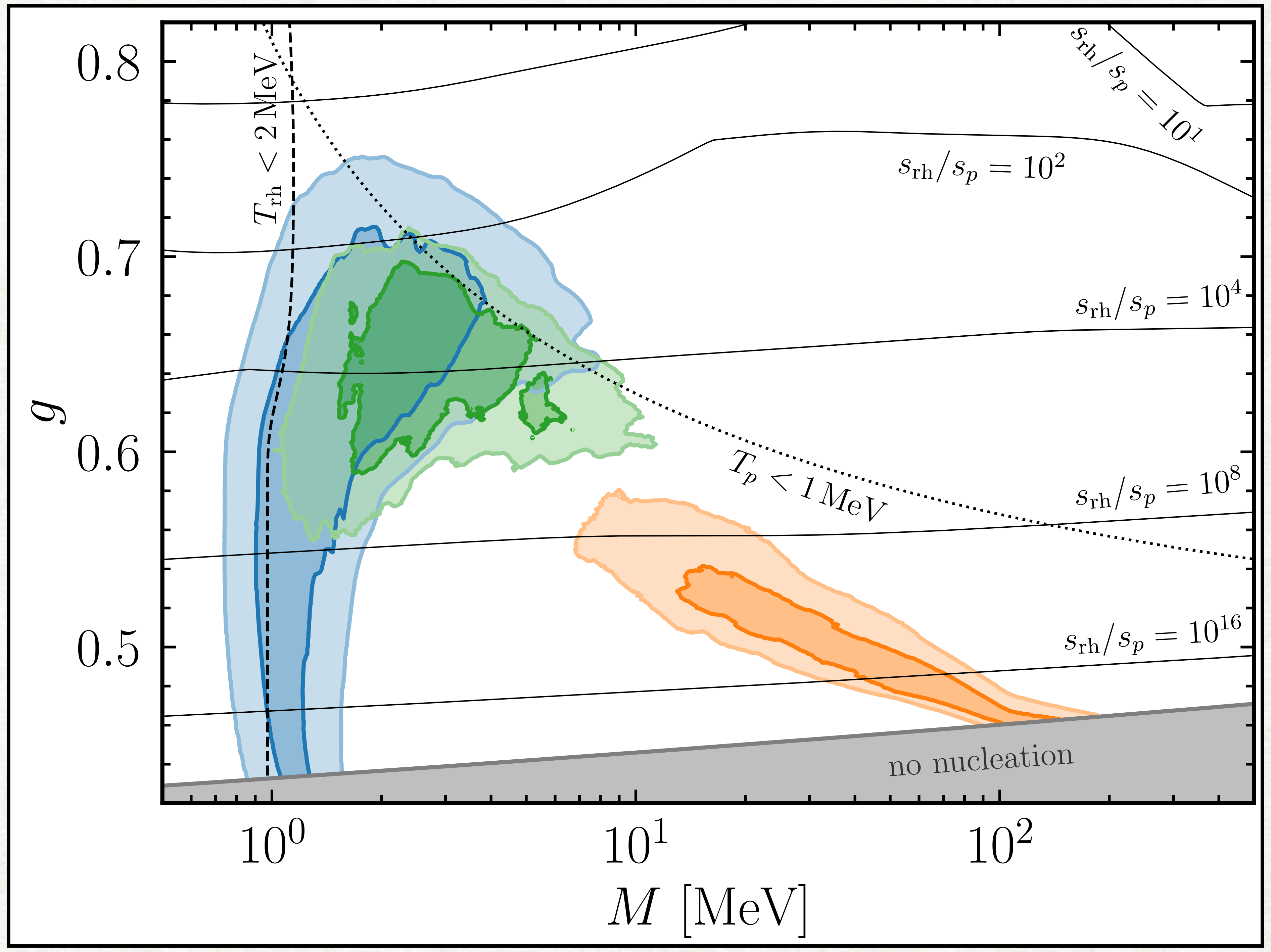
$$V_1(\phi, T) = \frac{m^2(T)}{2}\phi^2 - \frac{\delta(T)}{3}\phi^3 + \frac{\lambda(T)}{4}\phi^4$$

$\log(\phi)$ cancels

Neglect Daisy contributions !

$$\kappa \equiv \frac{\lambda(T)m^2(T)}{\delta^2(T)} = \frac{\log(T/M)}{6}$$

→ **OPA**



📖 From https://arxiv.org/pdf/2306.14856 by E. Madge, E. Morgante, **CPI**, N. Ramberg, W. Ratzinger, S.Schenk, P. Schwaller

📖 For more details look https://arxiv.org/abs/2212.08085 by N.Levi, T. Opferkuch, D. Redigo

What we wanna explore now?

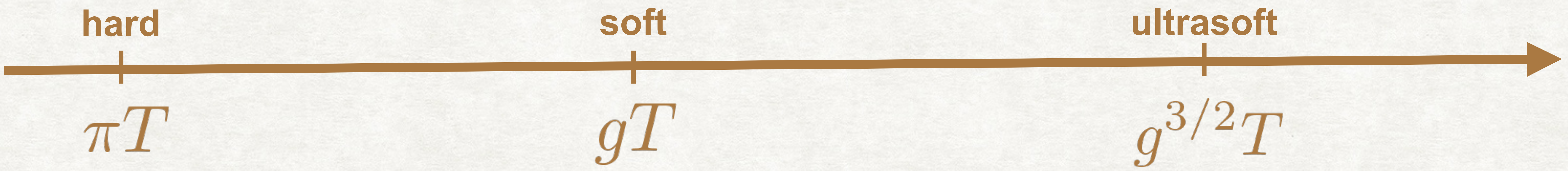
- OPA vs. HT expansion and full form of the 4D potential adding Daisy resumation.
- Daisy resumation vs. Dimensional reduction (DR) EFTs
- Effects of 2-Loop corrections using DR.

Perturbation theory breakdown → IR divergences

DAISY RESUMATION

$$V_{\text{ring}}(\phi, T) = -\frac{T}{12\pi}\sum_i n_i\left[\left(m_i^2(\phi,T)\right)^{3/2} - \left(m_i^2(\phi)\right)^{3/2}\right]$$

DIMENSIONAL REDUCTION (DR)



Integrate out non-zero Matsubara modes

Integrate massive temporal scalars

SET UP

Dark photon: Extend SM with an additional U(1).

Supercooling

1-Loop potencial in the MS scheme:

$$V_{1-\text{loop}}(\phi) = -\frac{m^2}{2}\phi^2 + \frac{\lambda}{4}\phi^4 + \frac{3(g^2\phi^2)^2}{64\pi^2}\left[\log\frac{g^2\phi^2}{\mu^2} - \frac{5}{6}\right] + \frac{(3\lambda\phi^2 - m^2)^2}{64\pi^2}\left[\log\frac{|3\lambda\phi^2 - m^2|}{\mu^2} - \frac{3}{2}\right] + \frac{(\lambda\phi^2 - m^2)^2}{64\pi^2}\left[\log\frac{|\lambda\phi^2 - m^2|}{\mu^2} - \frac{3}{2}\right]$$

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Breaking parameter $\epsilon = m^2/v^2$

$$\lambda = \frac{g^4}{16\pi^2}\left(1 - 3\log\frac{g^2v^2}{\mu^2}\right) + \epsilon + \frac{\lambda}{16\pi^2}\left[3\,(3\lambda - \epsilon)\left(1 - \log\frac{|3\lambda - \epsilon|}{\mu^2/v^2}\right) + (\lambda - \epsilon)\left(1 - \log\frac{|\lambda - \epsilon|}{\mu^2/v^2}\right)\right]$$

If $\epsilon \ll \dfrac{g^4}{16\pi^2}$ $\qquad \lambda \sim \dfrac{g^4}{16\pi^2}$

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But DRalgo power counting

$$g^2 \sim \lambda \sim m^2$$

MATCHING 3D AND 4D THEORY

couplings

$$g_{3d}^2 = g^2 T - \frac{g^4 L_b T}{48\pi^2} \qquad \lambda_{3d} = \lambda T + \frac{T}{16\pi^2}\left[g^4\left(2-3L_b\right) + 6g^2\lambda L_b - 10\lambda^2 L_b\right]$$

$$\lambda_A = 0 + \frac{g^4 T}{\pi^2} \qquad \lambda_{A\Phi} = 2g^2 T + \frac{g^2 T}{24\pi^2}\left[24\lambda - g^2\left(L_b - 4\right)\right]$$

masses

$$m_{3d}^2 = m^2 - \frac{T^2(3g^2+4\lambda)}{12} + \frac{L_b\left(3g^2-4\lambda\right)m^2}{16\pi^2} + \frac{g^4(8+216c_+ +39L_b)T^2}{576\pi^2} + \frac{\lambda^2(12c_+ +5L_b)T^2}{24\pi^2}$$
$$- \frac{g^2\lambda(1+12c_+ +3L_b)T^2}{24\pi^2} - \frac{8g_{3d}^4 - 16g_{3d}^2\lambda_{3d} + 16\lambda_{3d}^2 + \lambda_{A\phi}^2}{32\pi^2}\log\frac{\mu_3}{\mu_R}$$

$$\mu_D^2 = \frac{g^2 T^2}{3} + \frac{g^2}{144\pi^2}\left[12\lambda T^2 + g^2(7-L_b)T^2 - 36m^2\right]$$

constants $c_+ = \frac{1}{2}\left(\gamma_E - L_b - 12\log A\right)$ $\qquad L_b = \log\frac{\mu_R^2 e^{2\gamma_E}}{16\pi^2 T^2}$

DIFFERENT APPROACHES

$$\phi_{3d} \to \phi/\sqrt{T}$$

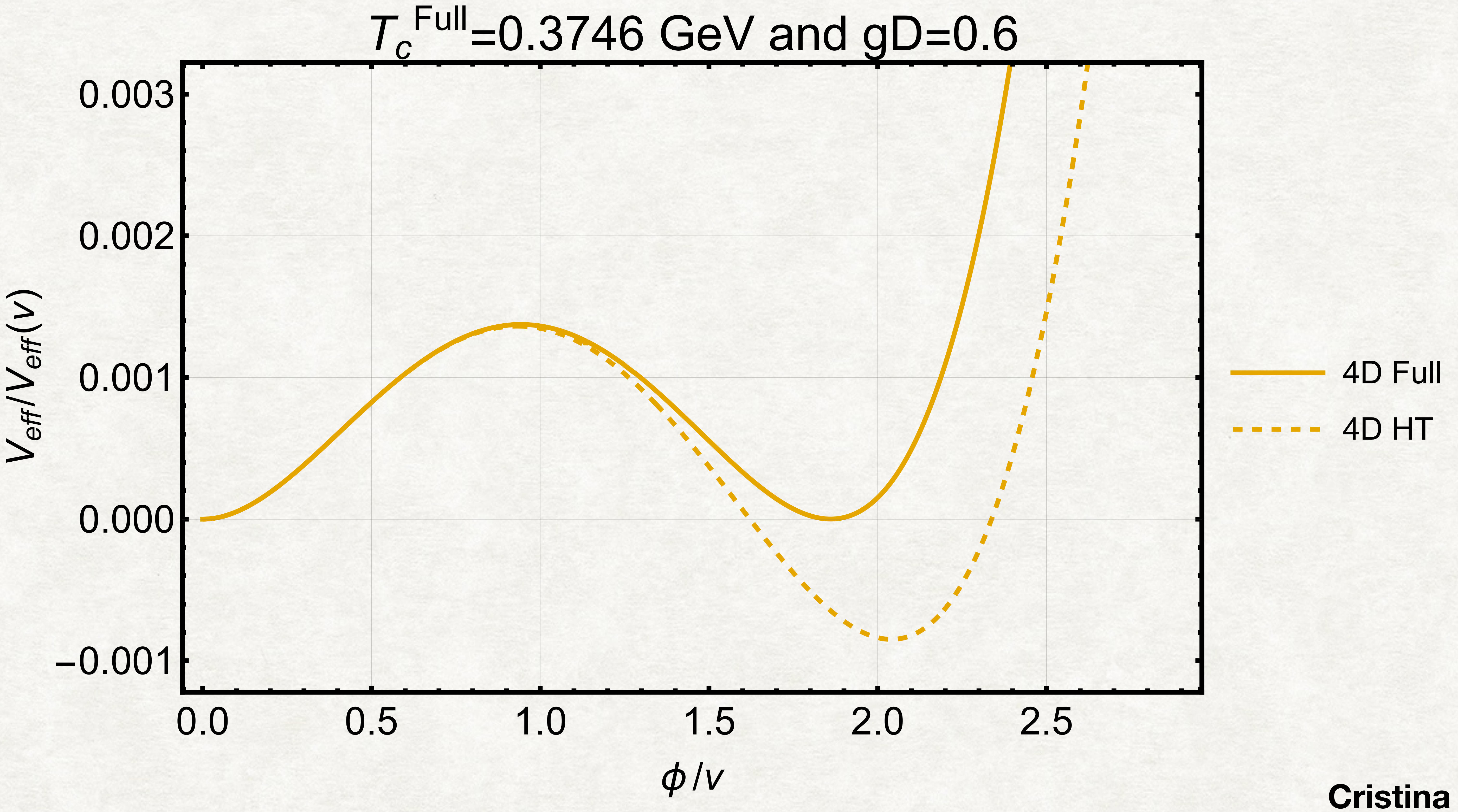
4D: $V_{eff}(\phi, T) = V_{tree}(\phi) + V_{CW}(\phi) + V_T(\phi, T) + V_{daisy}(\phi, T)$

Model parameters

$\mu_R = v = 1$

$\lambda = 10^{-10}$

$m^2 = \lambda v$



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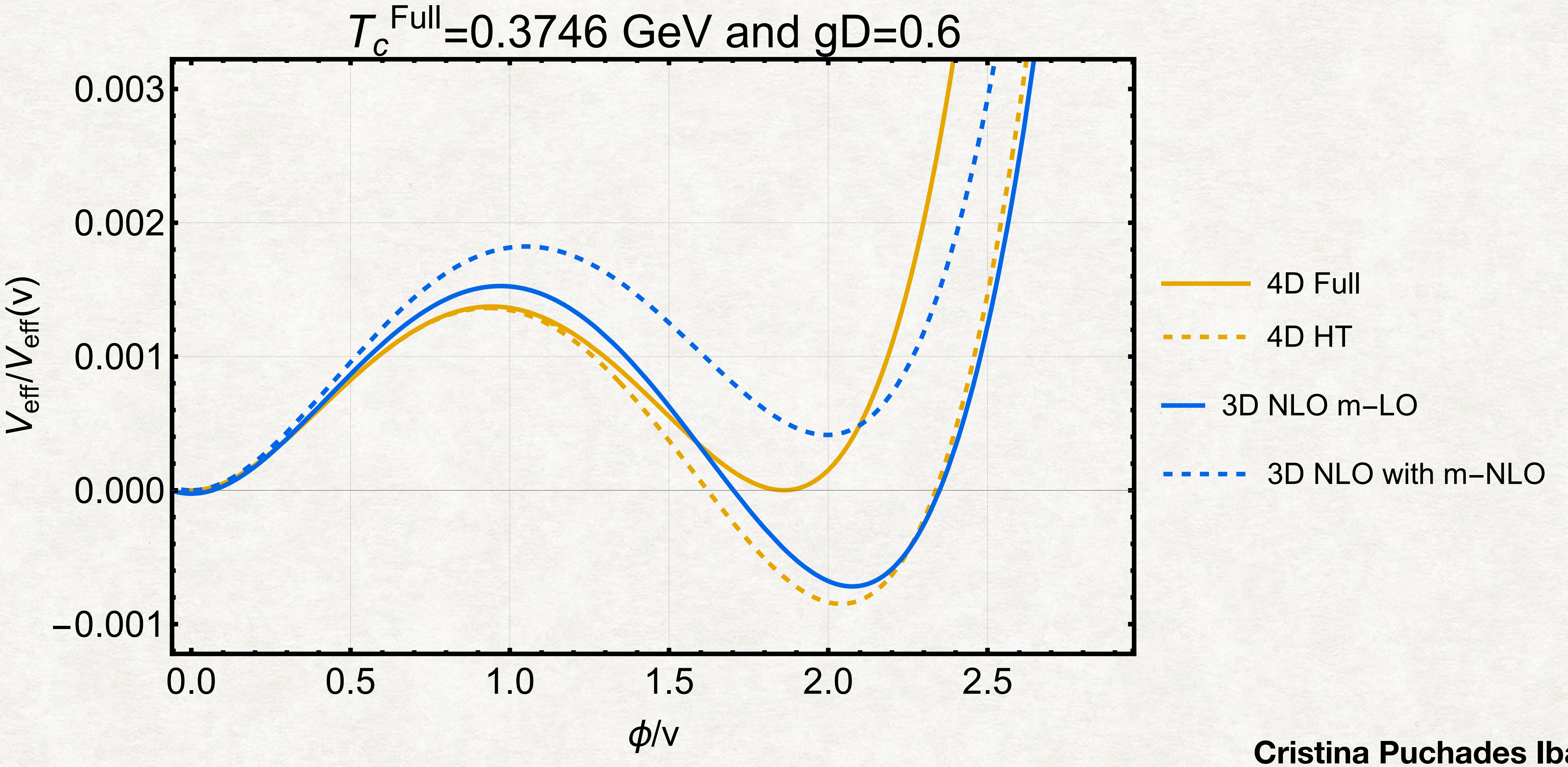
3D soft: **NLO (1-Loop):**

- masses at LO + couplings NLO
- masses at NLO with 3d couplings NLO+couplings NLO



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3D soft:

NLO (1-Loop):

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- masses at NLO with 3d couplings NLO+couplings NLO

NNLO (2-Loop):

- masses at NLO with 3d couplings NLO+couplings NLO
- V-LO masses NLO with couplings at LO + V-NLO+V-NNLO masses LO and couplings LO

Model parameters

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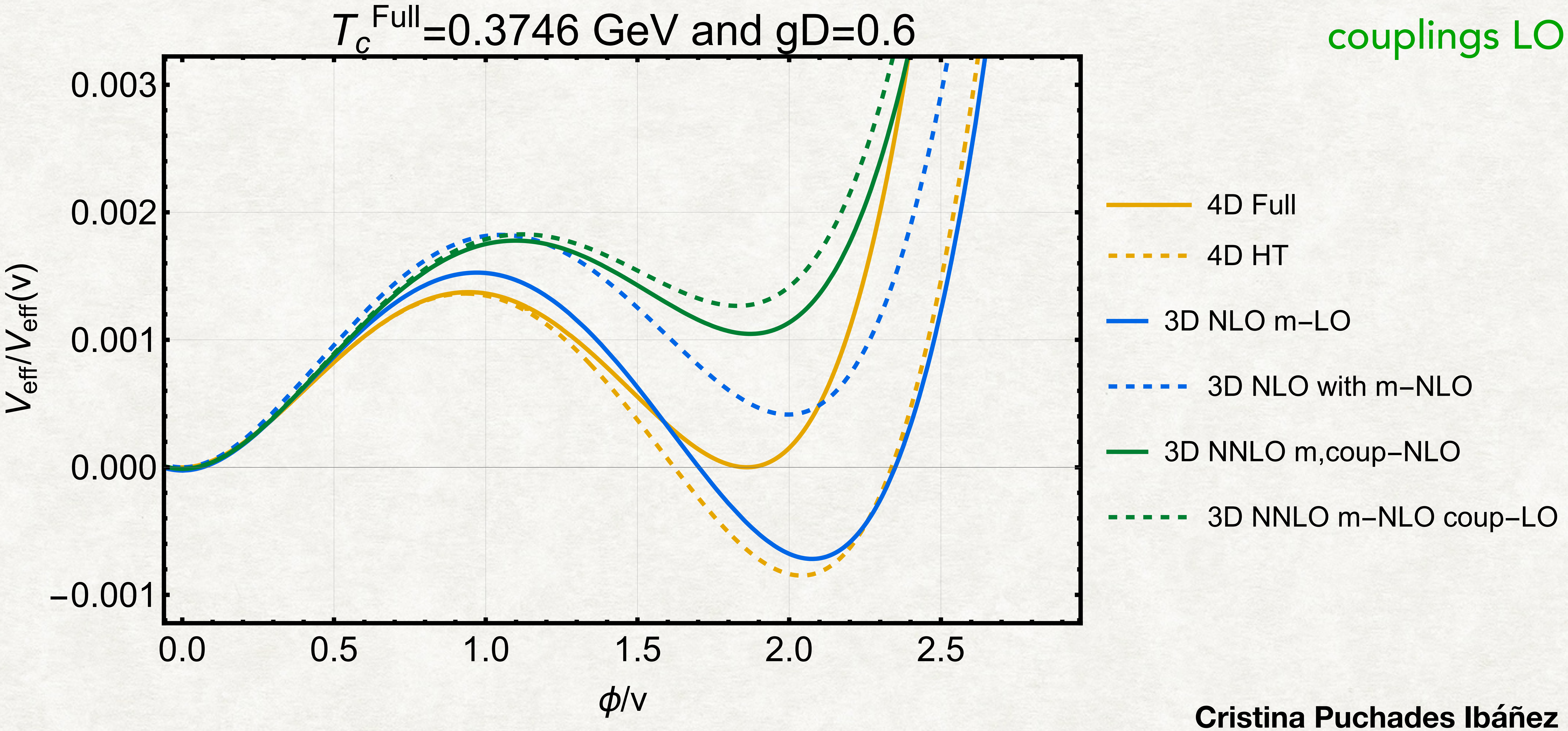


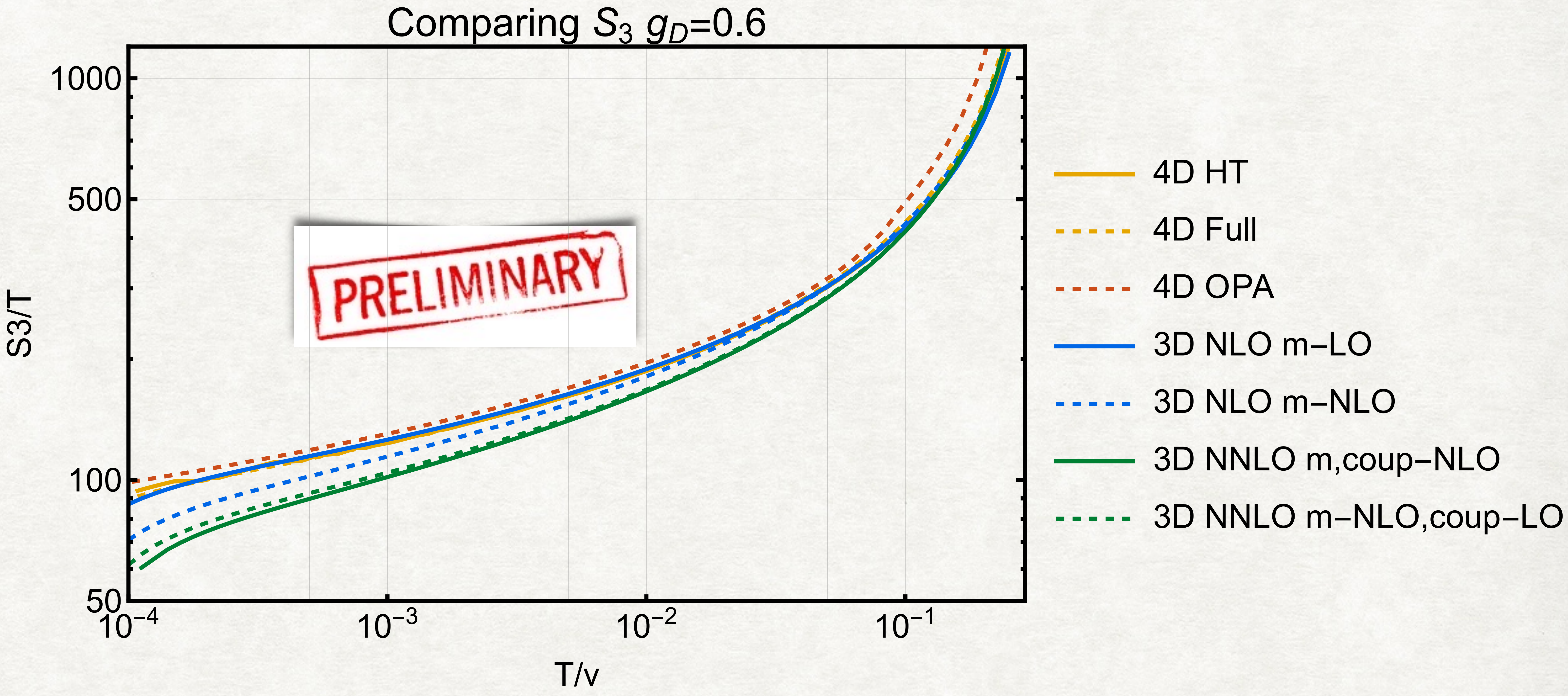
FOPT PARAMETERS

Espinosa:

$$\frac{S_3}{T} = \frac{32\pi\sqrt{2}}{3}\int_0^{\phi_0} d\phi\, \frac{\left[V_{\rm eff}(\phi,T) - V_t(\phi)\right]^{3/2}}{\left(\frac{dV_t}{d\phi}\right)^2}$$

$$\alpha \simeq \frac{\Delta V_{CW}^{NLO}}{\rho_*} \qquad \rho_* = \frac{\pi^2}{30}\, g_*\, T_*^4$$

$$\frac{\Gamma(T_n)}{H(T_n)^4} = 1 \qquad \frac{\beta}{H_*} = T_* \frac{d}{dT}\left(\frac{S_3}{T}\right)\Big|_{T=T_*}$$



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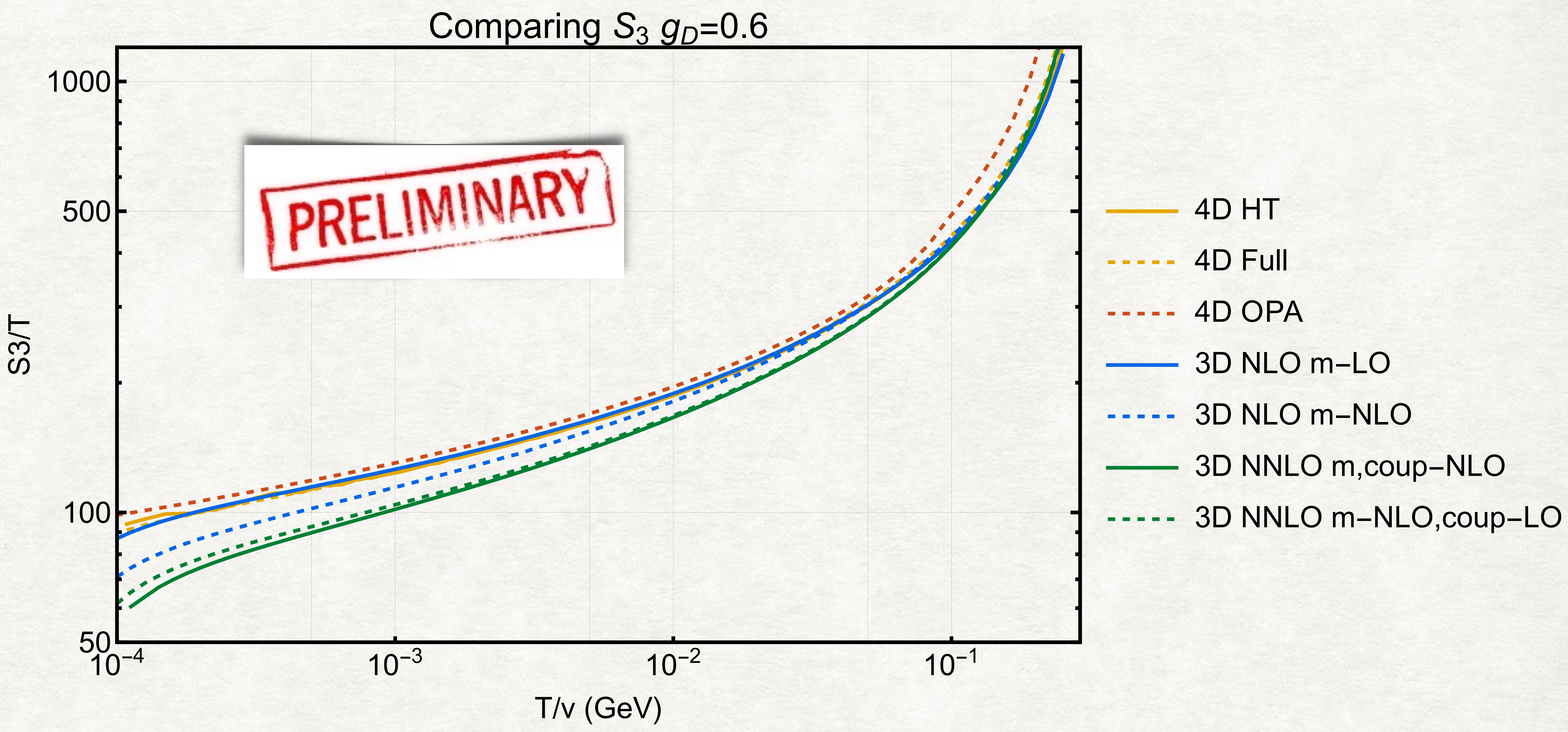
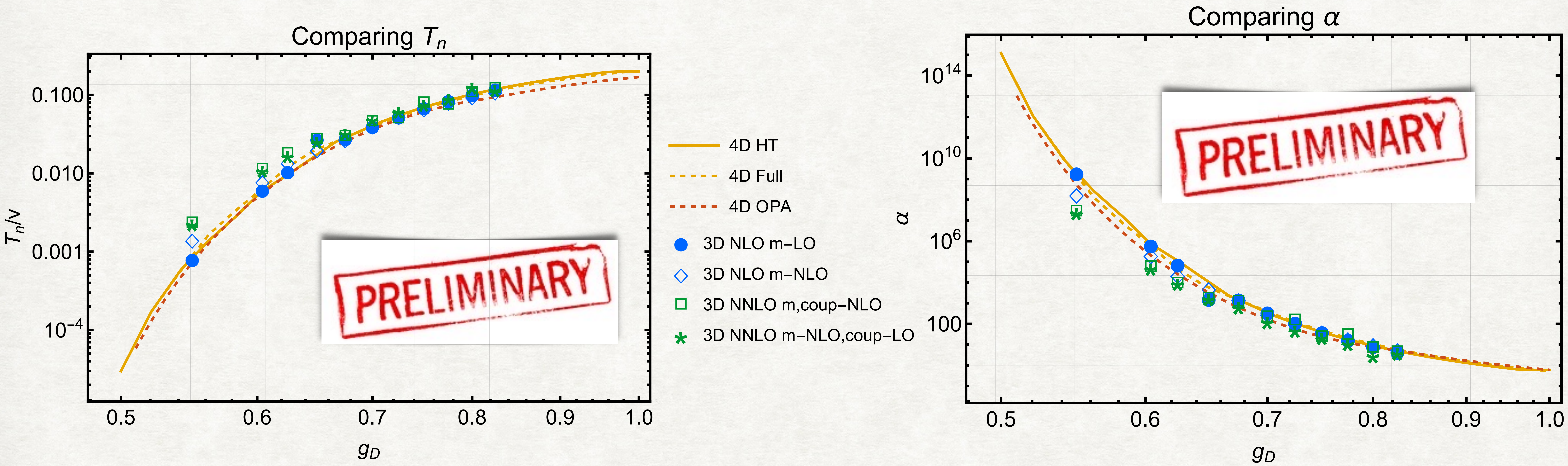


📖 For more details look https://arxiv.org/pdf/2312.12413

M. Kierkla, B. Siezewska, T. V. I. Tenkanen and J. van de Vis

RESULTS

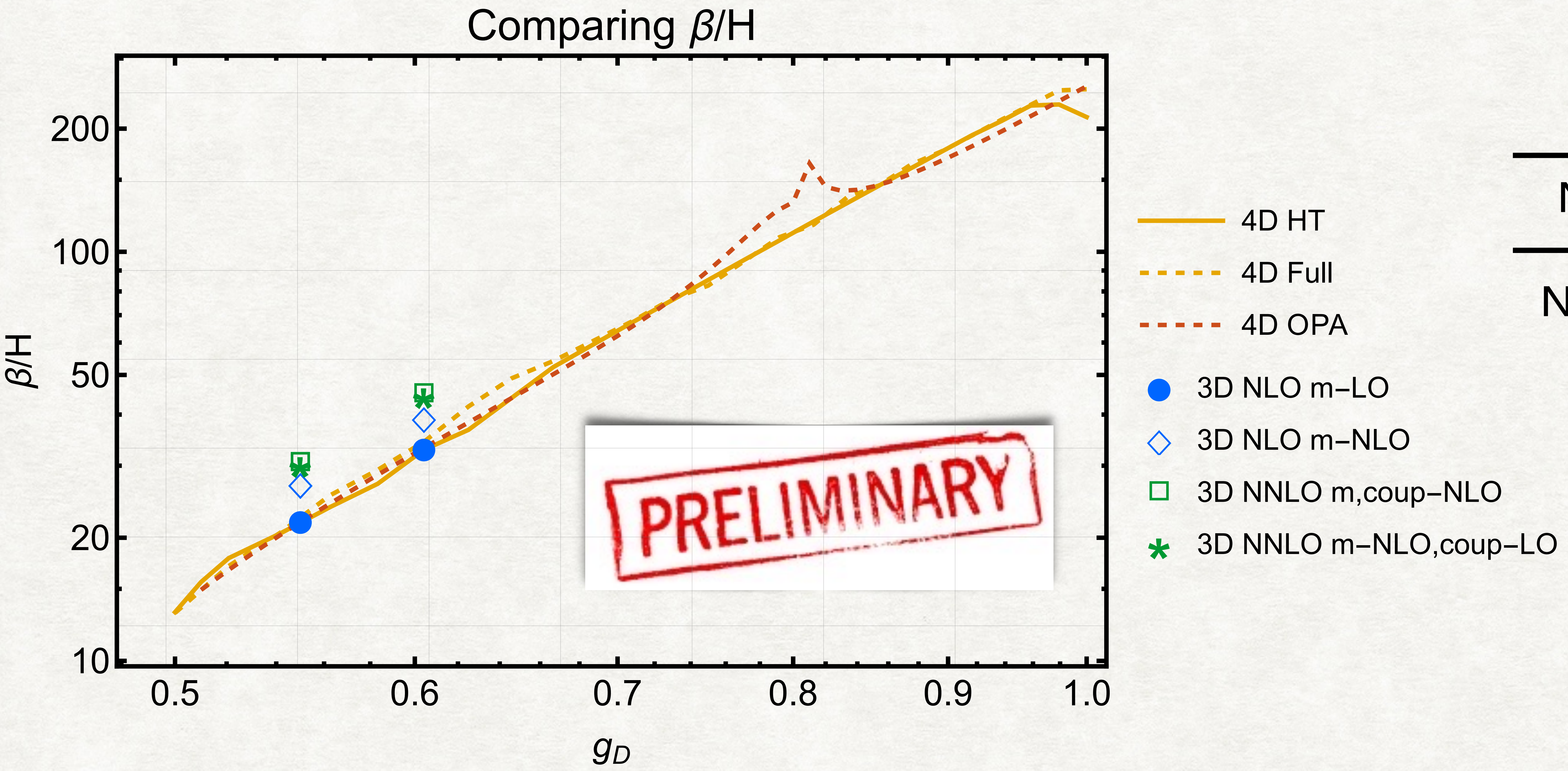


- Numeric problems.

- The LO with NLO masses and the NNLOs make a difference for small couplings.

- LO with LO masses is very close to the 4D theory: DR includes daisy

RESULTS



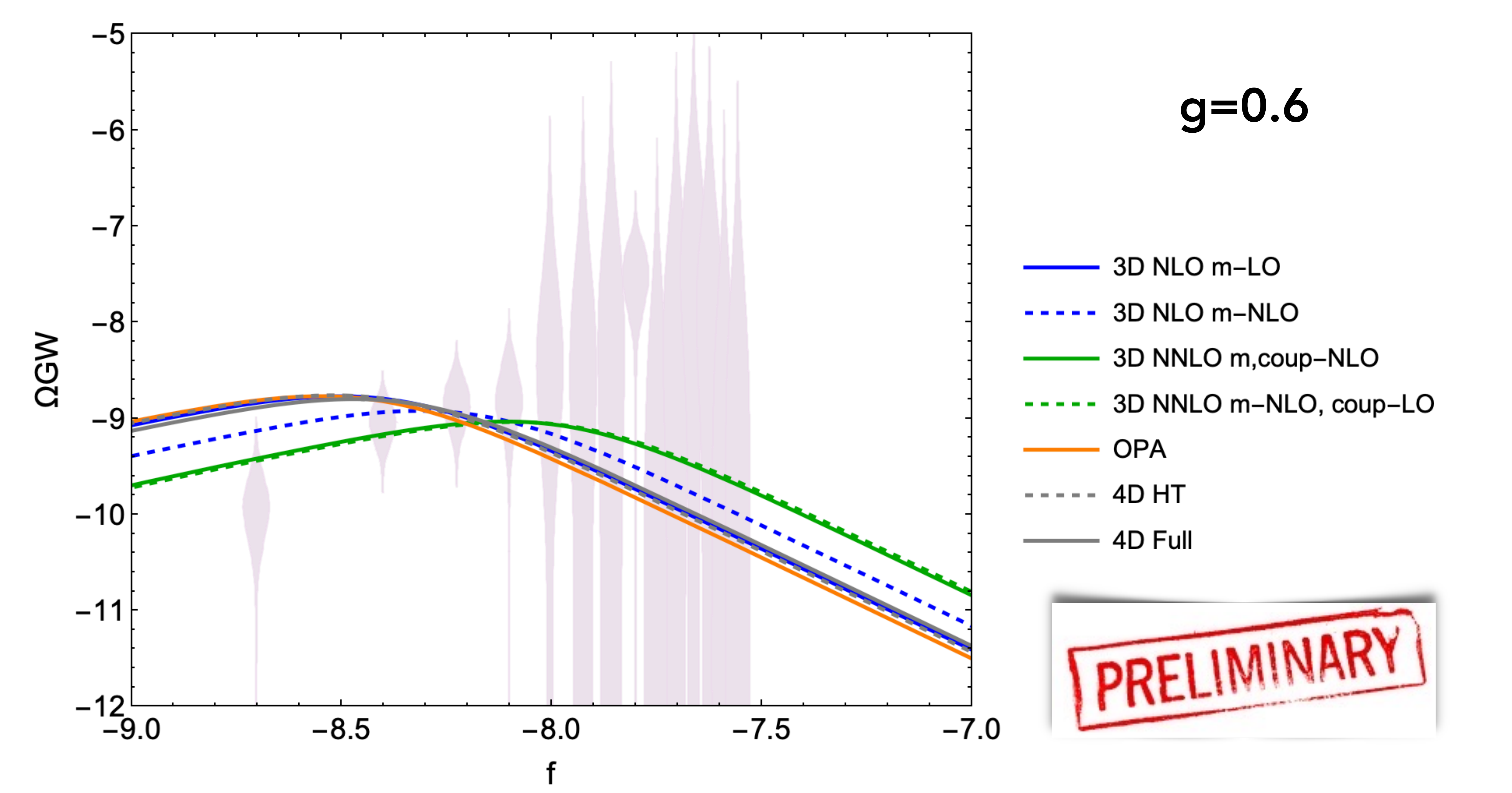
	$g = 0.6$	$g = 0.55$
NLO	$\sim 18\%$	$\sim 31\%$
NNLO	$\sim 32\%$	$\sim 45\%$

- “artificial” peak

-Due to numeric problems we have only 2 points.

- Smaller the coupling more impact have DR corrections

CONCLUSIONS AND IMPROVEMENTS



Improve numerics.

Explore the effect of the running.

Consistent power counting

Effect of V-NLO with LO masses negligible.

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GW spectrum in the relativistic limit from

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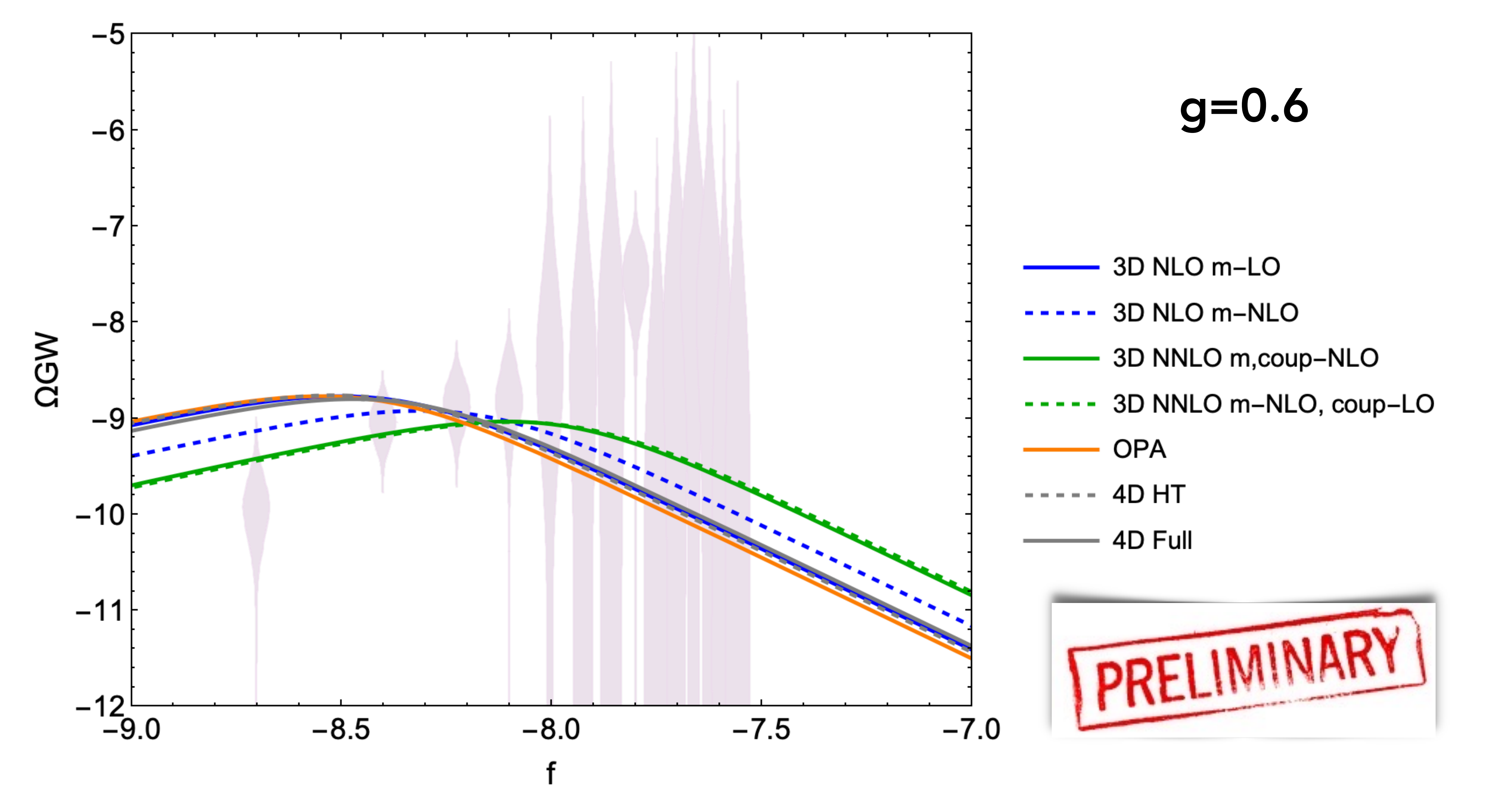

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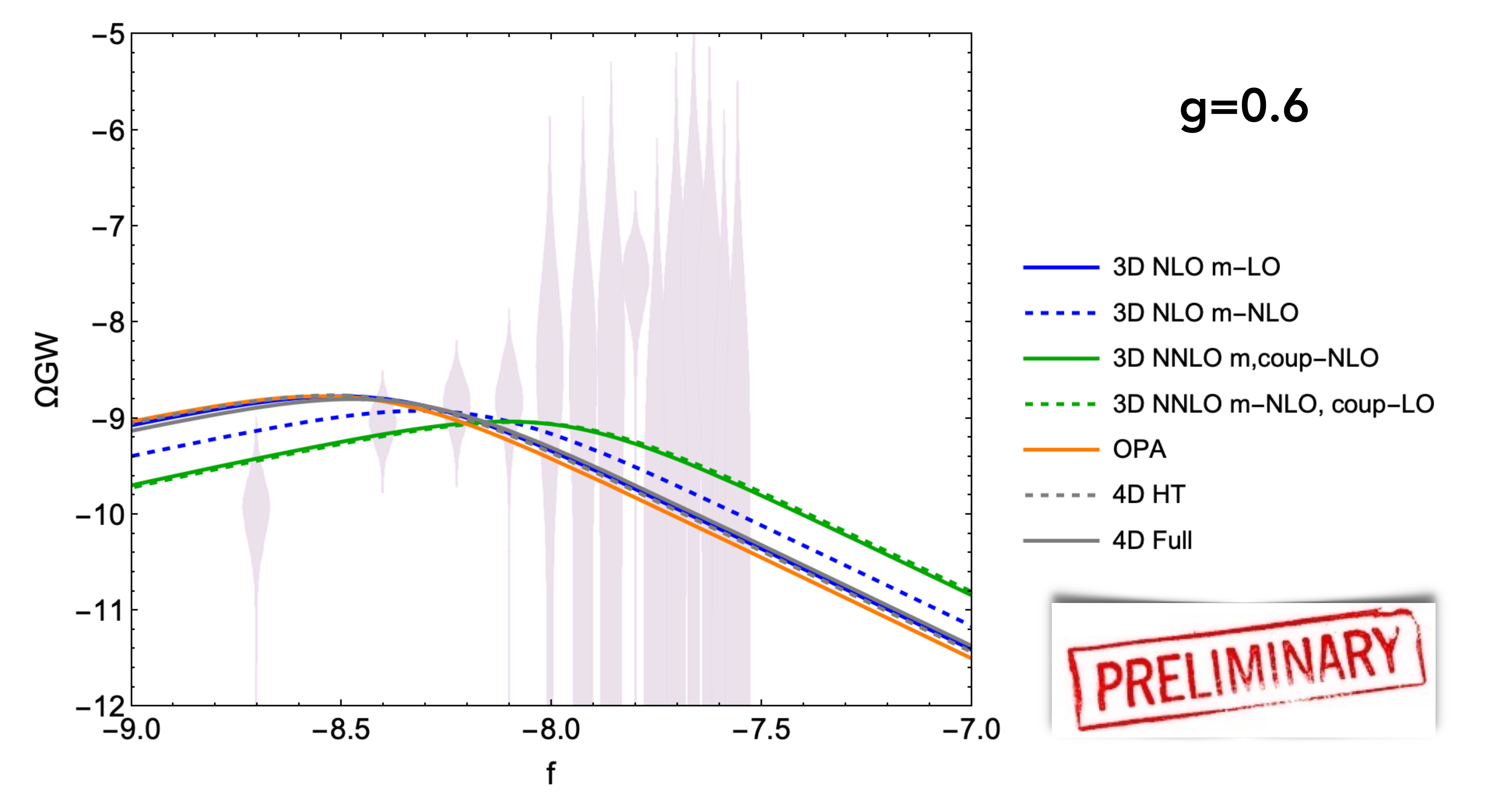

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