

The (complicated) story of Primordial Black Holes production in cosmological Phase Transitions

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in collaboration with
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Part I – Bright prospects of PBHs



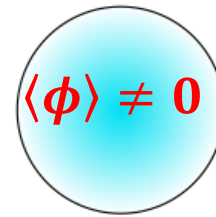
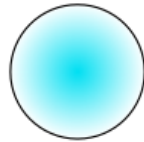
Why would we expect PBHs to form?

"old" phase

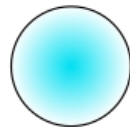
"old" phase

"new" phase

$$\Gamma(t) = H_I^4 e^{\beta t}$$

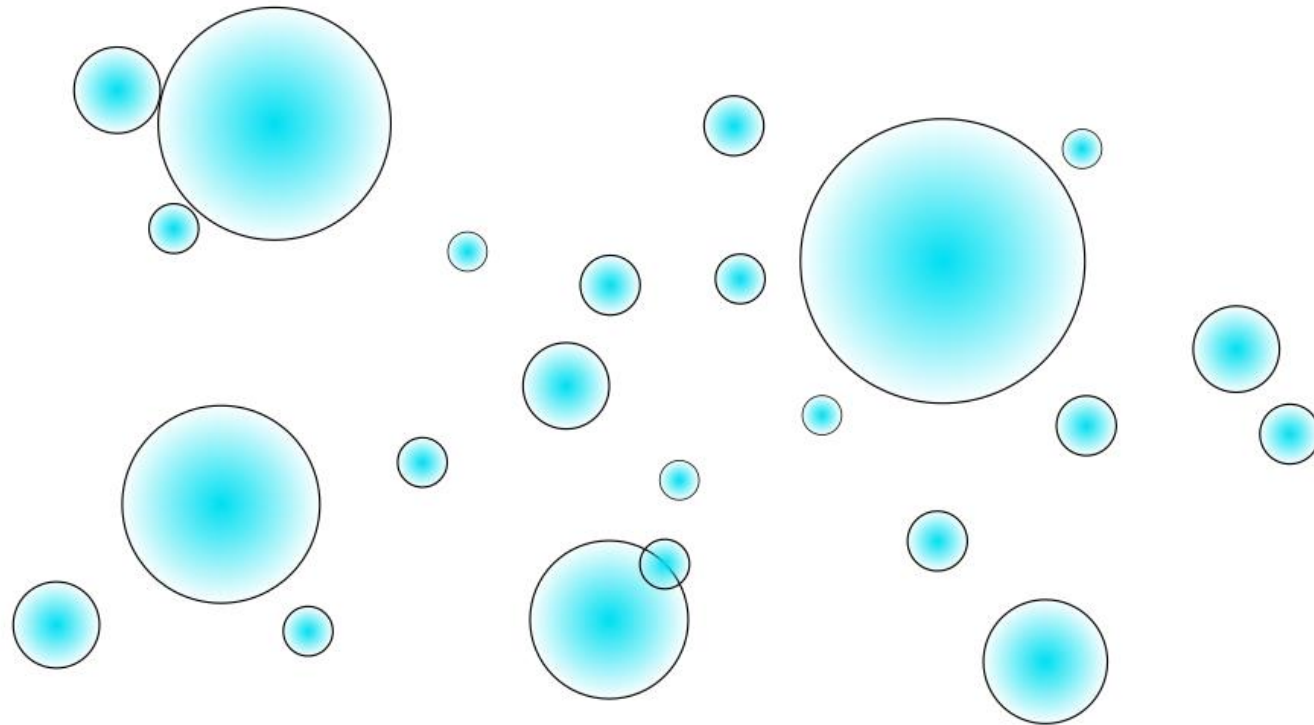


$$\langle \phi \rangle = 0$$



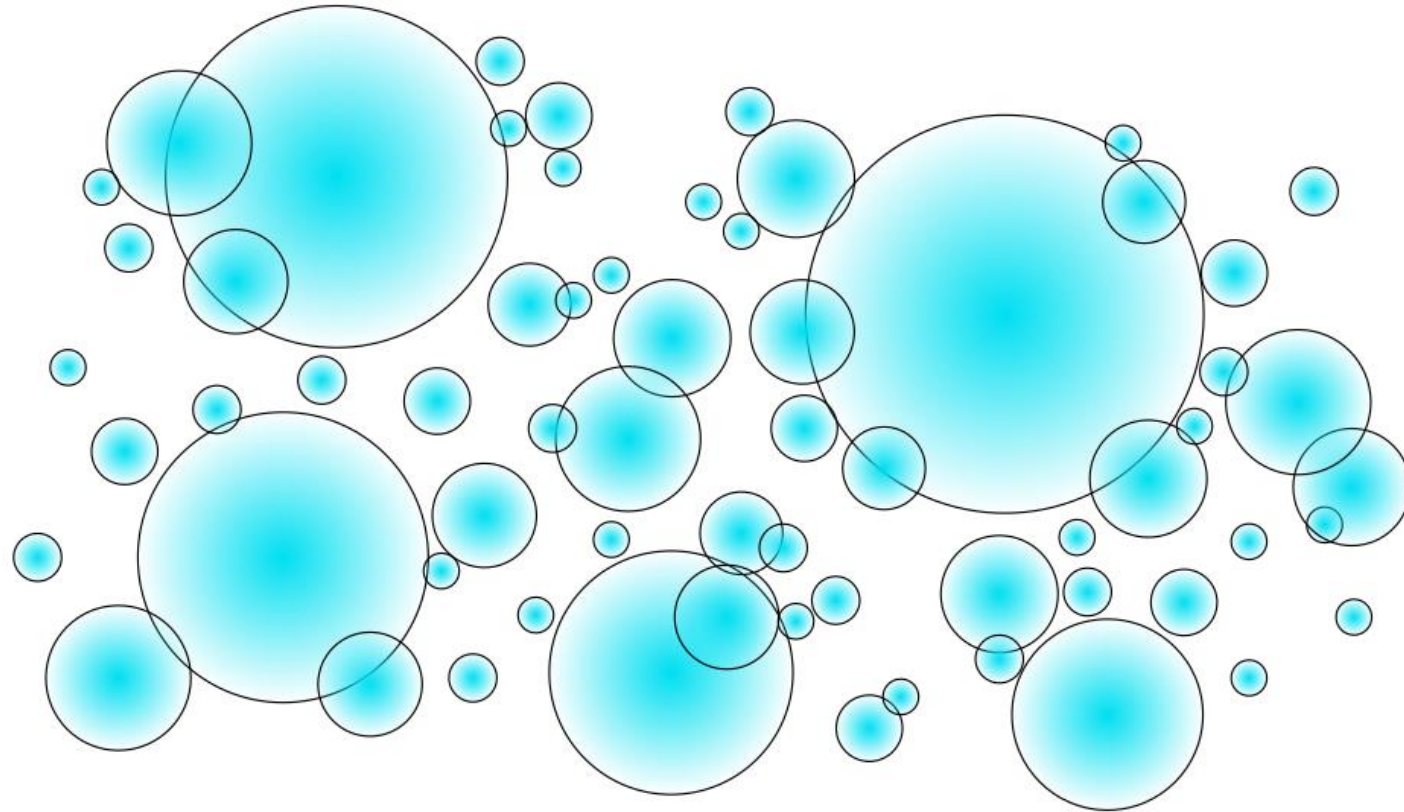
"old" phase

"new" phase



"old" phase

"new" phase



BLACK HOLE FORMATION

Statistical nature of bubble
nucleation



inhomogeneities

Slow, supercooled transition



period of thermal inflation

BLACK HOLE FORMATION

Statistical nature of bubble
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inhomogeneities

Slow, supercooled transition

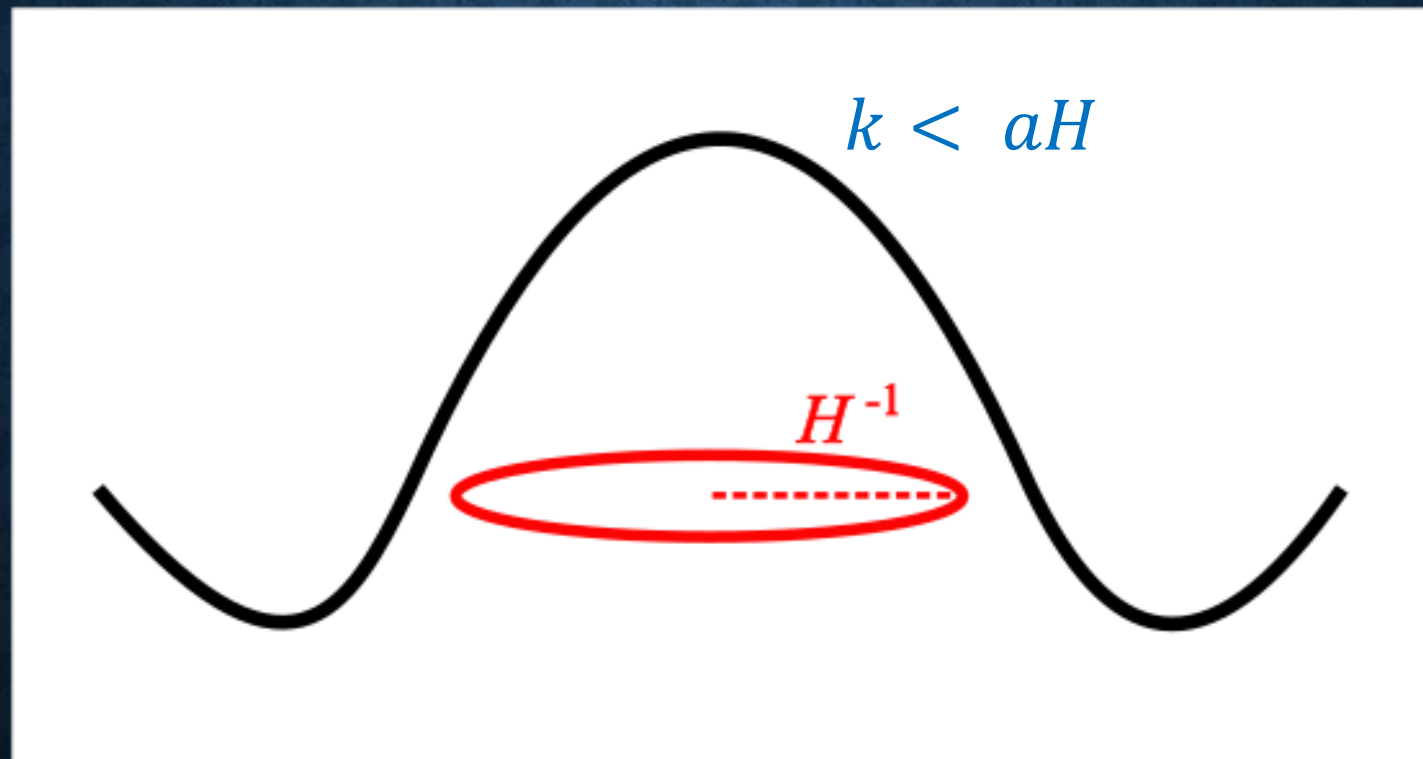


period of thermal inflation

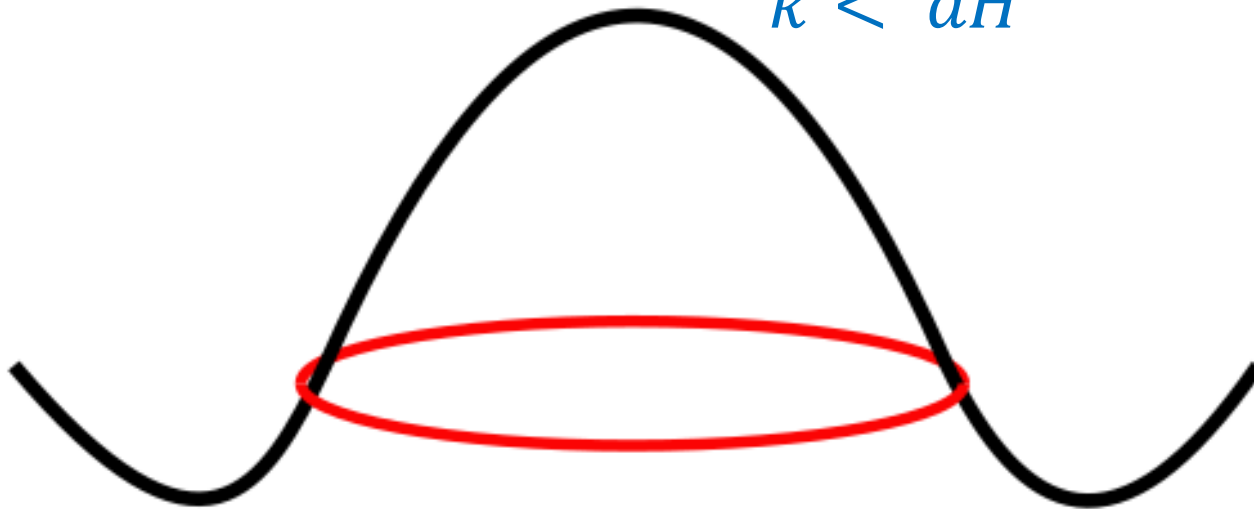
$$\dot{\rho}_r + 4H\rho_r = -\dot{\rho}_v$$

Large fluctuations of
energy density

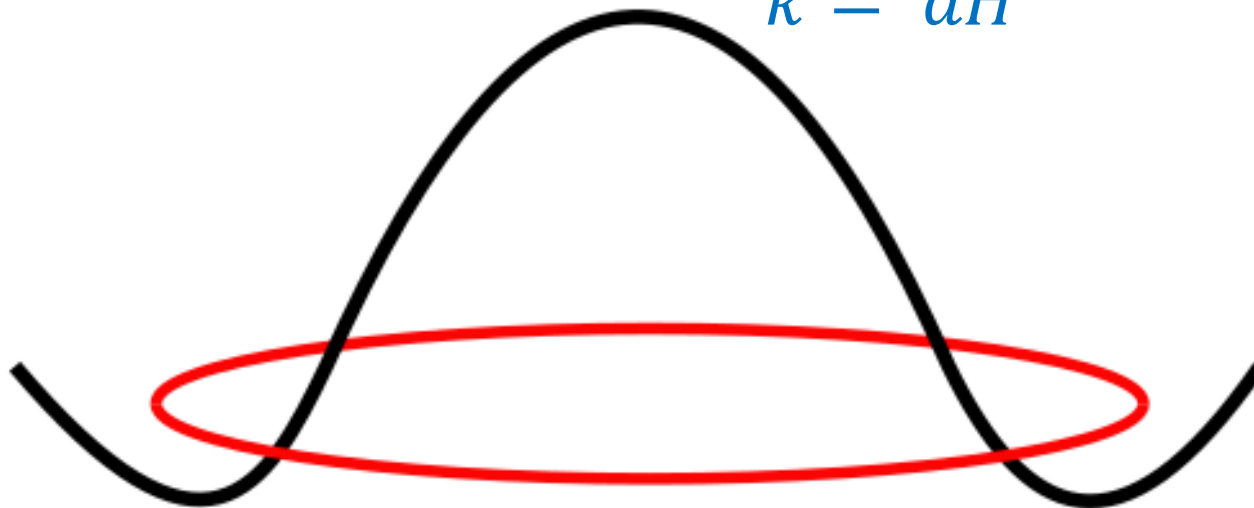
$$\delta = \frac{\rho - \rho_b}{\rho_b}$$



$$k < aH$$



$$k = aH$$

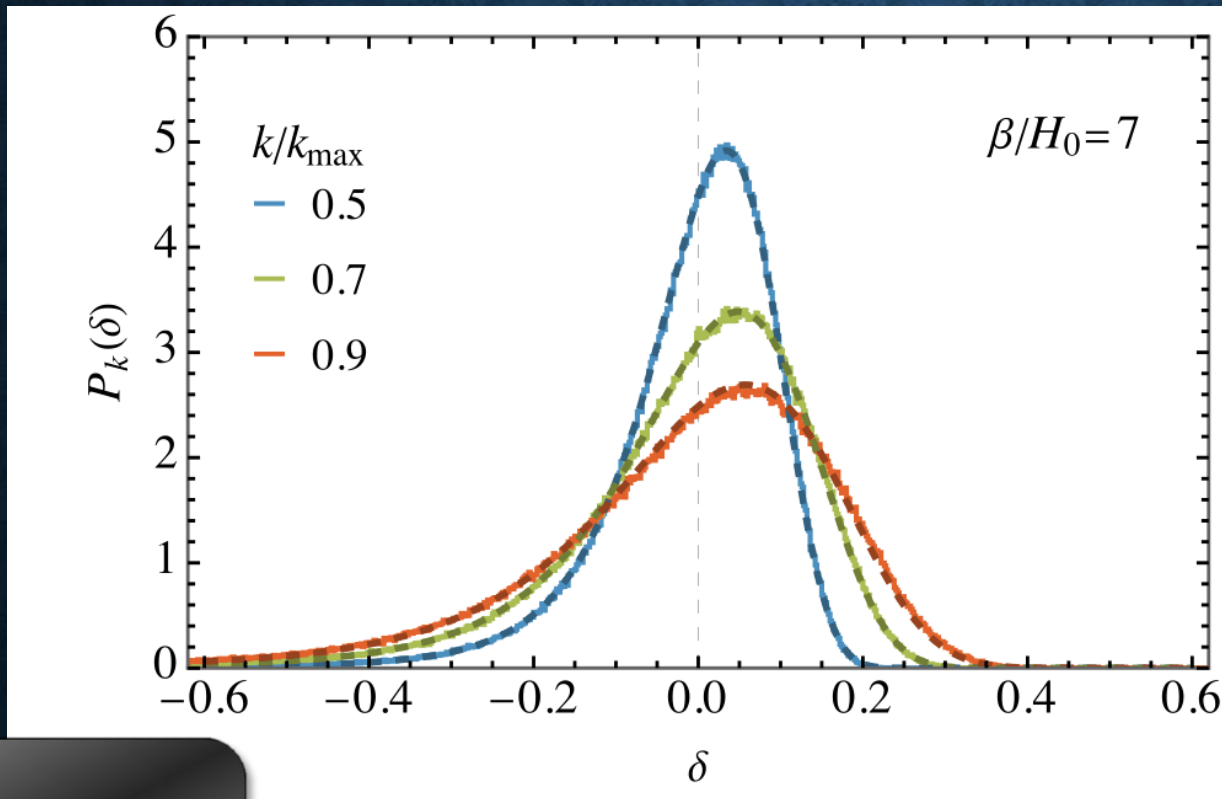


BLACK HOLE FORMATION

Strategy:

perform the evolution of different-sized patches
described by the wavenumber k

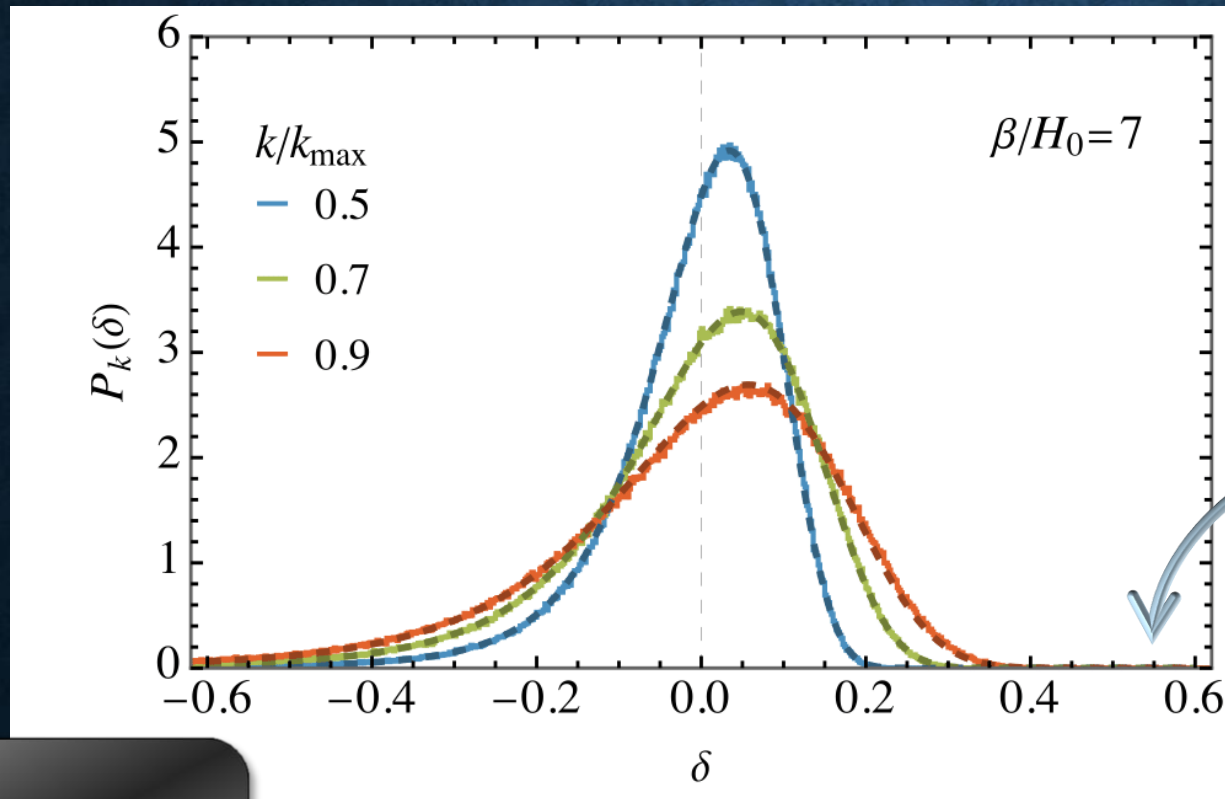
BLACK HOLE FORMATION



$$\Gamma(t) \approx H_I^4 e^{\beta t}$$

$$\delta = \frac{\rho - \rho_b}{\rho_b}$$

BLACK HOLE FORMATION



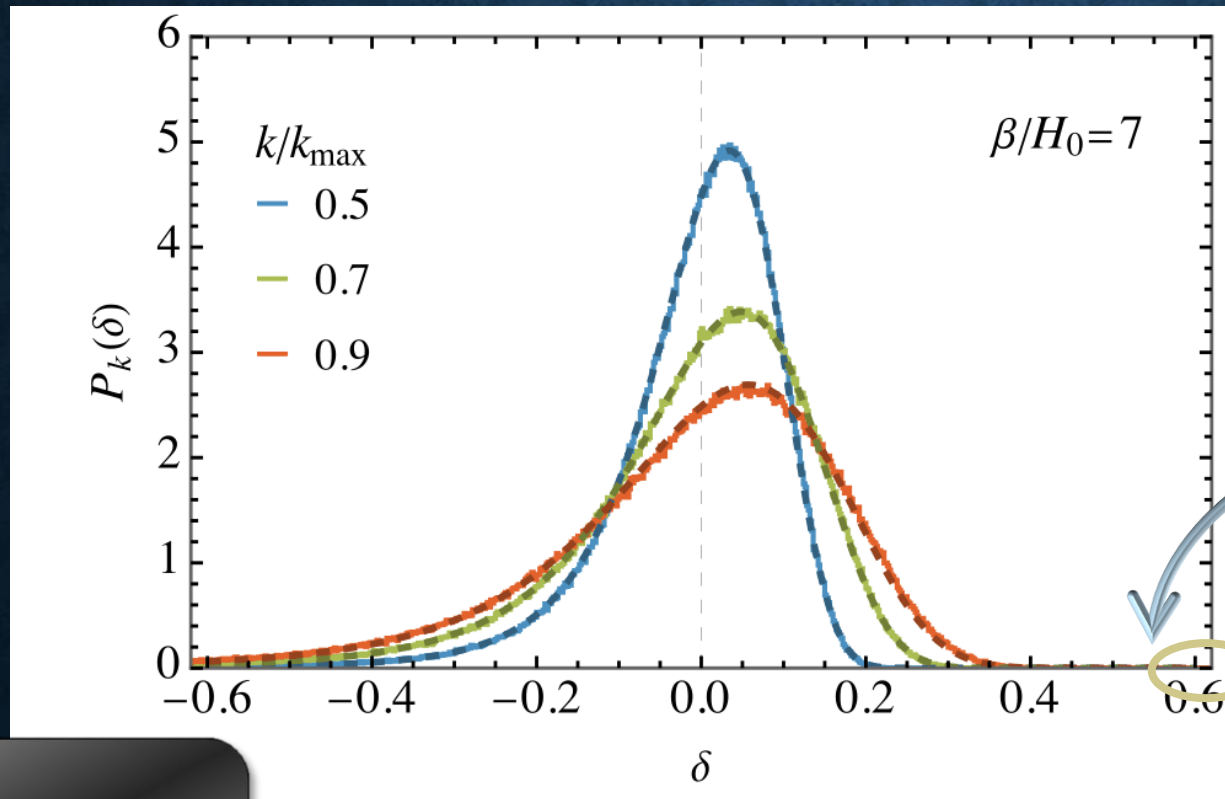
Critical scaling law

$$M(\delta) = \kappa M_k (\delta - \delta_c)^\gamma$$

$$\delta_c = 0.55$$

$$\delta = \frac{\rho - \rho_b}{\rho_b}$$

BLACK HOLE FORMATION



Critical scaling law

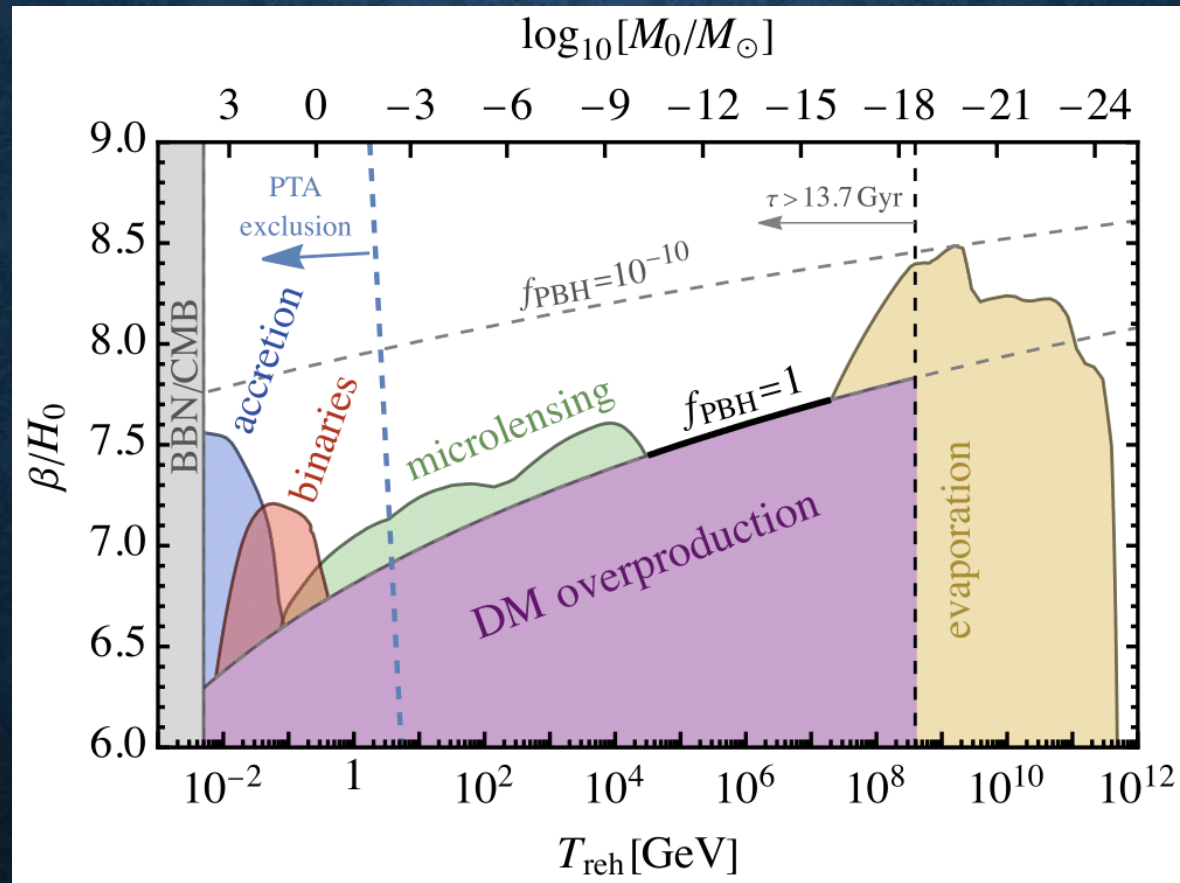
$$M(\delta) = \kappa M_k (\delta - \delta_c)^\gamma$$

$$\delta_c = 0.55$$

PBH formation

$$\delta = \frac{\rho - \rho_b}{\rho_b}$$

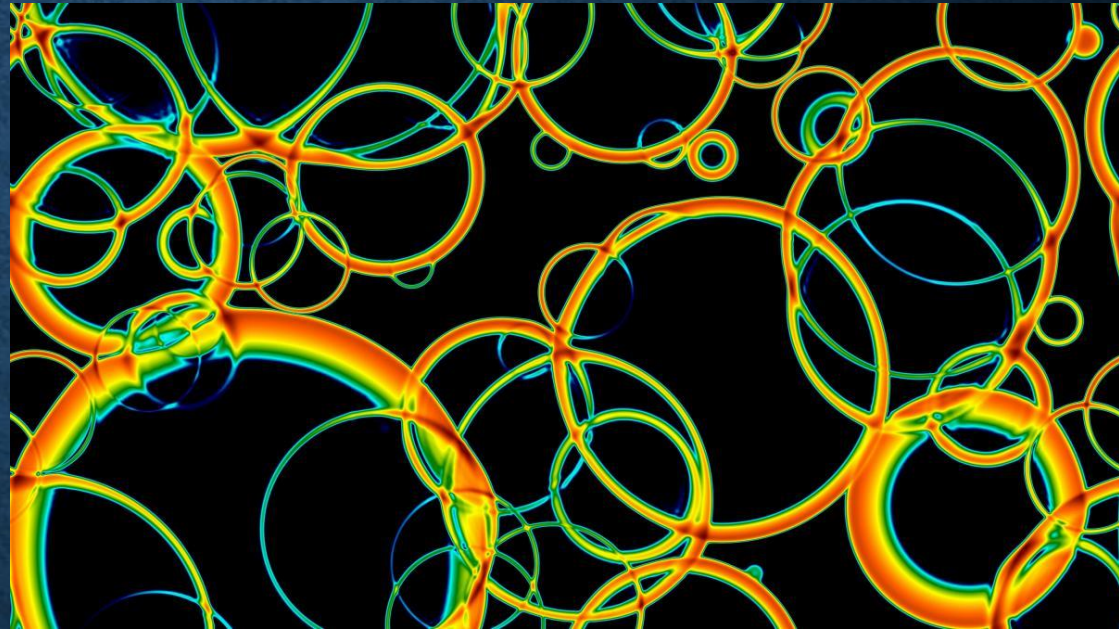
BLACK HOLE FORMATION



GRAVITATIONAL WAVES

During phase transition:

- bubble collisions
- sound waves in plasma



D. Weir, University of Helsinki

GRAVITATIONAL WAVES

During phase transition:

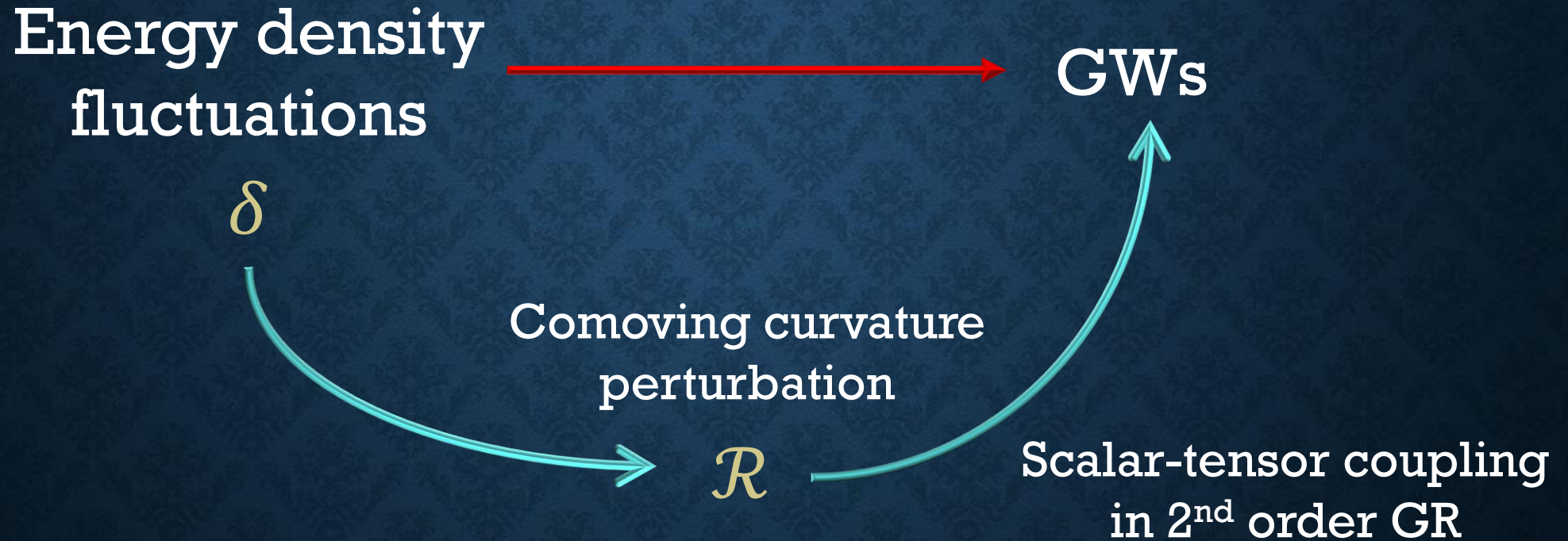
- bubble collisions
- sound waves in plasma

Second order effects?

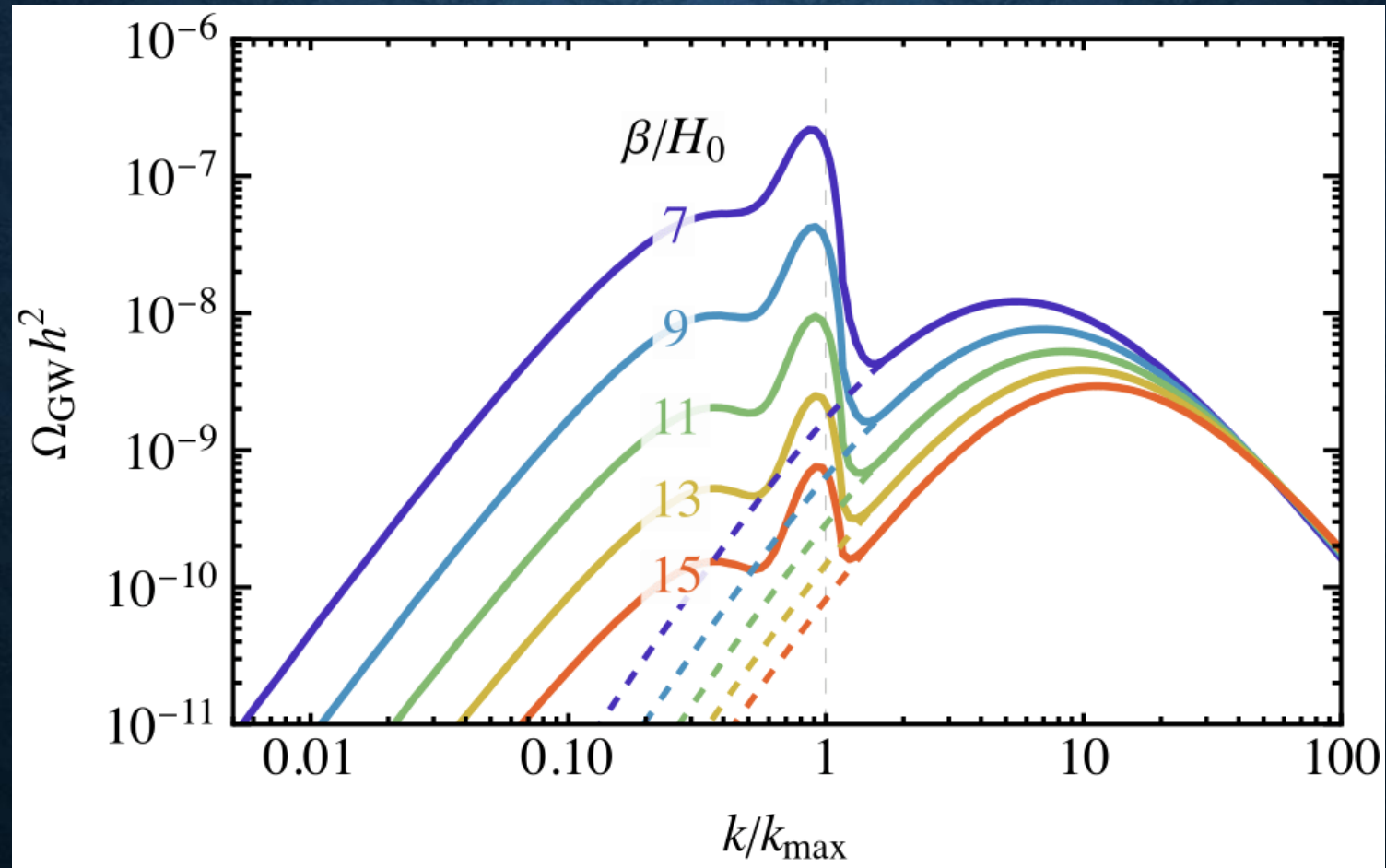
- scalar induced gravitational waves

Energy density
fluctuations  GWs

SCALAR INDUCED GRAVITATIONAL WAVES



GRAVITATIONAL WAVES



Part II – The tragedy is revealed



Based on work by
G. Franciolini, Y. Gouttenoire and R. Jinno
[2503.01962]

THE TROUBLE BEHIND IT ALL

Discussing energy density fluctuations, we introduce

$$\rho_k = \bar{\rho} + \delta\rho_k$$

Background

Linear
perturbation

THE TROUBLE BEHIND IT ALL

We are working with objects
in linear regime of GR
perturbation theory



THE TROUBLE BEHIND IT ALL

We are working with objects
in linear regime of GR
perturbation theory



We have to employ
full linearized GR
formalism!

LINEARIZED GENERAL RELATIVITY

Gauge dependence

```
graph TD; A[Gauge dependence] --> B[Gauge invariant quantities  
ΦB, ΨB, ζ, ℛ, δρm]; A --> C[Gauge freedom  
• longitudinal gauge  
• comoving gauge  
...];
```

Gauge invariant quantities

$\Phi_B, \Psi_B, \zeta, \mathcal{R}, \delta\rho_m$

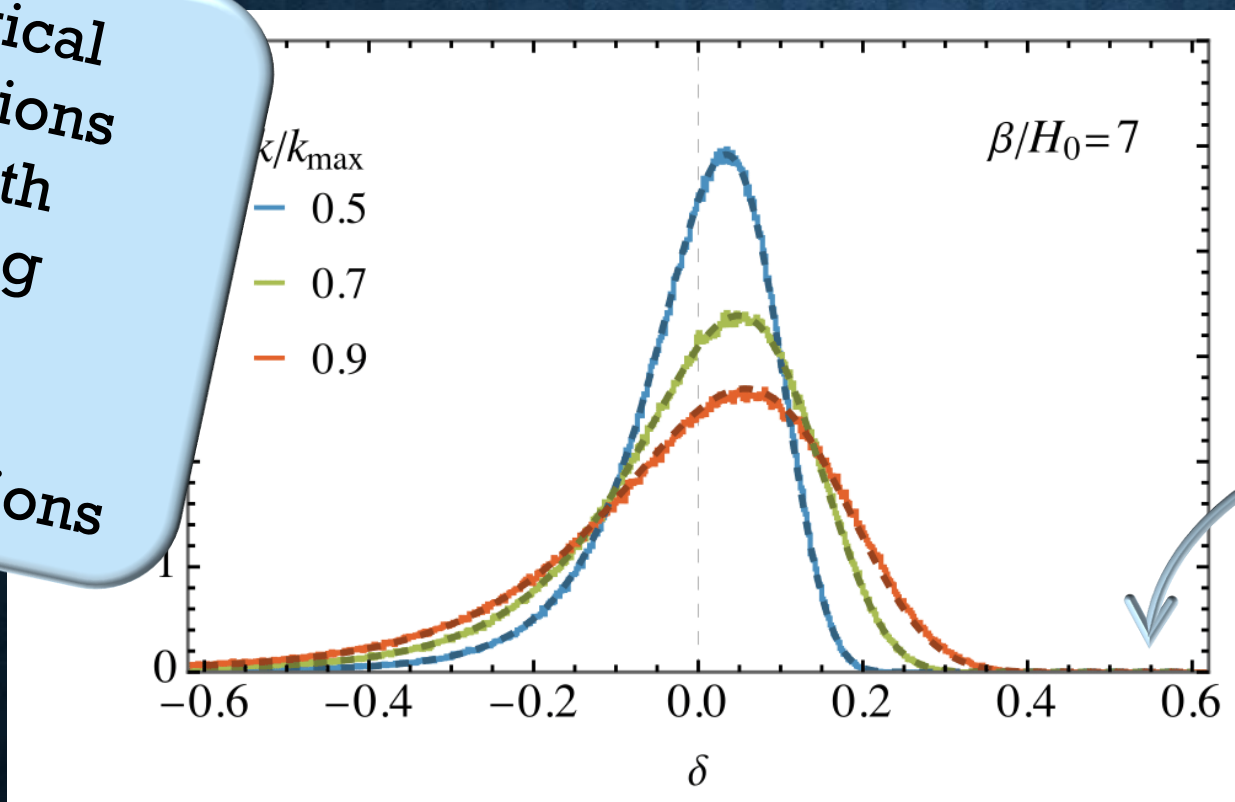
Gauge freedom

- longitudinal gauge
- comoving gauge

...

BLACK HOLE FORMATION

Numerical simulations work with comoving energy density perturbations



Critical scaling law

$$M(\delta^{(c)}) = \kappa M_k \left(\delta^{(c)} - \delta_c^{(c)} \right)^\gamma$$

$$\delta_c^{(c)} = 0.55$$

$$\delta^{(c)} = \frac{\rho^{(c)} - \rho_b}{\rho_b} = \frac{\delta \rho_m}{\rho_b}$$

STRATEGY

Solve the dynamics in different Hubble patches during the transition and find the distribution of $\delta^{(c)}$ working in the comoving gauge

OR

Compute the evolution in the gauge of your choice and translate the density contrast $\delta^{(*)}$ into $\delta^{(c)}$

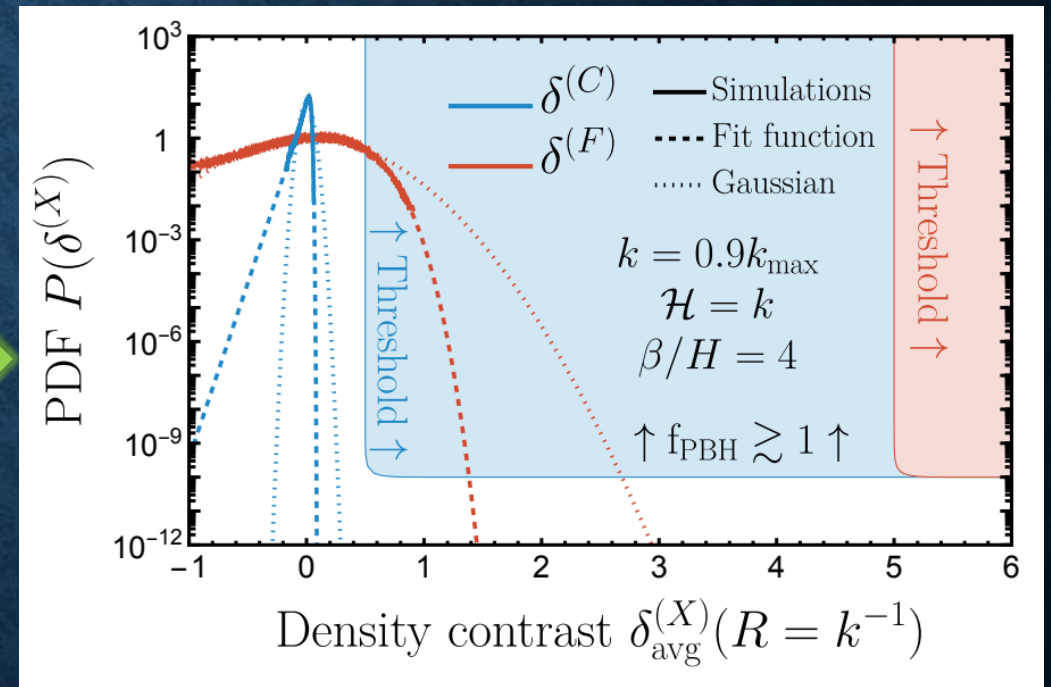
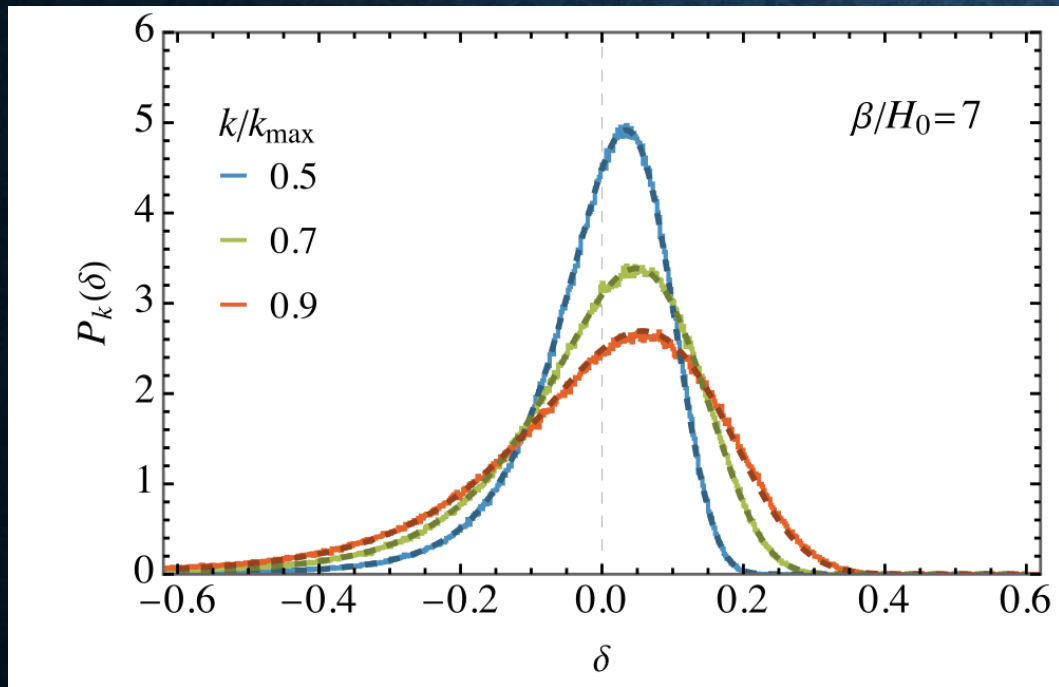
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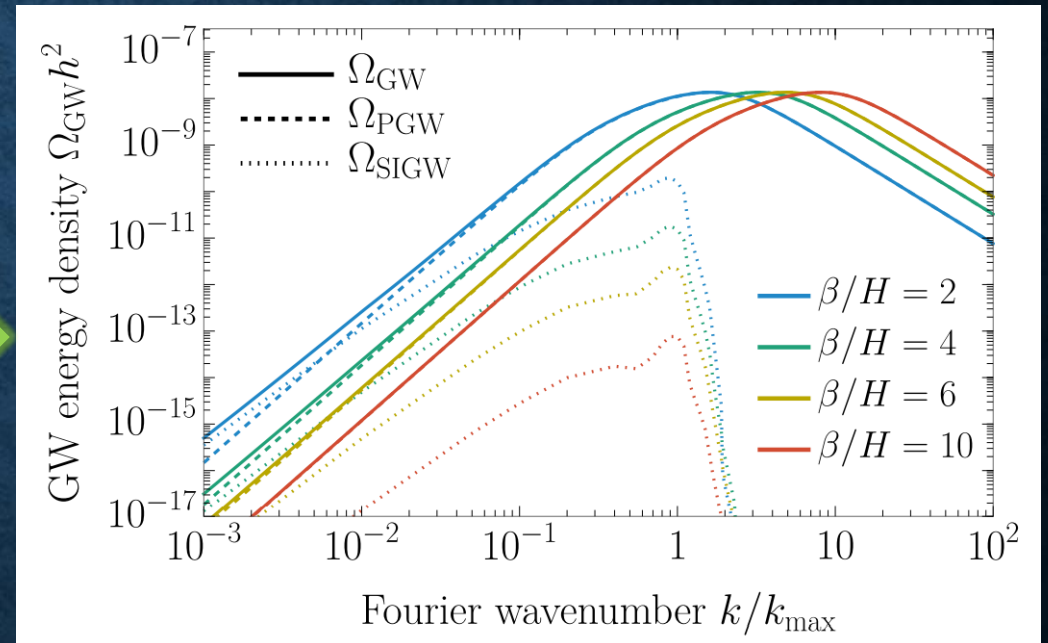
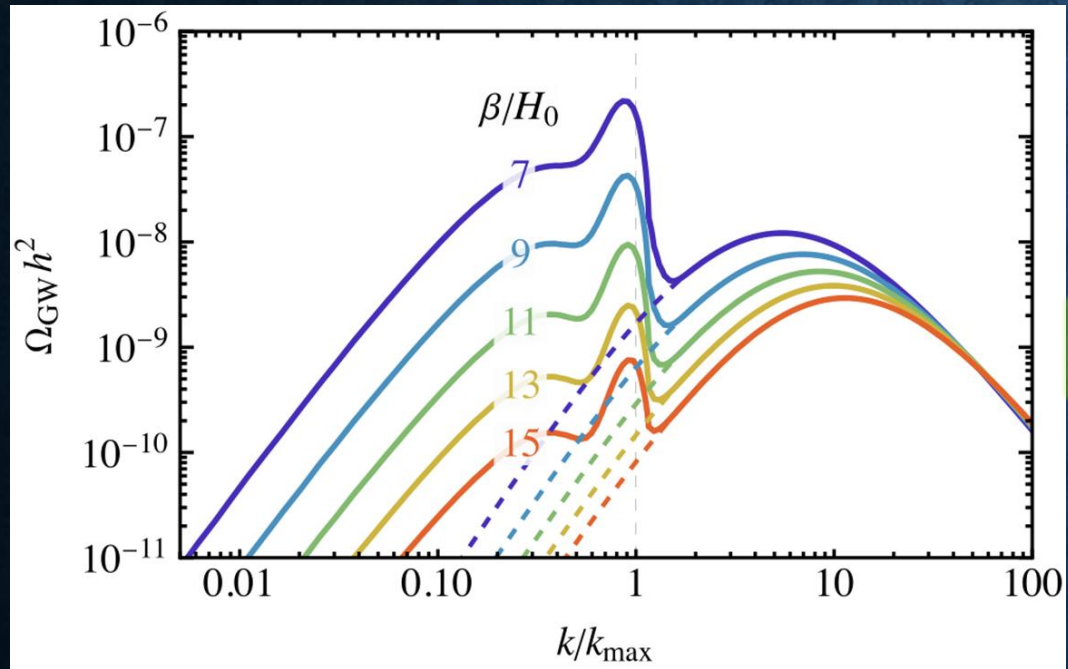
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RESULTS



G. Franciolini, Y. Gouttenoire,
R. Jinno, arXiv: 2503.01962

RESULTS



G. Franciolini, Y. Gouttenoire,
R. Jinno, arXiv: 2503.01962

CONCLUSIONS

No PBHs from
first-order phase
transitions

No visible
secondary GW
signals

IS THERE ANY HOPE?

- Expand the nucleation rate to 2nd order

$$\Gamma(t) \approx H_I^4 e^{\beta t - \gamma^2 t^2}$$

IS THERE ANY HOPE?

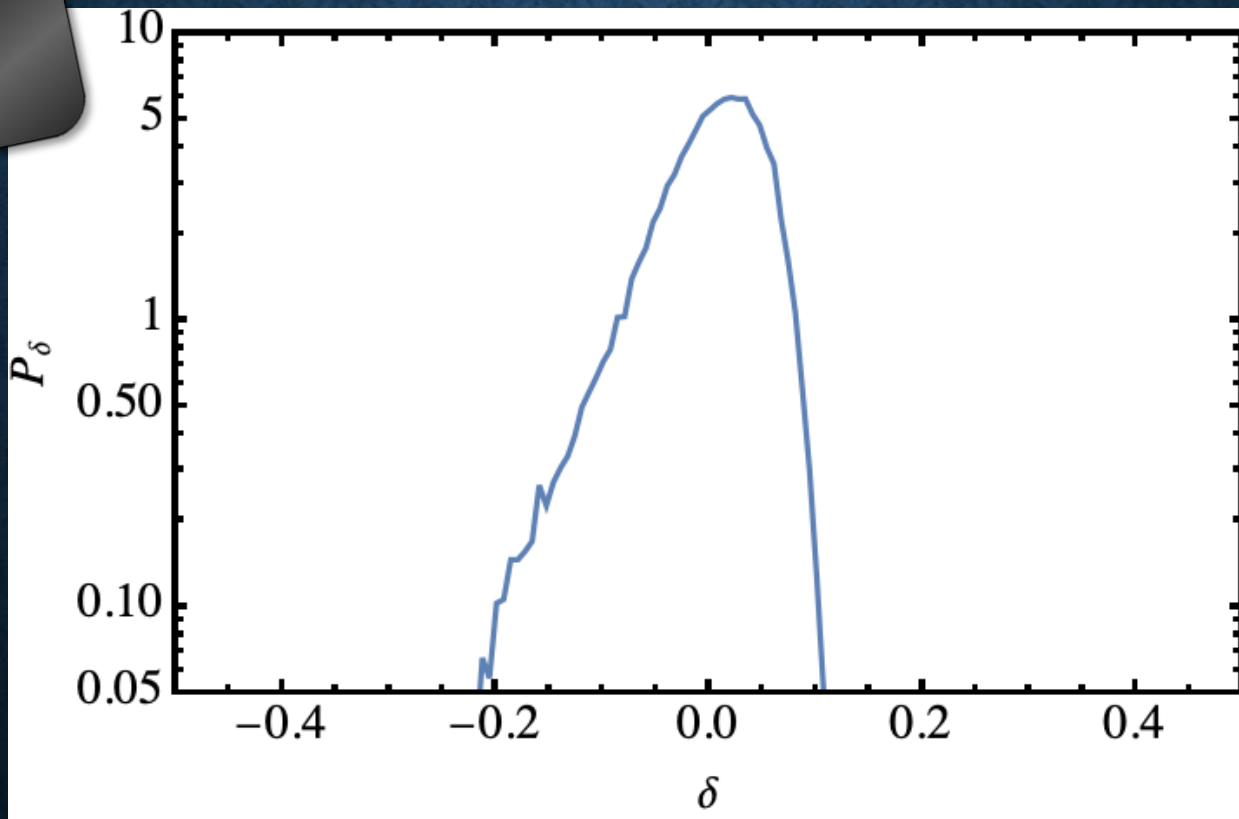
- Expand the nucleation rate to 2nd order

$$\Gamma(t) \approx H_I^4 e^{\beta t - \gamma^2 t^2}$$

Does not help



$$\begin{aligned}\beta/H_I &= 4 \\ \gamma/\beta &= 0.4\end{aligned}$$



PBH
formation

IS THERE ANY HOPE?

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$$\Gamma(t) \approx H_I^4 e^{\beta t - \gamma^2 t^2}$$

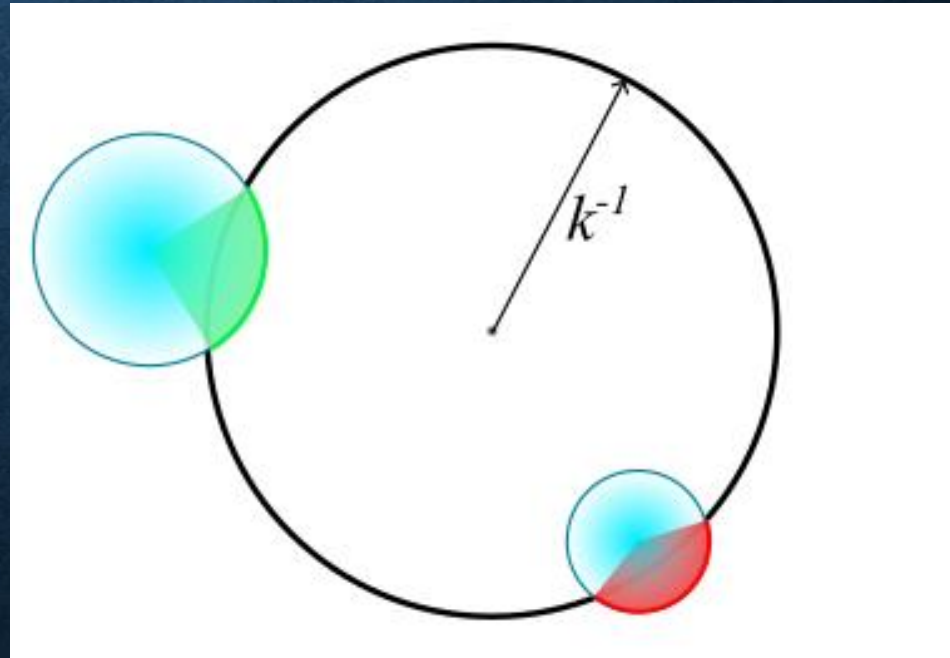
- Include the energy flux from bubble walls travelling inside/outside the patch

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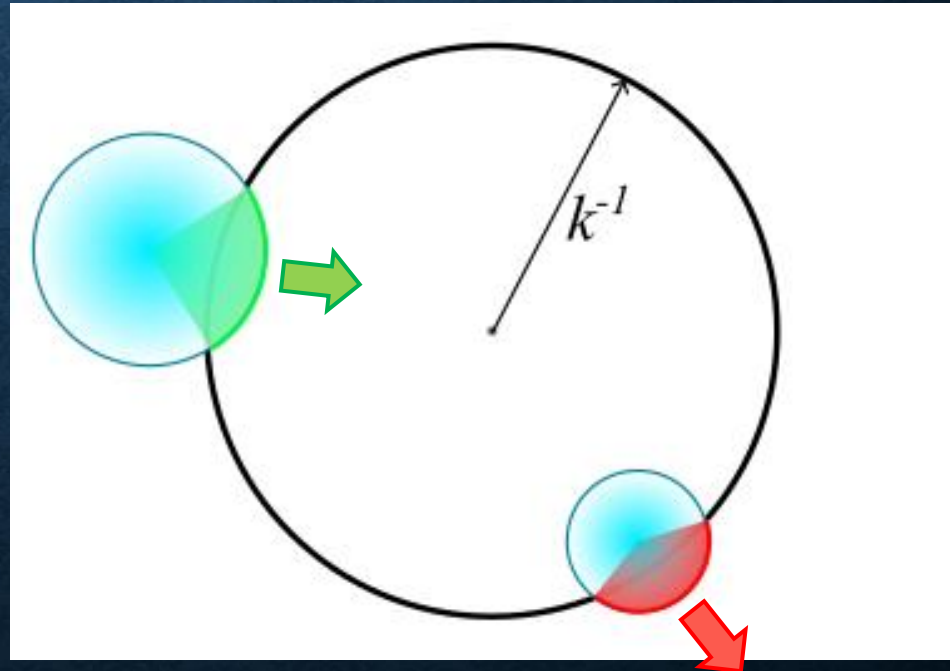


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Work in progress

IS THERE ANY HOPE?

- Expand the nucleation rate to 2nd order

$$\Gamma(t) \approx H_I^4 e^{\beta t - \gamma^2 t^2}$$

- Include the **energy flux** from bubble walls travelling **inside/outside** the patch
- Consider different EoS after the transition:
Scalar field oscillation \longrightarrow early MD
QCD phase transition

IS THERE ANY HOPE?

- Expand the nucleation rate to 2nd order

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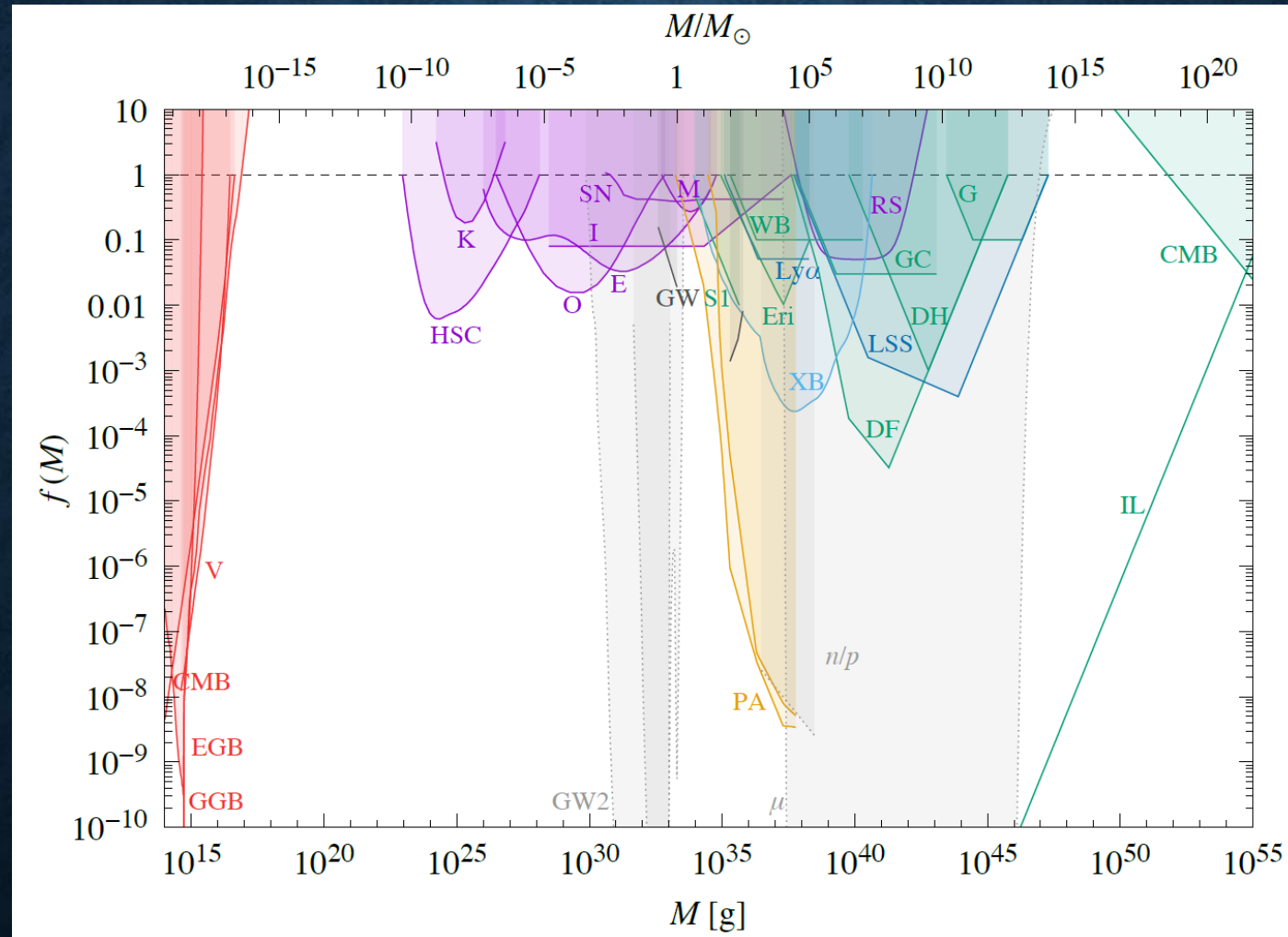
Scalar field oscillation \longrightarrow early MD

QCD phase transition

Distant (?)
Future work

Thank you!

PBHs AS DARK MATTER CANDIDATES



B. Carr, K. Kohri, Y. Sendouda,
J. Yokoyama, *Rept. Prog. Phys.* **84**
(2021) 11, 116902

SUPERCOOLED PHASE TRANSITION

nucleation of true vacuum
bubbles

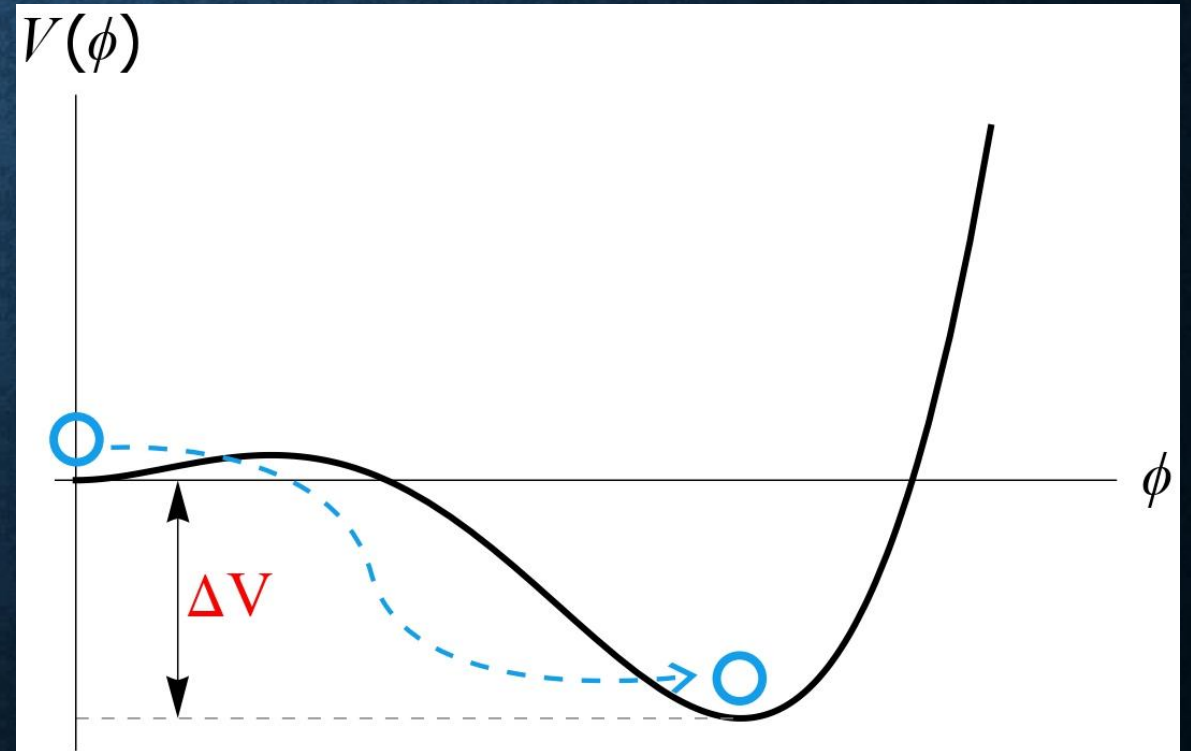
$$\Gamma(t) \approx H_I^4 e^{\beta t}$$

Transition strength

$$\alpha \approx \frac{\Delta V}{\rho_r} \gg 1$$

bubble wall velocity

$$v_w \approx 1$$



BLACK HOLE FORMATION

density distribution $P_k(\delta)$

fraction of total energy density collapsing to PBHs

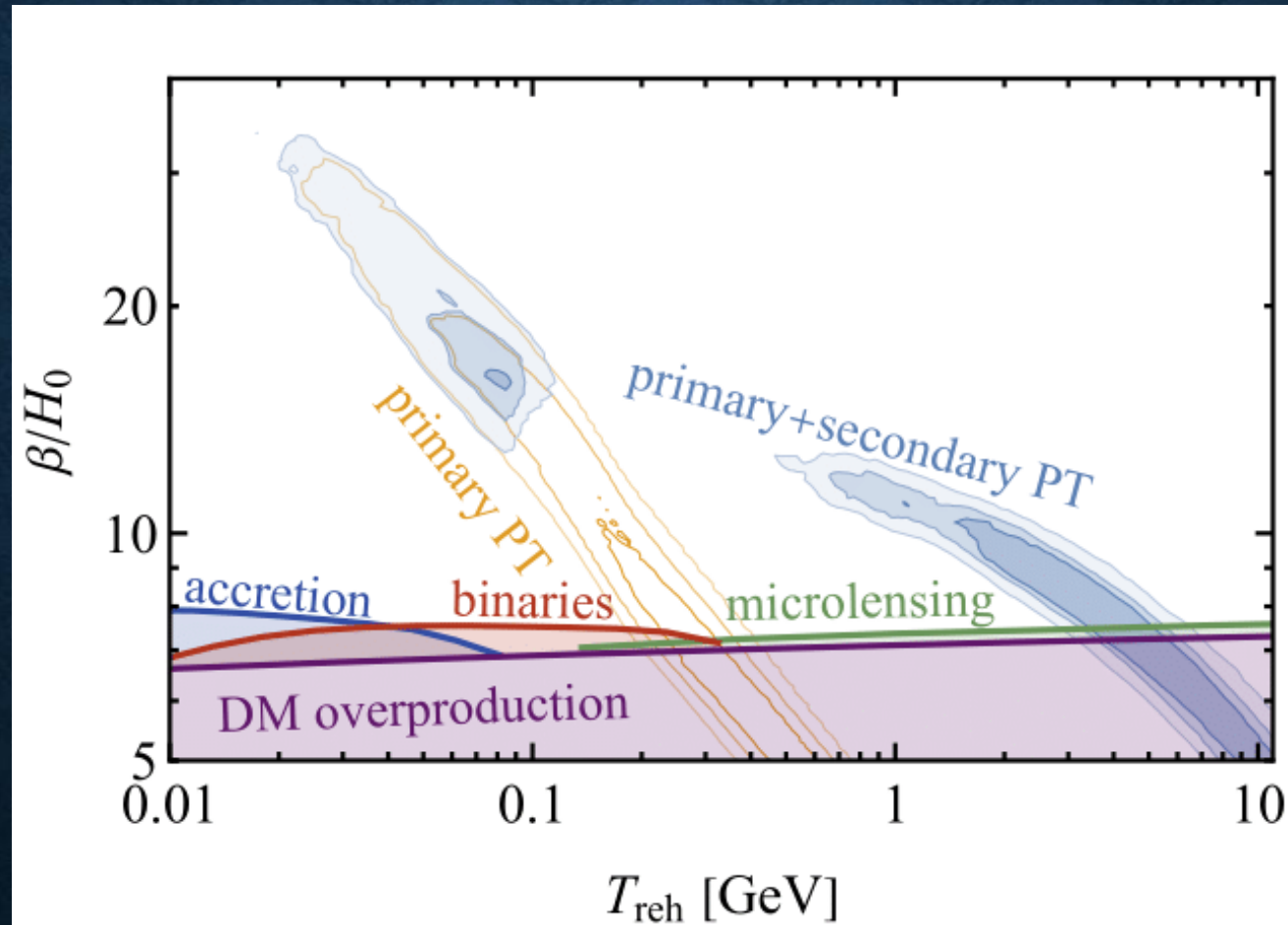
$$\beta_k(M) = \int_{\delta_c} d\delta \frac{M}{M_k} P_k(\delta) \delta_D \left(\ln \frac{M}{M(\delta)} \right)$$

present PBH mass function

$$\psi(M) = \int d \ln k \beta_k(M) \frac{\rho_r(T_k)}{\rho_c} \frac{s(T_0)}{s(T_k)}$$

PBH abundance $\Omega_{PBH} = \int d \ln M \psi(M)$

GRAVITATIONAL WAVES



GRAVITATIONAL WAVES

