# Gravitational wave archaeology from an early period of matter domination, theory and applications

 

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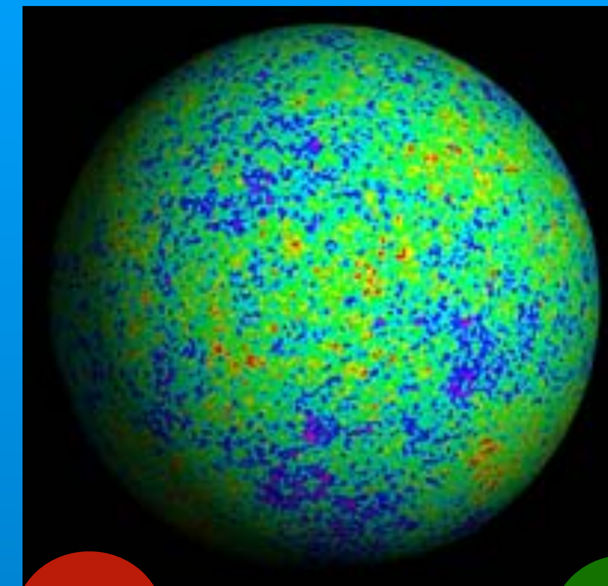
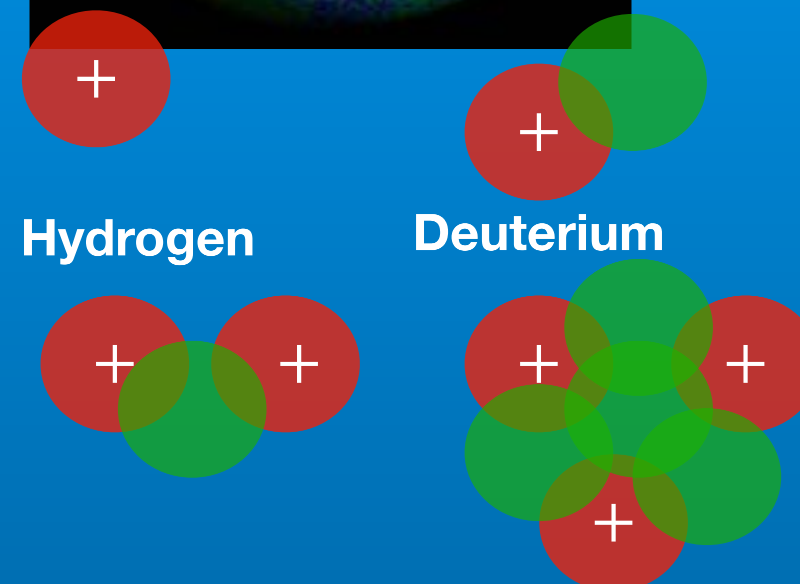
## Why Gravitational waves are amazing for cosmology

$t = 10^6$ years

$t = 1$ min

$t = 0!$





Hydrogen

Deuterium

Helium-3

Lithium-7

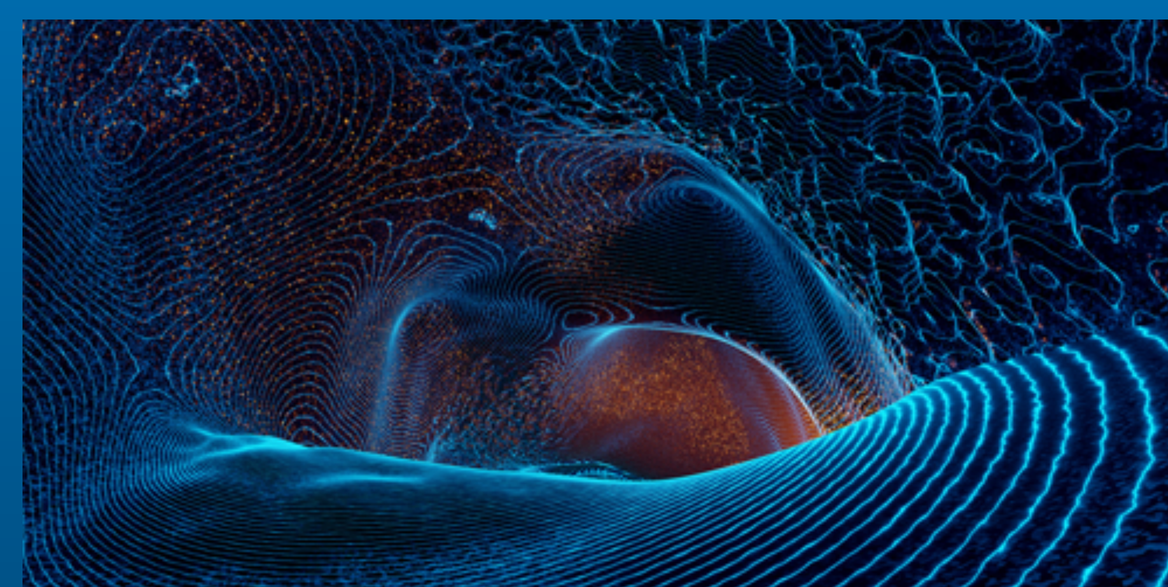


Image credits: WMAP,
Phys. Rev. Lett. 124, 041804

## New physics is hard to test!

**Electroweak Baryogenesis scale** $10^2 - 10^3$ GeV
**Leptogenesis scale:** $10^9 - 10^{14}$ GeV
**Affleck Dine Scale:** $\sim 10^{12} - 10^{15}$ GeV
**Dark matter scale ~ ???**

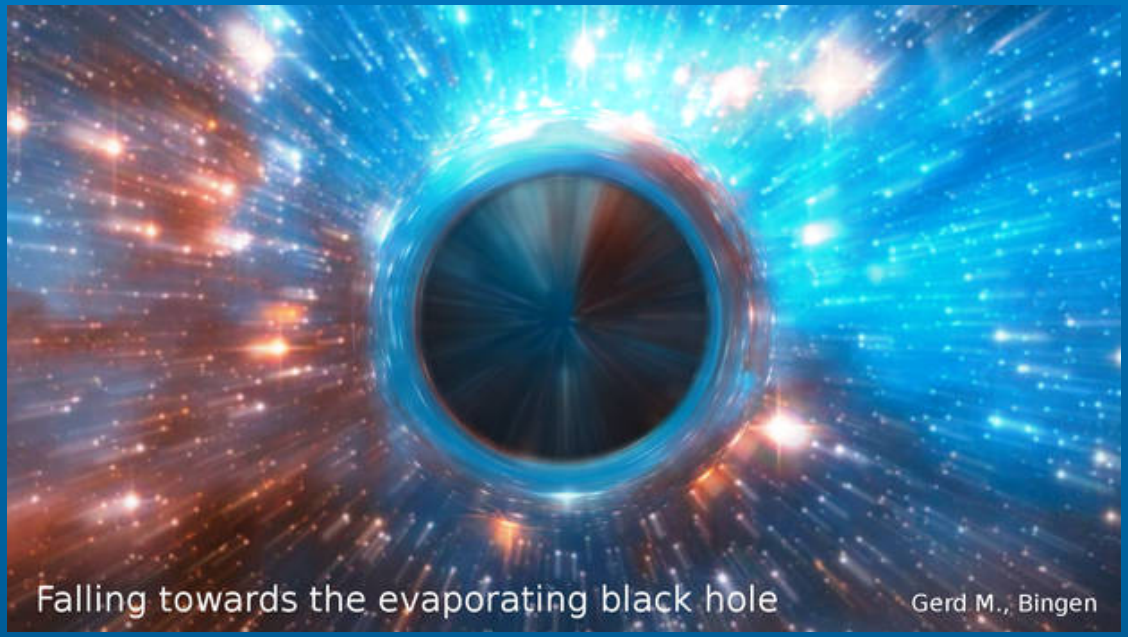
## Scalar induced gravitational waves

- Scalar perturbations are a source for tensor perturbations at second order

$$\Omega_{\rm GW}^{\rm induced}h^2 \sim 10^{-1}\Omega_{r,0}h^2\mathscr{P}_{\mathscr{R}}^2 \sim 10^{-6}\mathscr{P}_{\mathscr{R}}^2$$

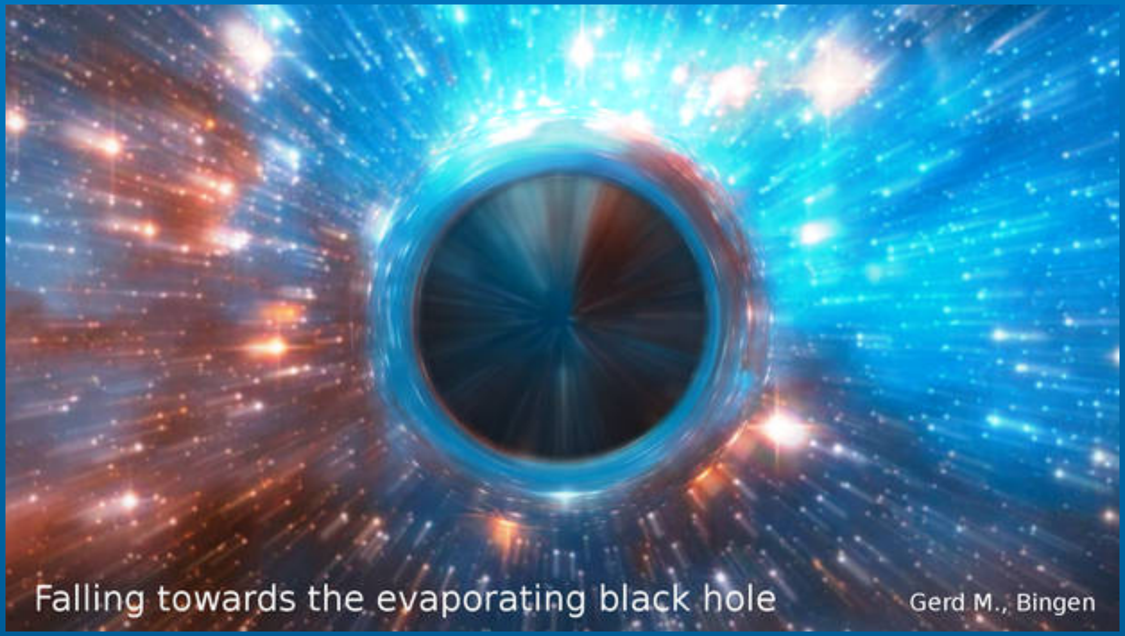
**CMB extrapolation**

$\mathscr{P}_{\mathscr{R}} \sim 10^{-9} \rightarrow \Omega_{\rm GW}^{\rm induced}h^2 \sim 10^{-24}$



## How to enhance $\mathscr{P}_{\mathscr{R}}$

1) Ultra slow roll
2) Sudden change in the equation of state



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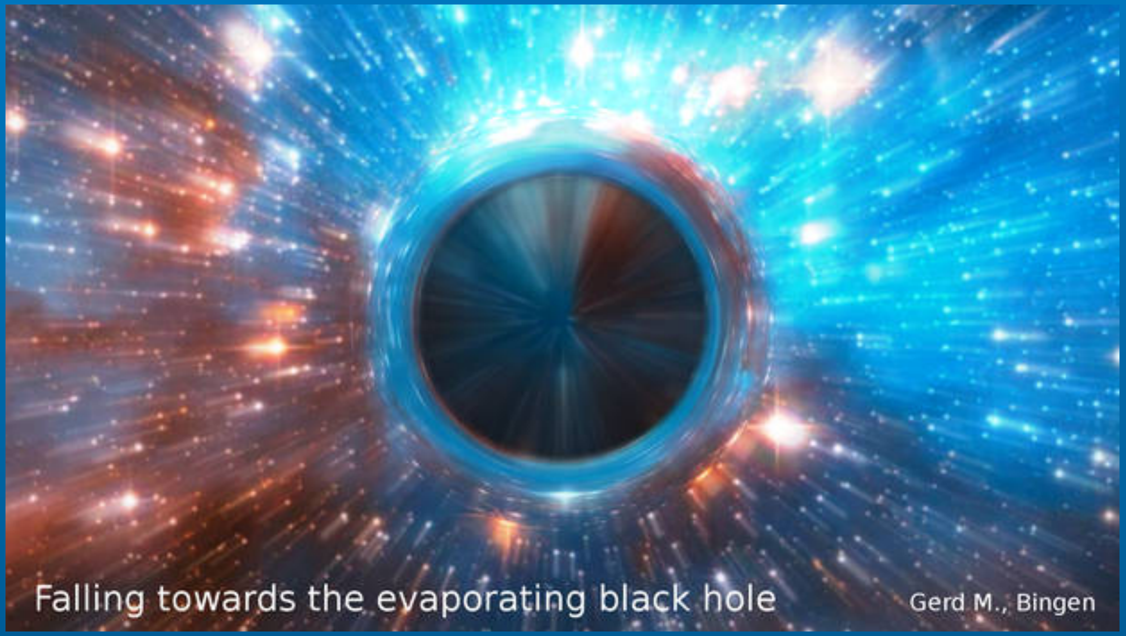
2) Sudden change in the equation of state
   - Matter domination to radiation domination



## How to enhance $\mathscr{P}_{\mathscr{R}}$

### Perturbations in conformal newtonian gauge

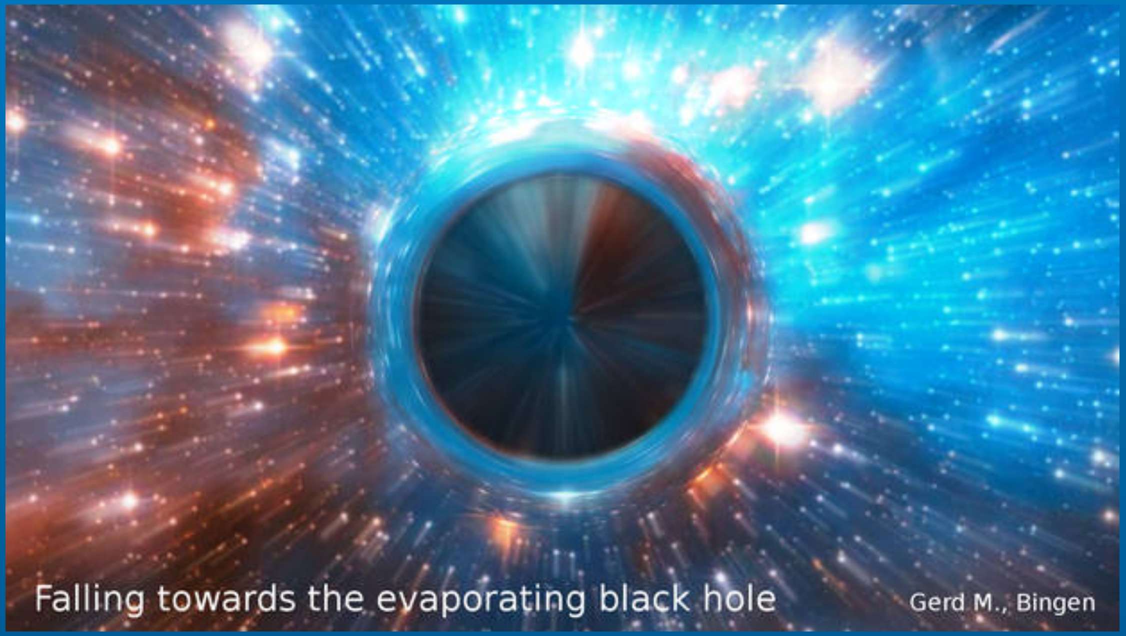
$$ds^2 = -a^2(1+2\Phi)d\eta^2 + a^2\left((1-2\Psi)\delta_{ij} + \frac{1}{2}h_{ij}\right)dx^i dx^j$$

### Equations of motion

$$h'' + 2\mathscr{H}h' + k^2 h = 4S$$

$$S = \int \frac{d^3q}{(2\pi)^{3/2}} e_{ij} q_i q_j \left(2\Phi\Phi + \frac{4}{3(1+w)}(\mathscr{H}^{-1}\Phi' + \Phi)(\mathscr{H}^{-1}\Phi' + \Phi)\right)$$

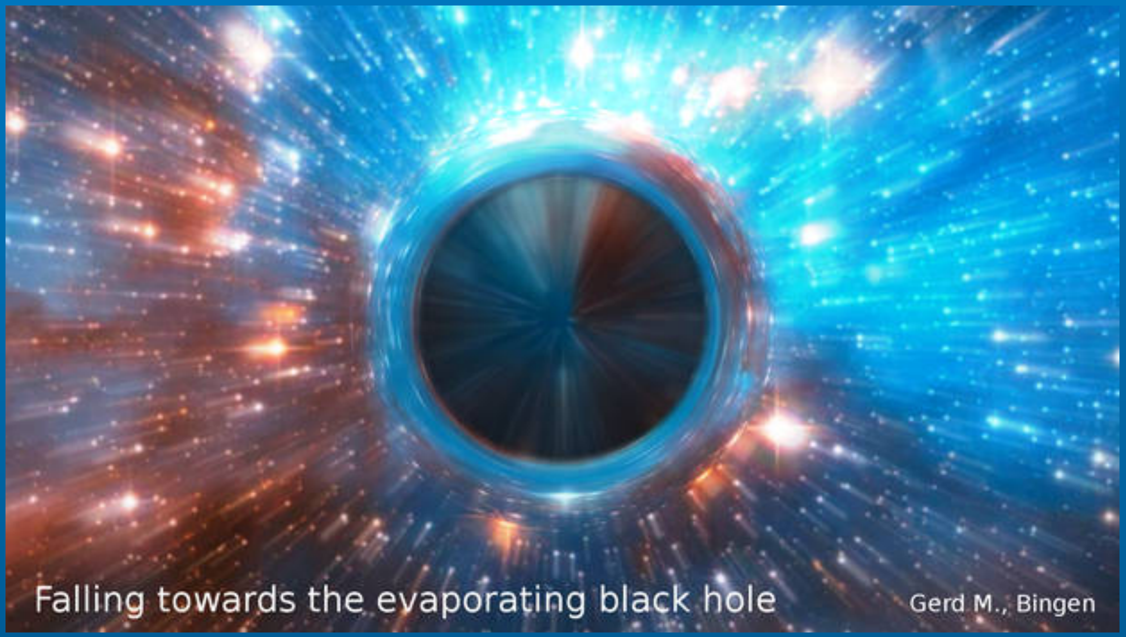
$$\langle hh \rangle = \frac{2\pi^2}{k^3}\delta(\Delta k) P_{\text{tensor}}$$



## How tensor modes are sourced by scalar modes

$$\langle hh \rangle = \frac{2\pi^2}{k^3}\delta(\Delta k)P_{\rm tensor}$$

$$\bar{\mathscr{P}}_{\rm tensor} = 4\int_0^\infty dv \int_{|1-v|}^{1+v} du \left( \frac{4v^2 - (1+v^2-u^2)^2}{4vu} \right)^2 \bar{I}^2 \mathscr{P}_{\mathscr{R}}(uk)\mathscr{P}_{\mathscr{R}}(vk)$$



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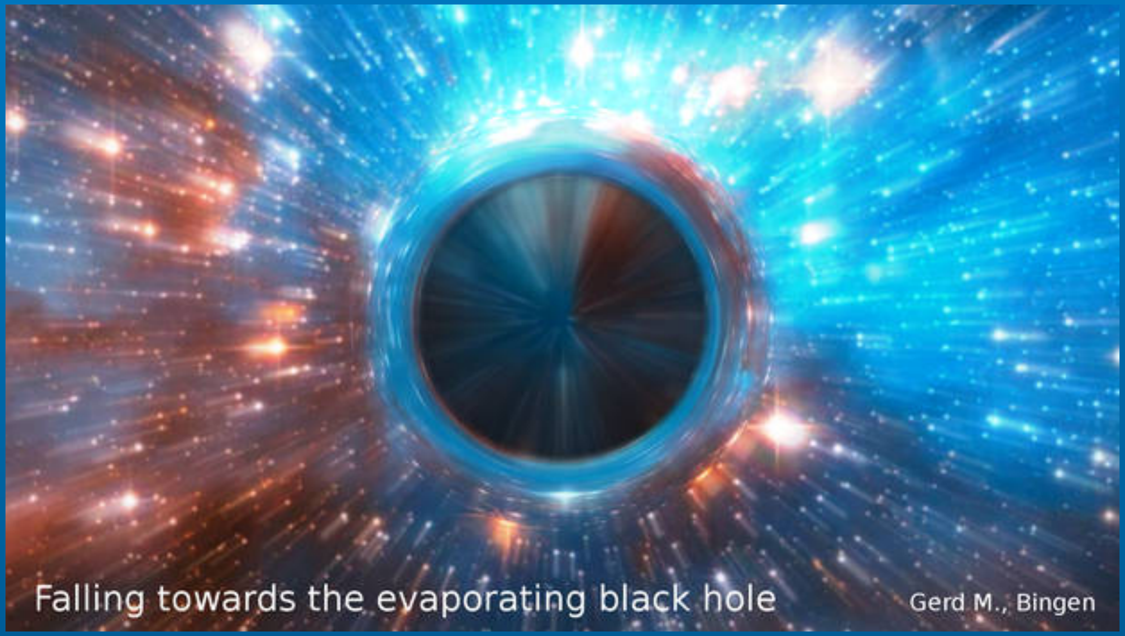
$$I = \int_0^x d\bar{x} \frac{a(\bar{\eta})}{a(\eta)} kGf \qquad\qquad x = k\eta$$



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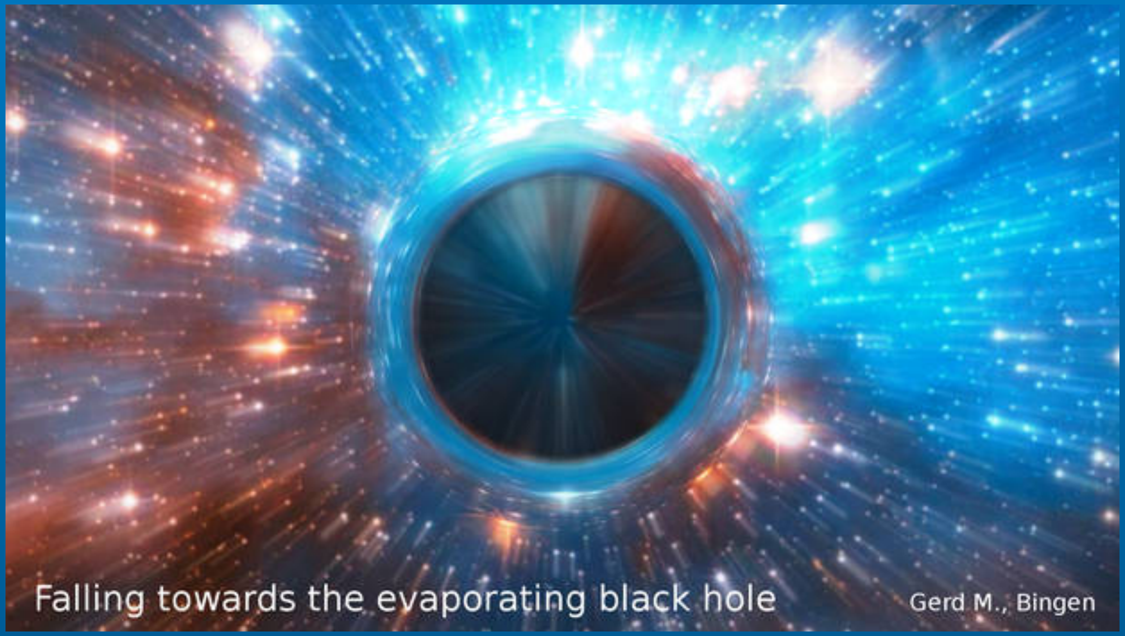
$$f = \frac{3(2(5+3w)\Phi\Phi + 4\mathscr{H}^{-1}(\Phi'\Phi + \Phi\Phi') + 4\mathscr{H}^{-2}\Phi'\Phi'}{25(1+w)}$$



## Modeling the transition from matter to radiation domination

$$\Gamma(\eta) = \Gamma_{\max}(\tanh(\beta(\eta - \eta_R)) + 1)/2$$

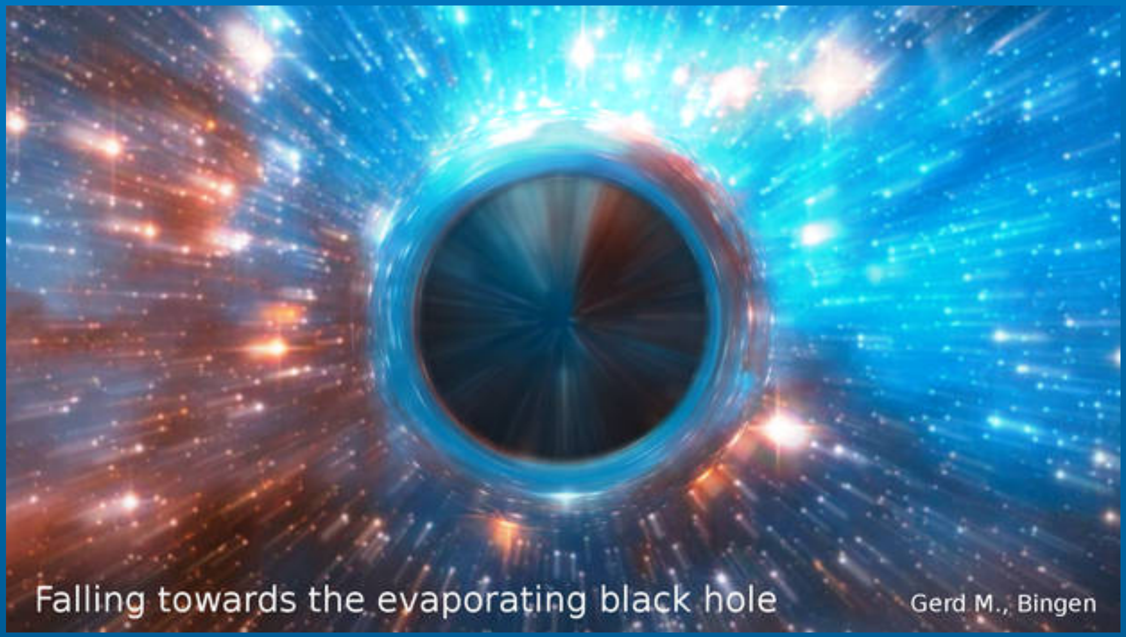
$$\delta_m' = -\theta + 3\Phi' - a\Gamma(\eta)\Phi$$

$$\theta_m' = -\mathscr{H}\theta_m + k^2\Phi$$

$$\delta_r' = -\frac{4}{3}(\theta_r - 3\Phi') + a\Gamma(\eta)\frac{\rho_m}{\rho_r}(\delta_m - \delta_r + \Phi)$$

$$\theta_r' = \frac{k^2}{4}\delta_r + k^2\Phi - a\Gamma(\eta)\frac{3\rho_m}{4\rho_r}\left(\frac{4}{3}\theta_r - \theta_m\right)$$

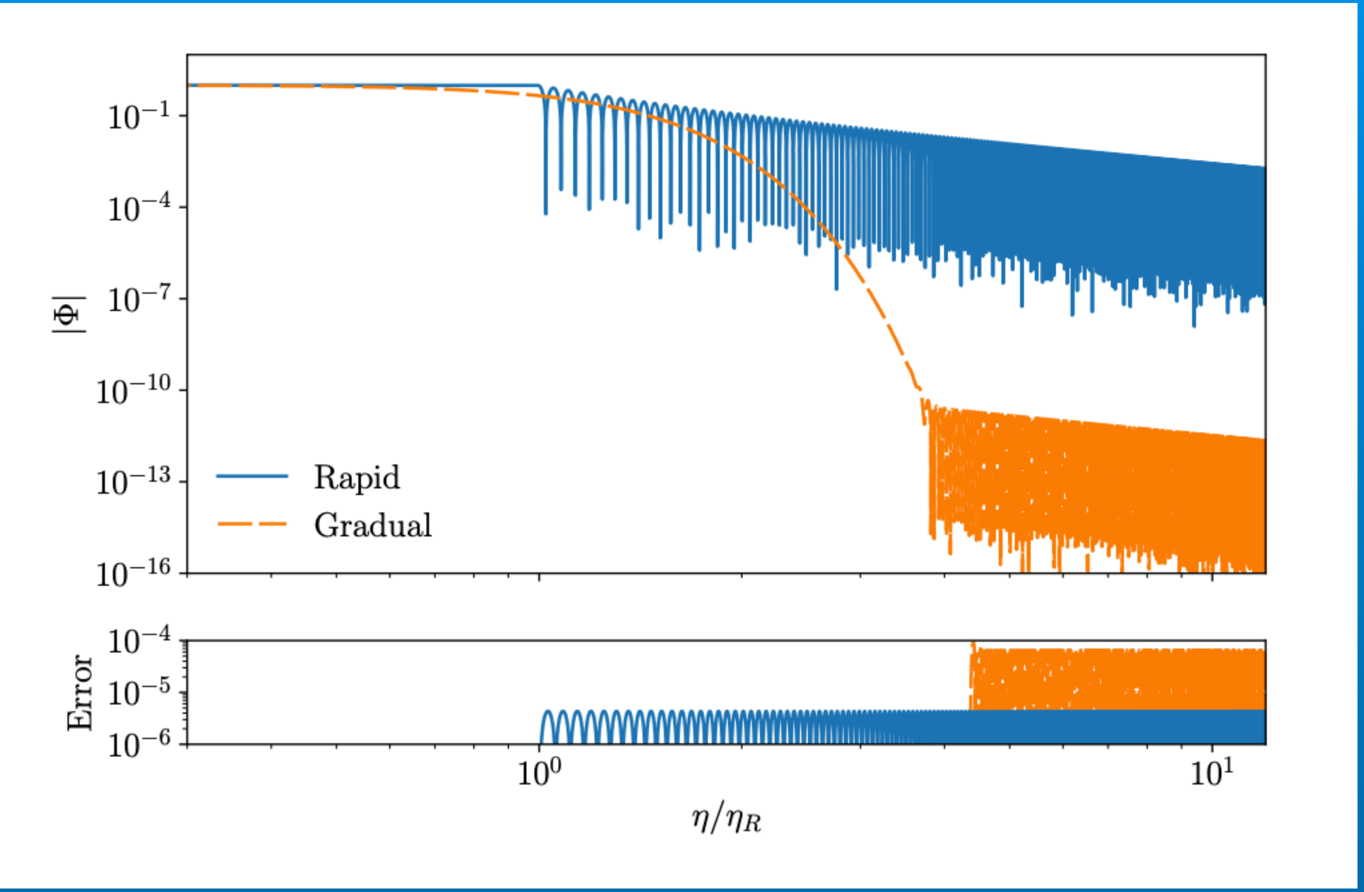
$$\Phi' = \frac{k^2\Phi + 3\mathscr{H}^2\Phi + \frac{3}{2}\mathscr{H}^2\left(\frac{\rho_m\delta_m + \rho_r\delta_r}{\rho_{\mathrm{tot}}}\right)}{3\mathscr{H}}$$



## Modeling the transition from matter to radiation domination

### Initial power spectrum

$$P_\zeta(k) = \Theta(k_{\rm inf} - k) A_s \left(\frac{k}{k_*}\right)^{n_s - 1}$$




**Balazs, Pearce, Pearce, White 2311.12340**

## Fast vs gradual transitions

$$I = \int kGf = \int kG^{\text{MD}}f + \int kG^{\text{RD}}f \rightarrow \Omega_{\text{GW}} \sim I^2 = \Omega_{\text{MD}} + \Omega_{\text{RD}} + \Omega_{\text{cross}}$$

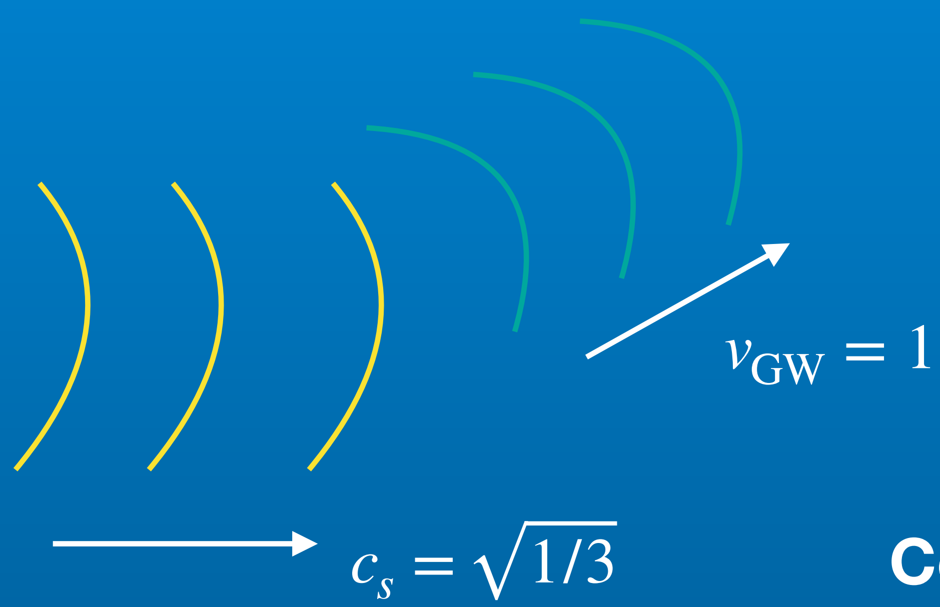
$$\Omega_{\text{cross}} < 0$$

**So this is reason why instantaneous transitions are stronger. Mathematical explanation**

1) $\Phi'$ **does not vanish as quickly, therefore neither does f**
2) **Negative definite cross term vanishes**

## Physical explanation why the instantaneous transitions have larger GWs

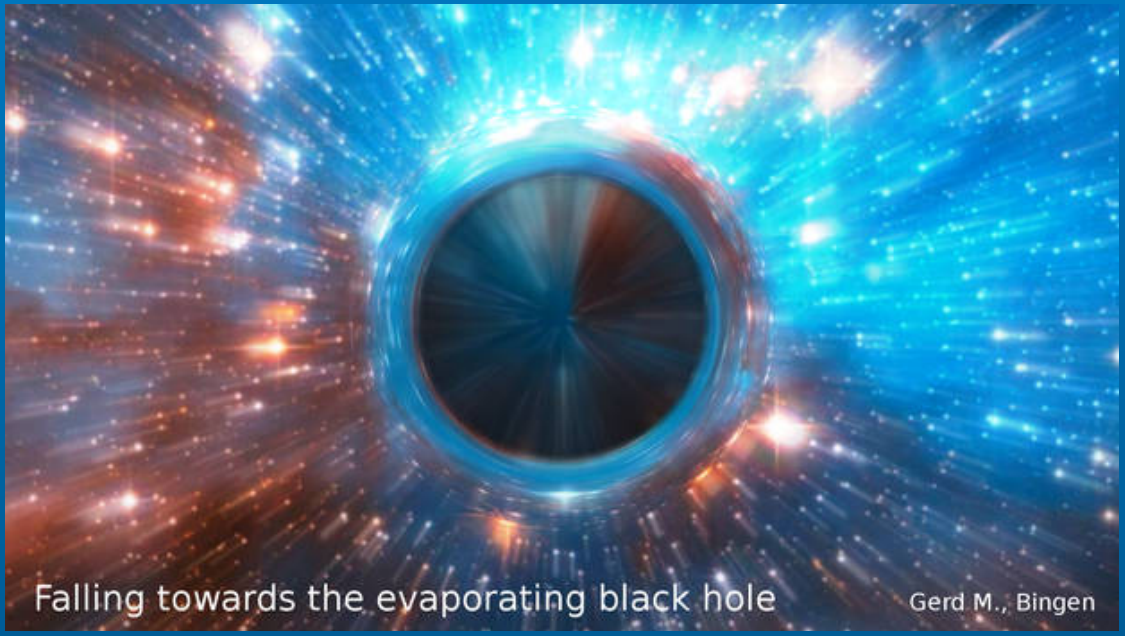
- During matter domination perturbations grow
- Suppose the transition to radiation is very fast
- Perturbations cannot melt away in this case but produce sound waves




**Constructive interference**

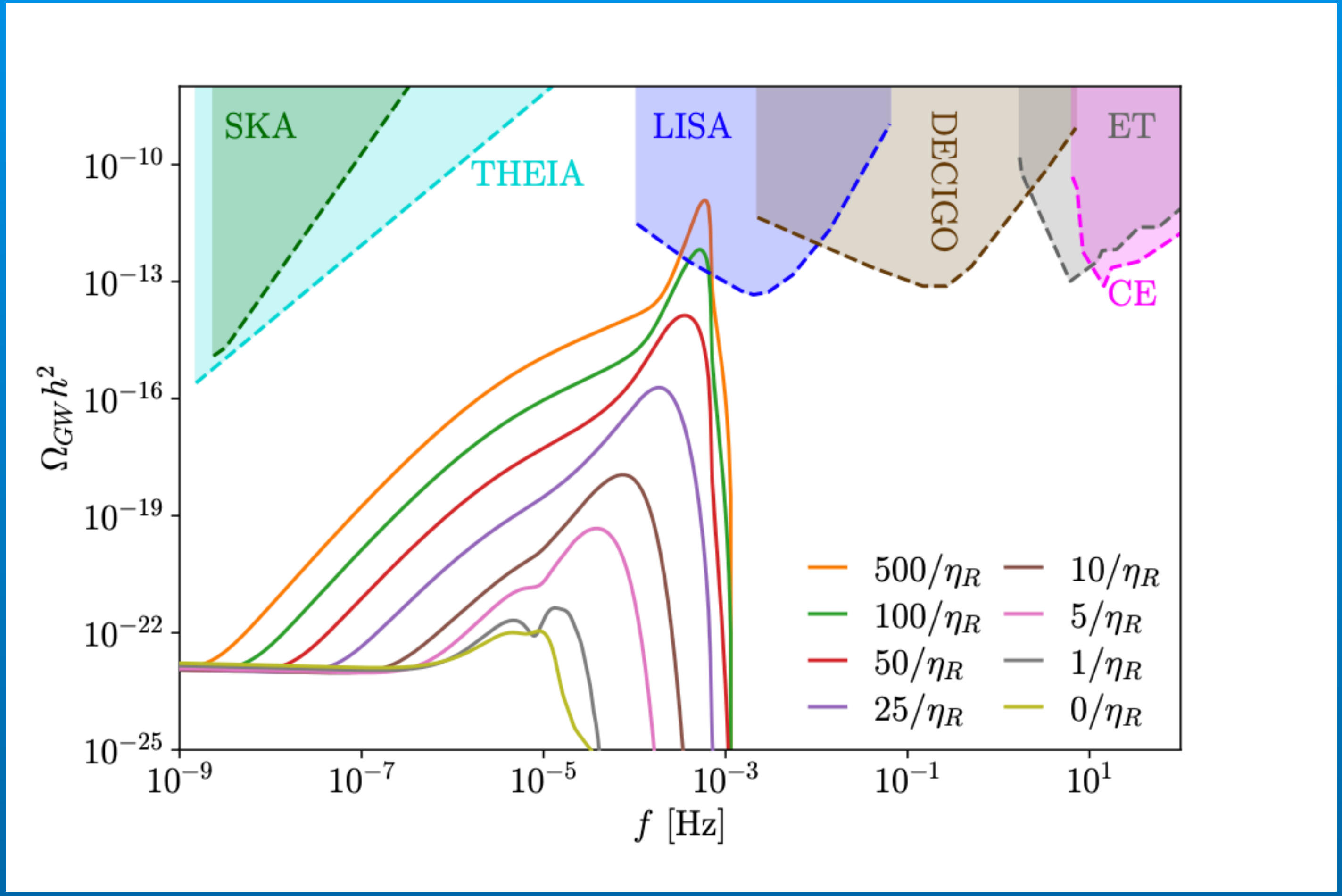
Possible reasons why there is a sudden change from matter to radiation

Very light primordial black holes
Clumps of field (Q-balls)
Particle whose decay is forbidden by a symmetry
Coherent scalar field



Distinguishing between different sources:

Model independent case

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## Model dependent case:

**Model**

1) Primordial black holes
2) Delayed Q-balls
3) "Thin wall" Q-balls
4) "Thick wall" Q-balls

**Mass spectrum**

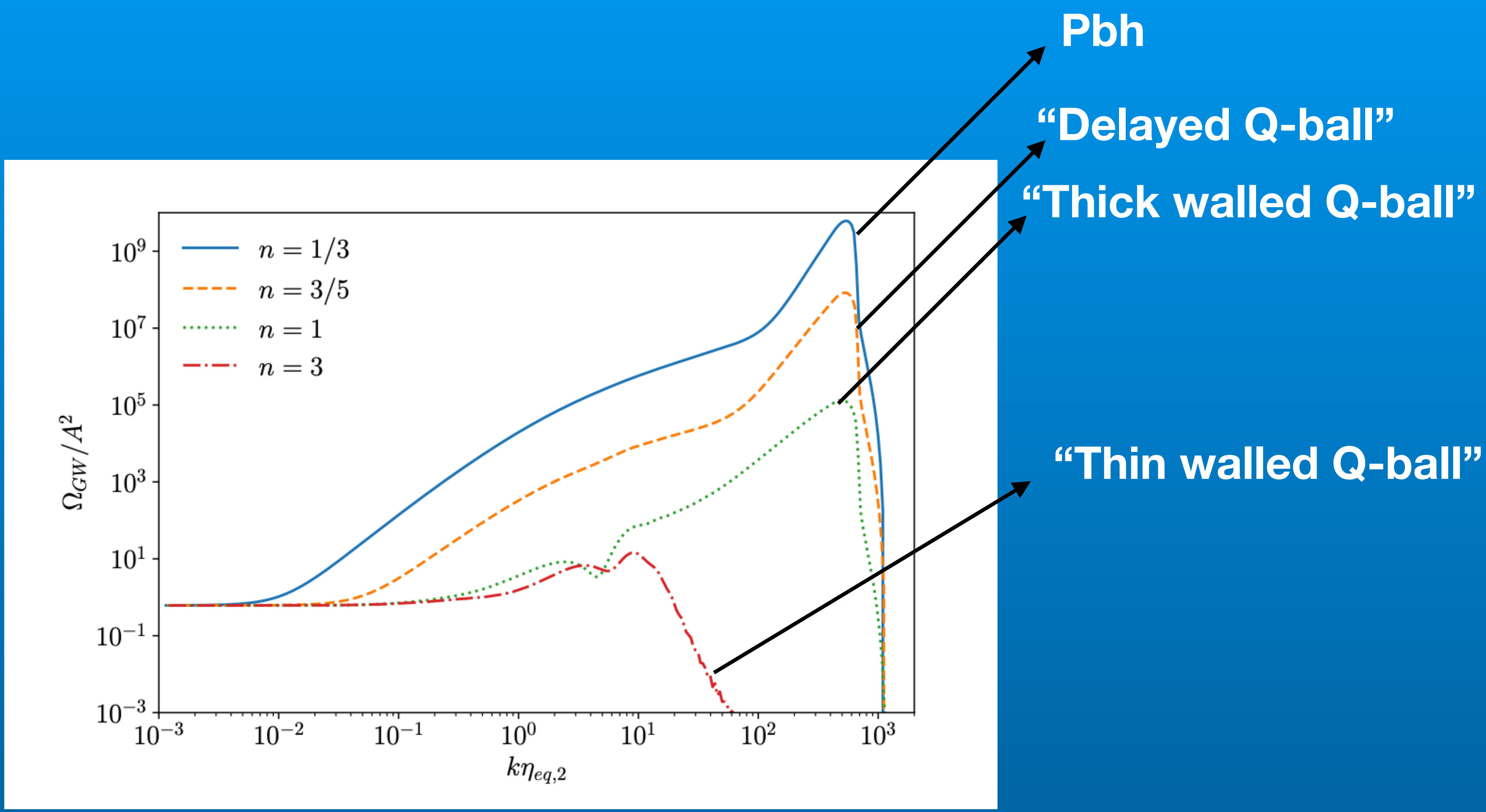
1) Monochromatic
2) Log normal

### "Delayed Q-balls"

$$
\begin{aligned}
V(\phi) &= V_{\text{gauge}} + V_{\text{grav}} + V_A \\
&= M_F^4 \left[ \log \left( \frac{|\phi|^2}{M_m^2} \right) \right]^2 + m_{3/2}^2 |\phi|^2 \left( 1 + K \log \frac{|\phi|^2}{M_*^2} \right) + V_A
\end{aligned}
$$

For K>0 Q-ball formation is "delayed" until $\phi$ gets small enough for $V_{\text{grav}}$ to *not* dominate

## Results for monochromatic mass distributions

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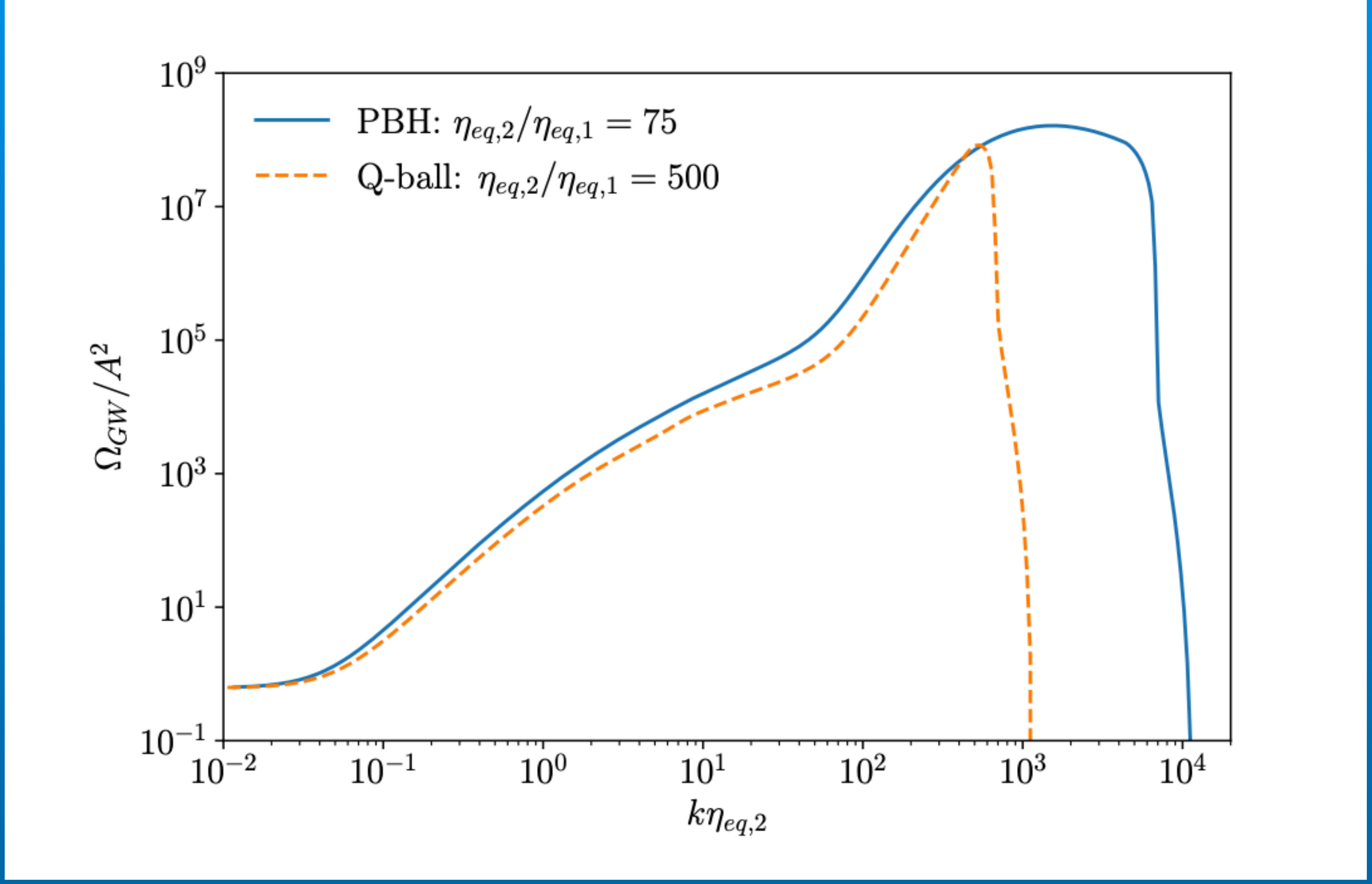
## The pbh looked a similar shape to delayed Q-balls, can you distinguish in other scenarios?

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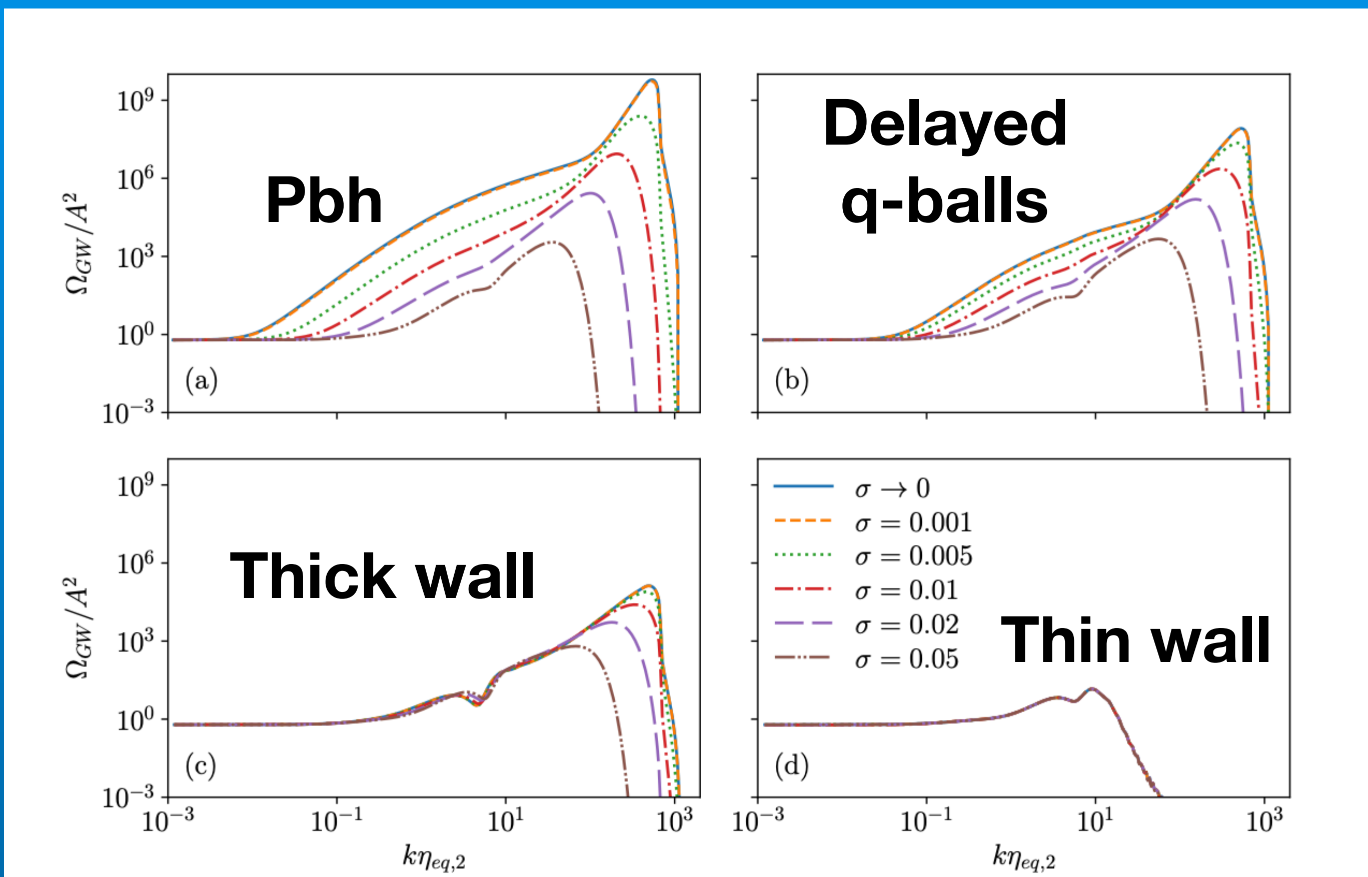
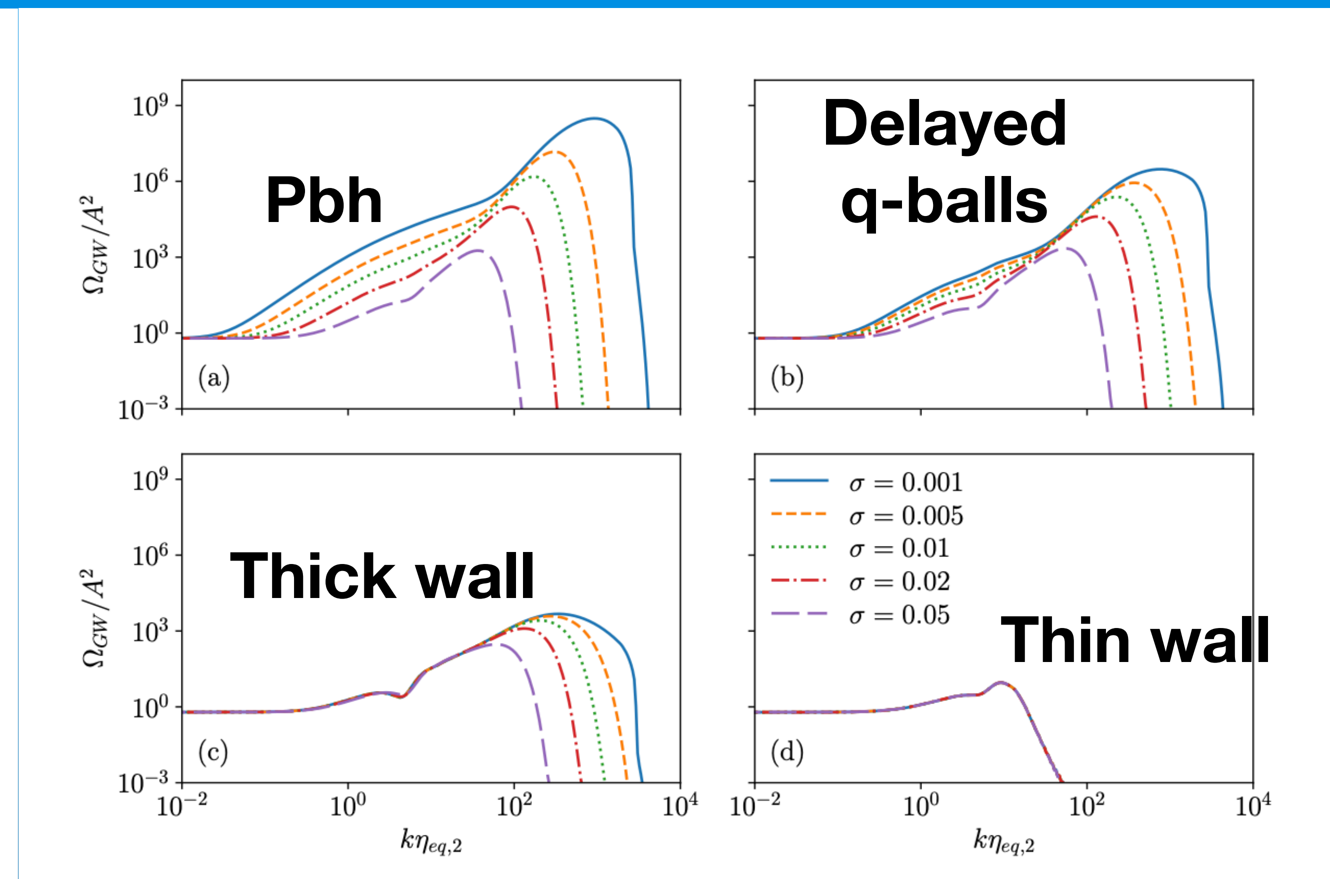
# Non monochromatic mass distributions:




Long era of emd

Short era

$$\rho_m(t) = \rho_{m,i} \int \beta(M_i, t)\, d\ln M_i.$$

$$\beta_i = \beta(t=0) = \frac{\mathcal{N}}{\sqrt{2\pi}\sigma} \exp\left(-\frac{(\ln(M_i/M_0))^2}{2\sigma^2}\right),$$

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## Conclusion

An early period of matter domination that suddenly transitions to radiation can leave a detectable signal in the gravitational wave background

There are three time scales that determine the shape of the background

Different causes of matter domination have different predictions and can in principle be distinguished